Environ. Sci., Hokkaido University	11 (2)	125～139	Dec. 1988

Tests of Stability for Spatial Price Equilibrium Model

Akio Itoh
Department of Agricultural Policy, Division of Social Environment, Graduate School of Environmental Science, Hokkaido University, Sapporo, 060, Japan

Fusao Itoh
Department of Agricultural Development, Graduate School of Agriculture, Hokkaido University, Sapporo, 060, Japan

Toshio Kuroyanagi
Department of Agricultural Policy, Division of Social Environment, Graduate School of Environmental Science, Hokkaido University, Sapporo, 060, Japan

Abstract

The spatial price equilibrium model is an analytical tool for equilibrating respective markets in many regions, and in political using, which relie much on the madel's stability.

Although, in recent years, attemps to test the stability of optimal solution utilizing sensitivity analysis have been made, they are limited to discussing the theoretical developments by utilizing assumed values or to the stability of optimal solution given by the perturbations of specific regions.

This paper tests the stability and usefulness of the model by analyzing the influences over the optimal solution created by all parameter perturbations in all regions in terms of the sensitivity analysis of specific agricultural products.

Key Words: Spatial Price Equilibrium, Sensitivity Analysis, Quadratic Programming.

1. Introduction

A number of spatial equilibrium analyses have been applied to as methods dealing with equilibrium of respective markets in various regions ever since their formulation by T. Takayama and G. G. Judge. [10].

As the optimal value of spatial equilibrium model, however, is determined by the intercepts and slopes of demand and supply functions and the value of unit transportation cost; not only should demand and supply functions in spatial equilibrium analysis be estimated accurately, but also the stability of optimal solution to be worked out.

As for the sensitivity analysis for stability, until now, studies by Boot (1963), Tobin (1987) and the like have been availed, all of which, however, discuss only the theoretical developments based on assumed values. On the other hand, studies on coal by Irwin and Young (1982) and Ueji (1986) are based on real values.

The sensitivity analyses made by them, however, are too partial in discussing the stability of optimal solution in the specific regions.

To test the stability of optimal solution, in our study, the theory of Irwin and Young (1982) has further been developed, that is, sensitivity analyses for all parameter perturbations has been made.

From the viewpoint of usefulness of the model, a specific agricultural product, carrot from which the real value can easily be found, has been taken up as an object to be analyzed.

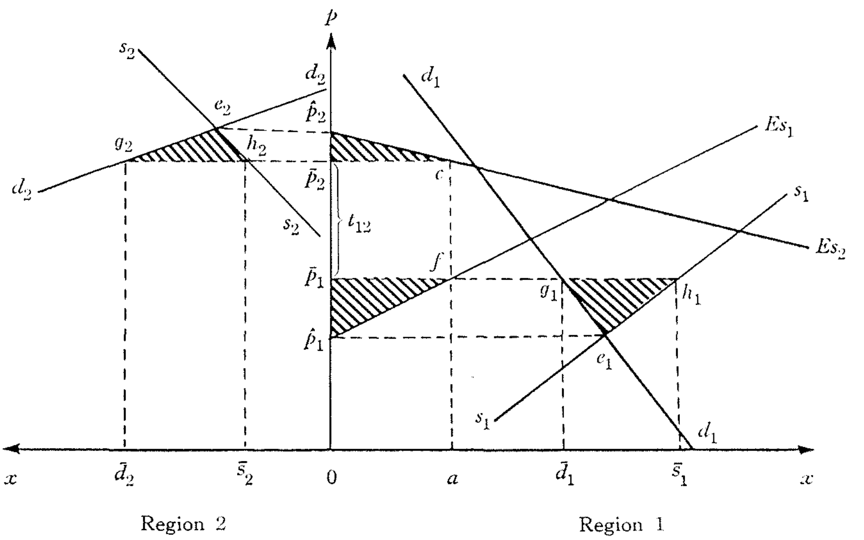
In the following section two, the concept of spatial equilibrium model and the theoretical framework of sensitivity analysis are detailed, and in section three clause, sensitivity analyses is made utilizing specific agricultural products. Subsequently, in section four the stability of optimal solution worked out of spatial equilibrium analysis is evaluated.

2. Framework of Analysis

2-1. *Spatial equilibrium model*

Here we quote the model of T. Takayama and G. G. Judge. This model is built up as a quadratic programming problem utilizing the concept of Net Social Payoff (hereafter referred to as NSP) advocated by P. A. Samuelson. [1]
The model is summarized as follows:

Figure 1 shows NSP between two markets. When markets are competitive, each other an equilibrium is set up at the points (e. g. e_1, e_2) where demand and



NOTE d_i-d_i : Demand Function in the Region $i(i=1,2)$
s_i-s_i : Supply Function in the Region $i(i=1,2)$
E_{si} : Excess Supply Function in the Region $i(i=1,2)$
$\hat{p}_1 f c \hat{p}_2$: Social Payoff
$\bar{p}_1 f c \bar{p}_2$: Total Transportation Cost
$\bar{p}_1 f \hat{p}_1 + \bar{p}_2 c \hat{p}_2$: Net Social Payoff

Figure 1. NSP between Regional Markets.

supply curves are crossed. Between the two markets, any specific or technological restrictions are not supposed to exist. At the time, commodities are drawn to the higher price market, as for as the equilibrium price difference is higher than the unit transportation cost ($\hat{p}_2 - \hat{p}_1 > t_{12}$) between the two markets, and transferred from the region 1 to the region 2. Through this process, price difference is reduced and an interregional equilibrium is performed at the points (e. g. $\bar{p}_1$, $\bar{p}_2$) where the price difference is equal to the unit transportation cost. In case the price difference is lower than the unit transportation cost, commodities are not transfered and an equilibrium can maintained just as it is.

Social Payoff in each region is defined geometically as lower portion than the extended demand curve or the extended supply curve. Further, the difference between social payoff and total interregional transportation cost is defined as NSP[1]. In other words, when demand and supply curves are linear, NSP is shown as Figure 1, $\bar{p}_1 f \hat{p}_1 + \hat{p}_2 c \bar{p}_2$ (a quadratic formula for the transportation quantity or the price).

When the formula is developed and applied in n region, mathematical model follows:

First, demand function:

$$d_i = \alpha_i - \beta_i P_i \qquad (i = 1, \cdots, n) \qquad (1)$$

$$\text{for } (\alpha_i, \beta_i > 0)$$

supply function:

$$s_j = \theta_j + \gamma_j P^j \qquad (j = 1, \cdots, n) \qquad (2)$$

$$\text{for } (\theta_j \gtreqless 0, \ \gamma_j > 0)$$

d_i and s_j are demand and supply quantities, and p_i and p^j are demand and supply prices. Further, when t_{ij} and x_{ij} show respectively the unit transportation cost and the transportation quantity from the i region to the j region, it comes out as follows:

$$\begin{aligned} \text{NSP} = F(X) &= \sum_i \int_0^{\bar{d}i} P_i d(d_i) - \sum_j \int_0^{\bar{s}i} P^j d(s_j) - \sum_i \sum_j t_{ij} x_{ij} \\ &= \sum_i \frac{\alpha_i}{\beta_i} \sum_j x_{ij} - \frac{1}{2} \sum_i \frac{1}{\beta_i} (\sum_j x_{ij})^2 - \sum_j \left(- \frac{\theta_j}{\gamma_j} \right) \sum_i x_{ij} \\ &\qquad - \frac{1}{2} \sum_i \frac{1}{\gamma_j} (\sum_i x_{ij})^2 - \sum_i \sum_j t_{ij} x_{ij} \end{aligned} \qquad (3)$$

$$X = (x_{11}, x_{12}, \cdots, x_{nn})'$$ [2]

$\bar{d}_i$ and $\bar{s}_i$ are respectively regional equilibrium demand and supply quantities in the i region. When maximizing the above (3) in terms of $x_{ij} \geqq 0$, a neccessary condition should be:

From Kuhn — Tucker Condition:

$$\frac{\partial F(X)}{\partial x_{ij}} = \frac{\alpha_i}{\beta_i} - \frac{1}{\beta_i} \sum_j x_{ij} - \left(- \frac{\theta_j}{\gamma_j} + \frac{1}{\gamma_j} \sum_i x_{ij} \right) - t_{ij} \leqq 0$$

$$\frac{\partial F(X)}{\partial x_{ij}} x_{ij} = 0\,, \qquad x_{ij} \geqq 0$$

is worked out[3]. These terms are shown below when utilizing (1) and (2) as follows:

$$P_i - P^j \leqq t_{ij}\,, \qquad \frac{\partial F(x)}{\partial x_{ij}} x_{ij} = 0 \tag{4}$$

As shown above, spatial equilibrium model utilized to maximize (3) under restriction (4) can be developed not only for the transportation quantity, but also for the price terms.

NSP can be worked out in price terms as follows:

$$\begin{aligned} \mathrm{NSP} = F(P) &= \sum_i \int_{\hat{P}_i}^{\bar{P}_i} (d_i)\, dp_i - \sum_j \int_{\hat{P}_j}^{\bar{P}_j} (s_j)\, dp^j + \sum_i \sum_j t_{ij} X_{ij} \\ &= \sum_i \alpha_i p_i - \frac{1}{2} \sum_i \beta_i (p_i)^2 - \sum_j \theta_j P^j - \frac{1}{2} \sum_j \gamma_j (P^j)^2 + \sum_i \sum_j t_{ij} X_{ij} \end{aligned} \tag{5}$$

for $P = (P_1 \cdots P_n,\ P^1 \cdots P^n)'$

$\hat{P}_i$: Pre-trade equilibrium price in the i region.

$\bar{P}_i$: Post-trade equilibrium price in the i region.

From the above information, the spatial equilibrium model in the price terms should be formulated as a maximized problem of (6) quadratic programming problem, restricted by (7) as follows:

$$F(P) = \begin{bmatrix} \alpha \\ -\theta \end{bmatrix}' P - \frac{1}{2} P' \begin{bmatrix} \beta & 0 \\ 0 & \gamma \end{bmatrix} P = A' P - \frac{1}{2} P' BP \tag{6}$$

$$G' P \leqq T,\ P \geqq 0 \tag{7}$$

for $A = (\alpha_1 \cdots \alpha_n,\ -\theta_1 \cdots -\theta_n)'$

$$B = \begin{bmatrix} \beta_1 \cdots 0 & 0 \cdots 0 \\ \vdots \ \ddots \ \vdots & \vdots \quad \vdots \\ 0 \cdots \beta_n & 0 \cdots 0 \\ 0 \cdots 0 & \gamma_1 \cdots 0 \\ \vdots \quad \vdots & \vdots \ \ddots \ \vdots \\ 0 \cdots 0 & 0 \cdots \gamma_n \end{bmatrix}$$

$$T = (t_{11},\ t_{12} \cdots t_{nn})'$$

$$G = \begin{bmatrix} 1 & & 1 & \cdots\cdots & 1 & \\ & \ddots & & \ddots & & \ddots \\ & 1 & & 1 & \cdots\cdots & 1 \\ -1 \cdots -1 & & & & & \\ & & -1 \cdots -1 & & & \\ & & & \cdots\cdots & & \\ & & & & -1 \cdots -1 & \end{bmatrix}$$

Incidentally, the quadratic programming problem is reduced to the linear programming problem on the premise that an optimal solution exists[4]. Then the optimal value of equilibrium price and interregional transportation quantity are worked out by means of the modified P. Wolfe Simplex method. The Symplex tableau is shown as follows:

	P_0	$X' \geqq 0$	$P' \geqq 0$	$V' \geqq 0$
Z_1	T		G'	I
Z_2	A	G	B	

$Zi(i=1, 2)$: basic solution vector
V: slack solution vector corresponding to X'
I: identical matrix

2-2. *Framework of sensitivity analysis*

The optimal value of spatial equilibrium model is determined by the intercepts and slopes of linear demand and supply functions and the estimated unit transportation cost.

In considering the stability of spatial equilibrium model, the problem should be how the parameter perturbation affects the optimal value[5].

Here, we would like to show a theoretical framework in carrying out sensitivity analysis of spatial equilibrium model.

First of all, let us think about how to maximize (6) with the restrictions (7). Under the achievement of interregional equilibrium, an equality is accurately set up between (7) and an optimal combination of i and j. At that time, our quadratic programming problem is translated into constrained optimal problem as follows:

Lagrange function:

$$L(P_s) = A' P_s - \frac{1}{2} P'_s BP_s + \lambda' (T - G' P_s) \tag{8}$$

$$P_s = (P_{1s} \cdots P_{ns},\ P^1_s \cdots P^n_s)$$
$$\lambda = (\lambda_{11},\ \lambda_{12} \cdots \lambda_{nn})$$

P_s is equilibrium price vector, and λ is lagrange multipliers vector.

The first order condition for maximization is:

$$\frac{\partial L(P_s)}{\partial P_s} = A - BP_s - G\lambda = 0 \tag{9}$$

$$\frac{\partial L(P_s)}{\partial \lambda'} = T - G' P_s = 0 \tag{10}$$

and, from (9) and (10), equilibrium price vector P_s is worked out as follows:

$$P_s = B^{-1} A - B^{-1} G (G' B^{-1} G)^{-1} (G' B^{-1} A - T) \tag{11}$$

Then we define "sensitivity" as follows respectively.

[Sensitivity of intercept]

$$\frac{\partial P_s}{\partial A} = B^{-1} - B^{-1}G(G' B^{-1} G)^{-1} G' B^{-1}$$

$$= \begin{bmatrix} \frac{\partial P_{1s}}{\partial \alpha_1} \cdots \frac{\partial P_{1s}}{\partial \alpha_n} & \frac{\partial P_{1s}}{\partial(-\theta_1)} \cdots \frac{\partial P_{1s}}{\partial(-\theta_n)} \\ \vdots \quad\quad \vdots & \vdots \quad\quad \vdots \\ \frac{\partial P_{ns}}{\partial \alpha_1} \cdots \frac{\partial P_{ns}}{\partial \alpha_n} & \frac{\partial P_{ns}}{\partial(-\theta_1)} \cdots \frac{\partial P_{ns}}{\partial(-\theta_n)} \\ \frac{\partial P_s^1}{\partial \alpha_1} \cdots \frac{\partial P_s^1}{\partial \alpha_n} & \frac{\partial P_s^1}{\partial(-\theta_1)} \cdots \frac{\partial P_s^1}{\partial(-\theta_n)} \\ \vdots \quad\quad \vdots & \vdots \quad\quad \vdots \\ \frac{\partial P_s^n}{\partial \alpha_1} \cdots \frac{\partial P_s^n}{\partial \alpha_n} & \frac{\partial P_s^n}{\partial(-\theta_1)} \cdots \frac{\partial P_s^n}{\partial(-\theta_n)} \end{bmatrix} \tag{12}$$

[Sensitivity of slope]

$$\frac{\partial P^s}{\partial B} = \frac{\partial P^s}{\partial B^{-1}} \cdot \frac{\partial B^{-1}}{\partial B}$$

$$= \begin{bmatrix} \frac{\partial P_{1s}}{\partial \beta_1} \cdots \frac{\partial P_{1s}}{\partial \beta_n} & \frac{\partial P_{1s}}{\partial(-\theta_1)} \cdots \frac{\partial P_{1s}}{\partial(-\theta_n)} \\ \vdots \quad\quad \vdots & \vdots \quad\quad \vdots \\ \frac{\partial P_{ns}}{\partial \beta_1} \cdots \frac{\partial P_{ns}}{\partial \beta_n} & \frac{\partial P_{ns}}{\partial(-\theta_1)} \cdots \frac{\partial P_{ns}}{\partial(-\theta_n)} \\ \frac{\partial P_s^1}{\partial \beta_1} \cdots \frac{\partial P_s^1}{\partial \beta_n} & \frac{\partial P_s^1}{\partial(-\theta_1)} \cdots \frac{\partial P_s^1}{\partial(-\theta_n)} \\ \vdots \quad\quad \vdots & \vdots \quad\quad \vdots \\ \frac{\partial P_s^n}{\partial \beta_1} \cdots \frac{\partial P_s^n}{\partial \beta_n} & \frac{\partial P_s^n}{\partial(-\theta_1)} \cdots \frac{\partial P_s^n}{\partial(-\theta_n)} \end{bmatrix}$$ (13)[7]

[Sensitivity of transportation cost]

$$\frac{\partial P_s}{\partial T} = B^{-1}G(G' B^{-1} G)^{-1}$$

$$= \begin{bmatrix} \frac{\partial P_{1s}}{\partial t_{11}} \cdots \frac{\partial P_{1s}}{\partial t_{nn}} \\ \vdots \quad\quad \vdots \\ \frac{\partial P_{ns}}{\partial t_{11}} \cdots \frac{\partial P_{ns}}{\partial t_{nn}} \\ \frac{\partial P_s^1}{\partial t_{11}} \cdots \frac{\partial P_s^1}{\partial t_{nn}} \\ \vdots \quad\quad \vdots \\ \frac{\partial P_s^n}{\partial t_{11}} \cdots \frac{\partial P_s^n}{\partial t_{nn}} \end{bmatrix} \tag{14}$$

However, when judging the stability of spatial equilibrium model by utilizing the sensitivity of above (12)-(14), we should face a difficult problem that there are no theoretical criteria for judging this stability. So, we regard the stability of model as the case that sensitivities in (12), (13), (14) are equal to or less than one against unit perturbation.[8] Our judgement in empirical analysis is based on this concept.

3. Empirical Analysis

3-1. *Analytical Object*

The objects to be analyzed should desirably be provided with as many of characteristics of spatial equilibrium model as possible, that is, competitive markets, full information, homogeneity of commodities and so on. The objects heretofore studied for spatial equilibrium analysis are such as coal, cow's milk, daily products and so on. They are succeeded in selections which put stress on the homogeneity of commodities. But we select vegetables, which are not very satisfactory in homogeneity but form relatively competitive markets. Out of a variety of vegetables, the carrot is selected as the object to be analyzed on the grounds that it is out of the scope "important vegetables demand and supply adjustment special project" enforced by the government and the Keito-Nokyo (systematic agricultural cooperation) since 1980 and that it fluctuates a little in its market price[9]. The demand and supply regions for analysis are divided into ten blocks in consideration of geographical and climatic similarities. The blocks are Hokkaido, Tohoku, Kita Kanto, Keihin, Tokai, Chubu-Hokuriku, Keihansin, Shikoku and Kyushu[10]. The period of sensitivity analysis is for the year 1985.

3-2. *Estimate of regional demand and supply function, Unit transportation cost*

Linear demand and supply functions should be estimated as accurately as possible in consideration of the characteristics of spatial equilibrium model. Up until now, most studies have been carried out by estimating directly formulas (1) and (2) which created such problems as too low R^2 and the like.

So, to find linear demand and supply functions, we take the following procedures based on Ueji [13].

First, the following functions are set in order to get demand and supply elasticity.

(Estimated demand function)

$$\text{In}\, Q^t = a_0 + a_1 \text{In}\, P^t + a_2 \text{In}\, Q^{t-1} + a_3 DT + c_i DR + U_t \qquad (15)^{11)}$$

for $(a_1 = ha,\ a_2 = 1-h)$

(Estimated supply function)

$$\text{In}\, Q_t = b_0 + b_1 \text{In}\, P_t + b_2 \text{In}\, Q_{t-1} + b_3 DT + d_i DR + V_t \qquad (16)$$

for $(b_1 = kb,\ b_2 = 1-k)$

P^t : Real price of wholesale markets of "period t" deflected by overall consumers' price index in 1980 as 100

P_t : Real farmers' price of "period t" deflected by agricultural products indexes in 1980 as 100

Q^t : Wholesale quantity of "period t"
Q_t : Shipment quantity of "period t"
DT : Weather dummy
DR : Regional dummy
a_1, b_1 : Short-run price elasticity
a, b : Long-run price elasticity
h, k : Adjustment coefficient
a_i, b_i, c_i, d_i : Estimated parameter
U_t, V_t : Error term

Next, linear demand and supply functions are worked out by availing long-run price elasticity obtained from (15) and (16) and the average price and quantity in each region for the past three years[12].

The data utilized for estimating (15) and (16) are "The Survey Report on Wholesale Markets for Vegetables and Fruits", "The Survey Report of Production and Shipment for Vegetables" and "The Survey Report on Prices and Wages in Rural Villages" published annually by the Japanese Government, Ministry of Agriculture, Forestry and Fisheries. The period of estimation is 11 years from 1975 through 1985[13].

Table 1 shows the results of estimation for (15) and (16). The estimation results are mostly good except for parts of supply functions, and both the short-run and long-run elasticities show reasonable values.

The unit transportation cost can be estimated by the multi-interregional transportation distance multiplied by freight rate. Since the freight rate reflecting the real rate in the market could not be availed, we found the unit transportation cost from the "automobile distance system freight rate" (Kanto Transportation Bureau, 1985) as substitute for the real rate[14].

3-3. *Solution of spatial price equilibrium*

Table 2 shows the results provided by following the parameter and the unit transportation cost obtained from the preceeding section into the Simplex tableau. From the viewpoint of interregional transportation quantity, the regional consumption in each region and the influx from chief producing districts to the huge consumption areas are regarded as optional distribution.

When the equilibrium prices and the real prices are compared, the market prices are all shifted downward, while the real farmers' prices are partially shifted by as much as 30-40%. These price differences suggest that there is market inefficiencies such as institutional trade or commercial usage. But these differences which are caused model limitations are also included.

The major reasons for model limitations are the following two points. One is the fact that despite high real farmers' price of Kyo-ninjin (carrot produced in Kyoto region), that of the Keihanshin region (Kyoto-Osaka-Kobe districts) is actually reflected in the estimation, it is not clearly indicated in the model because of the assumed equality of commodities. The other point is, because of the restricted data used for estimation, the seasonal factors of the product are not reflected in

Table 1. Estimation results of reginal demand function and supply function of carrot

	Demand							Supply						
	Parameter			R^2	DW	Adjustment coefficient	Long-run elasticity	Parameter			R^2	DW	Adjustment coefficient	Long-run elasticity
	a_0	a_1	a_2					b_0	b_1	b_2				
Hokkaido	3.6334	−0.3188	0.5121	0.9920	1.825	0.4879	−0.6534	−0.1603	0.4946	0.4913	0.9807	1.886	0.5087	0.9657
	(6.88)***	(−4.93)***	(6.87)***					(−0.34)	(2.96)**	(2.54)**				
Tohoku	3.4849	−0.4140	0.5589	0.9875	1.765	0.4411	−0.9387	−1.1707	0.5226	0.5946	0.9777	2.151	0.4054	1.2890
	(6.76)***	(−6.32)***	(7.89)***					(−2.13)**	(6.62)***	(6.22)***				
Kita Kanto	3.6281	−0.4142	0.5619	0.9536	2.223	0.4381	−0.9454	2.8711	0.1959	0.2375	0.9962	1.838	0.7625	0.2569
	(5.33)***	(−4.35)***	(7.51)***					(4.12)***	(3.76)***	(2.12)**				
Keihin	3.5000	−0.3054	0.3852	0.9988	1.904	0.6148	−0.4967	1.3846	0.4309	0.3589	0.9966	1.772	0.6411	0.6721
	(9.89)***	(−7.52)***	(5.94)***					(2.02)	(6.43)***	(3.14)***				
Tokai	3.1935	−0.3302	0.4721	0.9901	1.885	0.5279	−0.6254	−0.5754	0.3218	0.6521	0.9700	0.612	0.3479	0.9249
	(8.68)***	(−7.83)***	(7.20)***					−(1.44)	(4.41)***	(8.31)***				
Chubu・Hokuriku	2.7478	−0.2468	0.6943	0.9986	1.636	0.3057	−0.8073	2.4149	0.1002	0.1847	0.9985	1.797	0.8153	0.1228
	(4.14)***	(−3.85)***	(8.28)***					(5.32)***	(2.73)**	(1.57)				
Keihanshin	3.7116	−0.2555	0.4722	0.9978	1.596	0.5278	−0.4840	−0.9595	0.1949	0.7664	0.9699	1.677	0.2336	0.8343
	(8.73)***	(−5.01)***	(9.21)***					(−1.53)	(1.94)*	(10.37)***				
Chugoku	3.0454	−0.3943	0.5451	0.9955	1.574	0.4549	−0.8648	0.3336	0.1123	0.6684	0.9841	2.176	0.3316	0.3386
	(7.09)***	(−6.14)***	(5.78)***					(1.01)	(2.17)**	(5.64)***				
Shikoku	4.6125	−0.4522	0.2949	0.9857	2.256	0.7051	−0.6413	−1.0560	0.2494	0.8837	0.9917	1.576	0.1163	2.1444
	(11.07)***	(−8.43)***	(5.65)***					(−1.28)	(2.02)*	(7.92)***				
Kyushu	1.9878	−0.1425	0.6931	0.9912	2.328	0.3069	−0.4643	0.0048	0.2433	0.6205	0.9851	1.991	0.3795	0.6411
	(4.13)***	(−2.54)**	(11.30)***					(0.01)	(3.77)***	(7.08)***				

Notes) 1. ***, ** and * are respectively 1%, 5% and 10% significant level.
2. Parentheses are "t avlue".
3. R^2 is R-square. DW, is Durbin-Watson ratio.

Table 2. Estimation of optimal value of spatial price equilibrium (Unit : 100 t, Yen/kg)

Consumer region / Producer region	Hokkaido	Tohoku	Kita Kanto	Keihin	Tokai	Chubu・Hokuriku	Kei-hanshin	Chugoku	Shikoku	Kyushu	Total supply	Real farmers' price
Hokkaido	201.08 148.02	19.26 32.54	0.00 21.51	332.57 356.44	108.36 48.29	0.00 66.86	595.51 198.62	0.00 45.90	0.00 29.87	0.00 77.97	1,256.79 1,026.02	77.42 62.60
Tohoku	0.00 2.17	298.22 136.73	0.00 3.53	0.00 67.24	0.00 1.94	0.00 47.37	0.00 15.94	0.00 5.97	0.00 5.47	.000 20.92	298.22 307.28	93.21 95.10
Kita Kanto	0.00 7.60	0.00 22.30	124.30 43.49	0.00 66.21	0.00 9.06	0.00 3.52	40.06 2.87	0.00 0.00	0.00 1.91	0.00 0.00	164.35 156.99	94.46 77.40
Keihin	0.00 24.23	0.00 109.26	0.00 45.57	1,441.28 1,132.84	0.00 69.62	103.78 40.23	0.00 63.08	0.00 0.00	0.00 0.00	0.00 0.00	1,545.06 1,484.83	98.62 93.00
Tokoi	0.00 0.00	0.00 0.00	0.00 0.00	0.00 16.46	89.46 47.10	0.00 18.60	0.00 11.14	0.00 1.90	0.00 3.69	0.00 0.00	89.43 98.89	100.94 117.00
Chubu・Hokuriku	0.00 0.00	0.00 0.00	0.00 0.00	0.00 5.90	0.00 13.58	402.91 251.64	0.00 113.49	0.00 8.65	0.00 4.39	0.00 0.00	402.91 397.65	104.03 93.30
Keihanshin	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	50.01 62.65	0.00 0.00	0.00 0.00	0.00 0.00	50.01 62.65	107.11 139.40
Chugoku	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 0.00	0.00 9.86	14.51 2.63	0.00 0.00	0.00 0.00	14.51 12.49	104.17 65.60
Shikoku	0.00 0.00	0.00 5.66	0.00 4.09	0.00 64.07	0.00 4.65	0.00 36.63	0.00 115.72	0.00 18.74	110.32 36.96	0.00 0.00	110.32 286.52	101.49 114.90
Kyushu	0.00 0.00	0.00 1.42	0.00 0.00	0.00 44.73	0.00 0.86	0.00 7.85	23.82 75.92	125.51 47.16	0.00 19.58	264.75 126.70	414.07 324.22	94.40 65.30
Total demand	201.08 182.02	317.49 307.94	124.30 118.19	1,773.95 1,753.89	197.80 195.10	506.69 472.70	709.39 669.29	140.02 130.95	110.32 101.87	264.75 225.59	4,345.67 4,157.54	
Market price	77.42 92.10	93.21 97.30	94.46 100.90	98.62 109.00	100.94 105.70	104.03 113.40	107.11 122.20	104.17 112.80	101.49 117.10	94.40 101.70		

Notes) 1. Shipment quantity is estimated for designated consumer regions.
2. Upper column shows calculated value. Lower column shows actual average value form 1983 through 1985.

the model. The differences in the transportation cost can be explained to have been created for exactly the same reason[15].

When the above-mentioned points are taken into consideration, the results of Table 2 are not at all lacking actual reasonability but can be utilized sufficiently for sensitivity analysis[16].

3-4. *Sensitivity Analysis*

Before showing the results of sensitivity analysis, let us go back once again to (12), (13) and (14). As already mentioned, sensitivity anlysis means finding out the degree of influence of equilibrium price P_s over the small perturbation of given parameters A, B and T. When the two points of $P_{is}=P_s^i\,(i=1\cdots n)$ and of $t_{ij}=t_{ji}$ assumed in our estimation of the unit transportation cost are taken into consideration, (12), (13) and (14) are reduced to (12)′, (13)′ and (14)′ which in our case is as follows:

$$\frac{\partial P_s}{\partial A}=\begin{bmatrix} \frac{\partial P_{1s}}{\partial \alpha_1} \cdots \frac{\partial P_{1s}}{\partial \alpha_{10}} & \frac{\partial P_{1s}}{\partial(-\theta_1)} \cdots \frac{\partial P_{1s}}{\partial(-\theta_{10})} \\ \vdots & \vdots \\ \frac{\partial P_{10s}}{\partial \alpha_1} \cdots \frac{\partial P_{10s}}{\partial \alpha_{10}} & \frac{\partial P_{10s}}{\partial(-\theta_1)} \cdots \frac{\partial P_{10s}}{\partial(-\theta_{10})} \end{bmatrix} \qquad (12)'$$

$$\frac{\partial P_s}{\partial B}=\begin{bmatrix} \frac{\partial P_{1s}}{\partial \beta_1} \cdots \frac{\partial P_{1s}}{\partial \beta_{10}} & \frac{\partial P_{1s}}{\partial \gamma_1} \cdots \frac{\partial P_{1s}}{\partial \gamma_{10}} \\ \vdots \quad\quad \vdots & \vdots \quad\quad \vdots \\ \frac{\partial P_{10s}}{\partial \beta_1} \cdots \frac{\partial P_{10s}}{\partial \beta_{10}} & \frac{\partial P_{10s}}{\partial \gamma_1} \cdots \frac{\partial P_{10s}}{\partial \gamma_{10}} \end{bmatrix} \qquad (13)'$$

$$\frac{\partial P_s}{\partial T}=\begin{bmatrix} \frac{\partial P_{1s}}{\partial t_{11}} \cdots \frac{\partial P_{1s}}{\partial t_{1010}} \\ \vdots \quad\quad \vdots \\ \frac{\partial P_{10s}}{\partial t_{11}} \cdots \frac{\partial P_{10s}}{\partial t_{1010}} \end{bmatrix} \qquad (14)'$$

Now, we assume 10% for the perturbation ratio of given parameter. When reducing the unit transportation cost by 10% and expecting the case to satisfy $P_i-P_j\leqq t_{ij}\,(i,j=1\cdots n)$, (14)′ is worked out and as shown:

$$\frac{\partial P_s}{\partial T}=\begin{bmatrix} \frac{\partial P_{1s}}{\partial t_{12}} & \frac{\partial P_{1s}}{\partial t_{14}} & \frac{\partial P_{1s}}{\partial t_{15}} & \frac{\partial P_{1s}}{\partial t_{16}} & \frac{\partial P_{1s}}{\partial t_{17}} & \frac{\partial P_{1s}}{\partial t_{34}} & \frac{\partial P_{1s}}{\partial t_{35}} & \frac{\partial P_{1s}}{\partial t_{36}} & \frac{\partial P_{1s}}{\partial t_{37}} & \frac{\partial P_{1s}}{\partial t_{710}} & \frac{\partial P_{1s}}{\partial t_{810}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial P_{10s}}{\partial t_{12}} & \frac{\partial P_{10s}}{\partial t_{14}} & \frac{\partial P_{10s}}{\partial t_{15}} & \frac{\partial P_{10s}}{\partial t_{16}} & \frac{\partial P_{10s}}{\partial t_{17}} & \frac{\partial P_{10s}}{\partial t_{34}} & \frac{\partial P_{10s}}{\partial t_{35}} & \frac{\partial P_{10s}}{\partial t_{36}} & \frac{\partial P_{10s}}{\partial t_{37}} & \frac{\partial P_{10s}}{\partial t_{710}} & \frac{\partial P_{10s}}{\partial t_{810}} \end{bmatrix} (14)''$$

The results of sensitivity analysis of (12)′, (13)′ and (14)′ are showed in a matrix form.

In their, the perturbation of equilibrium prices to 10% perturbation of unit transportation cost is all less than 10% in absolute value, and the most of them shows less than 5%. The larger the difference of interregional equilibrium prices before perturbation is, the more sensitive trend is seen.

As for 10% perturbation of intercepts and slopes of linear demand and supply

[In case intercepts are shifted upward by 10%]

$$
\frac{\partial P_s}{\partial A}=\left(\begin{array}{rrrrrrrrrrrrrrrrrrrr}
-0.30 & -0.99 & -1.96 & 9.00 & -0.37 & 1.87 & 1.72 & -0.72 & -1.30 & -0.43 & -4.65 & -1.59 & -0.92 & -4.16 & -1.32 & -1.89 & -1.33 & -1.32 & -1.55 & 0.66 \\
-0.25 & -0.82 & -1.46 & 7.67 & -0.30 & 1.55 & -1.42 & -0.60 & -1.08 & -0.36 & -2.87 & -1.32 & -0.76 & -6.31 & -1.10 & -4.42 & -1.10 & -1.10 & -1.28 & 0.05 \\
-0.25 & 1.95 & 1.32 & 10.14 & 2.46 & 1.53 & 4.18 & -0.59 & -1.07 & -0.36 & -1.05 & 1.46 & -0.75 & -3.41 & -1.08 & -1.55 & -1.09 & -1.08 & -1.27 & 2.81 \\
-0.24 & -0.78 & -1.38 & 7.06 & -0.29 & 1.47 & 1.35 & -0.56 & -1.02 & -0.34 & -3.65 & -1.25 & -0.72 & -3.26 & -1.04 & -1.48 & -1.04 & -1.04 & -1.21 & 0.05 \\
-0.23 & -0.76 & -5.53 & 6.90 & -0.28 & 1.43 & 1.32 & -0.55 & -1.00 & -0.33 & -9.96 & -1.22 & -0.70 & -3.19 & -1.01 & -1.45 & -1.02 & -1.01 & -1.19 & 0.04 \\
-0.23 & -0.74 & -1.31 & -0.74 & -0.27 & 1.39 & 1.28 & -0.53 & -0.97 & -0.32 & -11.57 & -1.18 & -3.25 & -3.09 & -0.98 & -3.97 & -0.99 & -0.98 & -1.15 & -10.24 \\
-0.22 & -0.72 & -1.27 & -6.12 & -0.26 & 1.35 & 1.25 & -0.52 & -0.94 & -0.31 & -5.45 & -1.15 & -0.66 & -3.01 & -0.96 & -1.37 & -0.96 & -0.96 & -1.12 & -7.46 \\
5.56 & 1.60 & 4.48 & -9.01 & 5.52 & 1.39 & -0.97 & -2.89 & -0.97 & -2.68 & -7.97 & -1.18 & -0.68 & 2.70 & -0.98 & -3.66 & -0.99 & -0.98 & -1.15 & -2.17 \\
2.38 & 0.63 & 1.27 & -0.64 & 2.33 & 0.00 & 7.61 & 0.00 & 3.39 & 0.00 & 0.72 & 0.00 & -3.87 & 0.00 & 0.00 & 4.85 & 0.00 & 0.00 & -4.35 & 0.00 \\
6.14 & 22.47 & 21.84 & -7.34 & 6.09 & 1.53 & 1.41 & -0.59 & -1.07 & -0.36 & -5.21 & -1.30 & -0.75 & 19.87 & -1.08 & -1.55 & -1.09 & -1.08 & -1.27 & -2.39
\end{array}\right)
$$

[In case slopes are shifted non-elastically by 10%]

$$
\frac{\partial P_s}{\partial B}=\left(\begin{array}{rrrrrrrrrrrrrrrrrrrr}
-1.54 & -3.99 & -0.91 & -3.80 & 0.77 & 0.43 & -3.82 & -1.56 & -1.30 & -1.49 & -3.14 & -2.23 & -1.29 & -3.54 & 1.01 & -1.65 & -1.37 & -1.34 & -4.10 & -1.28 \\
-1.28 & -3.31 & 5.71 & -3.16 & -0.28 & 0.35 & -3.17 & -1.29 & -1.08 & -1.24 & -2.61 & -1.85 & -1.08 & -2.92 & 0.84 & -1.37 & -1.14 & -1.11 & -3.40 & -1.06 \\
-1.26 & -0.51 & 0.95 & -3.12 & 3.40 & 3.11 & -0.37 & 1.49 & -1.07 & -1.22 & 0.19 & 0.93 & 1.70 & -2.90 & 0.83 & -1.35 & -1.13 & 1.67 & -0.60 & -1.72 \\
-1.21 & -3.13 & -0.72 & -2.98 & 0.61 & 0.33 & -3.00 & -1.22 & -1.02 & -1.17 & -2.46 & -1.57 & -1.02 & -2.78 & 0.79 & -1.29 & -1.08 & -1.05 & -3.22 & -1.00 \\
-1.18 & -7.24 & -0.11 & -2.92 & 0.59 & 0.33 & -2.93 & -1.19 & -1.00 & -1.14 & -2.41 & -1.74 & -0.99 & -2.71 & 0.77 & -1.26 & -1.05 & -1.03 & -7.23 & -0.98 \\
-1.15 & -2.97 & -0.11 & -2.83 & -9.71 & -9.97 & -6.15 & -1.16 & -0.97 & -1.11 & -4.90 & -1.66 & -3.53 & -2.63 & -9.54 & -3.79 & -1.02 & -1.00 & -3.05 & -3.52 \\
-1.12 & -2.88 & -7.43 & -2.75 & -6.94 & -7.19 & -3.48 & -1.13 & -0.94 & -1.08 & -2.27 & -1.61 & -0.94 & -2.56 & -6.77 & -1.19 & -0.99 & -0.97 & -2.96 & -0.92 \\
-1.15 & 2.82 & -7.06 & 2.96 & -9.50 & -7.39 & 1.92 & -3.52 & -0.97 & -3.47 & -2.33 & -1.66 & -0.96 & -2.63 & -6.96 & -3.48 & -1.02 & -3.36 & 1.55 & -3.31 \\
0.21 & 0.00 & -0.95 & 0.00 & -1.14 & -1.29 & 0.00 & 0.00 & -1.02 & 0.00 & -5.57 & 0.00 & 0.00 & 0.00 & -0.85 & 5.04 & 0.00 & 0.00 & -1.74 & -4.15 \\
-1.27 & 20.01 & -7.16 & 3.27 & -7.87 & -8.16 & 20.15 & -1.28 & -1.07 & -1.22 & -2.57 & -1.83 & -1.06 & -2.90 & -7.68 & -1.35 & -1.03 & -1.10 & 19.92 & -1.05
\end{array}\right)
$$

functions, although both of them partially show more than 10% reactions, the most of them show values of less than 5%[17].

[In case transportation cost is reduced by 10%]

$$\frac{\partial P_s}{\partial T}=\begin{pmatrix} 2.59 & 0.80 & 0.01 & 0.23 & 0.54 & 0.30 & 0.76 & 0.35 & 1.30 & -0.34 & 0.02 \\ -0.87 & 0.67 & 0.01 & 0.19 & 0.45 & 0.25 & -2.39 & 0.29 & 1.08 & -3.14 & 0.02 \\ 2.12 & 3.42 & 2.77 & 0.19 & 0.45 & 0.69 & 0.95 & 0.82 & 1.07 & 2.48 & 0.02 \\ 2.03 & -1.52 & 0.01 & 0.18 & 0.43 & 0.24 & 0.60 & 0.27 & 1.02 & -0.27 & 0.02 \\ -2.49 & 0.62 & -2.33 & 0.17 & 0.42 & 0.23 & 0.43 & 0.27 & 1.00 & -0.26 & 0.02 \\ 1.93 & 0.60 & 0.01 & -2.40 & 0.40 & 0.22 & 0.57 & 0.26 & 0.97 & -2.82 & 0.02 \\ -3.93 & 0.58 & 0.01 & 0.16 & -2.38 & 0.22 & 0.55 & 0.25 & -7.30 & -0.25 & 0.02 \\ -6.41 & 0.60 & 0.01 & 0.17 & -2.45 & 0.22 & -1.79 & -2.10 & -4.95 & 5.25 & -0.92 \\ -7.28 & 0.00 & 0.00 & 0.00 & 0.00 & -2.94 & -2.59 & -2.91 & 0.00 & 2.05 & 0.00 \\ -4.40 & 0.66 & 0.01 & 0.19 & -2.70 & 0.25 & 0.63 & 0.29 & -5.46 & 5.79 & 0.02 \end{pmatrix}$$

Although the relationship with the level of intercepts before perturbation is not neccessarily clear, the trend is that the less elastic slopes a region shows relatively, the more sensitive it reacts.

When all the above things are judged comprehensively, the sensitivities of equilibrium prices to the perturbation of given parameter is generally insensitive. These can be interpreted as a factor which supports the stability of optimal solution worked out of the spacial price equilibrium model.

4. Conclusion

According to this analysis, it has been sufficiently comfirmed that the spatial equilibrium model with agricultural products as a object is stabilized enough to be analyzed. This supports the fact that the spatial equilibrium model can be utilized as a guide post in policy judgment, however, it does leave quite a few unsolved problems to be dealt with.

One of the problems is that commodities available for the spatial equilibrium models are restricted. The more perturbable the price and the quantity of commodities are, the more risks of isolating the model from the real situations are caused, which necessarily makes the kinds of commodities limited. Also, the problem which specifies linear demand and supply functions is an essential factor in controlling the accuracy of the model itself, including aggregation of data, and more studies on this subject should be made in the time to come.

Note

1) See P. A. Samuelson [1]
2) ′mark shows transposition of matrix.
3) Sufficient maximization is guaranteed by the concavity of $F(x)$.

4) See Kozo Sasaki [8]
5) Note that this stability relates to that of the model itself, and does not mean the market stability for the degree of market adjustment function.
6) In the following empirical work, the development (11) is not utilized, but estimated on the basis of matrix form of (12)-(14). The process is not at all lacking theoretical coordination. As for the development of (11), refer to Boot [4].
7) Following two points refer to why vector expression is deleted. One is that it is lengthily complicated, although vector expression is possible to develop of (11) if B is diagonal matrix. Another is that in empirical study Sensitivities are calculated not by the use of vector expression but matrix expression. However, it never violates both consistency.
8) In this respect, we are not free criticism for "ambiguous" or "subjective". These comments, so to speak, are fatal to sensitivity analysis.
9) Important vegetables are white rape (Chinese cabbage), growing in autumn and winter, radish growing in autumn and winter, cabbage and onion. Carrot is one of the designated vegetables, but the degree of adjustment of demand and supply is relatively low.
10) Metropolitan and respective districts correspond to the supply blocks. On the other hand, 70 markets out of the first and second city groups listed in "The Survey Report on Wholesale Markets for Vegetables and Fruits" (the Japanese Government, Ministry of Agriculture, Forestry, and Fisheries) correspond to the demand blocks. Niigata, however, is included in the North Kanto block, and the Tokai block consists of Tozan and Shizuoka Prefecture.
11) To capture the demand in the wholesale markets, a functional form with P^t as dependent variable and Q^t as independent variable is considered. The data for estimation, however, is the annual data, and setting up of (15) is judged to be more reasonable.
12) See Kozo Kasahara [7]
13) However, the supply function for Hokkaido is estimated for the years from 1966 through 1985.
14) Marine transportation is not considered here. Also, we use the standard freight ratio and assume the unit transportation cost may possibly bias upward.
15) Besides the two above-mentioned reasons, it would somewhat affect our model to have set the unit transportation cost t_{ij} inside its own region as "zero".
16) Without interregional systematic and technological restrictions and with competitive markets as trade medium, the real prices of traditional vegetables like carrot would not be separated much from the optimal prices. If separation does happen to be much, reliability of the model is lowered and there would be no reason for carrying out sensitivity analysis.
17) Many cases in which equilibrium prices do not at all react to the given perturbation are seen in the Shikoku block, which should have been created by estimating too elastically the long-run price elasticity of supply to the Shikoku

region. On the other hand, many cases of over-sensitive reactions are seen in the Kyushu region, in which the unit transportation cost should possibly have been estimated as too low.

References

1) P. A. Samuelson (1952): "Spatial Price Equilibrium and Linear Programming", The American Economic Review, vol. XLII.

2) C. L. Irwin and C. W. Yang (1982): "Iteration and Sensitivity for a Spatial Equilibrium Problem with Linear Supply and Demand Functions", Operations Research, vol. 30, No. 319-335.

*3) Iichiro Takahasi (1985): "Theory of Agricultural Market", published by Meibun Shobo.

4) J. C. G. Boot (1963): "On Sensitivity Analysis in Convex Quadratic Programming Problems", Operations Research **11**: 771-786.

*5) Kazuo Mutoh and Masaru Morishima (1979): "Interregional Agricultural Plan", published by Meibun Shobo.

*6) Kazuo Mutoh (1977): "A Theoretical and Empirical Study of the Application of Operations Research Method for Regional Planning in Agriculture" published Tokyo Agricultural University.

*7) Kozo Kasahara (1982): "Location of Agricultural and Relative Industries", published by Meibun Shobo.

*8) Kozo Sasaki (1969): "Interregional Equilibrium of Cow's Milk Market in East Jpan", vol. 41, No. 3 Journal of Rural Economics.

9) R. L. Tobin (1987): "Sensitivity Analysis for General Spatial Price Equilibrium", Journal of Regional Science, vol. 27, No. 1, 77-102.

10) T. Takayama and G. G. Judge (1964): "Spatial equilibrium and Quadratic Programming", Journal of Farm Economics, Vol. 46, pp. 67-93.

11) T. Takayama and G. G. Judge (1971): "Spatial and Temporal Price and Allocation Models", North-Holland.

*12) Toshiaki Kohno and Akira Mori (1984): "How to Reorganize Vegetable Product Regions and Correspond to Markets", published by Meibun Shobo.

*13) Toshio Ueji (1986): "Perturbation of Vegetable Prices and Reaction of Procedure", published by Meibun Shobo.

*: in Japanese

(*Received 11 October 1988*)