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Introduction of Spatial Organization of Office Activities

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Abstract

This paper is one of the urban land use theories. The pioneering work by Alonso in the early 1960's. It's a monocentricity model and the CBD is assumed to have pre-specified center of production activities. Dr. Ogawa improved on it in 1978[3]. His model is one of the general equilibrium models of urban land use. He did not assume a priori the location of either employment or households, nor the direction of each commuting trip. He determined the location of employment, households, and the direction of commuting trips within his model.

The purpose of this paper is to extent the model by considering recent growth of telecommunication technology. The telecommunication technology makes possible that firms have their separate offices. We can show that equilibrium patterns of this case are exist under proper assumptions and conditions.

Key Words: telecommunication, urban land use theories, general equilibrium models,

1. Introduction

The recent growth of telecommunication technology is changing decision of office locations of firms. The decreasing of communication cost among offices makes separating the locations of them. For example, now many firms have a front office is in Manhattan, and a back office is in suburban New Jersey. I want find the relationship such phenomena and the recent telecommunication technology.

2. The Model

Dr. Ogawa did not pre-specified the CBD is in the center of production activities. The model has two sectors which are a business firm and a household sector. He showed that existence of the equilibrium patterns and the optimal patterns.

This model has three sectors. The firm has two offices. One is a front office which does consumer services, collection of information, and interaction among other firms. It is called headquarters. Another is a back office which does not any interaction among other firms. It is called branch office.

I assume that behavior of each sector is as follows. Each front office interacts with each other front office. It's relate to the profit of firms. The back offices have no interac-

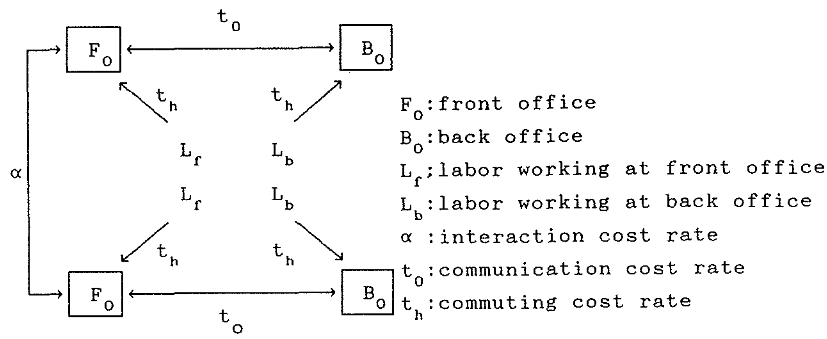


Figure 1. The model structure.

tion. Each firm must communicate between front office and back office. Each labor commutes from his/her residential place.

I assume that A closed economy in which the following assumptions are satisfied.

A1.(No Relocation Costs)

All agents are free to choose locations.

A2.(Perfect Markets)

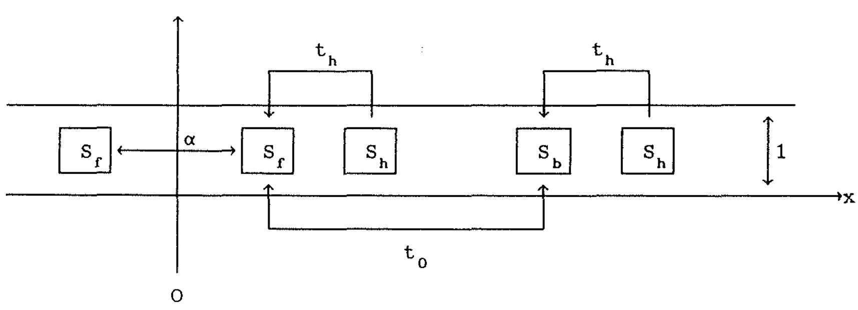
There exist complete markets for all goods and services at all locations.

A3.(Homogeneous Space)

Utility functions of households and technologies of firms are independent of location, and initial endowments of resources are equal over space.

I consider a one-dimensional city which consists of three sectors.

Profits of a firm which locates its front office at x and its back office at y are given by



S_f = the lot size of each front office

S_b = the lot size of each back office

S_h = the lot size of each household

S_f, S_b, and S_h are fixed to some positive constants.

Figure 2. The model structure (one dimensional space)

$$\max_{x,y} \pi(x,y)=A(x)-R(x)S_f-R(y)S_b-W(x)L_f-W(y)L_b-T(x,y). \quad (1)$$

$$T(x,y)=t_0 \mid x-y \mid +\delta(x,y)c \quad (2)$$

$$\text{where } \delta(x,y)= \begin{cases} 0 \text{ for } x=y \\ 1 \text{ for } x\neq y \end{cases}$$

$A(x)$=the degree of spatial accessibility of location x

$T(x,y)$=communication cost

L_f=no. of people who are working at front office

L_b=no. of people who are working at back office

The each firm has the front office at x and the back office at y. It gets $A(x)$ as a revenue, and pays each rent $R(x)S_f$, $R(x)S_b$, and each wage $W(x)L_f$, $W(y)_b$, and communicating cost $T(x,y)$. It tries to maximize the profit p by choosing the locations of each office.

The objective of the household is equivalent to choosing the residential location, x, and the job site, x , on order to maximize the amount of composite commodity:

$$\max_{x,x_w} Z=W(x_w)-R(x)S_h-T_h(x,x_w). \quad (3)$$

z=the amount of composite commodity consumed

$W(x_w)$=wage paid by a business firm location at x_w

$R(x)$=land rent for a unit of land at x

x=living place

x_w=working place

$$T_h(x,x_w)=t_h \mid x-x_w \mid, \; T_h=\text{commuting cost} \quad (4)$$

The each household lives at x, pays land rent $R(x)S_h$, works at x_w, pays $T_h(x,x_w)$, and gets $W(x_w)$. The each household tries to maximize the amount of composite commodity consumed by choosing x and x_w.

We want to achieve profit maximizing of each firm by using two profit functions which of front and back office. It means that we can get the same result as by using the profit function of each firm. Suppose that:

$$p(x^*,y^*)=\max_{x\epsilon X, y\epsilon Y} \pi(x,y) \Rightarrow \begin{cases} \pi_f(x^* \mid y^*)=\max_{x\epsilon X} \pi_f(x \mid y^*) \\ \pi_b(y^* \mid x^*)=\max_{y\epsilon Y} \pi_b(y \mid x^*) \end{cases} \quad (5)$$

Now, (x^*,y^*) is a global maximum for π (x^*,y^*) is a Nash equilibrium.

When the each office of a firm freely choose their locations, how can we find profit functions $\pi_f(x \mid y)$ and $\pi_b(y \mid x)$ for achieving the optimal location pair (x^*,y^*). One of the answers is that the each office must pay the communication cost.

$$\pi_f(x \mid y)=A(x)-R(x)S_f-W(x)L_f-T(x,y)-I(y) \quad (6)$$

$$\pi_b(x \mid y)=I(x)-R(x)S_b-W(x)L_b-T(x,y) \quad (7)$$

where

$$\begin{cases} I(y) : \text{any function of } y \\ I(x) : \text{any function of } x \end{cases}$$

$$\pi_f(x \mid y)+\pi_b(y \mid x)=\pi(x,y)$$

I(x) is any function of x. We can think this as an internal transfer of each firm. This is an account rule of each firm for achieving a Nash equilibrium.

I am using these bit rent curves for this analysis. bit rent curve for each front office:

$$\Phi_f(x,y)=\frac{1}{S_f}(A(x)-W(x)L_f-T(x,y)-I(y)-\pi_f^*) \tag{8}$$

bit rent curve for each back office:

$$\Phi_b(x,y)=\frac{1}{S_b}(I(x)-W(y)L_b-T(x,y)-\pi_b^*) \tag{9}$$

bit rent curve for each household :

$$\Psi(x,x_w)=\frac{1}{S_h}(W(x_w)-T_h(x,x_w)-p_zZ^*) \tag{10}$$

Equilibrium Conditions :

For an urban configuration U to be equilibrium, it is necessary and sufficient that it satisfies the following conditions.

(i) business firm equilibrium condition

$$(\pi^*-\pi(x,y))f^*(x)b^*(y)=0 \tag{11}$$

where $\pi^*\equiv\max_{x,y}\ \pi(x,y)$

(ii) household equilibrium condition

$$(U^*-U(S_h,z^*))h^*(x)=0 \tag{12}$$

where $U\equiv\max^*_{z,x,x_w}\{U(S_h,Z)\mid W(x_w)=R(x)S_h+p_zZ+T_h(x,x_w),\ Z\geqq 0\}$

$h(x)$ = density function of household

(iii) land market equilibrium condition

$$R^*(x)=\max\ \{\Phi_f^*(x),\Phi_b^*(x),\ \Psi^*(x),R_A\} \tag{13}$$

$$(R^*(x)-\Phi_f^*(x))f^*(x)=0 \tag{14}$$

$$(R^*(x)-\Phi_b^*(x))b^*(x)=0 \tag{15}$$

$$(R^*(x)-\Psi^*(x))h^*(x)=0 \tag{16}$$

$$R^*(x)=R_A \text{ at } x=f^-,\ f^+ \tag{17}$$

f^-, f^+: urban fringes

$$S_hh^*(x)+S_ff^*(x)+S_bb^*(x)\leqq 1 \tag{18}$$

where $\Phi_f^*(x)=\Phi_f(x\mid W^*(x),\ \underset{\sim}{f}^*(x),\ \pi^*)$

$\Phi_b^*(x)=\Phi_b(x\mid W^*(x),\ \underset{\sim}{b}^*(x),\ \pi^*)$

$\Psi^*(x)=\Psi(x\mid \underset{\sim}{W}^*(x),\ U^*)$

$\underset{\sim}{W}(x)$ = wage profile over all x

$W(x)$ = the value of $\underset{\sim}{W}(x)$ at x

$\underset{\sim}{f}(x)$ = distribution of front office over all x

$\underset{\sim}{b}(x)$ = distribution of back office over all x

(iv) labor market equilibrium condition

$$f^*(x)L_f + b^*(x)L_b = \int_{f-}^{f+} h^*(y)P^*(y,x)dy \tag{19}$$

where P(y,x)=commuting pattern between residential location y and job sites x

(v) total activity unit number constraints

$$\int_{f-}^{f+} h(x)dx = N \tag{20}$$

$$\int_{f-}^{f+} f(x)dx = \frac{N_f}{L_f} = M \tag{21}$$

$$\int_{f-}^{f+} b(x)dx = \frac{N_b}{L_b} = M \tag{22}$$

(vi) nonnegative constraints

$$h^*(x),\ f^*(x),\ b^*(x),\ R^*(x),\ W^*(x),\ P^*(x,x_w) \geqq 0 \tag{23}$$

where f^- and f^+ are urban fringe distance

$$\begin{cases} f^- = \inf\ \{x \mid f(x)>0 \text{ or } b(x)>0 \text{ or } h(x)>0\} \\ f^+ = \sup\ \{x \mid f(x)>0 \text{ or } b(x)>0 \text{ or } h(x)>0\} \end{cases}$$

3. An Example Pattern of the Model

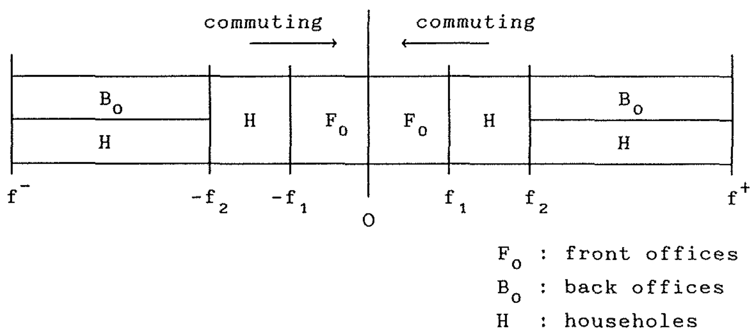


Figure 3. The model structure of the example.

The most likely case is as follows. The pattern of this case has front offices at center of the city. Its households live around the front office. And, back offices and its households are in the suburbs. It is one of the most interest pattern in this model.

From (1), (2) in this pattern behavior of firms is shown below.

Firms :

$$\max_{x,y}\ \pi = A(x) - R(x)S_f - R(y)S_b - W(x)L_f - W(y)L_b - T(x,y) \tag{24}$$

$$T(x,y) = t_0 \mid x-y \mid + \delta(x,y)c \tag{25}$$

where

$$\delta(x,y) = \begin{cases} 0 \text{ for } x=y \\ 1 \text{ for } x \neq y \end{cases}$$

in this pattern $x \neq y$, so $\delta(x,y)=1$

$A(x)$=the degree of spatial accessibility of location x

S_f=the lot size of each front office

S_b=the lot size of each back office

S = the total land area of each firm
N = total population
N_f = no. of people who are working at front office
N_b = no. of people who are working at back office
L = no. of labor per a firm
L_f = no. of labor who working at front office
L_b = no. of labor who working at back office
M = no. of firms
M_f = no. of front office
M_b = no. of back office

From (3) in this pattern behavior of households is as follows.
Households :

$$\max_{x,x_w} Z = W(x_w) - R(x)S_h - t_h \mid x - x_w \mid \tag{26}$$

x = the residential location
x_w = the job site
$R(x)$ = land rent for a unit of land at x
$W(x_w)$ = wage paid by a business firm location at x_w
S_h = the lot size of each household

$$M = M_f = M_b = \frac{N_f}{L_f} = \frac{N_b}{L_b} = \frac{N}{L} \tag{27}$$

This pattern must be symmetric. So, it suffices to examine the right half of the city. We can get boundary and fringe distances.

$$\int_{-f_1}^{f^1} f(x)dx = M_f = M,\ f(x) = \frac{1}{S_f} \tag{28}$$

$$f_1 = \frac{S_f M}{2} \tag{29}$$

$$\int_{f_1}^{f^2} h(x)dx = \frac{1}{2}N_f,\ h(x) = \frac{1}{S_h} \tag{30}$$

$$f_2 = \frac{(S_f + S_h{}^{L}{}_f)M}{2} \tag{31}$$

$$\int_{f_2}^{f^3} b(x)dx = \frac{1}{2}M \tag{32}$$

$$f_3 = \frac{(S + S_h L)M}{2} \tag{33}$$

We can represent the bid rent functions as follows.
bid rent functions:

$$\Phi_f(x,y) = \frac{1}{S_h}(A(x) - W(x)L_f - T(x,y) - I(y)\pi_f^*) \tag{34}$$

$$\text{def. } I(y)=\begin{cases} R_b-t_0y & (x<y) \\ R_b & (x=y) \\ R_b+t_0y & (x<y) \end{cases} \tag{35}$$

when $y>x$ then

$$\Phi_f(x)=\frac{1}{S_f}(A(x)-W(x)L_f+t_0x-R_b-\pi_f^*) \tag{36}$$

$$\Phi_b(x)=\frac{1}{S_h}(R_b-W(x)L_b-t_0x-\pi_b^*) \tag{37}$$

$$\Psi(x)=\frac{1}{S_h}(W(x_w)-T_h(x,x_w)-p_zZ^*) \tag{38}$$

The degree of spatial accessibility at location x is defined as follows.

$$A(x)=\int_{f-}^{f+}f(x)(K-\alpha\ |\ x-y\ |\ )dy \tag{39}$$

This is a linear accessibility function.
where

$$f(x)=\begin{cases} 0 & (x\leqq -f_1,\ f_1\leq x) \\ \frac{1}{S_{f-}} & (f_1\leqq x\leqq f_1) \end{cases} \tag{40}$$

$$A(x)=\begin{cases} \alpha Mx+KM & (x\leq -f_1) \\ -\frac{1}{S_f}x^2+KM-\frac{\alpha S_f M^2}{4} & (-f_1\leq x\leq f_1) \\ -\alpha Mx+KM & (f_1\leq x) \end{cases} \tag{41}$$

・ $f_2\leqq x\leqq f^+$

The bid rent functions $\Phi_b(x)$ and $\Psi(x)$ must be equal to agricultural rent R_A at urban fringe f^+.

$$\Phi_b(f^+)=\Psi(f^+)=R_A \tag{42}$$

The wage W(x) of the back offices at x is as follows.

$$W(x)=-w_bx+W_b \tag{43}$$

where W_b is the wage paid by the back offices at O (which is yet unknown). And, the bid rent functions of the back office and the household are equivalent in this area.

$$\Psi(x)=-\frac{1}{S_h}w_bx+\frac{1}{S_w}(W_b-p_zZ^*) \tag{44}$$

$$\Phi_b(x)\equiv\Psi(x) \tag{45}$$

$$w_b=\frac{S_h}{S_b+S_hL_b}t_0 \tag{46}$$

We can get this bid rent curve by using above equations.

$$\Phi_b(x)=-\frac{1}{S_b+S_hL_b}t_0x+\frac{S+S_hL}{S_b+S_hL_b}\frac{M}{2}t_0+R_A=\Psi(x) \tag{47}$$

$$\pi_b^*=R_b-W_bL-\left[\frac{S+S_hL}{S_b+S_hL_b}\frac{M}{2}t_0+R_A\right]S_b \tag{48}$$

$$p_zZ^*=W_b-\left[\frac{S+S_hL}{S_b+S_hL_b}\frac{M}{2}t_0+R_A\right]S_h \tag{49}$$

・ $f_1 \leqq x \leqq f_2$

To get the value of $\Psi(f_2)$ we substitute f_2 into (47).

$$\Psi(f_2)=\frac{M}{2}t_0+R_A \tag{50}$$

$$W(x)=-w_fx+W_f \tag{51}$$

where W_f is the wage paid by the front offices at O (which is yet unknown). Wage rate w_f must be equal to commuting rate t_h. (See References 5).)

$$w_f=t_h \tag{52}$$

We can get this bid rent curve by solving above equations.

$$\Psi(x)=-\frac{1}{S_h}t_hx+\frac{M}{2}\left[\frac{S_f+S_hL_f}{S_h}t_h+t_0\right]+R_A \tag{53}$$

$$p_zZ^*=W_f-\frac{(S_f+S_hL_f)M}{2}t_h-\left[\frac{M}{2}t_0+R_A\right]S_h \tag{54}$$

$$W_b=W_f-\frac{(S_f+S_hL_f)M}{2}t_h+\frac{S_f+S_hL_f}{S_b+S_hL_b}\frac{M}{2}S_ht_0 \tag{55}$$

・ $0 \leqq x \leqq f_1$

From (41) the accessibility function is as follows.

$$A(x)=-\frac{1}{S_f}x^2+KM-\frac{\alpha S_fM^2}{4} \tag{56}$$

From (51) and (52) the wage function is

$$W(x)=-t_hx+W_f. \tag{57}$$

By substituting f_1 into (53) we get

$$\Phi_f(f_1)=\Psi(f_1)=(L_ft_h+t_0)\frac{M}{2}+R_A \tag{58}$$

and this bid rent curve by solving above equations.

$$\Phi_f(x)=-\frac{\alpha}{S_f^2}x^2+\frac{1}{S_f}(L_ft_h+t_0)x+\alpha M^2 4+R_A \tag{59}$$

$$\pi_f^*=KM-\frac{\alpha S_fM^2}{2}-W_fL_f-R_b+R_AS_f \tag{60}$$

$$\pi^*=\pi_f^*+\pi_b^*$$

$$\pi^*=KM-\frac{\alpha S_fM^2}{2}-(W_fL_f+W_bL_b)-R_AS-\frac{S+S_hL}{S_b+S_hL_b}S_b\frac{M}{2}t_0 \tag{61}$$

The result of the example

The result of this example is as follows.

(a) Wage profile

The wage profile is shown by Figure 4.

$$W(x)=\begin{cases} \dfrac{S_h}{S_b+S_hL_b}t_0x+W_b & (f^-\leqq x\leqq -f_2) \\ t_hx+W_f & (-f_1\leqq x\leqq 0) \\ -tx+W_f & (0\leqq x\leqq f_1) \\ -\dfrac{S_h}{S_b+S_hL_b}t_0x+W_b & (f_2\leqq x\leqq f^+) \end{cases} \tag{55}$$

$$W_b=W_f-\frac{(S_f+S_hL_f)M}{2}t_h+\frac{S_f+S_hL_f}{S_b+S_hL_b}\frac{M}{2}S_ht_0 \tag{56}$$

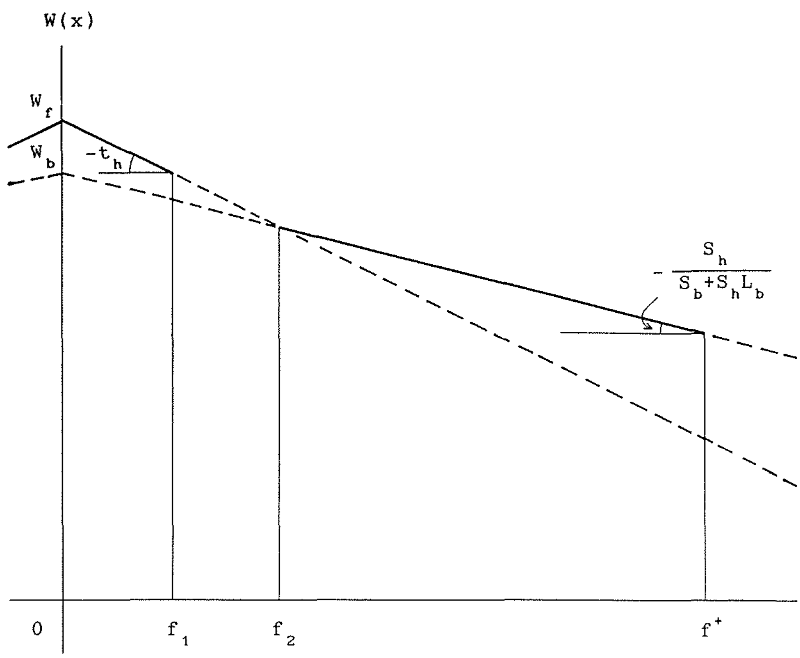


Figure 4. Wage progile.

(b) Bid rent curves

We got these bid rent curves.

$$\Phi_b(x)\equiv\Psi(x)=\frac{1}{S_bS_hL_b}t_0x+\frac{S+S_hL}{S_b+S_hL_b}\frac{M}{2}t_0+R_A \quad (f^-\leqq x\leqq -f_2) \tag{47'}$$

$$\Psi(x)=\frac{1}{S_h}t_hx+\frac{M}{2}\left[\frac{S_f+S_hL_f}{S_h}t_h+t_0\right]+R_A \quad (-f_2\leqq x\leqq -f_1) \tag{53'}$$

$$\Phi_f(x)=-\frac{\alpha}{S_f^2}x^2+\frac{1}{S_f}(L_ft_h+t_0)x+\frac{\alpha M^2}{4}+R^A \quad (-f_1\leqq x\leqq f_1) \tag{57}$$

$$\Psi(x)=-\frac{1}{S_h}t_h x+\frac{M}{2}\left(\frac{S_f+S_h L_f}{S_h}t_h+t_0\right)+R_A \quad (f_1\leqq x\leqq f_2) \tag{53}$$

$$\Phi_b(x)\equiv\Psi(x)=-\frac{1}{S_b+S_h L_b}t_0 x+\frac{S+S_h L}{S_b+S_h L_b}\frac{M}{2}t_0+R_A \quad (f_2\leqq x\leqq f^{+}) \tag{47}$$

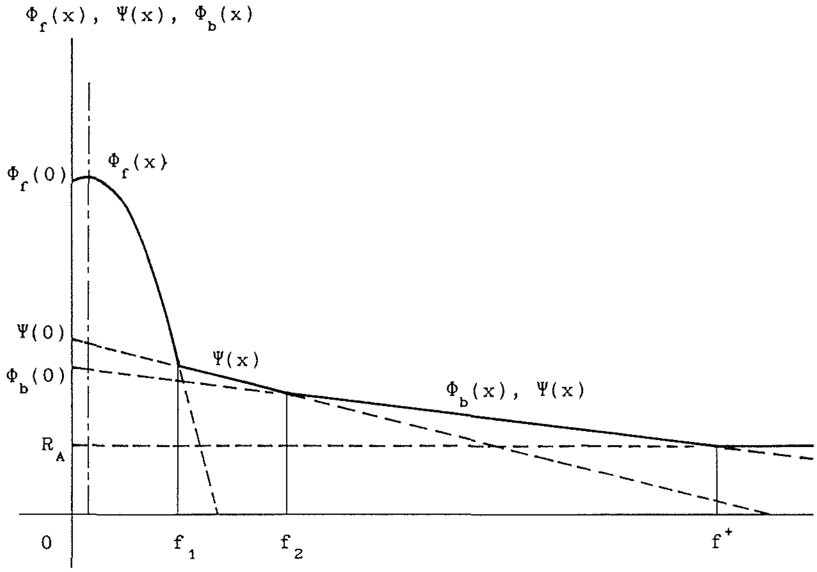


Figure 5. Bid Rent Curves.

By the symmetry of the configuration, I draw the figure only right hand side.

(c) Equilibrium conditions

For achieving this equilibrium pattern we must satisfy these conditions as follows.

at f_1

$$\Phi_b(f_1)<\Phi_f(f_1)-c \tag{58}$$

$$t_0<\frac{S_b+S_h L_b}{S_h}\left(t_h-\frac{2}{N_f}\right)c \tag{59}$$

at 0

$$\Phi_f(0)>\Psi(0) \tag{60}$$

$$t_0<-\frac{S_f+S_h L_f}{S_h}t_h+\frac{\alpha M}{2} \tag{61}$$

range of C

$$0\leqq c<\frac{\alpha N_f M}{4}\frac{S_h}{S_f+S_h L_f} \tag{62}$$

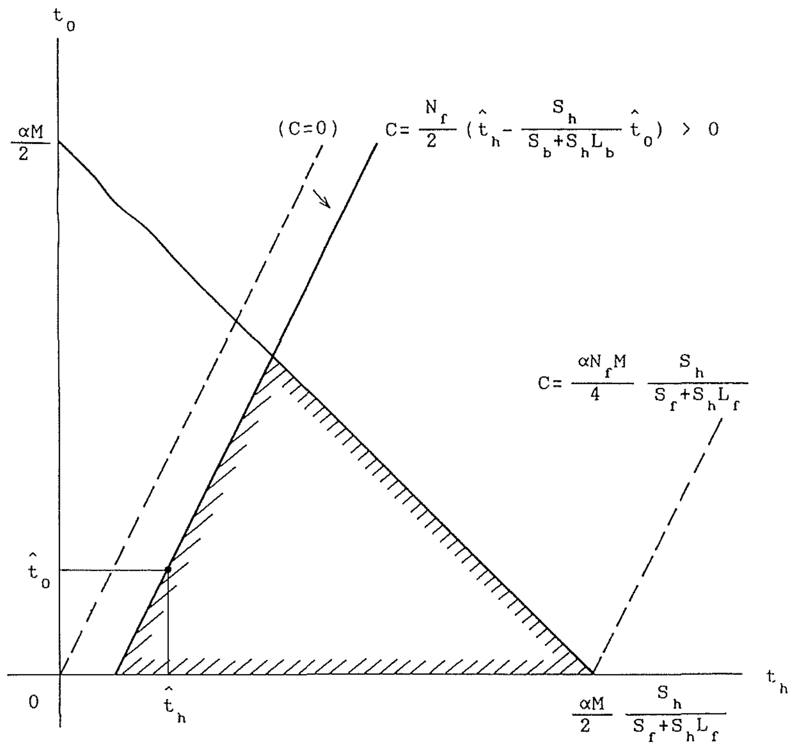


Figure 6. Equilibram Conditions.

4. Conclusions

We could find one equilibrium pattern of this model. The existence of this equilibrium pattern is showing that the firms can have two offices (the front office and the back office) in proper conditions. For separating offices from this example the communication cost(t_0) needs to be smaller than the conditions of the equations (59) (61). This means for the separation of offices cost-down of the communication is needed.

In future work, we should find all equilibrium patterns and optimizing patterns. The result will show that behaviors of the firms and efficient land use patterns under new technology of telecommunications.

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