Experimental Investigations on the Internal Granular Movements of Sand.

By

Prof. Fukuhei TAKABEYA, *Kogakuhakushi.*

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It is well to recognise that the present earth mechanics cannot compute anything in a manner approaching the accuracy usually possible in stress computation for steel works, especially of frames and beams.

Earth work engineering, however, occupies the chief part of civil engineering practice and the computations concerned with soil problems must always be worked out beyond the range of our knowledge.

The classical theories of granular masses formulated by Coulomb and Rankine still play an important rôle in the computation of granular earth pressure, practically no progress having been made since their day. However, such powerful arguments have accumulated during the last half century against the validity of those theories, that many new test results obtained independently on the old earth-pressure theories, have been gained by various methods and in regard to various points by many authors, among whom K. Terzaghi may be worthy of special mention.(1)

In the present note the internal granular movements of dry sands obtained experimentally under several conditions of boundaries will be reported with special regard to the partial yielding of the base of a sand pile, the effect of horizontal slips(2) and angular displacements of a side wall.

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(1) Karl Terzaghi: Erdbaumechanik auf bodenphysikalischer Grundlage, 1925.
Old Earth-Pressure Theories and New Test Results, Engineering News-Record, 1920.
Principles of Soil Mechanics, Engineering News-Record, 1925.

(2) A similar experimental investigation and some others were reported by Terada and Miyabe in connection with a study on the phenomena of land-slide. See Bulletin of the Earthquake Research Institute, Tokyo Imperial University, Vol. IV, March, 1928.

The results obtained in the experiments are sufficient to show that the old earth pressure theories may be quite inaccurate even for obtaining the dry sand pressure, because the angle of repose, which is a very important element in the old theories, seems in our experiments to be less effective than the angle of plane which has been called by the author *plane of actual rupture.*

The propagation phenomena of the internal granular movements are essentially new to the best of the author's knowledge and may be found to have a certain application to earth work engineering concerned with culvert work in sandy soil as well as to mining works in similar soil, serving to suggest some tendencies of the properties of the earth in question.

Experiment and Results.

The granular substances used in the present experiments are chiefly of river sands and have been completely dried up, having been sieved to the size of 0,86 mm dia. mixed, in some measure, with the finer ones.

In order to observe the mutual similarity of sand and other granular substances, we have tried by different kind of grains and sometimes by powdered clay to the special study of the property of different kinds. The apparatus of the experiment consists of a rectangular box, whose one of the side walls being always kept by glass plate to the facility of observation to the internal granular movements. Horizontal lines of black sands were used for observing the deformation of the internal mass; the black sand was applied only adjacent to the glass side wall, so that it may not disturb the characteristics of the sand mass as a whole.

The experiments which have been done by the author may be classified as follows:

(a). Effect of Partial Sinking of Bed
(b). Effect of Rotation of Side Wall
(c). Effect of Sliding of Side Wall
(d). Effect of Sidewards Displacement

I. Effect of Partial Sinking.

For the investigation of the effect of partial sinking of the bed, we made a slit varying from 1.0 cm up to 10 cm in width, i.e. successively in ten different cases; the rectangular boxes used in this experiment are as shown in the following photograph (*Fig. 1*).

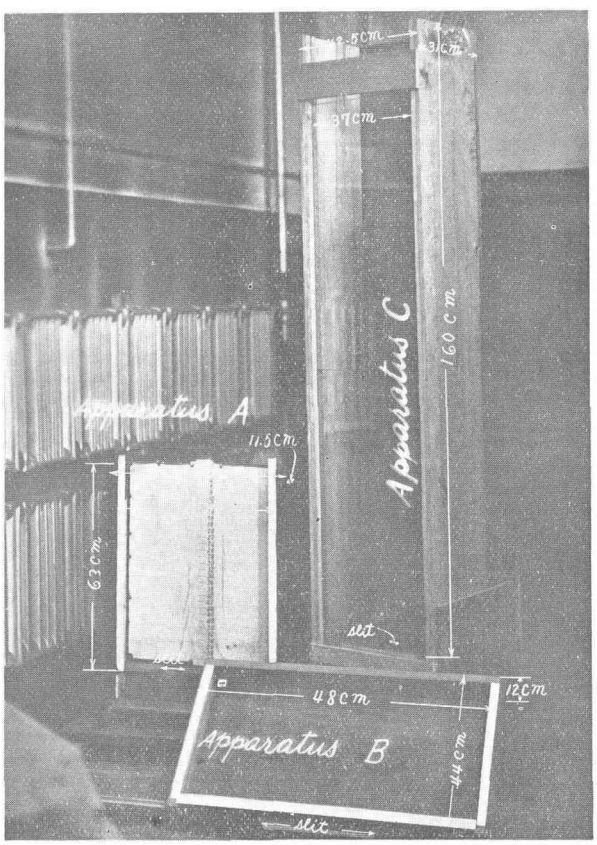

Fig. 1.

Sand is filled loosely in the vessel always in horizontal layers and the different horizontal surfaces are marked by a number of horizontal lines of black sand, making the relative movement of the granular masses easier to observe. This simple method of observation shows, so to speak, the history of the deformation in a very short time.

In order to study the propagation phenomena of the granular movement we filled sand very loosely in the vessel and gradually caused a partial sinking in the bed, piling up the falling sand grains from the bed slit in the natural state and removing them little by little.

The results of the experiments by Apparatus A are shown in Photographs No. 1 to No. 30, in which Photographs No. 1 to No. 12 show the different state of successive granular movements in the case of the 7 cm slit and Photographs No. 13 to No. 24 the similar configuration for the 8 cm slit.

The internal granular deformation begins in the vertical direction and propagates as shown in Photographs No. 1 to No. 6. The accompanying movement continues only in the vertical direction, gradually extending its effective boundary (Photo. No. 7—No. 12).

Photographs No. 13 to No. 24 show the similar phenomena for the case of the 8 cm slit. For the sake of more precise study, the early granular movement for the 10 cm slit is given in Photographs No. 25 to No. 30.

It is noticed from these experiments that the movement of sand grains relatively early is confined, even in the case of granular sand completely dried up, within the boundary of the sand tube which stands above the slit. In the relatively later movement the sideward sand grains take part, especially the free grains on the upper surface.

More precise observations may enable us to investigate the interesting phenomena of granular deformation in a very short period.

Photographs No. 31 to No. 36 are of similar experiments using red beans and soy-beans. The effective boundary has a tendency to extend sidewards, in comparison with the foregoing cases.

Next we have investigated the granular movement caused by gradual widening of the slit, using Apparatus B. Photographs No. 43 and 44 show some initial and nearly final stages respectively of the deformation; this may be studied again by the author more precisely later on. Photographs No. 37 to No. 42 show the irregular

movement of sand completely saturated with water; this experiment was done by giving shocks intermittently. This shows a complicate but regular form of movement in lump state.

Again, some of the foregoing experiments using Apparatus A were repeated with Apparatus C, in order to investigate the same phenomena on a larger scale. In these sand was completely dried and filled loosely (Photo. No. 45—No. 49), while Photographs No. 50 to No. 55 show the similar characteristics of dry sand compactly packed and its photograph-number with suffix "a" gives the details enlarged. Further, Photographs No. 56 and No. 57 are of dry powdered clay.

The entire investigation above treated is concerned with the internal granular movement caused by bed sinking due to a single slit, while it has been next noticed what characteristics might be expected from two parallel slits; according to the experimental results each movement takes place independently and the one never hinders the other. See Photo. No. 58—No. 69.

II. Effect of Rotation and Sliding of Side Wall.

In order to investigate the effect of rotation of a side wall, we used a rectangular box, Apparatus D, both whose side walls were of glass plate (*Fig. 2*).

Sand was filled in the box always in horizontal layers and the different horizontal surfaces were marked as in the foregoing cases by a number of horizontal lines of black coloured sand. A vertical side wall hinged at the lower edge has been put in gradual motion of rotation and we observed the granular movement of the sand. Photographs No. 70 to No. 75 show the results.

With respect to *Fig. 3*, B-B' represents the fixed bed plate of the rectangular box, W-W' the rotatable wall hinged at the lower end W, which is driven in clockwise direction as shown in the sketch, and a-b is the initial horizontal surface of the loosely packed sand. When the rotatable wall is put in motion and the point W'

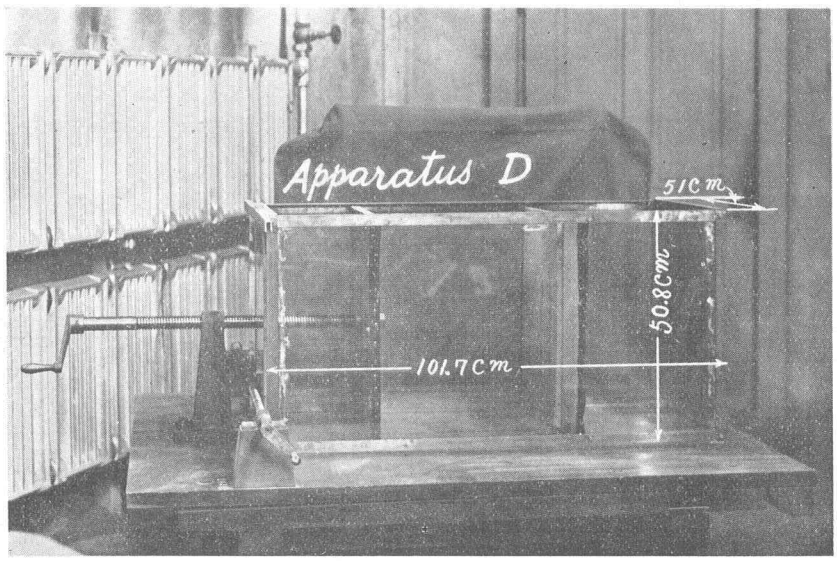

Fig. 2.

comes to the point W'', the horizontal surface near the wall makes a curved surface i,e. the horizontal surface between c and b makes a curved surface cb', while the surface between a and c keeps its initial position. The sand mass on the right side of cW is put in motion generally, while the mass on its left side seems to remain quite unaffected.

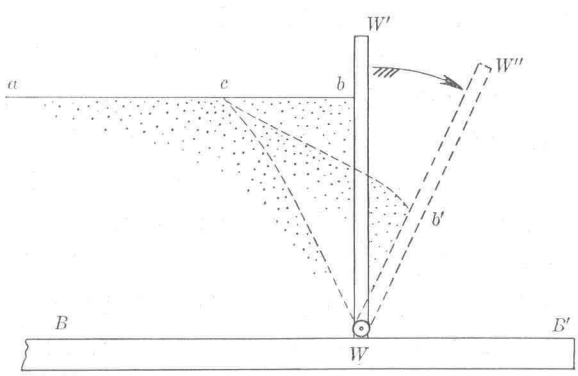

Fig. 3.

The same apparatus has been used to study the effect of sliding of the side wall, but in this case the vertical side wall was reconstructed to be movable in the horizontal direction.

On gradually sliding the movable wall along, the sand mass adjacent to the movable wall slips down along the plane of actual rupture.

Photographs No. 76 to No. 81 show the results.

With regard to *Fig. 4*, *B-B'* represents the fixed base of the rectangular box, *W-W'* the sliding wall which is driven in horizontal direction as shown in the sketch, and *a-b* is the initial horizontal surface of the sand loosely packed. When the sliding wall is slid

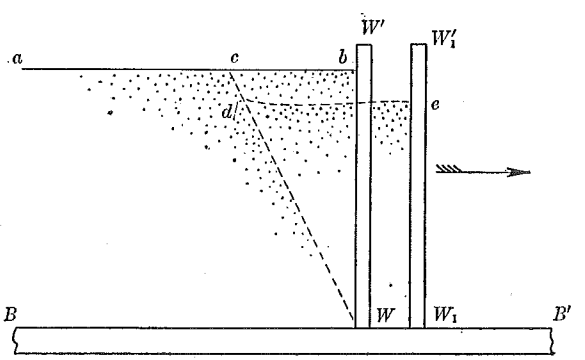

Fig. 4.

forward and the point W' comes to the point W'_1, the sand mass of wedge form cbW comes to be a sand mass of trapezoidal form deW_1W; at the start the horizontal surface *b-c* comes down keeping its horizontal state of initial surface as shown in Photographs No. 77 and No. 78; in the course of gradual sliding, the sand grains make a slope varying at every moment, finally to the so-called angle of repose for the given sand. Photograph No. 81 shows the final state of receding.

The above two cases are quite different in the boundary conditions; it is however noticed that the angle α marked in Photograph No. 72 shows approximately the similar value with the angle β in

Photograph No. 78. These angles have been called by the author "Plane of Actual Rupture," which always depend on the proper nature of the sand used in the experiment. The very important element in old earth pressure theories is the angle of repose for every kind of earth, while in the present note we may emphasize that the more important element in the calculation of actual earth pressure may be the plane of actual rupture, because the angle of repose play its rôle only when the free surface is in motion.

Photographs No. 82 to No. 87 serve for the explanation of the plane of actual rupture, which may not depend upon the length of the box but upon the characteristics of the sand itself.

III. Effect of Sidewards Displacement.

In the case of sidewards displacement of a small part of the vertical wall, we have studied two cases, of which the one is concerned with a single vertical slit of 4 cm width, and the other with two vertical slits of the same dimensions; the former is shown in Pl. XVI, Photographs No. 88—No. 96 and the latter in Pl. XVII, Photographs No. 97—No. 105.

The propagation phenomena of granular internal movement in this case bears a striking resemblance to those of the case, where the horizontal slits in a partial sinking have been investigated.

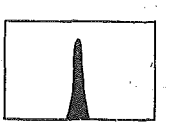

Fig. 5.

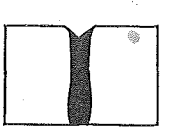

Fig. 6.

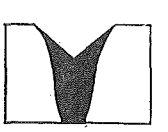

Fig. 7.

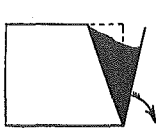

Fig. 8.

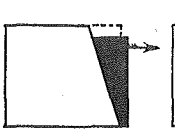

Fig. 9.

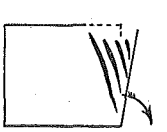

Fig. 10.

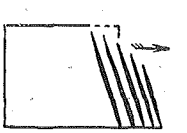

Fig. 11.

The following index may be convenient so as to make it possible to locate quickly the results obtained in the experiment.

I. Effect of Partial Sinking

Dry Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Width of Slit 7.0 cm, Apparatus A. (Pl. I, II).

Dry Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Width of Slit 8.0 cm, Apparatus A. (Pl. III, IV).

Dry Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Width of Slit 10 cm, Apparatus A. (Pl. V).

Beans Packed Loosely. Width of Slit 4 cm, Apparatus A. (Pl. VI).

Sand Packed and Saturated with Water Completely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Width of Slit 10 cm, Apparatus A. (Pl. VII).

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Granular Movement Caused by Gradual Widening of the Slit. Apparatus B. (Pl. VIII, No. 43, 44).

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Width of Slit 4 cm, Apparatus C. (Pl. VIII, No. 45-49).

Sand Packed Compactly, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Width of Slit 7 cm, Apparatus A. (Pl. IX, X, No. 50-55).

Dry Powdered Clay Packed Loosely. Width of Slit 1.0 cm, Apparatus A. (Pl. X, No. 56, 57).

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Width of Both Slits 2 cm, Apparatus A. (Pl. XI, No. 58-61).

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Width of Both Slits 6 cm, Apparatus A. (Pl. XI, No. 62, 63).

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Semicircular Two Holes of 5 cm Dia. Apparatus A. (Pl. XII).

II. Effect by Rotation and Sliding of Side Wall

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Angle of Rotation $7^1/_2$ degrees to 90 degrees. Apparatus D. (Pl. XIII).

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Distance of Receding 2 cm to 28 cm, Apparatus D. (Pl. XIV).

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Length of the Box 7 cm. Distance of Receding 2 cm to 22 cm, Apparatus D. (Pl. XV).

III. Effect of Sidewards Displacement

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Single Vertical Slit of 4 cm Width, Apparatus B. (Pl. XVI).

Sand Packed Loosely, after being Sieved to the Size of 0.86 mm Dia. Mixed with Finer Ones. Double Vertical Slits of 4 cm Width, Apparatus B. (Pl. XVII).

Summary and Conclusions

The general conclusions to be drawn from the results of the present investigation may be summarized as follows:

1. In regard to fine sands loosely packed, the internal granular movement caused by the sinking due to a horizontal slit in the bed plate confines itself at the start to the vertical direction and propagates in a characteristic feature as shown in Photographs No. 26 to No. 30 and there never comes any sidewards breakdown of the granular mass. This deformation appears in general at its start in a characteristic feature as shown in Fig. 5 and terminates as shown in Fig. 7, after taking the intermediate state of Fig. 6.

2. As we noticed in the case of horizontal double slits, the deformation phenomena of the sand mass which stands on the first horizontal slit seem to be unaffected by the other one due to the second slit (See Photographs No. 58—No. 69). It may be said that there is a characteristic of independent deformation in such case.

3. With respect to sands completely saturated with water, there occurs no sidewards breakdown whatsoever, but there come irregular movements of a lump state, only in a vertical tube of the sand mass which stands above the slit.

4. For sand compactly packed, the deformation phenomena are quite different from the case in which loosely packed sand is treated. Here we frequently observe deforming voids. (Photographs No. 51—No. 55)

5. Both for rotation and receding of the side wall, the plane of actual rupture may be approximately the same in amount; from the former operation there results the deformation feature shown in Figs. 8 and 10, and from the latter there comes the granular movement as shown in Figs. 9 and 11.

Sapporo, May, 1931.

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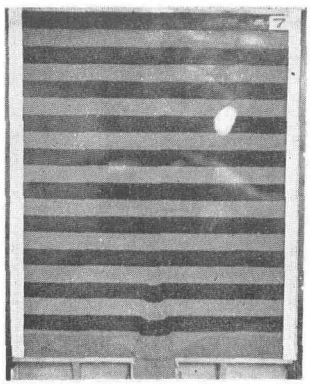

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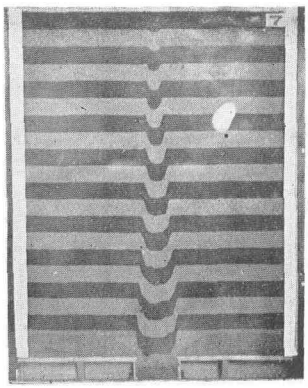

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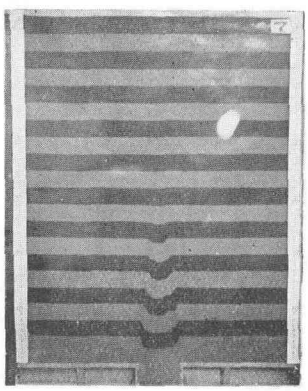

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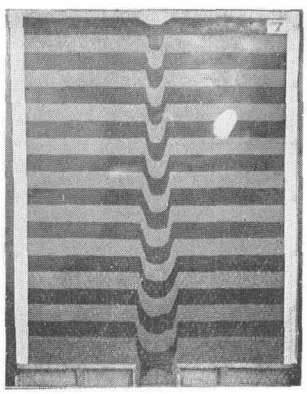

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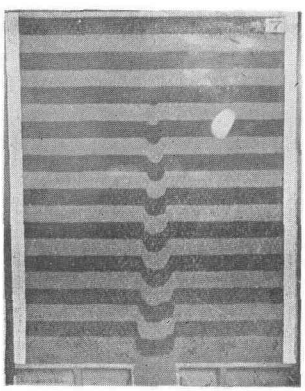

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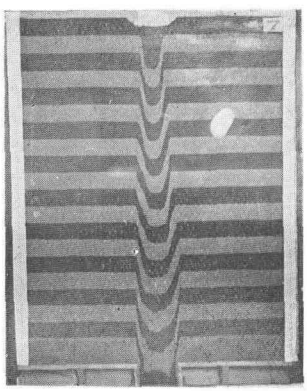

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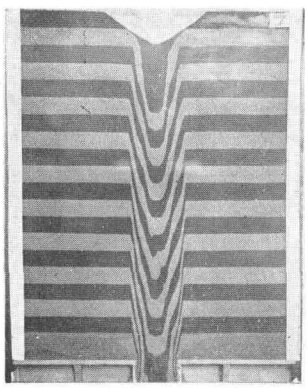

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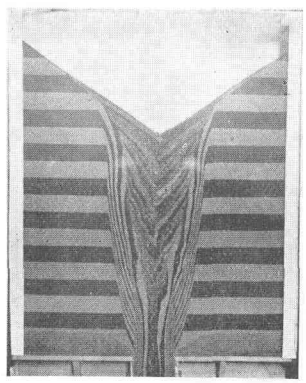

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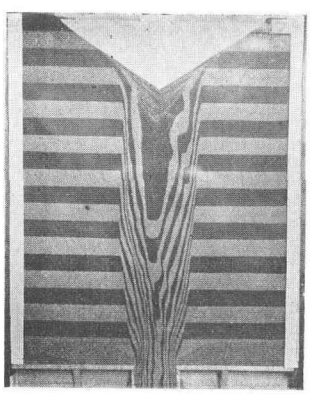

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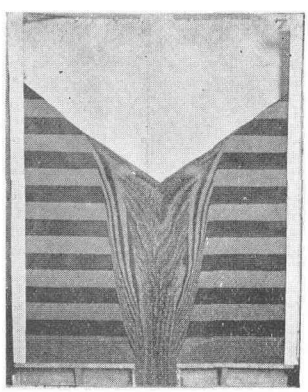

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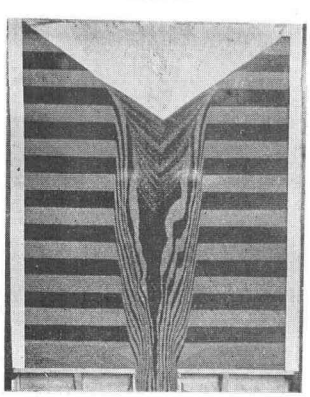

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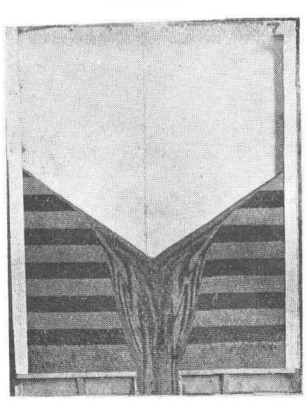

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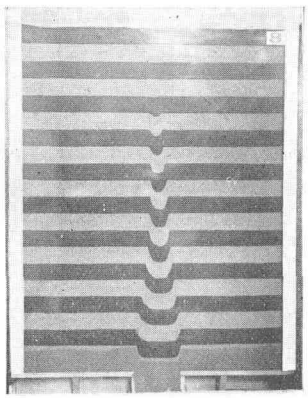

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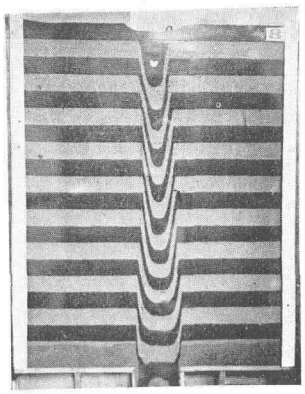

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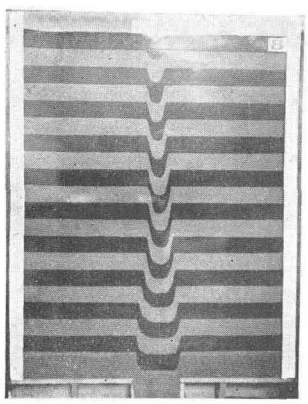

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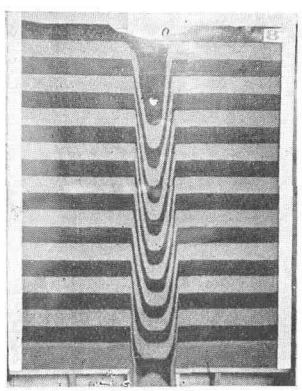

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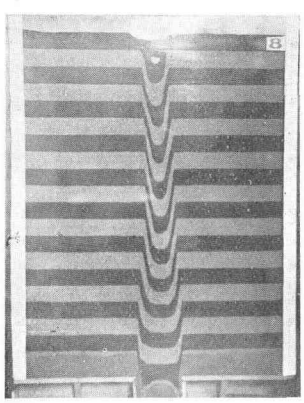

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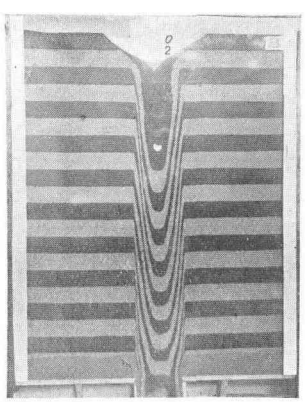

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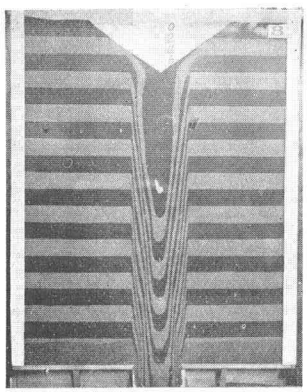

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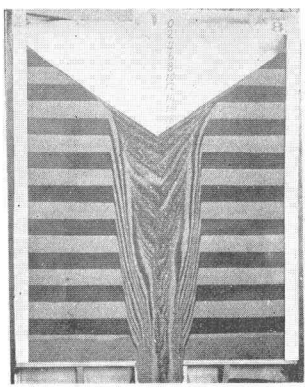

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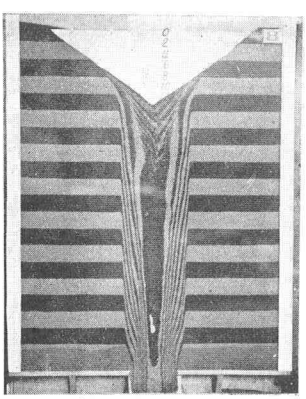

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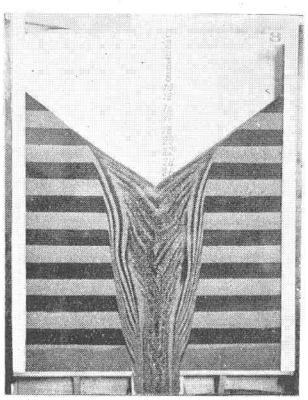
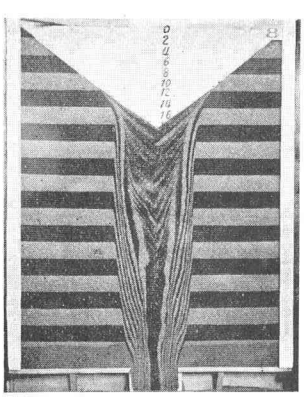
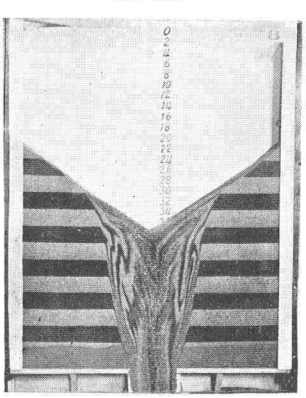


No. 21.



No. 24.



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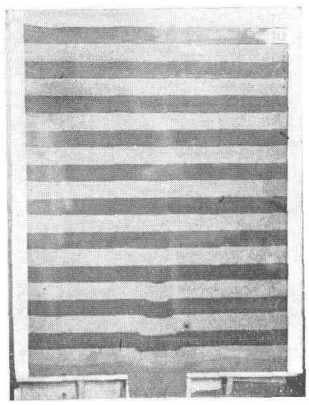
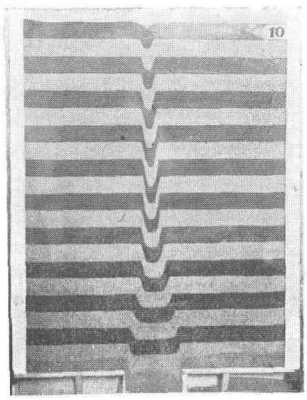
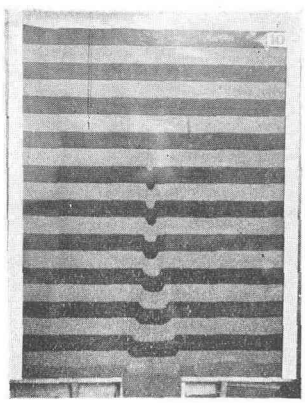
No. 25.



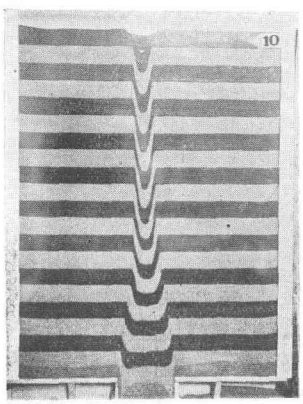
No. 28.



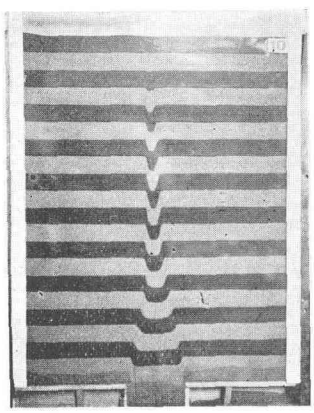
No. 26.



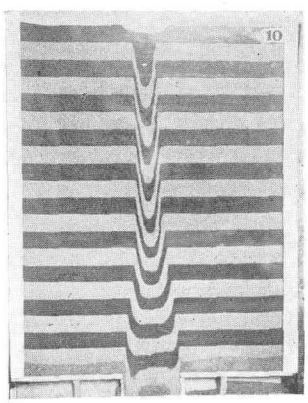
No. 29.



No. 27.



No. 30.



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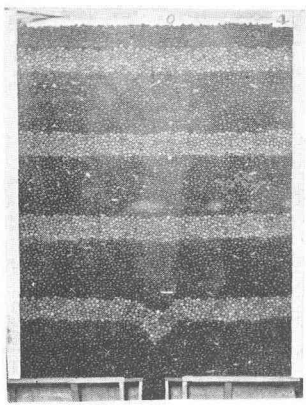
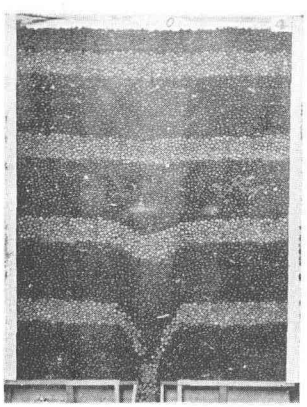
No. 31.



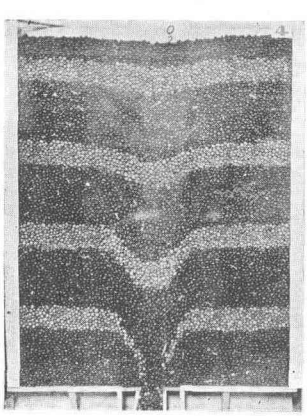
No. 34.

No. 32.

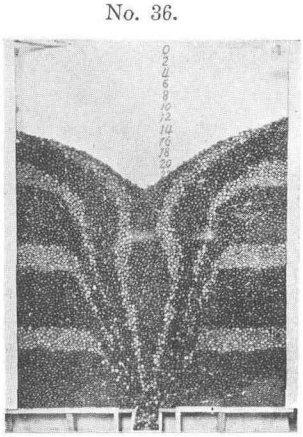


No. 35.

No. 33.



No. 36.



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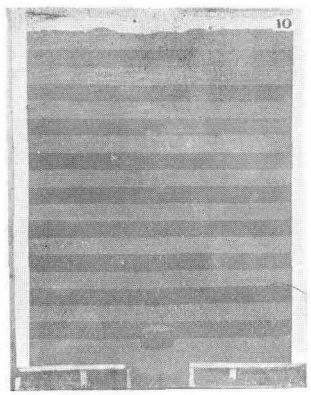
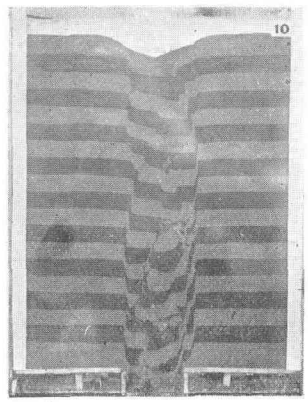
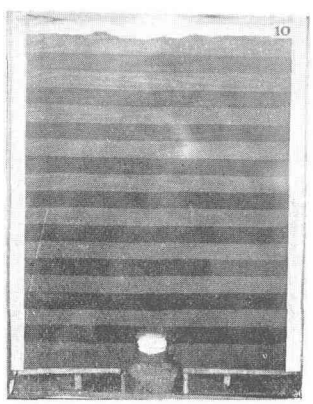
No. 37.



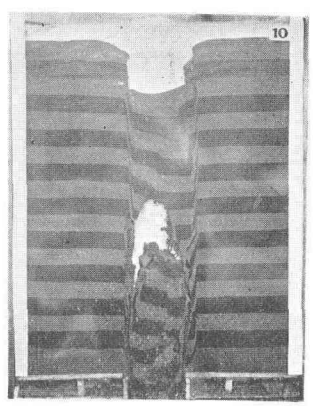
No. 40.



No. 38.



No. 41.



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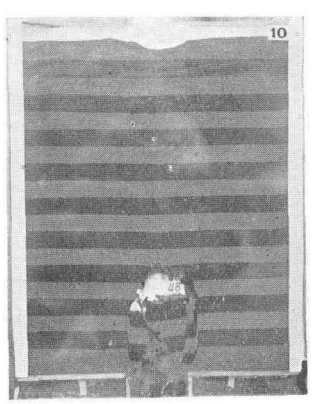
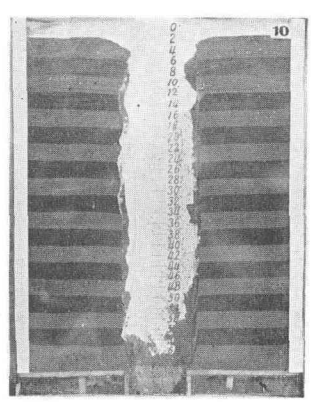


No. 42.



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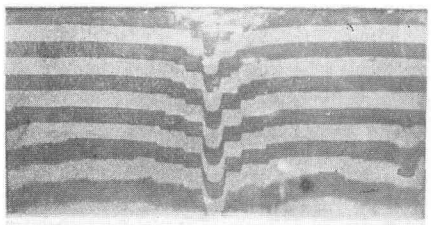
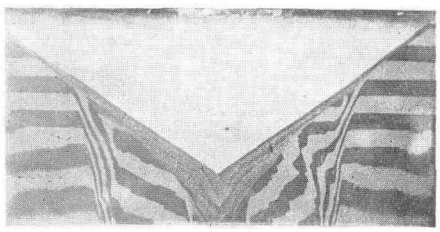
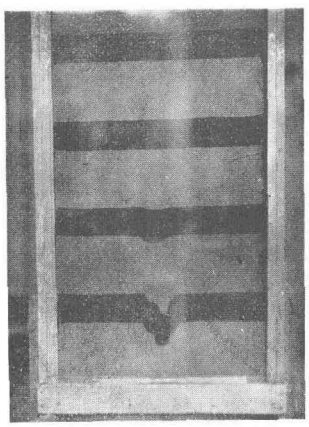
No. 43.



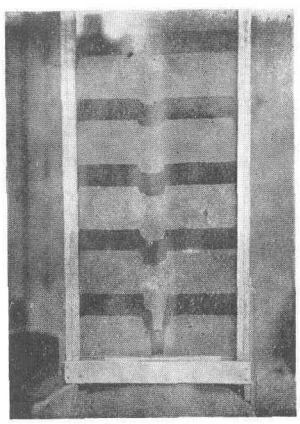
No. 44.



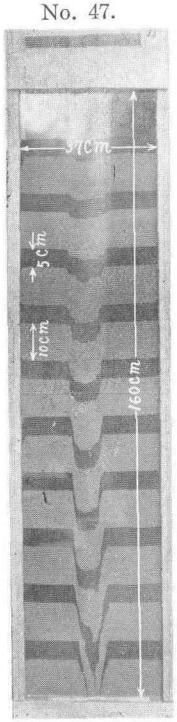
No. 45.



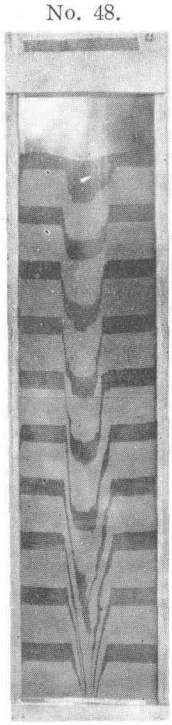
No. 46.



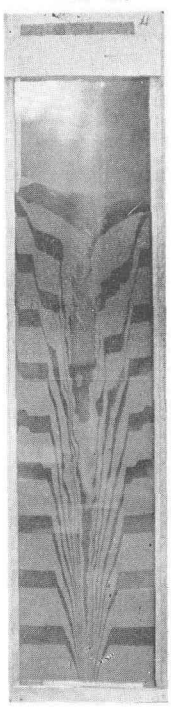
No. 47.



No. 48.



No. 49.



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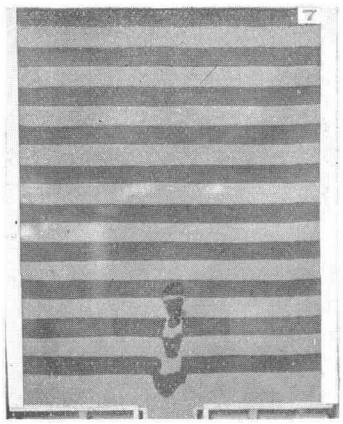
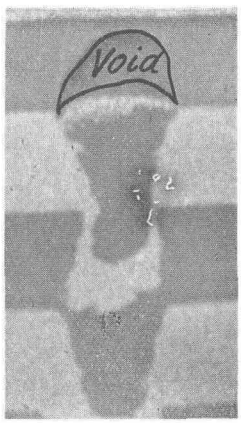
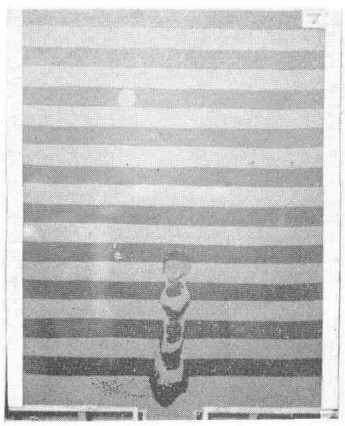
No. 50.



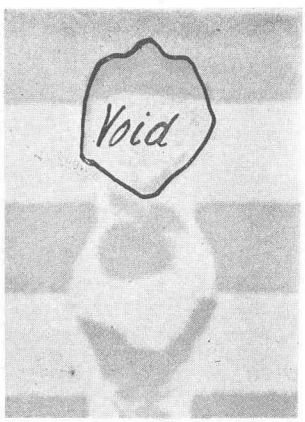
No. 50 a.



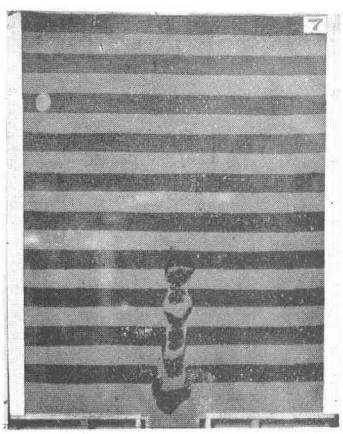
No. 51.



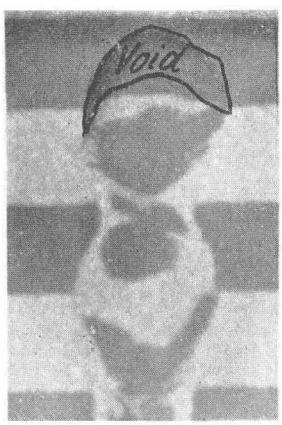
No. 51 a.



No 52.



No. 52 a.



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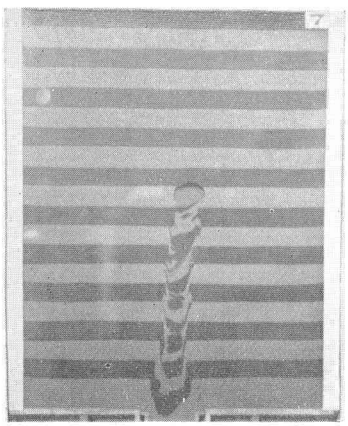
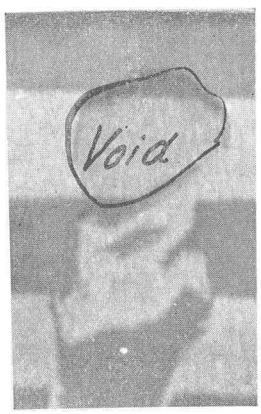
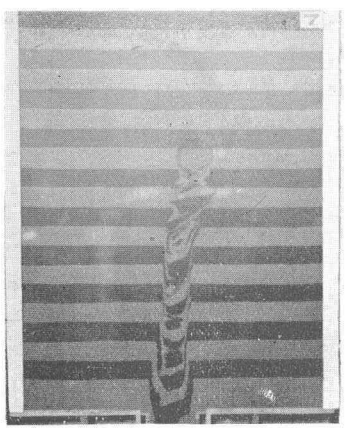
No. 53.



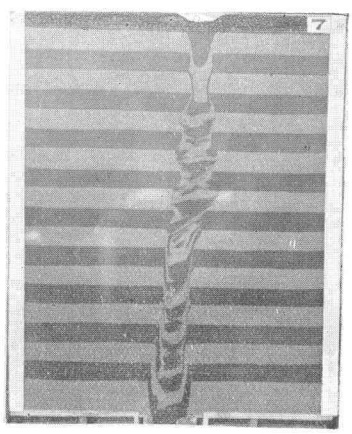
No. 53 a.



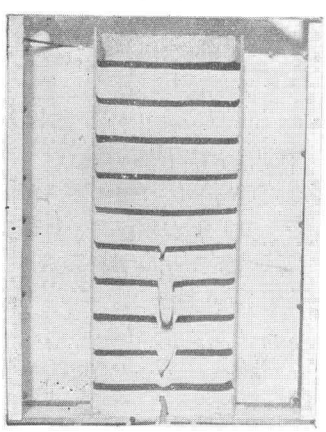
No. 54.



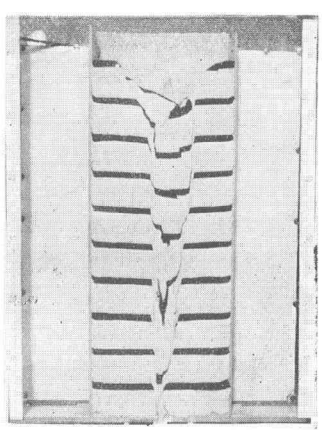
No. 55.



No. 56.



No. 57.



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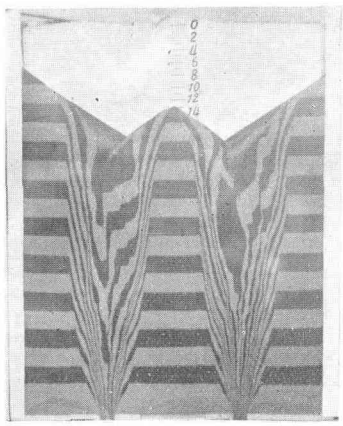
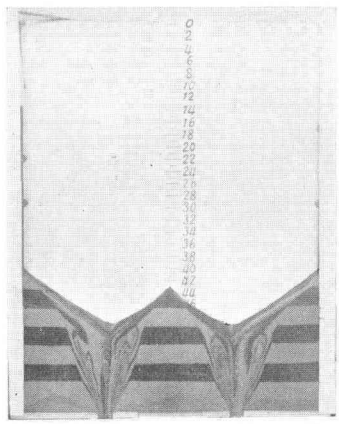
No. 58.



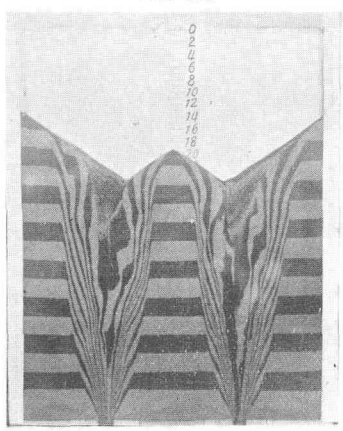
2 cm slit.

No. 61.



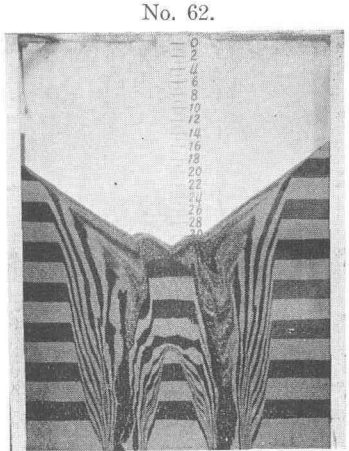
2 cm slit.

No. 59.



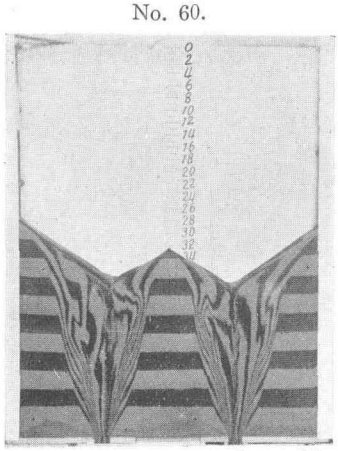
2 cm slit.

No. 62.



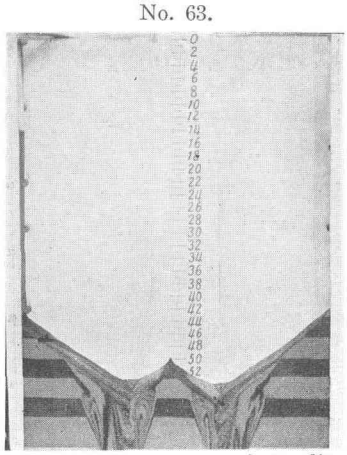
6 cm slit.

No. 60.



2 cm slit.

No. 63.



6 cm slit.

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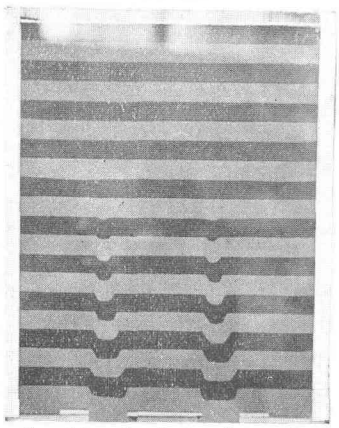
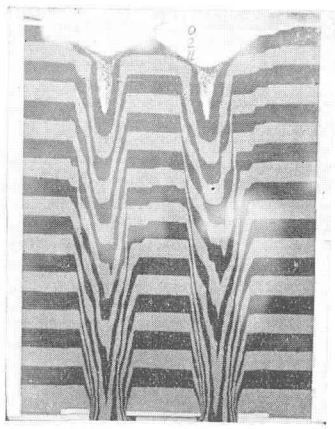
No. 64.



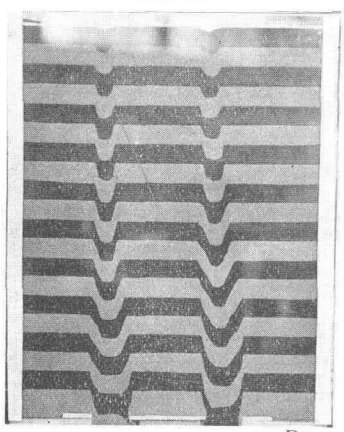
Semicircular hole of 5 cm dia.

No. 67.



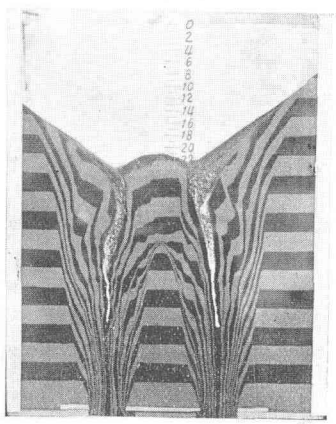
Do.

No. 65.



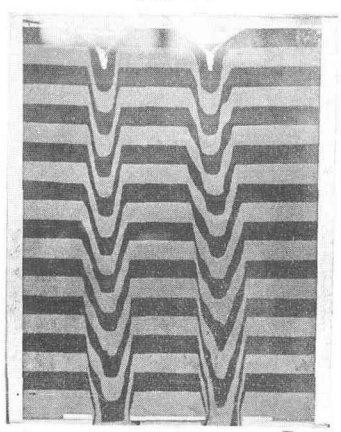
Do.

No. 68.



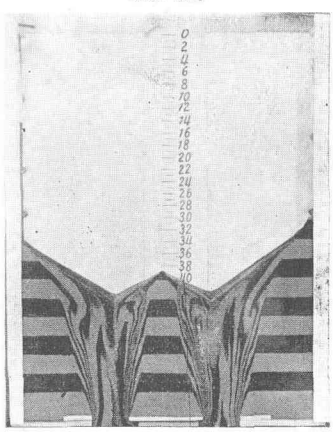
Do.

No. 66.



Do.

No. 69.



Do.

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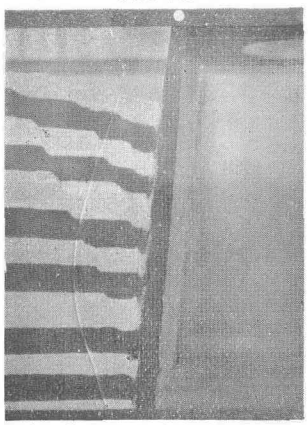
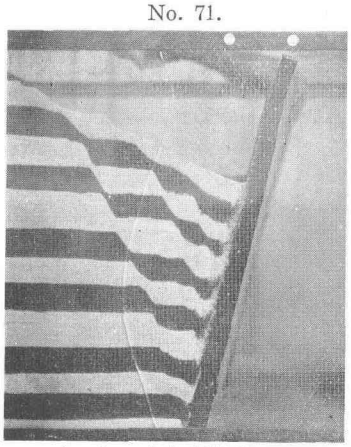
No. 70.



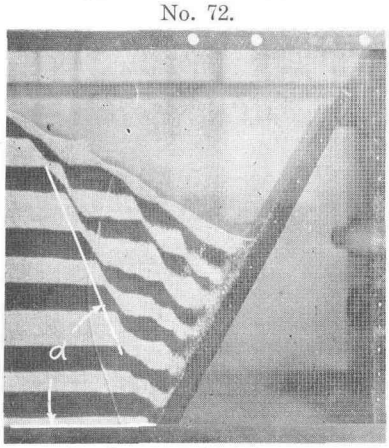
Angle rotated $7\frac{1}{2}$ degrees.

No. 71.



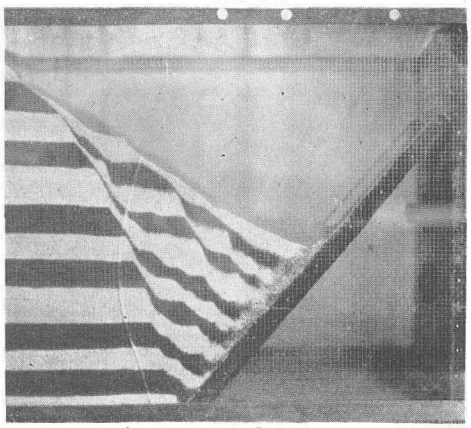
Angle rotated 15 degrees.

No. 72.



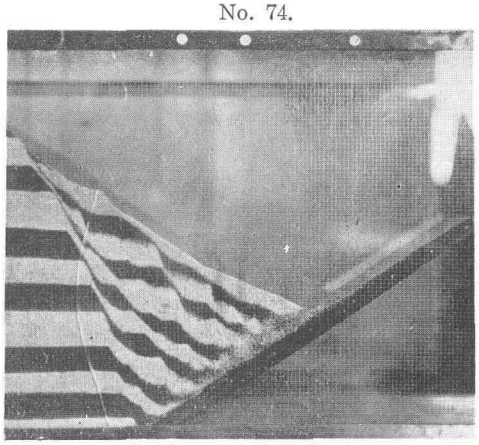
Angle rotated 30 degrees.

No. 73.



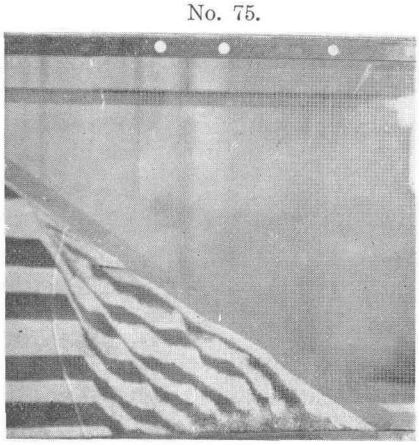
Angle rotated 45 degrees.

No. 74.



Angle rotated 60 degrees.

No. 75.



Rotation completed.

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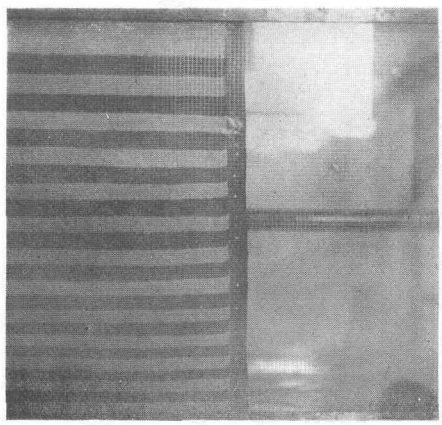
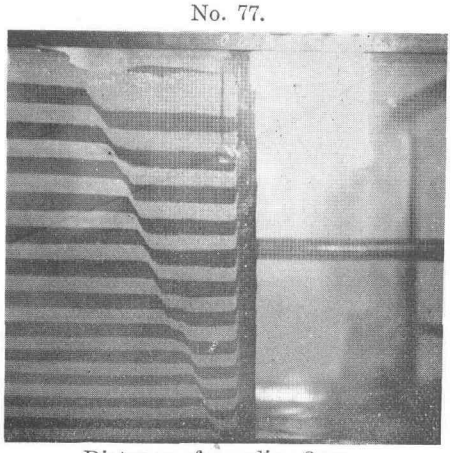
No. 76.



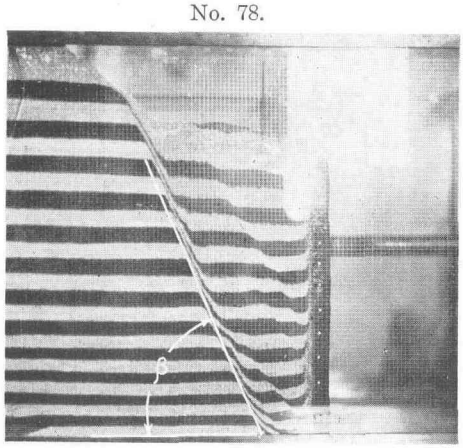
Distance of receding 0 cm.

No. 77.



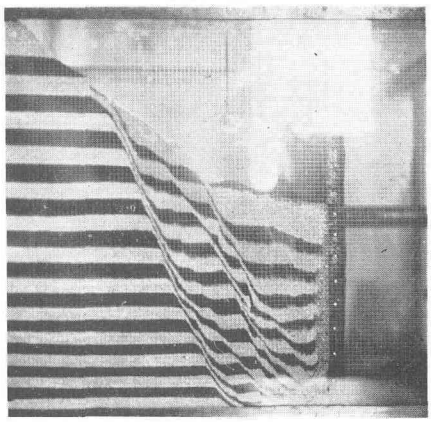
Distance of receding 2 cm.

No. 78.



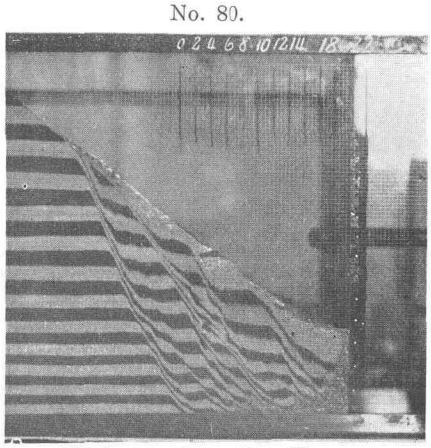
Distance of receding 6 cm.

No. 79.



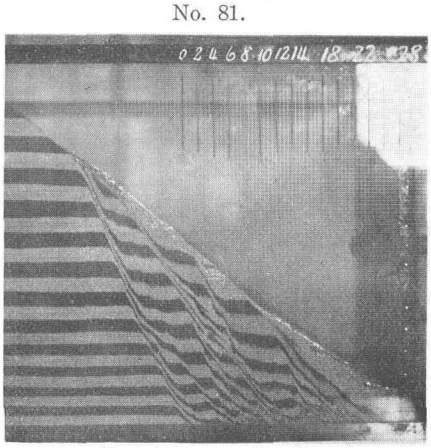
Distance of receding 12 cm.

No. 80.



Distance of receding 22 cm.

No. 81.



Receding completed.

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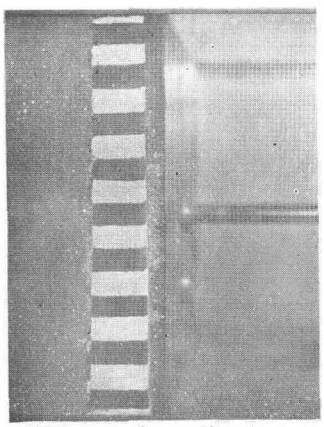
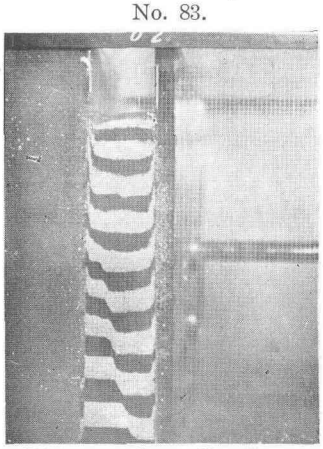
No. 82.



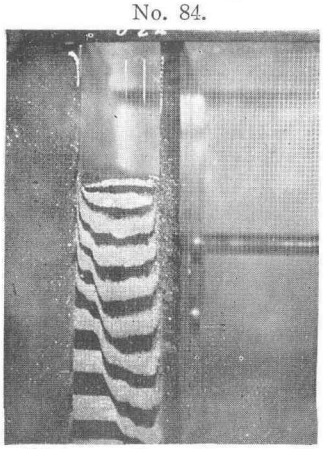
Distance of receding 0 cm.

No. 83.



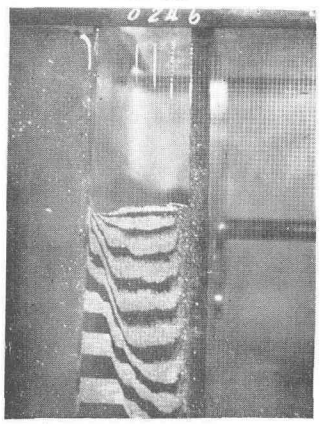
Distance of receding 2 cm.

No. 84.



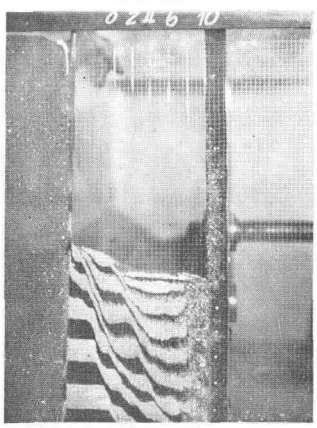
Distance of receding 4 cm.

No. 85.



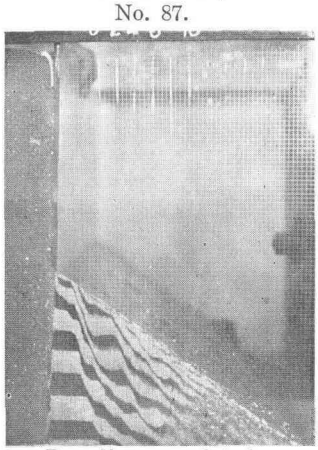
Distance of receding 6 cm.

No. 86.



Distance of receding 10 cm.

No. 87.



Receding completed.

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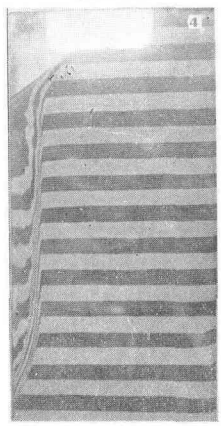
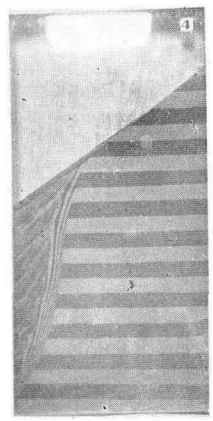
(Single Slit of 4 cm Width)

No. 88.

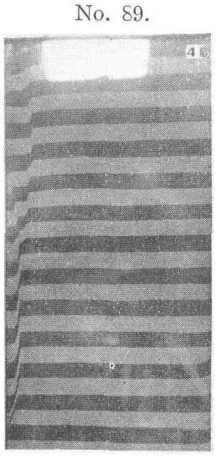
No. 91.



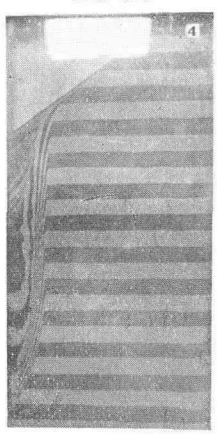
No. 94.



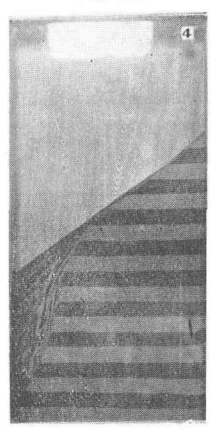
No. 89.



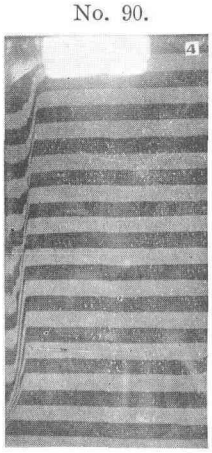
No. 92.



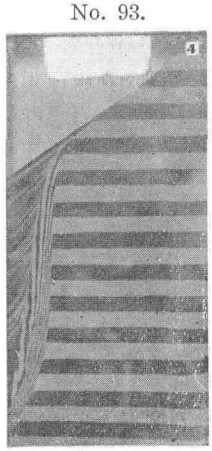
No. 95.



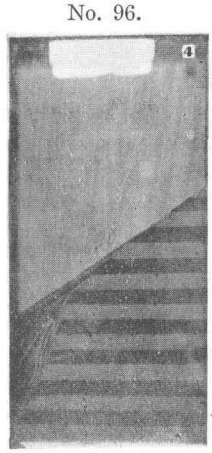
No. 90.



No. 93.



No. 96.



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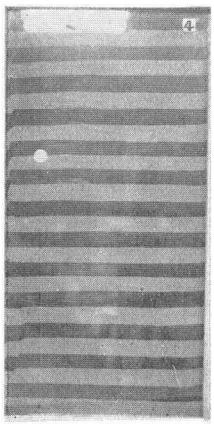
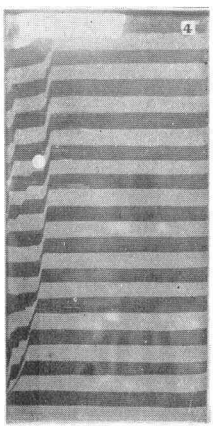
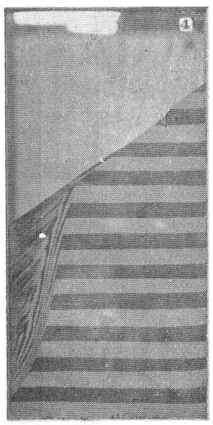
No. 97.



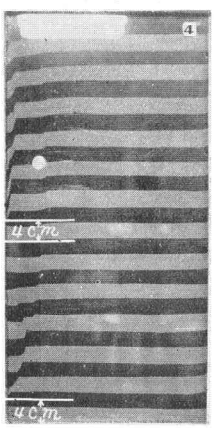
No. 100.



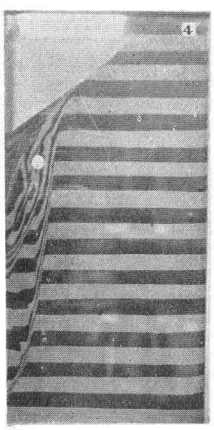
No. 103.



No. 98.

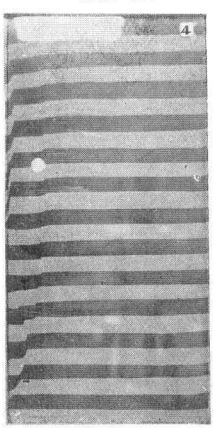


No. 101.

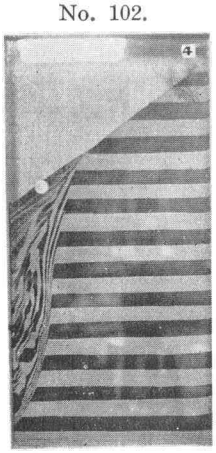


No. 104.

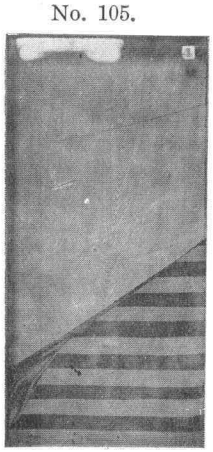
No. 99.



No. 102.



No. 105.



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