Correct New Formulas to Tracy's Procedure and a New Method of Adjustment of the Horizontal Hair in a Transit.

By

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Introduction.

Since the author's first lecture on Surveying for the students of the Faculty of Agriculture, Hokkaido Imperial University, 1925, he has found from their practical exercises, that the present formulas to Tracy's Procedure of the adjustment of the horizontal hair in a transit are in practice unreliable, and as he found large mistakes in principles and calculations upon looking through their proofs, he calculated correct new formulas in April, 1927.

Since that time, he has lectured upon his formulas to his students and let his colleagues know, but has not made public any paper upon the subject.

But again in April, 1931, he generalized his findings and further found that the reversed method of procedure is better than the original. In this paper the former point is proved in the first part and the latter demonstrated in the second.

Now, the error in a collimation line caused by its vertical deviation and inclination in reference to the position which produces no error, i.e., a certain position with respect to the optical axis of the objective lens system and the horizontal axis of the telescope, has been generally apt to be thought either negligibly small without any inspection of its magnitude, or mis-adjusted by the up-to-date corrections on account of the carelessness of theoretical and practical specialists over twenty-four years at the very least.

Up to the present the author has commonly experienced deviations over about 1/1000 or three minutes, a deviation of 13/60000 or forty-five seconds even in a Gurley's new five-inch transit which must be adjusted by its maker, and an exceptionally serious one of about 1/100 or thirty-four minutes in one of Gurley's old three-and-one-half-inch transits for students' use.

Notwithstanding this state of affairs, by the authors's new method and also to nearly the same degree by the authors's correct formulas for Tracy's Procedure, a man can usually correct the horizontal hair without any trouble by only a single adjustment to the extent of any error below 1/25000 or eight second, which need not be adjusted again for ordinary use because it does not exceed the errors of an eye-reading of telescope levels and plate levels generally.

Therefore the horizontal hair in a transit must be perfectly adjusted in direct levellings and other surveys which require precise vertical angles, as well as other adjustments.

Common Conventional Notations and Definitions.

We will define as follows:

O:	The horizontal axis of the telescope.
OC:	The vertical axis of the transit.
F and F':	The front and back foci of the objective lens system of the telescope.
H and H':	The front and back principal planes of the objective lens system.
K:	The real or imaginary cross-point of the cross-hairs.
$k = KM$:	The deviation of the horizontal hair from the axis of the objective lens system.
$\mathfrak{D}$:	The deviation of the collimation line from the axis of the objective lens system at the point A or B.

E: The distance of the point from the instrument station C.

f: The focal length of the objective lens system.

$C_p = OF,$
$= C + x_p,$
$= C + \dfrac{f^2}{E_p - C_p}$: The distance of the front focus of the objective lens system from the instrument center, when the point p is sighted. When E_p becomes infinity, C_p takes a certain constant value C, which is the so-called "Instrumental Constant."

$x_p = F'M,$
$= \dfrac{f^2}{E_p - C_p}$: The distance of the image of an object at p or the cross-hairs when focussed on the object, measured from the back focus of the objective, that is to say, the travelling distance of the objective to sight that object at p, measured from the back focus which corresponds to the sight of an infinitely distant point.

MT: The axis of the objective lens system.

e: The deviation of the axis of the objective from the horizontal axis of the telescope.

i: The inclination of the axis of the objective to a horizon.

a: The reading of the rod or the scale held at the point A or D.

b: The reading of the rod held at the point B in Tracy's Method, or a well-defined distant fixed point in the author's method.

I: The middle point of the first and the second readings of the rod or the scale.

r: The difference of the first and the second readings of the rod held at B, i.e., $|b_1 - b_2|$, in Tracy's Method.

s: The difference of the first and the second readings of the scale fixed at A or D, i.e., $|a_1 - a_2|$, in the author's method.

m: The distance of the crosshairs measured from the instrumental center.

$\mathfrak{C}$: The correction of the horizontal hair, by which it must be adjusted from the second reading in the same direction of the first reading in Tracy's Method or in the opposite direction to the first reading in the author's method.

Suffix A, D, or B shows a quantity when focussed there respectively.

Suffix 1 or 2 shows a quantity when the telescope is normal or inverted respectively.

Suffix o denotes the case in which $e_A = e_B = o$, $i_A = i_B$, and $k_A = k_B$.

Correct New Formulas to Tracy's Method of Adjustment of a Horizontal Hair in a Transit.

The procedure of the adjustment of the horizontal hair in a transit is described as follows:

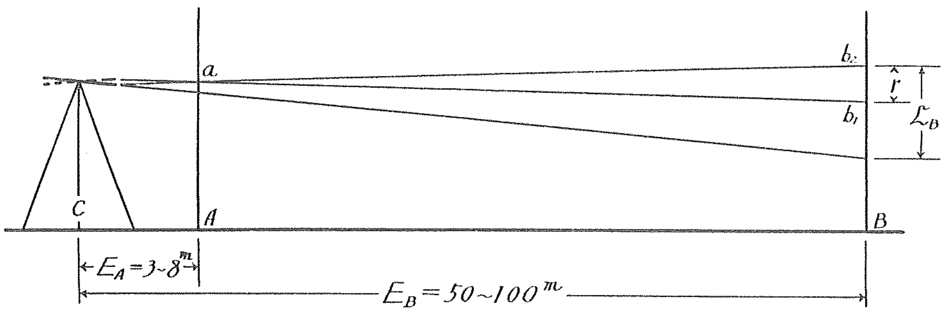

Fig 1.

Hold a levelling rod on a stake A, 3~8 m. away from the instrumental station C, clamp the telescope so that the line of sight is approximately level, and note the rod reading a. Without moving the telescope read a rod upon a second stake B, 50~100 m. away and in the straight line CA, and note its reading b_1. Unclamp, plunge, turn the plates, bring the inverted telescope to the former reading a on the first stake, and tighten all clamps. If the horizontal hair does not strike the former reading b_1 on the second stake but a new reading b_2, move the horizontal hair, with the telescope still clamped in the position it was in when the second reading b_2 was taken, from that reading in the direction of the first reading b_1 by the amount of the correction, which is given by the formulas (12) (14) and (16) below calculated, according to the type of the objective lens system and the magnitudes of the distances E_A and E_B. See Fig. 1.

Repeat the test and correction until the adjustment is perfected.

Strictly speaking, as the axis of the objective lens system and the horizontal axis of the telescope will not always lie in a straight line but the former may move along a certain curved path on account of minute errors in the workmanship of the instruments, the difference of the first and second readings of the rod held at B is more or less influenced. See Fig. 2.

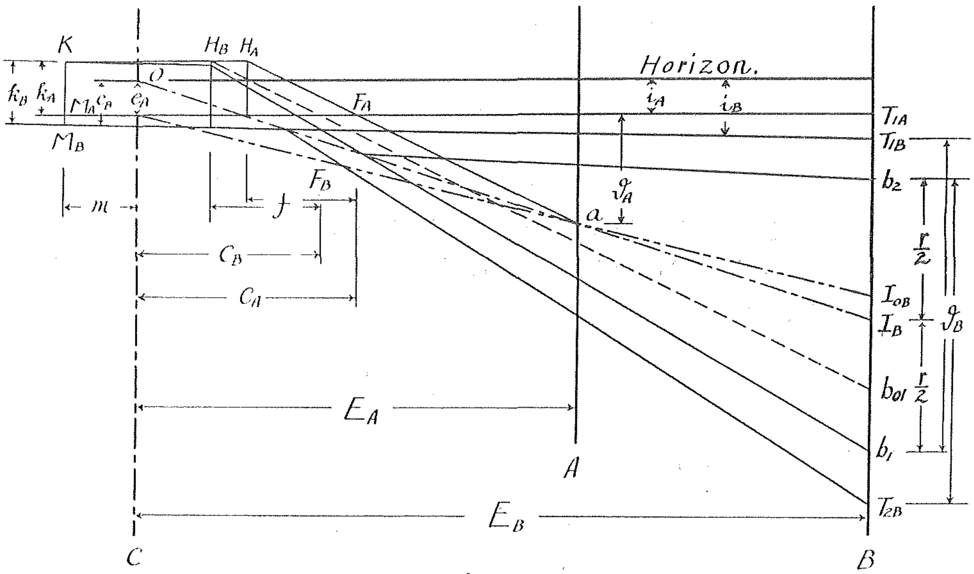

Fig. 2.

Because the collimation line through the fixed reading a upon the rod held at the point A is slightly varied by the above defects, when the rod held at the second point B is sighted, the increment of the first reading of the latter produced by this movement with respect to the first ideal case depicted in Fig. 3, where $e_B = e_A = o$ $i_B = i_A$ and $k_B = k_A$,

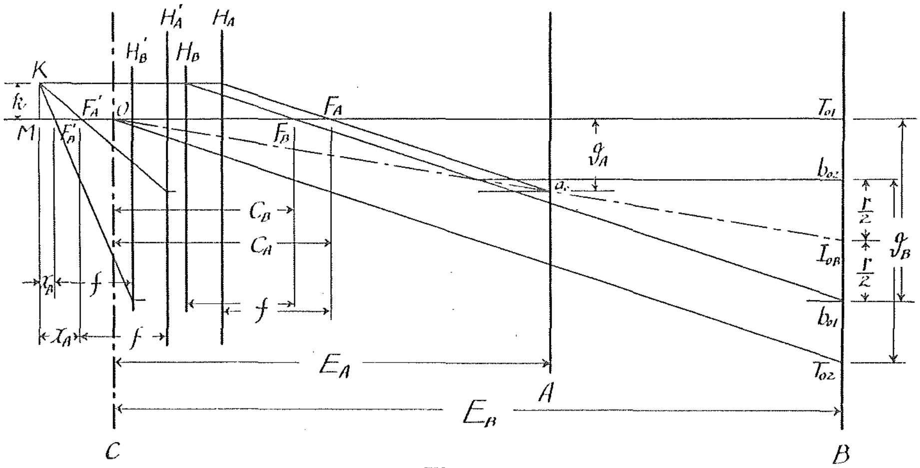

Fig. 3.

is, neglecting terms of the higher order,

$$\overline{b_{o1}b_1} = \frac{E_B - C_B}{f}\left\{e_B - e_A + (f-m)(i_B - i_A)\right\}. \tag{1}$$

Half the decrement of the difference of the first and second readings of the rod held at the point B in the second ideal case where $e_B = e_A$ $i_B = i_A$ and $k_B = k_A$, in reference to the first ideal, is from Fig. 2:

$$\overline{I_{oB}I_B} = \frac{E_B - E_A}{E_A} e_A. \tag{2}$$

But from Fig. 2, we obtain

$$\mathfrak{D}_B = \overline{T_{1B}b_1},$$

$$= \overline{T_{1A}b_{o1}} + \overline{b_{o1}b_1} - \overline{T_{1A}T_{1B}}. \tag{3}$$

The first term in the right hand side of Eq. (3) becomes

$$\overline{T_{1A}b_{o1}} = \frac{E_A(E_B - C_B)}{E_B C_A - E_A C_B} \overline{I_{oB}b_{o1}}\,, \tag{4}$$

where

$$\overline{I_{oB}b_{o1}} = \overline{I_B b_1} - \overline{b_{o1}b_1} + \overline{I_{oB}I_B}\,, \tag{5}$$

or by Eq. (1) (2) (5) and Fig. 2, we get

$$\overline{I_{oB}b_{o1}} = \frac{1}{2}r - \frac{E_B - C_B}{f}\left\{e_B - e_A + (f-m)(i_B - i_A)\right\} + \frac{E_B - E_A}{E_A}e_A\,. \tag{6}$$

Combining Eq. (3) (4) and (6) and neglecting terms of the higher order, we get

$$\mathfrak{D}_B = \frac{E_A(E_B - C_B)}{E_B C_A - E_A C_B}\frac{r}{2}$$

$$-\frac{E_A}{C}\frac{E_B - C_B}{f}\left\{e_B - e_A + (f-m)(i_B - i_A) - \frac{f}{E_A}e_A\right\}. \tag{7}$$

Here we affirm that if the second term in the right hand side of Eq. (7) is practically either proportional to the distance $E_B - C_B$ only or equal to zero, whatever value E_A or E_B may take, that is—

$$\frac{(\mathfrak{D}_B)_{r=0}}{E_B - C_B} = \text{a constant or zero,} \tag{8}$$

we can accurately adjust the horizontal hair in a transit by the formula for the correction

$$\mathfrak{C}_B = \frac{E_A(E_B - C_B)}{E_B C_A - E_A C_B}\frac{r}{2}\,, \tag{9}$$

which is required from Eq. (7), eliminating the terms related to the instrumental errors.

To find out experimentally that Eq. (8) or the condition of "Adjustability" holds good in ordinary transits, the author has first accurately adjusted the horizontal hair by his new formula Eq. (14), which is reduced below from Eq. (9); then he checked the adjustment, taking the readings upon the rod held at the point

B and the third D, which is taken instead of the point A at the distance $E_D = 2E_A$ from the instrumental station C. See Fig. 4.

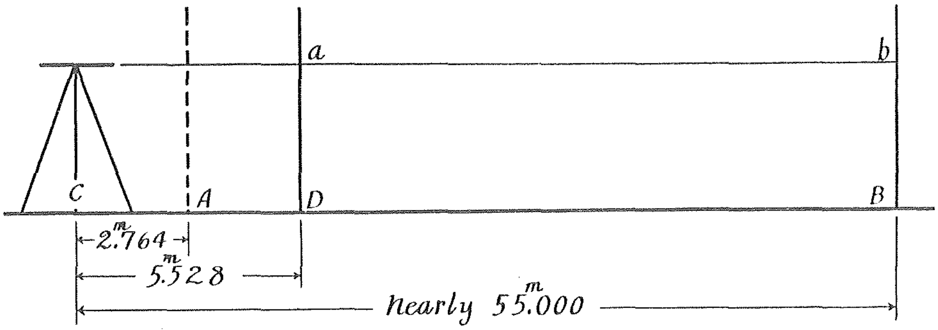

Fig. 4.

This serves as a double test for two distances, namely —— not only that between the points D and B in which D corresponds to A but also that between the points A and D in which D corresponds to B.

From the results given by Exp. 4 and Exp. 5 in Experimental Note I and correspondingly by Exp. 4 and Exp. 5 in Experimental Note 2, the author has learned considering the mechanical workmanship of the instruments, that if the horizontal hair is completely adjusted previously the difference of the first and second readings in the check becomes zero and accordingly the condition shown by Eq. (8) is fulfilled in ordinary transits, practically independent of the distances E_A and E_B which were used for the adjustment.

Now, for practical utility, we must transform Eq. (9) into convenient forms.

Neglecting the terms of the higher order, we obtain

$$\left.\begin{aligned} C_A &= C + \frac{f^2}{E_A}, \\ C_B &= C. \end{aligned}\right\} \qquad (10)$$

Thus by Eq. (10), Eq. (9) becomes

$$\mathfrak{C}_B = \frac{E_A(E_B - C)}{C(E_B - E_A) + \dfrac{f^2}{E_A} E_B} \cdot \frac{r}{2}. \qquad (11)$$

For the Porro's telescope in which C is very small or zero, we get from Eq. (11)

$$\mathfrak{C}_B = \frac{E_A^2}{f^2}\frac{r}{2}. \tag{12}$$

For the telescopes of Ramsden, Huygenian, and other types, in which C's are passably large:

If we estimate the distance E_B by eye-measurement or better by pacing, so that the condition

$$E_B = \frac{E_A^2}{f^2}C \tag{13}$$

is approximately satisfied; we get from Eq. (11) the following equation

$$\mathfrak{C}_B = \frac{E_A}{C}\frac{r}{2} \tag{14}$$

which is very convenient for practical purposes.

If the distance E_B is not large enough to satisfy Eq. (13) or

$$E_B \ll \frac{E_A^2}{f^2}C, \tag{15}$$

we get from Eq. (11), neglecting terms of the higher order,

$$\mathfrak{C}_B = \frac{E_A}{C}\left(1+\frac{E_A}{E_B}\right)\frac{r}{2}. \tag{16}$$

For a practical example, take

$$\left.\begin{aligned} E_A &= 10\ C, \\ E_B &= 200\ C \sim 250\ C, \end{aligned}\right\} \tag{17}$$

and then from Eq. (14), we obtain

$$\mathfrak{C}_B = 5\ r. \tag{18}$$

Several experiments made upon ordinary transits with Ramsden's telescopes are shown in Experimental Note 1.

A new Method of Adjustment of the horizontal Hair in a Transit.

The author's new method is the reverse of Tracy's method and is described as follows:

Select a fixed and well-defined point b, whose distance is over 50 meters from the instrumental station C and whose altitude does not exceed ± 5 degrees, and a second point A, whose distance is $2 \sim 8$ meters from the same point, arranging these three points in a straight line. See Fig. 1.

For point A, take a wooden post, a corner of a wooden house or fence, or such and affix a precise scale, which is graduated to 1/2 or 1 m.m., or for a more accurate adjustment, either a vernier scale or a micrometer, fastening it vertically with a few wooden pieces and nails.

Then first sight at b with the telescope normal, clamp the telescope in that position, read the scale fixed at the point A to a tenth of one graduation, and note its reading a_1. Sight at b again with the telescope inverted and tighten all clamps. If the horizontal hair does not strike the former reading a_1 on the scale at A but gives a new reading a_2, move the horizontal hair, with the telescope still clamped in the position it was in when the second reading a_2 was taken, from that reading a_2 in the direction opposite to the first reading a_1 to the extent of the correction, which is calculated from the formulas (12) (14) and (16) reduced below. See Fig. 1.

Repeat the test and correction until the adjustment is perfected.

Similarly as discussed minutely in the preceding article, the influence caused by the dis-alignment of the axis of the objective lens system and the horizontal axis of the telescope is explained in detail by sketch Fig. 2.

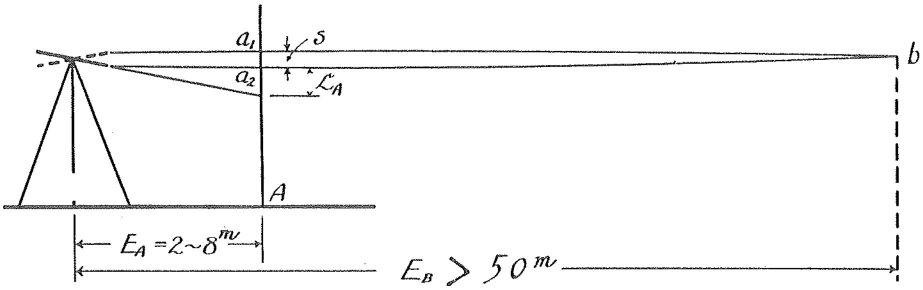

Fig. 1.

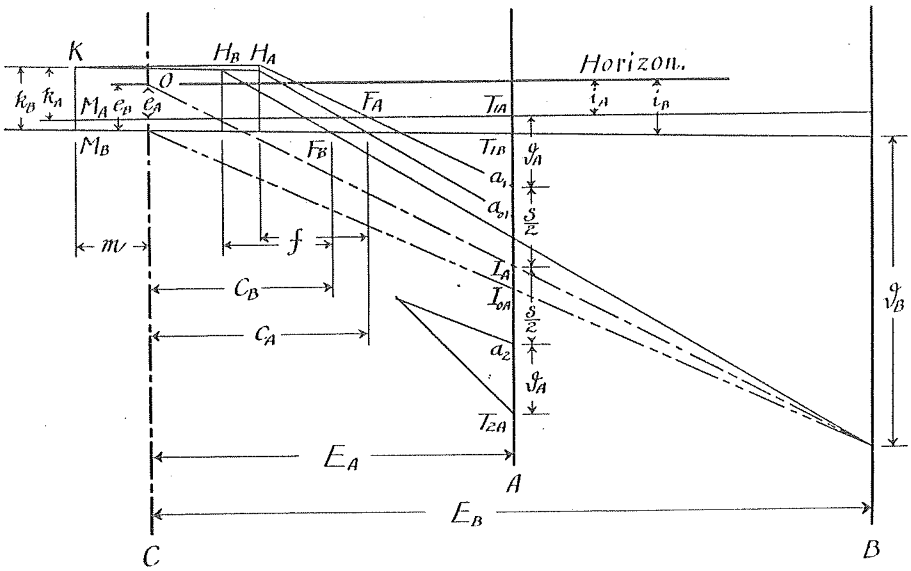

Fig. 2.

Likewise in the preceding article, the increment of the first reading of the scale at A produced by the displacement of the collimation line due to the defect in workmanship of the instrument, with respect to the first ideal case depicted in Fig. 3, where $e_A = e_B = 0$ $i_A = i_B$ and $k_A = k_B$,

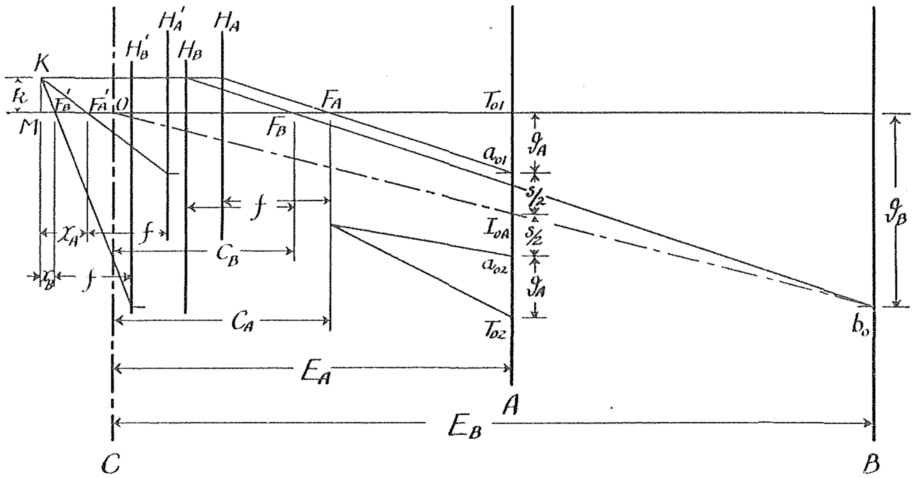

Fig. 3.

is, neglecting terms of the higher order,

$$\overline{a_1 a_{o1}} = \frac{E_A}{f}\left\{e_B - e_A + (f-m)(i_B - i_A)\right\}. \tag{1}$$

Half the decrement of the difference between the first and the second readings of the scale at A in the second ideal case where $e_A = e_B$ $i_A = i_B$ and $k_A = k_B$, with respect to the first ideal, is

$$\overline{I_A I_{oA}} = \frac{E_B - E_A}{E_A} e_B. \tag{2}$$

But from Fig. 2, we obtain

$$\begin{aligned} \mathfrak{D}_A &= \overline{T_{1A} a_1}. \\ &= \overline{T_{1A} T_{1B}} + \overline{T_{1B} a_{o1}} - \overline{a_1 a_{o1}}. \end{aligned} \tag{3}$$

The second term in the right hand side of Eq. (3) becomes

$$\overline{T_{1B} a_{o1}} = \frac{E_B(E_A - C_A)}{E_B C_A - E_A C_B} \overline{a_{o1} I_{oA}}, \tag{4}$$

where

$$\overline{a_{o1} I_{oA}} = \overline{a_1 I_A} - \overline{a_1 a_{o1}} + \overline{I_A I_{oA}}, \tag{5}$$

or by Eq. (1) (2) (5) and Fig. 2, we get

$$\overline{a_{o1} I_{oA}} = \frac{1}{2}s - \frac{E_A}{f}\left\{e_B - e_A + (f-m)(i_B - i_A)\right\} + \frac{E_B - E_A}{E_A} e_B. \tag{6}$$

Combining Eq. (3) (4) and (6) and neglecting terms of the higher order, we obtain

$$\mathfrak{D}_A = \frac{E_B(E_A - C_A)}{E_B C_A - E_A C_B} \frac{s}{2}$$

$$- \frac{E_A - C_A}{C} \frac{E_A}{f} \left\{ e_B - e_A + (f-m)(i_B - i_A) - \frac{f}{E_A} e_B \right\} . \tag{7}$$

But in like manner as in the preceding article, from the results given by Exp. 4 and Exp. 5 in Experimental Note 2 and correspondingly by Exp. 4 and Exp. 5 in Experimental Note 1, the author has learned considering mechanical workmanship of the instruments, that the condition of "Adjustability"

$$\frac{(\mathfrak{D}_A)_{S=0}}{E_A - C_A} = \text{a constant or zero} \tag{8}$$

is satisfied in ordinary transits, practically independent of the distances E_A and E_B which were used for the adjustment. See Fig. 4.

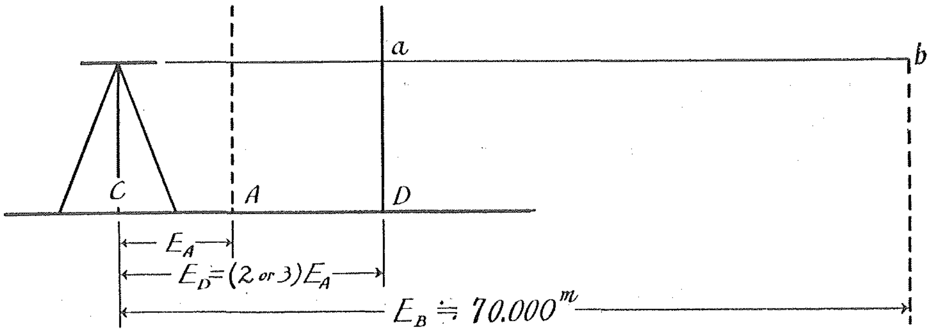

Fig. 4.

The author has taken the distances of the check stations from the instrumental station C, $E_D = 2E_A$ in Exp. 4 and $E_D \doteqdot 3E_A$ in Exp. 5 in Experimental Note 2.

All these experiments served as double checks, namely—checks for the stations D and B and simultaneously checks for the stations A and B.

By the above verification, we obtain the formula for correction of the horizontal hair

$$\mathfrak{C}_A = \frac{E_B(E_A - C_A)}{E_B C_A - E_A C_B}\frac{s}{2}. \tag{9}$$

But neglecting terms of the higher order, we get

$$\left.\begin{aligned} C_A &= C + \frac{f^2}{E_A - C}, \\ C_B &= C. \end{aligned}\right\} \tag{10}$$

Accordingly by Eq. (10), Eq. (9) becomes

$$\mathfrak{C}_A = \frac{E_B(E_A - C)}{C(E_B - E_A) + \frac{f^2}{E_A - C}E_B}\frac{s}{2}. \tag{11}$$

For Porro's telescope in which C is very small or zero, we get from Eq. (11)

$$\mathfrak{C}_A = \frac{E_A^2}{f^2}\frac{s}{2}. \tag{12}$$

For the telescopes of the Ramsden, the Huygenian, and other types, in which C's are passably large:

If we can select such an instrumental station C that the distances E_A and E_B satisfy approximately the condition

$$E_B = \frac{E_A(E_A - C)}{f^2}C, \tag{13}$$

we get from Eq. (11), neglecting terms of the higher order, the following formula for correction

$$\mathfrak{C}_A = \frac{E_A - C}{C}\frac{s}{2} \tag{14}$$

which is very convenient for practical purposes.

If the distance E_B is not so large as to satisfy Eq. (13) or

$$E_B \ll \frac{E_A(E_A - C)}{f^2}C, \tag{15}$$

we get from Eq. (11), neglecting terms of the higher order,

$$\mathfrak{C}_A = \frac{E_A - C}{C}\left\{1 + \frac{E_A}{E_B}\right\}\frac{s}{2}\,. \tag{16}$$

For a practical example, select so that

$$\begin{aligned} E_A &= 11\ C\,, \\ E_B &= 200\ C \sim 300\ C\,, \end{aligned} \tag{17}$$

then by Eq. (14), we get

$$\mathfrak{C}_A = 5\ s. \tag{18}$$

The results of several experiments made upon the ordinary transits with Ramsden's telescopes are shown in Experimental Note 2.

Conclusion.

First we will discuss the present methods described in the works of the authorities.

W. Norman Thomas' Method.—His operation is verbose and labourous, for we must determine the heights of three points and read three rods held upon them. Moreover his correction is not estimated by any standard formula but his adjustment is performed by trials.

John Clayton Tracy's Method.—Prof. J. C. Tracy's procedure of adjustment is a standard one, which is now used and also modified by the author in his new method, but Tracy has made a fatal mistake by applying the principle of single reversion to the analysis of the correction.

Because his correction of the horizontal hair is given by the equation

$$\mathfrak{C}_B = \frac{1}{2}r\,, \tag{A}$$

an error of 90% remains in every adjustment even when $E_A = 10\,\mathrm{C}$, so that 59% remains after successive five operations, 12% after twenty, and 4% after thirty.

Consequently these numbers show that his correction is impracticable.

Perhaps, for the remedy of this defect, the new formulas for correction to Tracy's Procedure, namely—

by moderate approximation:

$$\mathfrak{C}_B = \frac{E_A(E_B - C)}{C(E_B - E_A)} r\,, \tag{B}$$

and by further approximation:

$$\mathfrak{C}_B = \frac{E_A}{C} r\,, \tag{C}$$

would be calculated, but because their magnitudes are respectively nearly double those of the true corrections on account of the wrong approximations, which are given by Eq. (11) and Eq. (14) in the first article, the horizontal hair is carried out to just the opposite side of the right position at every adjustment and accordingly shall never be adjusted even by the successive operations.

Hereupon, as described in the Introduction, the author has newly calculated the correct formulas explained in the first article, by which the horizontal hair is usually adjusted by only a single operation to the extent of no interference for ordinary utility.

But the author's modified method described in the second article is superior to Tracy's Method.

In other works, the adjustment of the horizontal hair in a transit is entirely neglected so far as the present author knows.

Further the author has experimentally verified that the adjustment is practically not disturbed by the disalignment of the axis of the objective lens system and the horizontal axis of the telescope, but still remained after the maker's adjustment.

Therefore if the adjustment is not completed, such an instrument must be rejected because of its poor workmanship.

The author's method has the following special merits:

(1). The error in collimation is very small, because we are satisfied to sight only twice the distant fixed point b, even when it is inaccessible and also when there is more or less gossamer in the air. Moreover the scale at the near point A is perfectly fixed and the readings are not affected by the condition of the atmosphere. But on the contrary, in Tracy's Method, as it is neccessary to set up a rod at a distant point and read it twice, the readings are apt to be influenced by atmospheric conditions and also to be disturbed by the backward and forward movement of the rod at A, which must be held vertically fixed.

Therefore the readings in the author's method are more accurate than those in Tracy's Method.

(2). As the reading difference s does not exceed 7 m.m. usually, we can choose any precise scale, for instance, a scale with a vernier, a micrometer, etc.

(3). A man can adjust the horizontal hair accurately, not only in the open air but also indoors upon a rigid floor, sighting at a distant fixed point through a window even in undesirable weather or at night.

(4). The angle subtended by the reading difference s to the horizontal axis O is greater by C/E_A time than that subtended by r in Tracy's Method.

The author ventures to advise all theoretical and practical specialists to check horizontal hairs of new transits and at times of old transits, and adjust them according to circumstances.

The author avails himself of this opportunity to express his greatful thanks to Prof. T. Yoshimachi and Prof. F. Takabeya who have kindly afforded him great facilities for the preparation of this paper, to Assistant Prof. S. Gondaira who has experimentally inspected the formula of the author's new method, and to Assistant Prof. T. Sakai who has experimentally checked the author's correct formula to Tracy's Method and let his students of the Faculty of Engineering, Hokkaido Imperial University, adjust the horizontal hairs in practical exercises for the first time by others than the author.

EXPERIMENTAL NOTE 1.

(June~August, 1931.)

Designation.	Exp. 1.	Exp. 2.	Exp. 3.	Exp. 4.	Exp. 5.
Size, Transit, In.	4	4	$3\frac{1}{2}$	4	4
C, cm.	28.00	28.00	25.30	27.64	27.64
Maker	Tamaya Co.	Tamaya Co.	Gurley	Tamaya Co.	Tamaya Co.
Age, years	6	6	?, very old	6	6
Observer	T. Sakai	T. Sakai	students	T. Shingo	T. Shingo
E_A, m.	5.6	5.6	2.53	2.764	2.764
E_B, m. nearly	55.00	55.00	55.00	55.000	55.000
Correction $\mathfrak{C}_B$	$10\ r$	$10\ r$	$5\ r$	$5\ r$	$5\ r$
First Observation					
b_1, m.	1.076	1.786	0.4025	1.564	1.515
b_2, m.	1.062	1.775	0.4825	1.564	1.510
r, m.	0.014	0.011	0.0800	0.000	0.005
$\mathfrak{C}_B$, m.	0.140	0.110	0.4000	0.000	0.025
Moved to	1.202	1.885	0.0825	—	1.535
Second Observation					
b_1, m.	1.060	1.780	0.6925		1.508
b_2, m.	1.061	1.780	0.6825		1.508
r, m.	0.001	0.000	0.0100		0.000
$\mathfrak{C}_B$, m.	0.010	0.000	0.0500		0.000
Check Observation					
E_D, m.				5.528	5.528
E_B, as it was				55.000	55.000
b_1, m.				1.5925	1.408
b_2, m.				1.5925	1.408
r, m.				0.0000	0.000
$\mathfrak{C}_B$, m.				0.0000	0.000
Deviation, corrected					
Radian	0.002 55	0.002 00	0.006 40	0.000 00	0.000 46
Minute	8.8	6.9	21.9	0.0	1.6

Remark: The transits used in Exp. 4 and Exp. 5 were previously adjusted by the students, applying the author's correct formula.

EXPERIMENTAL NOTE 2.

(June~August, 1931.)

Designation.	Exp. 1.	Exp. 2.	Exp. 3.	Exp. 4.	Exp. 5.
Size, Transit, In.	4	$3\frac{1}{2}$	5	4	4
C, cm.	27.64	25.45	34.70	27.64	27.64
Maker	Tamaya Co.	Tamaya Co.	Gurley	Tamaya Co.	Tamaya Co.
Age, years	6	new	new	6	6
Observer	T. Shingo	S. Gondaira	T. Shingo	T. Shingo	T. Shingo
E_A, m.	2.764	2.800	3.817	3.588	3.036
E_B, m. nearly	65.000	65.000	80.000	70.000	70.000
Correction $\mathfrak{C}_A$	4.5 s	5 s	5 s	6 s	5 s
First Observation					
a_1	7.08	9.55	2.57	1.17	6.80
a_2	7.18	9.65	2.60	1.05	6.74
s	0.10	0.10	0.03	0.12	0.06
Correction $\mathfrak{C}_A$	0.45	0.50	0.15	0.72	0.30
Moved to	7.63	10.15	2.75	0.33	6.44
Second Observation					
a_1	7.15	0.00	2.59	1.12	6.75
a_2	7.15	0.00	2.59	1.12	6.75
s	0.00	0.00	0.00	0.00	0.00
Correction $\mathfrak{C}_A$	0.00	0.00	0.00	0.00	0.00
Check Observation					
E_D, m.				7.176	8.280
E_B, as it was				70.000	70.000
a_1				9.71	3.05
a_2				9.71	3.05
s				0.00	0.00
Correction $\mathfrak{C}_D$				0.00	0.00
Deviation, corrected					
Radian	0.000 94	0.000 98	0.000 22	0.001 09	0.000 54
Minute	3.1	3.4	0.74	3.7	1.9

Remark: A platting scale graduated to $\frac{1}{2}$ m.m. is used at point A for this adjustment.