"The Condition of Adjustability" and the Precision of the Author's Correct New Formulas to Tracy's Procedure and the Author's Method in Adjustment of the Horizontal Hair in a Transit.

By

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Introduction.

Because the author in his preceding paper* has only given "the Condition of Adjustibility" of the horizontal hair in a transit without further detailed explanation and moreover has not inspected the precision of his formulas; it is proposed in the present paper to clarify the contents and substantiate the accuracy of the formulas presented in the paper.

A man who knows the limitations of the machine tools, used in making a transit, and the workmanship of a telescope-tube and its objective lens, can not overlook the existence of eccentricity and inclination of the axis of the objective lens and their change due to travelling of the objective-slide along a curved path in the telescope tube.

But these defects will be made even more serious by irregular abraision and other accidental impediments.

For the reason of these facts, if "the Condition of Adjustability" Eq. (8), Eq. (8.0), Eq. (8b), or Eq. (8c) is not satisfied, the horizontal hair in a transit cannot be adjusted perfectly.

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*The author's paper: "Correct New Formulas to Tracy's Procedure and a New Method of Adjustment of the Horizontal Hair in a Transit.", in the Memoirs of the Faculty of Engineering, the Hokkaido Imperial University, Sapporo, Japan, Vol. 3, No. 1, March, 1932.

As for the precision of the author's formulas for the correction, diagrams of percentage of errors will be given for general examples.

Common conventional notations and definitions used in the preceding paper have not been altered except for the following:

OT_{oo}: The normal position of the axis of the objective lens system, i.e., its position when $e_A=e_B=o$, and $i_A=i_B=o$.

i : The inclination of the axis of the objective lens in reference to its normal position.

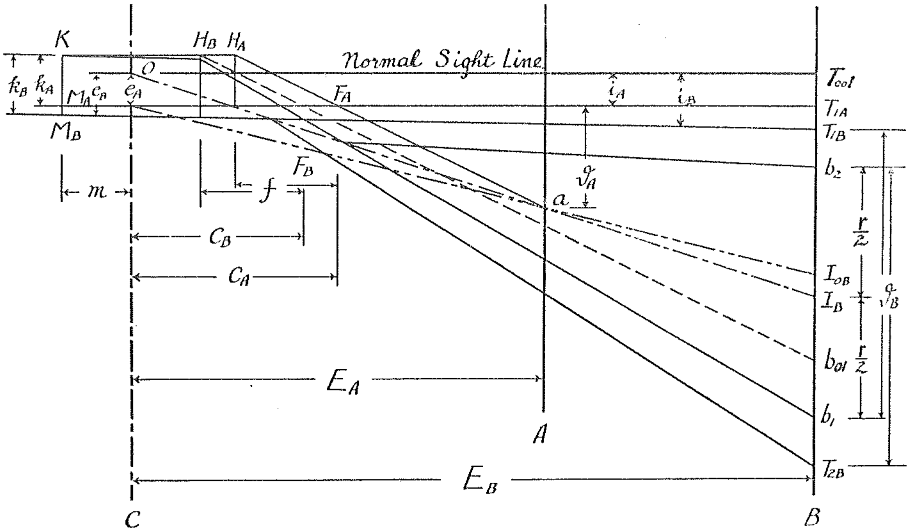

Fig. 2.

Below, the formulas will be expounded minutely, first in Tracy's Procedure and then in the author's method.

The Adjustment of the Horizontal Hair in a Transit by Tracy's Procedure.

1. "The Condition of Adjustability."

In order that the horizontal hair in a transit may be perfectly adjustable, the position of the symmetrical line $\overline{OaI_B}$ in Fig 2 must be absolutely invariant in the telescope independent of the positions of the points A and B, that is—after its adjustment is completed between a pair of points A and B, the conditions

$$1. \quad r = 0 \tag{8a}$$

and

$$2. \quad \overline{I_B T_{oo1}} \backsim E_B \tag{8b}$$

must be satisfied independent of the magnitude of the distances E_A and E_B.

Now, from Fig. 2, one obtains the relation

$$\begin{aligned} \mathfrak{D}_B &= \overline{T_{1B} b_1} \\ &= \overline{T_{1A} b_{o1}} + \overline{b_{o1} b_1} - \overline{T_{1A} T_{1B}}, \end{aligned} \tag{3}$$

from which, neglecting terms of the higher order, is derived

$$\begin{aligned} \mathfrak{D}_B = \frac{E_A(E_B - C_B)}{E_B C_A - E_A C_B} \frac{r}{2} - \frac{E_B(E_A - C_A)}{E_B C_A - E_A C_B} \frac{E_B}{f} \left\{ e_B - e_A + (f-m)(i_B - i_A) \right\} \\ + \frac{E_B - E_A}{E_B C_A - E_A C_B} (E_B - C_B) e_A - E_B(i_B - i_A), \end{aligned} \tag{7a}$$

Therefore, according to the second condition or Eq. (8b), one obtains neglecting terms of the higher order

$$\begin{aligned} - \frac{E_A - C_A}{E_B C_A - E_A C_B} \frac{E_B}{f} \left\{ e_B - e_A + (f-m)(i_B - i_A) \right\} \\ + \frac{E_B - E_A}{E_B C_A - E_A C_B} e_A + i_A = \text{a constant or zero}, \end{aligned} \tag{8.0}$$

Strictly speaking, Eq. (8.0) can not be satisfied generally, except $e_A = o = e_B$ and $i_A = o = i_B$, but because e_A, e_B, i_A, and i_B are really of very small magnitudes, it is not difficult to find the conditions by a close approximation which they must practically be fulfilled.

Now, the second term of Eq. (8.0) becomes, neglecting terms of the higher order,

$$\frac{E_B - E_A}{E_B C_A - E_A C_B} e_A \doteqdot \frac{e_A}{C} . \tag{8.1}$$

Therefore, putting together Eq. (8.0) and Eq. (8.1), there is obtained for "the Condition of Adjustability",

$$\left.\begin{aligned} & e_A = e_B = \text{a small constant or zero} \\ \text{and} \quad & i_A = i_B = \text{a small constant or zero} \end{aligned}\right\} \tag{8c}$$

theoretically, which must hold good practically whatever positions the points A and B may take, i.e., the objective lens must be simply translated along a straight path, having its axis in the same position, or along its optical axis, when the telescope is focussed.

Now, from condition 2 or Eq. (8b), one secures

$$\overline{I_B T_{oo1}} = \overline{I_B T_{1B}} + \overline{T_{1B} T_{oo1}} \backsim E_B , \tag{8.2}$$

where

$$\overline{I_B T_{1B}} = (\mathfrak{D}_B)_{r=o} \tag{8.3}$$

and

$$\overline{T_{1B} T_{oo1}} = e_B + E_B i_B . \tag{8.4}$$

Because the first term of Eq. (8.4) is considered negligibly small compared to the second term, one may take for "the Condition of Adjustability", from Eq. (8.3) instead of Eq. (8.2),

$$\frac{(\mathfrak{D}_B)_{r=o}}{E_B} = \text{a constant or zero} \tag{8}$$

whatever values E_A and E_B may have, which has been given in the author's preceeding paper.*

*The author's paper: "Correct New Formulas to Tracy's Procedure and a New Method of Adjustment of the Horizontal Hair in a Transit.", in the Memoirs of the Faculty of Engineering, the Hokkaido Imperial University, Sapporo, Japan, Vol. 3, No. 1.

Therefore, all the horizontal hairs in transits, which satisfy practically the condition shown by Eq. (8), Eq. (8b) or Eq. (8c), can be perfectly adjustable; while, on the other hand, if the conditions can not be fulfilled, or if the preceeding defects occur irregularly, the rod reading r_D taken for a point D, still clamped in position after the processes of adjustment have been completed for the points A and B, must amount up to some value, one half of which becomes analytically

$$\frac{r_D}{2}=\frac{E_D}{f}\left\{e_D-e_A+(f-m)(i_D-i_A)\right\}.$$

$$-\frac{E_D-E_A}{E_B-E_A}\frac{E_B}{f}\left\{e_B-e_A+(f-m)(i_B-i_A)\right\} \qquad (8d)$$

From Eq. (8d), "the Condition of Adjustability" shown by Eq. (8c) can be derived again, referring to condition 1 or Eq. (8a).

The magnitudes of such check readings r_Ds will be shown by the following examples:

Designation.	Example 1.	Example 2.
C, m.m.	276.4	276.4
f, m.m.	165.8	165.8
m, m.m.	55.3	55.3
E_A, m.	2.764	2.764
E_B, m. nearly	55	55
E_D, m.	5.528	5.528
i_B-i_A,	0	0
e_B-e_A, m.m.	0	0
i_D-i_A,	0.0010	0.0002
e_D-e_A, m.m.	0.1658	0.0332
r_D, m.m.	18.43	3.686
Deviation, sighted at D,		
Radian,	0.00167	0.00033
Minute,	5.73	1.15

In Example 2, we take $i_D - i_A = \frac{2}{10,000}$ or the geometric deviation of 0.001 mm. per 5 m.m. slide of the objective, which causes the error of 1.15 minutes in the collimation sighted at D.

Because such an error may often occur, it must not be passed over if the variation is acknowledged, whatever the case may be.

2. Precision of the Author's Correct New Formulas.

The author's correct new formulas for adjustment of transits with telescopes of Ramsden, Huygenian, and other types, in which Cs are passably large, are rewritten as follows:

The rigorous original correction, which is not approximated absolutely, is

$$\mathfrak{C}_B = \frac{E_A(E_B - C_B)}{E_B C_A - E_A C_B} \frac{r}{2} . \tag{9}$$

If the distance E_B be estimated by eye-measurement or better by pacing, so that the condition

$$E_B = \left(\frac{E_A - C}{f}\right)^2 C \tag{13}$$

is approximately satisfied, the formula (9) is gotten

$$\mathfrak{C}_B = \frac{E_A}{C} \frac{r}{2} , \tag{14}$$

which is very convenient for practical purposes.

If the distance E_B is not large enough to satisfy Eq. (13) or

$$E_B \ll \left(\frac{E_A - C}{f}\right)^2 C , \tag{15}$$

one gets from Eq. (9), neglecting terms of the higher order,

$$\mathfrak{C}_B = \frac{E_A}{C}\left(1+\frac{E_A}{E_B}\right)\frac{r}{2}\,, \tag{16}$$

Besides the above, for transits or ordinary size, the following two formulas are obtained, which are calculated accurately,

$$\mathfrak{C}_B = \left\{\frac{E_A}{C}\left(1+\frac{E_A}{E_B}\right)-0{,}4\right\}\frac{r}{2}\,, \tag{19}$$

and

$$\mathfrak{C}_B = \left\{\frac{E_A(E_B-C)}{C(E_B-E_A)}-0.4\right\}\frac{r}{2}\,. \tag{20}$$

In order to make easily understood the precision of the above formulas, they may be calculated for a general example, varying E_B from 15 m. to 200 m.;

$$C = 0.30 \text{ m.}$$
$$f = 0.18 \text{ m.}$$
$$E_A = 3.00 \text{ m.},$$

and plotting the results, Diagram 1 and Diagram 2 are secured as inserted in the next page, showing the relation between the distance E_B and $\phi_{(n)}$, where $\phi_{(n)}$ means the coefficient of a formula for correction

$$\mathfrak{C}_{B(n)} = \phi_{(n)}\frac{r}{2}\,, \tag{n}$$

and n its number, and also that between the distance E_B and the error of a correction in percentage, respectively.

Now, from the diagrams, it can be seen that if the condition shown by Eq. (13) is nearly satisfied by eye-measurement or better by pacing, the error in Eq. (14) or $\mathfrak{C}_{B(14)}$ falls only within one percent but when E_B is less than thirty meters, the error in $\mathfrak{C}_{B(16)}$ becomes less than three percent and is superior to the former.

The curves for $\mathfrak{C}_{(19)}$ or $\phi_{(19)}$ and $\mathfrak{C}_{B(20)}$ or $\phi_{(20)}$ coincide nearly with the rigorous curve $\mathfrak{C}_{B(9)}$ or $\phi_{(9)}$ and their errors fall within one percent, but $\mathfrak{C}_{B(19)}$ is slightly superior to $\mathfrak{C}_{B(20)}$ for the small value of E_B.

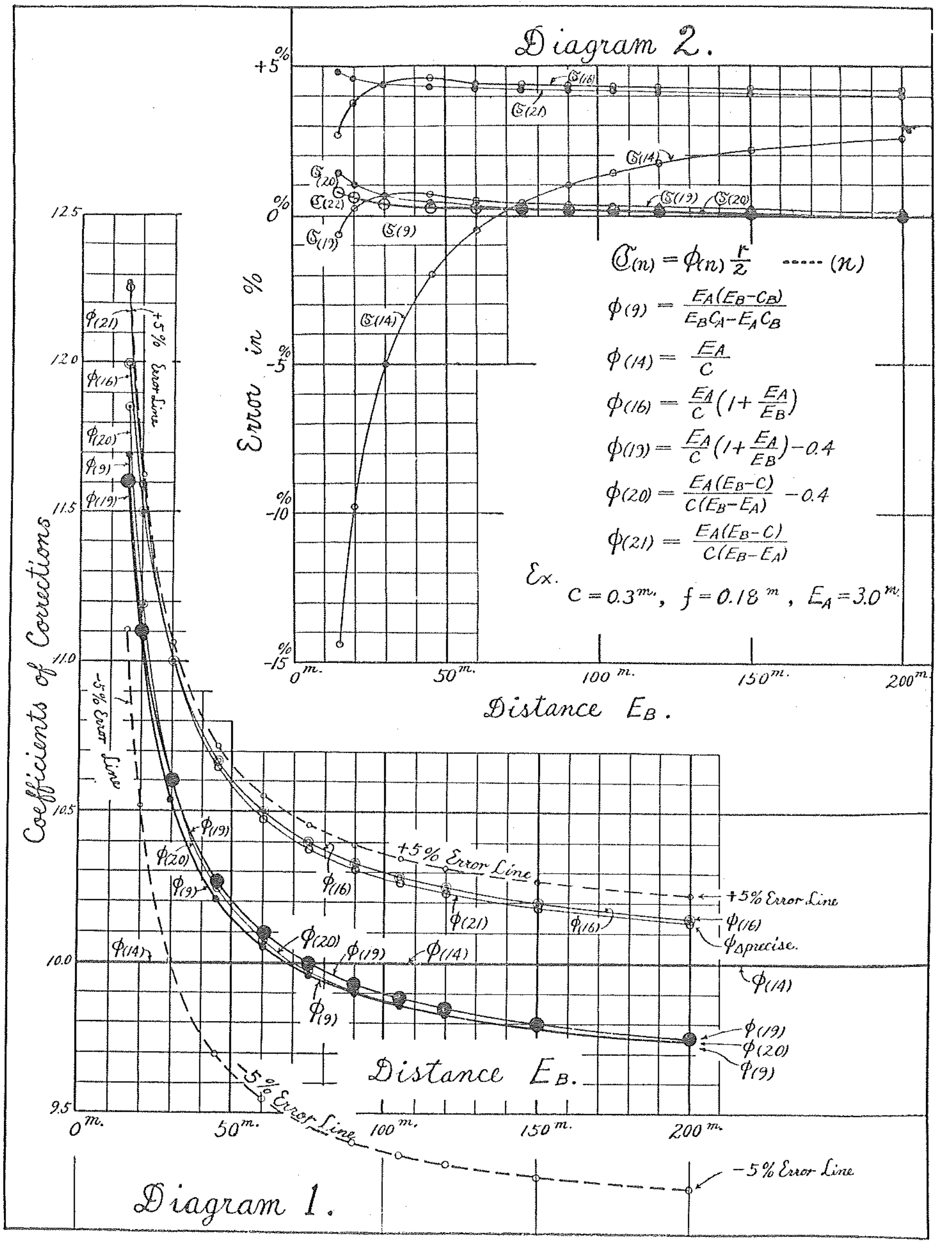
Diagram 2.
Error in %
Distance E_B.
$\mathfrak{C}_{(n)} = \phi_{(n)} \frac{r}{2}$ ----- (n)
$\phi_{(9)} = \frac{E_A(E_B - C_B)}{E_B C_A - E_A C_B}$
$\phi_{(14)} = \frac{E_A}{C}$
$\phi_{(16)} = \frac{E_A}{C}(1 + \frac{E_A}{E_B})$
$\phi_{(19)} = \frac{E_A}{C}(1 + \frac{E_A}{E_B}) - 0.4$
$\phi_{(20)} = \frac{E_A(E_B - C)}{C(E_B - E_A)} - 0.4$
$\phi_{(21)} = \frac{E_A(E_B - C)}{C(E_B - E_A)}$
Ex. $C = 0.3^m$, $f = 0.18^m$, $E_A = 3.0^m$
Coefficients of Corrections
Diagram 1.
Distance E_B.
+5% Error Line
−5% Error Line
$\phi_{(16)}$
$\phi_{\Delta precise}$
$\phi_{(14)}$
$\phi_{(19)}$
$\phi_{(20)}$
$\phi_{(9)}$
$\phi_{(21)}$

Now against the author's proof, the formula,† calculated by Assistant Professor K. Tanaka, to show a new process of calculation of the formulas for correction,

$$\mathfrak{C}_B = \frac{E_A(E_B - C)}{C(E_B - E_A)} \frac{r}{2} \tag{21}$$

is also plotted in the diagrams, which shows nearly the same results as formula (16), and whose error is not less than four percent in any case, because of its approximation.

Recently, Assistant Prof. K. Tanaka has again calculated the formula[1]

$$\mathfrak{C}_B = \frac{E_A(E_B - C)}{C_A(E_B - E_A)} \frac{r}{2} \tag{22}$$

which is slightly erroneous due to the indistinct approximation[2] as shown in Diagram 2. But it is recognized from the author's check[2]

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†The result, solved by Assistant Professor Kichiro Tanaka, was kindly sent to me by Prof. Hachiro Kimishima, the Kyushu Imperial University, Fukuoka, Japan, with his answer to my letter, in which the solution of my correct new formulas was written, July 23, 1932.

(1) 田中吉郎： 轉鏡儀横叉線整正ノ問題＝就イテ， 九州帝國大學工學彙報， 第八卷三號， 昭和八年八月.

(2) Asst. Prof. K. Tanaka's original solution and the author's parallel check are as follows:

K. Tanaka.		The author. (T. Shingo)			
$\frac{d_1}{d_2} = \frac{a}{b}$.	(a)			$\frac{d_1}{d_2} = \frac{a}{b}$	(A)
$\frac{d_1}{\Delta} = \frac{a - c_1}{b - c_1}$.	(b)	$\frac{d_1}{\Delta - \delta - \delta'} = \frac{a - c_1}{b - c_1}$	(B′)	$\frac{d_1}{\Delta} = \frac{a - c_1}{b - c}$.	(B)
$\Delta = d_2 + \frac{1}{2}d$.	(c)	$\Delta = d_2 + \frac{1}{2}d$.	(C′)	$\Delta = d_2 + \frac{1}{2}d$.	(C)
$\Delta = \frac{ad}{2c_1} \frac{b - c_1}{b - a}$.	(3)	$\Delta - \delta - \delta' = \left\{ \frac{d}{2} - \delta - \delta' \right\} \frac{a}{c_1} \frac{b - c_1}{b - a}$.	(III′)	$\Delta = \frac{ad}{2} \frac{b - c}{bc_1 - ac}$.	(III)

The author's reduction above is not approximated absolutely, in which the first formulas, notated by dash, are prepared to inspect the degree of approximation of Asst. Prof. K. Tanaka's and the second are the normal reduction.

Now, in formula (3) calculated by Tanaka, it is not made clear to what extent the error due to the neglect of $-\delta - \delta'$ in the first factor $(d/2 - \delta - \delta')$ of the right hand side in my first check formula (III′), shall extend; while, in the left hand side, it is explained.

Moreover, the labour of the calculation of the author's formula (III) or Eq. (9), which contains no error, is less than that of Asst. Prof. K. Tanaka's (3) at any event.

to Tanaka's solution that this approximation is not only unnecessary but also makes its explanation complex, because in the same way the exact formula (9) can be very easily obtained which contains absolutely no error.

The Adjustment of the Horizontal Hair in a Transit by the Author's Method.

1. "The Condition of Adjustability."

In order that the horizintal hair in a transit may be perfectly adjustable, the position of the line of symmetry $\overline{OI_Ab}$ in Fig. 2 must be absolutely invariant in the telescope no matter what may

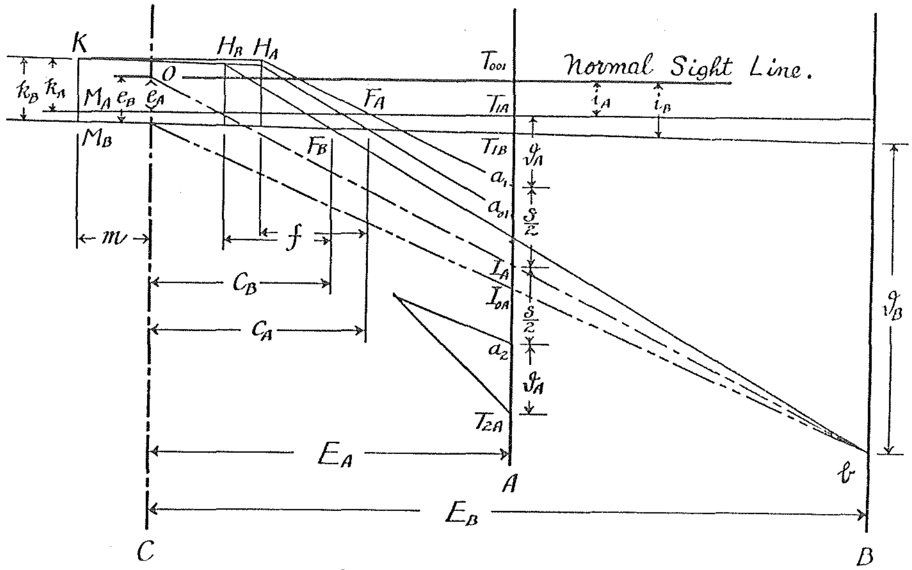

Fig. 2.

be the position of the points b and A, namely——after the adjustment is completed between a pair of points b and A, the conditions

$$1. \quad s = o \tag{8a}$$

and

$$2. \quad \overline{I_A T_{oo1}} \backsim E_A \tag{8b}$$

must hold good independent of the magnitude of the distances E_B and E_A.

Now, from Fig. 2, the relation is obtained

$$\mathfrak{D}_A = \overline{T_{1A}a_1}$$

$$= \overline{T_{1A}T_{1B}} + \overline{T_{1B}a_{o1}} - \overline{a_1 a_{o1}}, \tag{3}$$

from which, neglecting terms of the higher order, is gotten

$$\mathfrak{D}_A = \frac{E_B(E_A - C_A)}{E_B C_A - E_A C_B} \frac{s}{2} - \frac{E_A(E_B - C_B)}{E_B C_A - E_A C_B} \frac{E_A}{f} \left\{ e_B - e_A + (f-m)(i_B - i_A) \right\}$$

$$+ \frac{E_B - E_A}{E_B C_A - E_A C_B} (E_A - C_A) e_B + E_A (i_B - i_A). \tag{7a}$$

Accordingly, by the second condition or Eq. (8b), one obtains, neglecting terms of the higher order,

$$-\frac{E_B - C_B}{E_B C_A - E_A C_B} \frac{E_A}{f} \left\{ e_B - e_A + (f-m)\ (i_B - i_A) \right\}$$

$$+ \frac{E_B - E_A}{E_B C_A - E_A C_B} e_B + i_B = \text{a constant or zero.} \tag{8.0}$$

Theoretically, Eq. (8.0) can not be fulfilled, except $e_A = o = e_B$ and $i_A = o = i_B$, but as e_A, e_B, i_A, and i_B are really of very small magnitudes, the practical conditions can be easily found by a close approximation, which must be satisfied.

Now, the second term of Eq. (8.0) becomes appoximately

$$\frac{E_B - E_A}{E_B C_A - E_A C_B} e_B \doteqdot \frac{e_B}{C} \tag{8.1}$$

Therefore, putting Eq. (8.0) and Eq. (8.1) together, one gets for "the Condition of Adjustability"

$$e_A = e_B = \text{a small constant or zero}$$

and (8c)

$$i_A = i_B = \text{a small constant or zero,}$$

which must hold good practically whatever positions the points A and b may take, *i. e.*, the objective lens must practically travel along its optical axis, when the telescope is focussed.

Now, the condition 2 or Eq. (8b) becomes

$$\overline{I_A T_{oo1}} = \overline{I_A T_{1A}} + \overline{T_{1A} T_{oo1}} \backsim E_A \tag{8.2}$$

where

$$I_A T_{1A} = (\mathfrak{D}_A)_{s=o} \tag{8.3}$$

and

$$\overline{T_{1A} T_{oo1}} = e_A + E_A i_A \,, \tag{8.4}$$

Because the first term of Eq. (8.4) is negligiblly small compared to the second term, one may take for "the Condition of Adjustability" from Eq. (8.3) instead of Eq. (8.2),

$$\frac{(\mathfrak{D}_A)_{s=o}}{E_A} = \text{a constant or zero} \tag{8}$$

whatever values E_A and E_B may take, which has been given in the writer's preceding paper.

Now, "the Condition of Adjustability" Eq. (8c) will be obtained in another way.

In general, if "the Condition of Adjustability," shown by Eq. (8), Eq. (8.0), Eq. (8b) or Eq. (8c), can not be satisfied or the instrumental defects occur irregularly, the rod reading r_D taken for a point D, still clamped in the position, it was in when the operation of adjustment was completed for the points A and b, shall come to a real value, one half of which becomes analytically

$$\frac{r_D}{2}=\frac{E_D}{f}\left\{e_B-e_D+(f-m)\,(i_B-i_D)\right\}$$

$$-\frac{E_B-E_D}{E_B-E_A}\frac{E_A}{f}\left\{e_B-e_A+(f-m)\,(i_B-i_A)\right\}. \qquad \text{(8d)}$$

From this by the condition Eq. (8a), can be derived "the Condition of Adjustability" Eq. (8c), as in Tracy's Procedure.

Here numerical examples of the check reading may be shown as follows:

Designation.	Example 1.	Example 2.
C, m.m.	276.4	276.4
f, m.m.	165.8	165.8
m, m.m.	55.3	55.3
E_A, m.	3.036	3.036
E_D, m.	8.280	8.280
i_B-i_A,	0	0
e_B-e_A, m.m.	0	0
i_B-i_D,	0.0010	0.0002
e_B-e_D, m.m	0.1658	0.0332
r_D, m.m.	27.6	5.5
Deviation, sighted at D, Radian,	0.00167	0.00033
Minute,	5.73	1.15

Therefore it can be understood that it must not be passed by in any case when the variations are acknowledged, because such an error as shown in Example 2 may often occur.

2. Precision of the Formulas for Correction.

The formulas of correction for adjustment of the horizontal hair in an ordinary transit with a telescope of Ramsden, Hygenian, or other type, in which C is passably large, are rewritten as follows:

The theoretical original correction is accurately

$$\mathfrak{C}_A = \frac{E_B(E_A - C_A)}{E_B C_A - E_A C_B} \frac{s}{2}. \tag{9}$$

But if the point b is selected by eye measurement such that the distances E_A and E_B satisfy approximately the condition

$$E_B = \frac{E_A(E_A - C)}{f^2} C, \tag{13}$$

one gets from Eq. (9), neglecting terms of the higher order, the formula for correction

$$\mathfrak{C}_A = \frac{E_A - C}{C} \frac{s}{2}, \tag{14}$$

which is very convenient in practice.

If the distance E_B is not so large as to satisfy Eq. (13) or

$$E_B \ll \frac{E_A(E_A - C)}{f^2} C, \tag{15}$$

one gets from Eq. (9), neglecting terms of the higher order,

$$\mathfrak{C}_A = \frac{E_A - C}{C} \left\{1 + \frac{E_A}{E_B}\right\} \frac{s}{2}, \tag{16}$$

Besides the above formulas, one obtains for a general correction from Eq. (9), neglecting terms of the higher order,

$$\mathfrak{C}_A = \left\{\frac{E_A - C}{C}\left(1 + \frac{E_A}{E_B}\right) - 0.36\right\} \frac{s}{2}, \tag{19}$$

which gives a good approximate value of Eq. (9).

In order to make easily comprehensible the precision of the above formulas, their errors due to approximation will be shown in percentage for a general example:

$C \;\;= 0.30$ m.

$f \;\;= 0.18$ m.

$E_A = 3.00$ m. ,

varying E_B from 15 m. to 200 m., in Diagram 1.

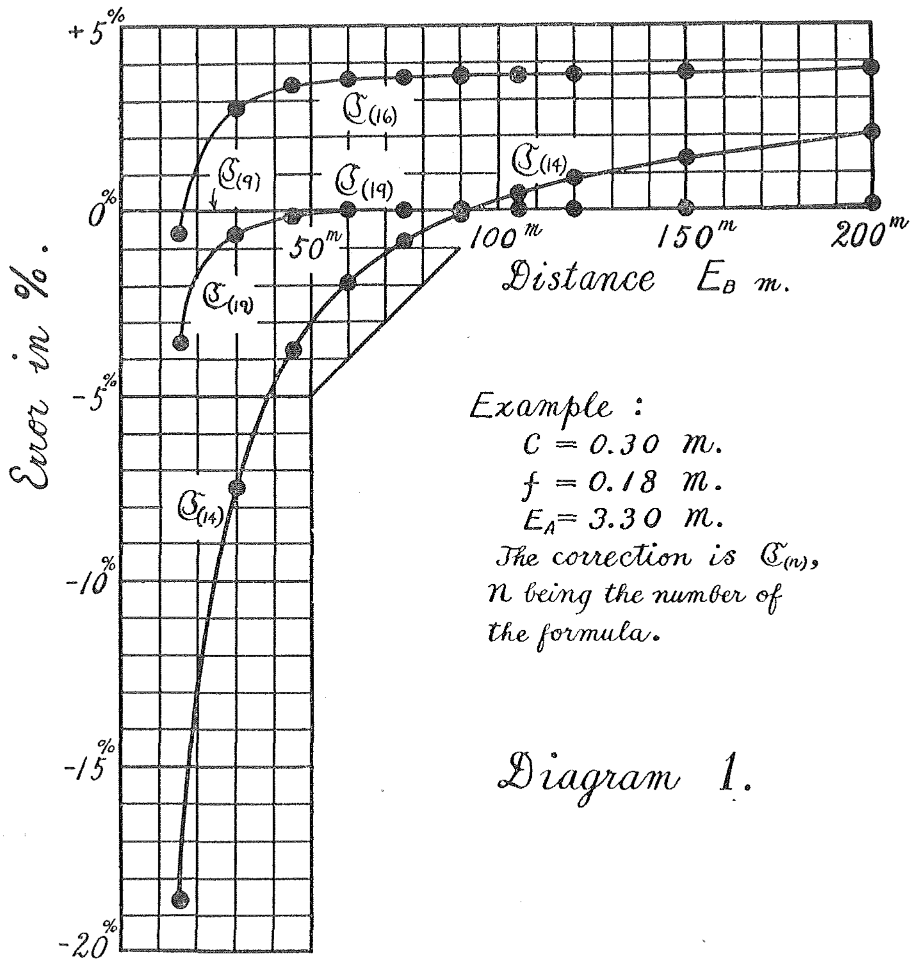

Diagram 1.

Now, it can be shown from Diagram 1, that if the condition shown by Eq. (13) is approximately fulfilled by eye measurement, the error of the formula shown by Eq. (14) falls within only $\pm 1\%$.

But whenever the point b is chosen over sixty meters away from the instrumental center C, its error does not surpass $\pm 3\%$.

Nextly, the formula shown by Eq. (16) is less erroneous than only $+3.5\%$ for $E_B < 50$ m. and -0.6% for $E_B = 15$ m., while that shown by Eq. (14) is erroneous over -3% for the former and reaches to -18.5% for the latter.

Lastly, the formula shown by Eq. (19) gives practically the correct value of adjustment for any value of E_B except $E_B < 30$ m., and even when E_B is taken as 15 m., which is the worst case, its error comes to only -3.6%.

In conclusion, putting the above results together, it must be certain that the formula shown by Eq. (14) is the most useful and convenient in practice.

Conclusion.

In the author's preceding paper, he exerted himself to simplify and condense the contents so as to be easily understood, but it seems now that all his toil came to nothing, because some parts of them were misunderstood. For that reason, this new paper has been prepared to explain those misapprehended points.

Here, again it is affirmed that because a transit belongs to the so-called "precise instrument," a man who knows the mechanical workmanship and the accidental defects of the objective lens and the telescope, must recognize not only the eccentricity and inclination of the optical axis of the objective lens, but also their variation due to the jolting and pitching motion of the objective slide when the objective goes out and in.

For this reason, the writer had proposed "the Condition of Adjustability" shown by Eq. (8) in his preceding paper, but here it has been possible to explain all its contents in detail, which mean that "the objective lens must be simply translated along a straight line, without changing the direction of its optical axis," namely—— that "the conditions

$e =$ the eccentricity of the axis of the objective referred to the horizontal axis of the telescope

$=$ a small constant or zero

and

$i =$ the inclination of the axis of the objective referred to its normal position

$=$ a small constant or zero"

must practically hold good, whatever point may be sighted.

It is the best when the check reading r_D, taken for any point D, can not be perceived, but whenever it is appreciated, it may be favorable if it does not cause any error in the reading of vertical angles.

Although, strictly speaking, there are no transits which have not the eccentricities and the inclinations of the axes of objectives, they may be rejected only upon test, which give impermissible values of r_D shown by Eq. (8d).

From the results of the experiments shown in Experimental Notes 1 and 2, one can admit that the values of r_D are too small to be perceived commonly.

But, again the writer ventures to advise all theoretical and practical specialists who adovocate and carry out the adjustment of the horizontal hairs in transits, that they must never overlook "the Condition of Adjustability" because they can appreciate not only the quality of new instruments and the disorder of old ones, but also prevent the errors coming into them previously upon test indicated by Eq. (8d.)

Further, the author has accurately explained the precision of the various formulas of correction in adjustment of the horizontal hair in an ordinary transit.

It has been shown that, in both methods, the formulas shown by Eq. (9) are theoretical results, those shown by Eq. (14) the most useful and convenient in practice, and the others auxilliary approximations.

A man must pay attention so as not to mix $\mathfrak{D}$, *i. e.*, the deviation of the collimation line from the axis of the objective lens system at the points A and B, and $\mathfrak{E}$, *i. e.*, the neccessary and sufficient correction of the horizontal hair.

Now, Tracy's Procedure was named conveniently, because Prof. J. C. Tracy's work "Plane Surveying" was very widely adopted.

None of the methods of adjustment of the horizontal hair in a transit proposed in other's works were worth mentioning in the writer's preceding paper, as they were inappropriate and inapplicable in practice.

The author's method is a systematical and universal one of adjustment of the cross hairs in a transit, because it can also be applied to adjustment of the vertical hair as well as to that of the horizontal hair, and is superior to the present one-fourth method, about which it is proposed to publish a paper in the very near future.

At the conclusion of the present paper, the writer would heartily advise those who study the adjustment of the horizontal hair in a transit, that they must understand "Geometrical Optics", "Optical Instruments", and "Machine Tools".

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