Integral Adjustment of the Cross-Hairs in a Transit and a Wye Level.

By

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Integral Adjustment of the Cross-Hairs

in

a Transit and a Wye Level

INTRODUCTION

A civil engineer or a lecturer on Surveying must not believe to excess in the precision and the substantial quality of transits, levels, and other surveying instruments without any rational test. They have many defects caused by imperfect workmanship and construction in manufactories, and irregular abraisions and accidental impediments while they are used in the field, of which the latter should properly in practice have much more influence on measurements than the former.

Therefore, these facts must not be dismissed from the mind, especially when the cross-hairs in a transit and a wye level are adjusted.

From these stand-points, the first and the second papers* were published by the present author under the dates of 16 Oct., 1931 and 1 March, 1934 respectively. In the former of those papers a new method and the correct new formulas for the then old method for adjustment of the horizontal cross-hair in a transit, completely solved from the standpoint of Geometrical Optics, were made public for the first time. In the latter the precision of the formulas and the conditions for adjustability were treated.

Before the publication of the first paper, the problems had been fundamentally mis-understood from want of knowledge in Geometrical Optics by all makers and practical and theoretical specialists for over twenty-four years at the very least.

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* References (1) and (2) respectively at the end of this paper.

Originally, it was first found out by the present author about May, 1925, that the then corrections in the old method were extremely erroneous.

Thereupon, a correct new formula for correction was tentatively calculated by the author in April, 1927, which of course, has been lectured to his students since that time but not made public till the issue of the first paper.

Then just previous to the publication of the first paper, **the correct new formulas were informed to Prof. of Civil Engineering, Hachirō Kimishima, of the Kyūshū Imperial University, Fukuoka, Japan and Prof. of Civil Engineering, Nobuo Seki, of the Tōkyō Imperial University, Tōkyō, Japan on 12 June, 1930, and willingly acknowledged by them.**

Moreover, **it was an un-deserved honour to the present author that his formulas were readily accepted by Prof. N. Seki and introduced by him in his new work on Surveying in Japan, namely—"測量學".†**

Therefore, this is an original unique work on Surveying in the world at present, in which the correct method of adjustment of the horizontal cross-hair is described.

Accordingly, ever since, the correct method has been lectured to the students of Civil Enginnering in the Tōkyō Imperial University, Tōkyō, the Kyūshū Imperial University, Fukuoka, and the Hokkaidō Imperial University, Sapporo, in Japan.

Subsequently, since it has been clearly proved by further theoretical researches that the present methods for the adjustment of the vertical cross-hair in a transit and the horizontal cross-hair in a wye level are of extremely doubtful accuracy, the herein reported general studies on the methods for "Integral Adjustment of the cross-hairs in a transit and a wye level" have been performed with aid of a subsidy by the **"Foundation for the Promotion of Scientific and Industrial Research of Japan"** from 19 May, 1934 to 18 May, 1935.

The present paper is a complete work, in which the later results of studies have been combined with those of the report, already presented to the above organization at the end of April, 1935.

Excepting the above, no thorough-going paper on the methods of adjustment of the cross-hairs in a transit and a wye level has ever been made public up to the present.

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† On pages 84-88.

Excepting the Introduction and Conclusion, the present paper is divided into five chapters, the outlines of whose contents are described as follows:

In Chapter I, the general optical relations in the objective lens system of the telescope in a surveying instrument are solved anew in detail for practical use, especially for a telescope of the internal focussing or the Wild-Zeiss type.

In Chapter II, the theories and the methods for adjustment of the horizontal cross-hair in a transit are generalized, compared with those in the first and the second, from which are newly computed the formulas for adjustment of the telescope of the Wild-Zeiss or the internal focussing type and the universal trial method, which needs no datum of a telescope. Thereupon, a few relevant experiments for the methods of adjustment were performed, whose results are shown in Table 1 in Art. 6 with the old examples from the first paper.

Up to the present, common deviations over 1/1,000 or 3′, and a deviation of 13/60,000 or 45″ even in a new foreign-made transit, only just purchased, have been experienced by the present author.

Notwithstanding this state of affairs, by the methods described in the present chapter, the horizontal cross-hair can usually be corrected without any trouble by a single adjustment to the extent of an error below 1/25,000 or 8″, which needs no further adjustment for ordinary use because it is generally beyond the capacity of the eye-reading of a telescope level and a plate level.

In Chapter III, **the Integral Adjustment of the vertical cross-hair in a transit** is theoretically, experimentally and practically studied to perfection by four illustrative experiments for finding out the original errors affecting the measured horizontal angle, by three criteria for grades of quality, and by the practical examples of Integral Adjustment.

Now, the methods of Integral Adjustment of the vertical cross-hair in a transit described here are the correct new ones for the so-called "Objective Slide Adjustment", exactly based on Geometrical Optics and precise calculation, and should properly be practised on a transit as needed in the field by a surveying engineer against the custom. All the old methods are not only fundamentally mistaken, but also they are performed only in the manufactory.

As found from the results of the first experiment listed in Table 7 of Art. 27 and in Table 8 of Art. 28, the common deviation 0.3 m.m. of the first principal point of the objective lens should cause an error of 20″ in an angle between the sides of 100 m. and 3 m. The deviations of

+0.621 m.m. at 2 m., +0.559 m.m. at 7 m. and +0.538 m.m. at 137.3 m. should likwise cause +63″ at 2 m. and +16″ at 7 m. Also a set of the maximum errors of the inclination of the optical axis $i = -1° \ 22' \ 44''$, its eccentricity at the rotational center of the telescope $e = +7.297$ m.m. and that at the front slide bearing $e_g = +5.209$ m.m. is found in Table 7.

Hereupon, it is also found that the principal origins of the detrimental error are the distortion of the standards due to accidents and the formal errors of the objective lens itself due to mis-centering.

Finally, the vertical cross-hairs in transits No. 5 and No. 23 were very successfully adjusted by the method of Integral Adjustment, with results as shown in Table 16 of Art. 43.

In Chapter IV, **The Integral Adjustment of the horizontal cross-hair in a wye level** is theoretically, experimentally and practically studied in nearly the same way as that for a transit in Chapter III, by three illustrative experiments, viz., those for determining the residual error remaining after the ordinary One-Half Adjustment, those due to the formal errors of the objective lens itself and the irregular variation of the deviation of collimation points from the level plane, also by two criteria and by practical examples for Integral Adjustment.

The methods of Integral Adjustment of the horizontal cross-hair in a wye level are just the same in theory and principle as those for the transit from its very nature, namely — they are the correct new methods of the so-called "Objective Slide Adjustment", exactly founded on Geometrical Optics and accurate computation, and should properly be performed on some extremely disordered wye level according to the need even by a civil engineer contrary to custom; while, on the other hand, the old method is not only theoretically mistaken, but also practised only in the manufactory.

Now, the maximum vertical deviations of the collimation points situated at distances of 3 m., 5 m. and 10 m. from the level plane were 34″, 18″, and 10″, respectively for level No. 1, in the results of the first experiment shown in Table 20 of Art. 61, which was performed on twenty-three wye levels, all kept in the original conditions as when bought. Also additional deviations were gotten by changing the aspect of the objective lens itself as it is in the holder correlative to the telescope-tube, amounting to 27″, 16″ and 8″ respectively for No. 7, in the results of the second experiment shown in Table 21 and Table 22 of Art. 62, when adjusted at 138.3 m. by the ordinary method of One-Half Adjustment.

Accordingly, all the above errors are very bad because an eye-reading of the telescope level is nearly 3″ in error.

In the third experiment, a set of extraordinarily large errors was found for level No. 7, namely, the eccentricity of the objective lens at the front slide bearing $e_g = -1.519$ m.m., that at the rear slide bearing $e_{gr} = -2.242$ m.m., that at the rotational center of the telescope $e = -1.354$ m.m., the inclination of the optical axis $i = +16' \ 08''$ and the irregular variations of e_g and e_{gr}, that is, $\varepsilon_{eg} = \pm 0.011$ m.m. and $\varepsilon_{egr} = \pm 0.044$ m.m. These are tabulated in Table 23C, Art. 63.

Subsequently, the methods of Integral Adjustment were very successfully performed on levels No. 1, No. 7 and No. 20; the results are given in Table 26 of Art. 66.

In Chapter V, the secondary problems are studied and discussed in relation to Integral Adjustment of the cross-hairs in a transit and a wye level, namely—the method of eliminating the influence of the formal errors of the objective lens itself, the qualifications of an excellent tripod and a superfine standard, the relative precision of the front and the rear slide-bearings, and the special telescope of the Ramsden or Huygenian type, in which the relation $\delta = g$ is fulfilled.

Now, the subject of the present paper consists in the perfect correction of the fundamentally mistaken methods of adjustment of the cross-hairs in a transit and a wye level and the complete elimination of the large errors coming into the measurements by the correct new methods of Integral Adjustment, but, in order to clear up the true nature and to appreciate "Integral Adjustment", the principal and superfluous problems were solved with the greatest possible exactness and miscellaneous extra experiments for finding the errors were illustratively performed.

Therefore, the nearer way to attain proficiency in Integral Adjustment is as follows:

First, if it is desired to comprehend only "the methods of Integral Adjustment", study from Art. 1 to Art. 13 and Art. 15 for cases of the horizontal cross-hair in a transit, Art. 1, Art. 2, Art. 16, Art. 17, Art. 19, Art. 20, Art. 21, from Art. 36 to Art. 43, and from Art. 54 to Art. 58 in cases of the vertical cross-hair in a transit, and Art. 1, Art. 2, Art. 16, Art. 17, Art. 19, Art. 20, Art. 21, from Art. 36 to Art. 43, Art. 59, Art. 66 and Art. 68 in cases of the horizontal cross-hair in a wye level.

Secondly, if it is only wished to adjust at once the cross-hairs in a transit and a wye level integrally, proceed according to the full-faced type descriptions and the full-faced type formulas in Art. 5, Art. 9 or Art. 13 for the horizontal cross-hair in a transit; according to those in

Art. 38, Art. 39, Art. 40, Art. 41 or Art. 42 for the vertical cross-hair in a transit; and according to those in Art. 66 and Art. 39, Art. 41 or Art. 42 for the horizontal cross-hair in a wye level.

Finally, the theories, the principles and the experiments for Integral Adjustment will be expounded minutely in the following.

COMMON CONVENTIONAL NOTATIONS AND DEFINITIONS

Notations and definitions will be defined as follows, in common to all the papers:

- a : The reading of the rod or the scale situated at point A.
- b : A well-defined distant point or the reading of the rod situated at a distant point B.
- C : The so-called "Instrumental Constant" or the instrumental center.
- $C_p = \overline{OF}$, $= C + \Delta_p$: The distance of the first or front focal point of the objective lens system from the center of rotation of the telescope, when point p is sighted. When p is infinitely distant, C_p takes a certain constant value C, namely—the so-called "Instrumental Constant."
- $\Delta_p = \overline{OF} - C$: The distance of the first or front focal point F of the objective, when focussed on p, measured from point F corresponding to the sight of an infinitely distant point, i.e., the point at the distance C measured from the center of rotation of the telescope.

  In the Ramsden, the Huygenian or the Porro telescope, it is the distance of the image of an object at p or the cross-hair when it is sighted, measured from the second or back focal point of the objective, that is to say, the travelling distance of the objective to sight the object at p, measured from the second focal point which corresponds to the sight of an infinitely distant point.
- $\mathfrak{C}$: The correction of a cross-hair.
- d : The distance between the optical centers of the two component lenses of the objective lens system.
- $\mathfrak{D}$: The deviation of the collimation point from the optical axis of the objective lens system.

δ : The distance of the first or front principal plane of the objective lens system from the center of rotation of the telescope.

Δ : The increment or variation of the quantity.

e : The eccentricity of the optical axis of the objective lens system from the horizontal axis of the telescope or the vertical axis of the instrument.

e_0 : Do., when the cross-hair is completely adjusted by the method of Integral Adjustment.

e_g : The eccentricity of the optical axis of the objective lens system from the meridian plane of a transit or the level plane of a wye level at the front slide bearing.

e_{gr} : The eccentricity of the optical axis of the objective lens system from the meridian plane of a transit or the level plane of a wye level at the rear slide bearing.

e_l : The eccentricity of the first principal point of the objective lens from the meridian of a transit or the level plane of a wye level.

(e) : The eccentricity of the first principal point of the objective lens system projected on the ξ-axis or $(e)=\xi_H=[e]\cos\theta$.

$[e]$: The absolute eccentricity of the first principal point of the objective lens system itself or the eccentricity referred to its ideal axis in the ξ-η plane.

$(E_p\varphi_p)$: The deviation of point p projected on the ξ-axis or ξ_p.

ε : The mean square error.

ϵ : The eccentricity of the locus of collimation points.

f, f_1 and f_2 : The focal lengths of the objective lens itself and its component lenses respectively.

F and F' : The first or front and the second or back foci of the objective lens system. They are sometimes called the anallactic points respectively.

g : The distance of the front slide bearing of the objective slide in the Ramsden, Huygenian or other telescope with an adjustable rear slide bearing from the center of rotation of the telescope in a transit or a wye level.

G and G_r : The front and the rear slide bearing of the objective slide.

H and H' : The first or front and the second or back principal planes of the objective lens system.

H_0 : The formal position of the first principal point of the objective lens system itself.

i : The inclination of the optical axis of the objective lens system referred to its normal position, that is to say, the angular deviation of that axis from the normal sight line, or from the meridian of a transit or the level plane of a wye level.

i_0 : Do., when the cross-hair is perfectly adjusted by the method of Integral Adjustment.

(i) : The inclination of the optical axis of the objective lens projected on the plane perpendicular to the ξ-η plane, including the ξ-axis, referred to the ideal optical axis, or $(i) = [i]\cos(\theta+\psi)$.

$[i]$: The absolute inclination of the optical axis of the objective lens system referred to the ideal.

I : The middle point of the first and the second readings of the rod or the scale.

$k = \overline{MK}$: The deviation of the horizontal or the vertical cross-hair from the axis of the objective lens system.

K : The crossing point of the cross-hairs.

L_1 and L_2 : The component first and second lenses respectively.

m : The distance of the cross-hairs from the center of rotation of the telescope.

$\overline{MT}$: The optical axis of the objective lens system.

O : The center of rotation of the telescope.

ϖ : The weight of the suffixed quantity in calculation by the Method of Least Squares.

$\overline{OC}$: The vertical axis of the instrument.

$\overline{OT_\infty}$: The normal position of the optical axis of the objective lens system, i.e., its position when $e_A = 0 = e_B$ and $i_A = 0 = i_B$.

p : Any point or the reading of the scale or the rod situated at point p or P.

$\wp$: The image point of the virtual collimation point p in the field through the objective lens system.

φ : The angular deviation of a collimation point from the meridian of the transit or the level plane of the wye level.

ψ : The angular divergence between $\overline{H_0H}$ and $\overline{H\wp}$ in the ξ-η plane.

r : The difference of the first and the second readings of the rod held at point B.

s : The difference of the first and the second readings of the scale situated at point A.

t : Twice the deviation of a collimation point from the meridian when the vertical cross-hair of a transit is adjusted by the ordinary method of One-Quarter Adjustment or the level plane when the horizontal cross-hair of a wye level is adjusted by the ordinary method of One-Half Adjustment.

τ : The distance of the optical center of the first lens from the center of rotation of the telescope.

θ : The direction angle of $\overline{H_0H}$ referred to the ξ-axis.

u : Twice the deviation of a point collimated at F. S., referred to the B. S. points A and B between which the cross-hair is adjusted by the method of Integral Adjustment of the horizontal cross-hair in a transit.

U : Do. at the points of the same distances as A and B.

V : The intersection point of the optical axis of the objective lens system and the straight line passing through a near and a distant point on the locus of collimation points.

$\hat{V}$: The intersection angle of the optical axis of the objective lens system and the straight line passing through a near and a distant point on the locus of collimation points at the intersection point V.

x : The distance of the intersection point V from the center of rotation of the telescope.

x and y : The coordinates of a point correlated to the objective lens system referred to the center of rotation of the telescope O as the origin and the meridian of the transit or the level plane of a wye level as the abscissa, that is to say—the distance of a point from O and its deviation from the meridian or the level plane. The suffix shows the referred point.

X—X : The horizontal axis of the telescope.

ξ and η : The abscissa and the ordinate referred to the rectangular coordinate axes perpendicular to the ideal optical axis of the objective lens system through the ideal position of its first principal point H_0.

z : The departure of the cross point of the cross-hairs from the meridian of the transit or the level plane of the wye level.

ζ : The interval between the front and the rear slide bearings.

Suffixes A, B, D, J, p, P, N, or ∞ show a quantity when focussed there respectively, where ∞ denotes an infinitely distant point.

Suffix H shows that the quantity is correlated to the horizontal cross-hair.

Suffix 0 denotes the case $e_A = e_B = 0$, $i_A = i_B$ and $k_A = k_B$, or in which the cross-hair is perfectly adjusted by the method of Integral Adjustment.

Suffix V shows that the quantity is correlated to the point V or the vertical cross-hair.

Suffix 1 or 2, attached to a or b, shows that it is taken with the telescope normal or inverted respectively.

Suffixes 1 and 2, attached to f, show the focal lengths of the component lenses of the objective.

Suffixes 1, 2, 3, etc., attached to e, i, t, and u, show the order of observation.

The parenthesized number (1), (2), etc., refers to literature listed at the end of the present paper.

The level plane, in this work, means a plane perpendicular to the vertical axis of rotation of the telescope through the center of its rotation.

The meridian or the meridional plane, in this work, means the vertical plane, which passes through the center of rotation of the telescope and is perpendicular to the horizontal axis of the telescope.

The method of One-Half Adjustment or simply, One-Half Adjustment of the cross-hair in a wye level means that method used at present, in which one-half the error found by rotating the telescope about its spindle as it is in the wyes is corrected by adjusting the position of the cross-hair.

The method of One-Quarter Adjustment or simply, One-Quarter Adjustment of the vertical cross-hair in a transit means that used at present, in which one-quarter the error found between the two equidistant conjugate points is corrected by adjusting the position of the vertical cross-hair.

The normal sight line means the sight line when $i = 0$, $e = 0$ and $k = 0$, that is to say, at any point sighted, $\varphi = 0$ or $t = 0$.

The normal sight plane means the sight plane when $i = 0$, $e = 0$ and $k = 0$, that is to say, at any point sighted, $\varphi = 0$ or $t = 0$. It is just the meridian for a transit or just the level plane for a wye level.

CHAPTER I

OPTICAL RELATION IN THE OBJECTIVE LENS SYSTEM OF THE TELESCOPE IN A SURVEYING INSTRUMENT

1. Fundamental Optical Relations in the Objective Lens System of the Telescope of a Surveying Instrument. From the standpoint of Geometrical Optics, the types of telescopes are generally divided into three classes, namely—the Ramsden and Huygenian, the Porro and the Wild-Zeiss types.

Their objective lens systems must generally be composed of more than two lenses at least for the purpose of necessary corrections of over three among the six optical errors of the third order.

The fundamental optical relations in the optical lens system of the telescope will plainly be exlained here, assuming that it is composed of two component lenses, as shown in Fig. 1, for convenience sake, in which the notations and the signs are as follows:

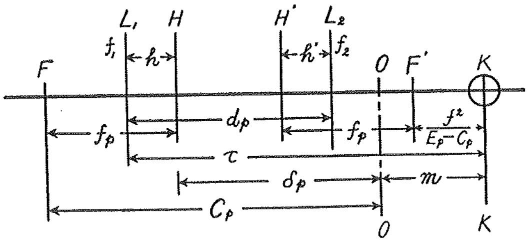

Fig. 1. General Optical Relations in the Objective Lens System.

C_p: The distance of the first focal point of the objective lens system from the center of rotation of the telescope when point p is sighted or $\overline{FO}$. When p is infinitely distant, it takes a certain constant value C, namely—the so-called "Instrumental Constant".

d: The distance between the optical centers of the two component lenses of the objective.

δ: The distance of the first or front principal plane of the objective lens system from the center of rotation of the telescope or $\overline{HO}$.

f, f_1 and f_2: The focal lengths of the objective lens itself and its two component lenses respectively.

F and F': The first or front and the second or back foci of the objective lens system.
H and H': The first or front and the second or back principal planes of the objective lens system.
h and h': $\overline{L_1H}$ and $\overline{L_2H'}$, respectively.
K: The cross-point of the cross-hairs.
L_1 and L_2: The component first and second lenses respectively.
m: The distance of the cross-hairs from the center of rotation of the telescope or $\overline{KO}$.
O: The cross-point of the vertical and the horizontal axes of the telescope, namely—the center of its rotation.
τ: The distance of the optical center of the first lens from the centre of rotation of the telescope.

Now, according to the above notations and definitions, the fundamental formulas are generally written down as follows:

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}=\frac{f_1+f_2-d}{f_2f_2}, \quad \text{(1)}$$

$$d=f_1+f_2-\frac{f_1f_2}{f}, \quad \text{(2)}$$

$$d-d_\infty=-\frac{f_\infty-f}{f_\infty f}f_1f_2,\quad \frac{f_\infty-f}{f_\infty f}=\frac{d_\infty-d}{f_1f_2}, \quad \text{(3)}$$

$$h=\frac{f_1d}{f_1+f_2-d}=\frac{f}{f_2}d, \quad \text{(4)}$$

$$h'=\frac{f_2d}{f_1+f_2-d}=\frac{f}{f_1}d, \quad \text{(5)}$$

$$f-h=\frac{f_1(f_2-d)}{f_1+f_2-d}$$

$$=f_1-\frac{f_1}{f_2}f, \quad \text{(6)}$$

$$C_p=\delta_p+f$$

$$=\delta_p+\frac{f_1f_2}{f_1+f_2-d}, \quad \text{(7)}$$

and

$$\tau=h+\delta_p+m$$

$$=d-\frac{f}{f_1}d+f+\frac{f^2}{E_p-C_p}. \quad \text{(8)}$$

Hereupon, for simplicity, further detailed formulas for respective types of telescopes will be reduced in order in the following articles.

2. The Ramsden and the Huygenian Telescopes. Now, in the first class of telescopes, those of the Ramsden and the Huygenian types are included. They are the most universally utilized and have such constructions that the objective lenses are drawn in or run out when focussing, as shown in Fig. 2 and Fig. 3, so that the so-called "Instrumental constant" C varies according to the distance of the point sighted.

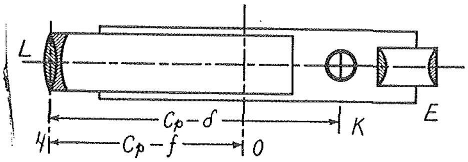

Fig. 2. Ramsden's Telescope.

Most telescopes of such types are commonly provided with hair or wire diaphragms which can be moved by four adjustable screws on them for adjustment of cross-hairs in fields.

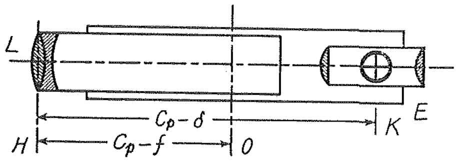

Fig. 3. Huygenian Telescope.

Moreover, for the purpose of correction of directions of optical axes of objectives, that is, the so-called Objective Slide Adjustment, telescopes of Gurley's type are commonly provided with the adjustable rear slide bearings and four adjustable screws respectively.

Now, since, for these telescopes, the relations

$$d = \text{a very small constant}, \quad f = \text{a constant}, \quad \ldots\ldots \quad (9)$$

$$\delta_p = \delta + \frac{f^2}{E_p - C_p} \quad \ldots\ldots \quad (10)$$

and

$$C = \delta + f \quad \ldots\ldots \quad (11)$$

hold good, the formula

$$C_p = C + \frac{f^2}{E_p - C_p}$$

and from it, the equation

$$(C_p - C)^2 - (E_p - C)(C_p - C) + f^2 = 0$$

are obtained.

Therefore, from these, **the formula**

$$
\begin{aligned}
C_p &= C+\frac{f^2}{E_p-C_p} \\
&= C+\frac{E_p-C}{2}-\sqrt{\left(\frac{E_p-C}{2}\right)^2-f^2} \\
&= C+\frac{f^2}{E_p-C}\left\{1+\left(\frac{f}{E_p-C}\right)^2+2\left(\frac{f}{E_p-C}\right)^4+5\left(\frac{f}{E_p-C}\right)^6\right. \\
&\qquad +14\left(\frac{f}{E_p-C}\right)^8+42\left(\frac{f}{E_p-C}\right)^{10} \\
&\qquad \left.+132\left(\frac{f}{E_p-C}\right)^{12}+\ldots\ldots\ldots\right\} \quad \ldots\ldots \quad (12)
\end{aligned}
$$

is derived, from which it can readily be seen that the so-called Instrumental Constant hyperbolically changes as the variable E_p-C.

3. **The Porro Telescope.**† In the second class, is Porro's telescope, as shown in Fig. 4, manufactured by Stark† in Wien, Austria, but the present author has not yet seen one. But if it is provided with the hair or wire diaphragm which can be moved by four adjustable screws, then the horizontal cross-hair can completely be adjusted in the field.

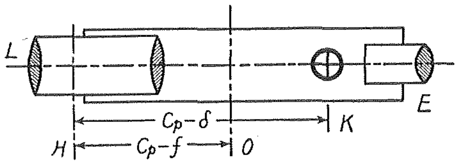

Fig. 4. Porro's Telescope.

Now, for this telescope, the relations

$$d = \text{a constant}, \quad f = \text{a constant}, \quad \ldots\ldots\ldots\ldots\ldots \quad (13)$$

$$\delta_p = \delta+\frac{f^2}{E_p-C_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

$$
\begin{aligned}
C &= \delta+f \\
&= \text{a very small constant or zero} \quad \ldots\ldots\ldots\ldots\ldots \quad (15)
\end{aligned}
$$

and accordingly

$$C_p = \frac{f^2}{E_p-C_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

are obtained.

† Jordan-Eggert: Handbuch der Vermessungs-kunde, Bd. II, 1914, s. 757.

4. The Wild-Zeiss or Internal Focussing Telescope. Now, to the third class belong the telescopes of the internal focussing type adopted for the instruments manufactured by Carl Zeiss† in Germany and Wild† in Switzerland, and for the precise wye level by Tamaya Co. and the "Fuji" transit and the "Hōkō" wye level by Sokkisha Co. in Japan, among which both the cross-hairs of only the last three are adjustable in the narrow scope of the present author's knowledge.

These telescopes are elaborately designed so that only the second members of the objective lens system are moved for focussing and accordingly they are completely sealed up in telescope tubes for safety. An example of a Wild-Zeiss telescope is depicted in Fig. 5.

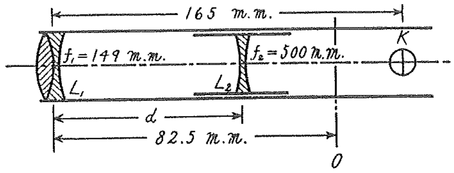

Fig. 5. Wild-Zeiss' Telescope.

For this telescope, the conditions

$$\boldsymbol{d} = \textbf{a variable,} \quad \boldsymbol{f} = \textbf{a variable,} \quad \tau = \textbf{a constant} \quad \ldots \text{ (17)}$$

hold good, from which, (2) and (3), the relation

$$\frac{f_p^2}{E_p - C_p} = \frac{f_\infty - f_p}{f_\infty} f_2 \frac{f_1^2 - f_\infty f_p}{f_1 f_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ (18)}$$

is obtained.

Accordingly, from (18), the new relations

$$f_\infty - f_p = \alpha_p \frac{f_\infty^2}{E_p - C_p}, \quad \alpha_p = \frac{f_1 f_p^3}{f_2 f_\infty (f_1^2 - f_\infty f_p)} \quad \ldots\ldots\ldots\ldots \text{ (19)}$$

and

$$\frac{f_\infty - f_p}{f_\infty} = \beta_p \frac{C}{E_p - C_p}, \quad \beta_p = \frac{f_1 f_p^3}{(f_1^2 - f_\infty f_p) f_2 C} \quad \ldots\ldots\ldots\ldots \text{ (20)}$$

are gotten.

Now, from (7) by (2), (3), (4), (5), (6), and (20), the distance of the first focal point of the objective lens system measured form the center of rotation of the telescope

† See Z. f. Instr., 1909, s. 334~340 and Jordan-Eggert's Handbuch der Vermessungskunde, Bd. II, 1914, s. 232~236 and s. 757~758.

$$C_p = C - \gamma_p \frac{C^2}{E_p - C_p}$$

$$= C + \frac{f_1}{f_2}(f_\infty - f_p) \quad \text{......................} \quad (21)$$

is obtained, where

$$C = \delta + f_\infty, \quad \gamma_p = -\beta_p \frac{f_1}{f_2} \frac{f_\infty}{C}.$$

In (19), (20) and (21), α, β and γ are the functions of f_p which vary only a few percent for its extreme values respectively, because f_p is very slightly variable.

Hereupon, it must be illustrated by a typical example.

Example.—For a Wild-Zeiss telescope as depicted in Fig. 5, whose optical data† are given as follows:

f_1	$= 149$ m.m. ,	$h+\delta$	$= 82.5$ m.m. ,
f_2	$= -500$ m.m. ,	f_1/C	$= 0.523$,
C	$= 284.60$ m.m. ,	δ/C	$= 0.289$,
τ	$= 165.00$ m.m. ,	$\frac{C-\delta-f_1}{C}$	$= 0.188$;
d_∞	$= 67.21$ m.m. ,		

the results are obtained as shown in Table 1 and Table 2.

Table 1.†

E, m.	d, m.m.	f, m.m.	$d-d_\infty$, m.m.	α.*	β.*	γ.*
∞	67.20	178.14	0	0.9931	0.62164	0.11595
50	68.69	177.51	1.49	—	—	—
5	83.43	171.49	16.23	—	—	—
2.28*	108.31	162.20*	41.11*	1.0664	0.66771	0.12455

Now, the optical formulas have already been calculated and the important coefficients α, β and γ found too, but since, in practice, the optical data except C are not made public by makers usually so that α, β, γ, etc. can not be computed from the preceding formulas, the new formulas for $(f_\infty - f_p)/f_\infty$, β, and γ should be reduced by which they can be calculated from the observed values of f_1, d and δ or more practically and simply d only, in the field.

† See Z. f. Instrumentenkunde, 1909, s. 334～340.

* Calculated by the present author.

Now, from formula (1) and (2), the relation

$$-\frac{f_\infty}{f_2}=\frac{f_1}{f_1+f_2-d_\infty} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots (22a)$$

is found and, on the other hand, from (6) and (21),

$$-\frac{f_\infty}{f_2}=\frac{C-(h+\delta)-f_1}{f_1}, \quad\cdots\cdots\cdots\cdots\cdots\cdots (22b)$$

because the relation

$$\begin{aligned} C-(h_p+\delta_p) &= C-(h+\delta) \\ &= f_1-\frac{f_\infty}{f_2}f_1 \end{aligned} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots (23)$$

should hold good, referred to Fig. 1, where $h_p+\delta_p=h+\delta$.

Therefore, the formula

$$-f_2=\frac{C-(h+\delta)}{C-(h+\delta)-f_1}f_1-d_\infty \quad\cdots\cdots\cdots\cdots\cdots (24)$$

is obtained from (22a) and (22b).

Putting this in (1), the focal length of the objective lens when point p is sighted

$$f_p=\frac{\dfrac{C-(h+\delta)}{C-(h+\delta)-f_1}f_1-d_\infty}{\dfrac{f_1^2}{C-(h+\delta)-f_1}+d_p-d_\infty}f_1 \quad\cdots\cdots\cdots\cdots\cdots\cdots (25)$$

is gotten, from which the formula

$$\frac{f_\infty-f_p}{f_\infty}=\frac{1}{\dfrac{f_1^2}{\{C-(h+\delta)-f_1\}(d_p-d_\infty)}+1} \quad\cdots\cdots\cdots (26)$$

is readily derived.

Accordingly, from (20) and (26), the coefficient

$$\beta_p=\frac{1}{\dfrac{f}{\{C-(h+\delta)-f_1\}(d_p-d_\infty)}+1}\,\frac{E_p-C_p}{C} \quad\cdots\cdots (27)$$

is obtained, and also, from (21), (22b) and (26), the coefficient

$$\gamma_p = \beta_p \frac{C-(h+\delta)-f_1}{C}. \quad \dots\dots\dots\dots\dots\dots\dots (28)$$

Therefore, from (21), (27) and (28), **the distance of the first focal point of the objective lens system from the center of rotation of the telescope**

$$\begin{aligned} \mathbf{C}_p &= \mathbf{C}\left\{1-\frac{f_\infty-f_p}{f_\infty}\frac{\mathbf{C}-(\boldsymbol{h}+\delta)-f_1}{\mathbf{C}}\right\} \\ &= \mathbf{C}\left\{1-\frac{1}{\dfrac{f_1^2}{\{\mathbf{C}-(\boldsymbol{h}+\delta)-f_1\}(d_p-d_\infty)}+1}\frac{\mathbf{C}-(\boldsymbol{h}+\delta)-f_1}{\mathbf{C}}\right\} \end{aligned} \quad \dots\dots\dots (29)$$

is exactly found.

Hereupon, referring to the preceding example of a Wild-Zeiss telescope and the results obtained from the direct observations of the internal focussing telescopes of the small and the large "Fuji" transits, manufactured by Sokkisha Co. in Japan, the ratios of the various members in (26), (27), (28), and (29), in respect to C, are computed and the results are given in Table 2, from which it can readily be seen that they are nearly invariant in practice.

Table 2.

Telescope	$\frac{f_1}{C}$	$\frac{h+\delta}{C}$	$\frac{C-(h+\delta)-f_1}{C}$	$\frac{C-(h+\delta)-f_1}{f_1^2}C$	$\left\{\frac{C-(h+\delta)-f_1}{f_1}\right\}^2$
a Wild-Zeiss'	0.523	0.289	0.188	0.687	0.129
"Fuji" transit, small	0.505	0.325	0.170	0.667	0.113
"Fuji" transit, large	0.485	0.352	0.164	0.697	0.114
Mean	0.504	0.322	0.174	0.684	0.119

Now, from (26), (27), (28) and (29), referring to Table 2, the practical formulas

$$\begin{aligned} \frac{f_\infty-f_p}{f_\infty} &= \beta_p \frac{C}{E_p-C_p} \\ &= \frac{2}{3}\frac{d_p-d_\infty}{C}, \end{aligned} \quad \dots\dots\dots\dots\dots\dots\dots\dots (30)$$

$$\gamma_p = \frac{d_p - d_\infty}{9C} \frac{E_p - C}{C}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (31)$$

and

$$\mathbf{C}_p = \mathbf{C} - \frac{d_p - d_\infty}{9} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (32)$$

are reduced respectively, neglecting terms of the higher order.

Example 1.——For a small "Fuji" transit, manufactured by Shokkisha Co., in which $C = 280$ m.m., if taken as $E_A = 10C$, then

$$d_A - d_\infty = 20.5 \text{ m.m.}$$

is gotton so that

$$C_A = 277.7 \text{ m.m.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (32a)$$

is obtained from (32).

Example 2.——For a large "Fuji" transit, manufactured by Sokkisha Co., in which $C = 340$ m.m., if taken as $E_A = 10C$, then

$$d_A - d_\infty = 21.1 \text{ m.m.}$$

is gotten so that

$$C_A = 33.77 \text{ m.m.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (32b)$$

is found from (32).

CHAPTER II

INTEGRAL ADJUSTMENT OF THE HORIZONTAL CROSS-HAIR IN A TRANSIT

SECTION I.—THE FIRST METHOD

5. **General Principle.** The formulas for the First Method of adjustment of the horizontal cross-hairs in transits, which have already been made public by the present author in his first and second papers,† titling the author's method, can be applied only to the Ramsden, the Huygenian and the Porro telescopes, but not to the Wild-Zeiss.

Therefore, their proofs should anew be generalized so as to be applied to all the existing telescopes without distinction of their types.

Now, the fundamental principle of this method consists in the fact that if two points, one distant and one near, are brought on the same straight line passing through the center of rotation of the telescope, then other collimation points should come on that of themselves, because it will do no harm that the locus of collimation points is simply considered as a straight line in practice.

Hence, the procedure of this method is again described as follows :

Select a well-defined fixed point b, whose distance is over 30 m. from the instrument station C and whose altitude does not exceed $\pm 5°$, and a second point A, whose distance is ten or twenty times that of the instrumental constant C from the same point, arranging these three points in a straight line. See Fig. 6.

At point A, affix a scale graduated to 1 m.m. vertically.

Then first sight at b with the telescope normal, clamp the telescope in that position, read the scale fixed at A to a tenth of one milimeter, and note its reading a_1. Sight at b again with the telescope inverted and tighten all clamps. If the horizontal cross-hair does not strike the former reading a_1 on the scale at A but gives a new reading a_2, move the horizontal cross-hair, with the telescope still clamped in the position it was in when the second reading a_2 was taken, from the reading a_2 in the direction opposite to the first reading a_1 to the extent of correction, which is given by (46) (47) or (48), (49), or (50) (51) or (52) reduced afterward. See Fig. 6.

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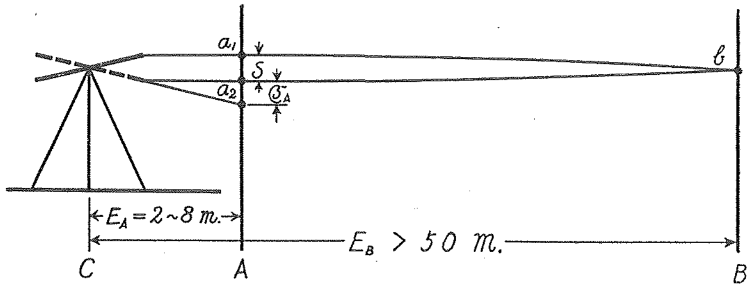
† See References (1) and (2) at the end of the present work.



Practical correction $\mathfrak{C}_A$.

E_A	Ramsden and Huygenian telescopes.	Porro's telescope.	The telescopes of the Wild-Zeiss' type.
E_A	$\frac{E_A - C}{C} \frac{S}{2}$	$\left(\frac{E_A}{f}\right)^2 \frac{S}{2}$	$\frac{E_A - C}{C - \frac{2}{3} \frac{d_A - d_\infty}{C} E_A} \frac{S}{2}$
$10\,C$	$5\,S$		$20\,S$ for a Wild-Zeiss' $10\,S$ for a Sokkisha's

Fig. 6. Illustration of the First Method of Integral Adjustment of the Horizontal Cross-Hair in a Transit.

Repeat the test and the correction until the adjustment in integrated.

Strictly speaking, as the axis of the objective lens system and the horizontal axis of the telescope will not always lie in a straight line but the former may move along a certain curved path on account of minute errors in the workmanship of the instruments, the difference of the first and second readings of the scale held at point A is more or less influenced. Refer to Fig. 7.

Hereupon, the notations are defined as follows :

E : The distance of the point sighted.

g : The distance of the front slide bearing from the center of rotation of the telescope.

i : The inclination of the optical axis of the objective lens system referred to its normal position or the line of normal sight when $i = 0$ and $e_g = 0$.

e_g : The eccentricity of the optical axis of the objective lens system at the front slide bearing, referred to the line of normal sight when $i = 0$ and $e_g = 0$.

e : Do. at the center of rotation of the telescope, or $e = e_g - gi$.

φ : The angular deviation of the point sighted, referred to the line of normal sight when $i = 0$ and $e = 0$.

C: The distance of the first focal point of the objective lens from the center of rotation of the telescope.

k: The distance of the cross point of the cross-hairs from the optical axis of the objective lens system.

f: The focal length of the objective.

The suffixes A, B, D, J, N, P, p, or ∞ show the point sighted.

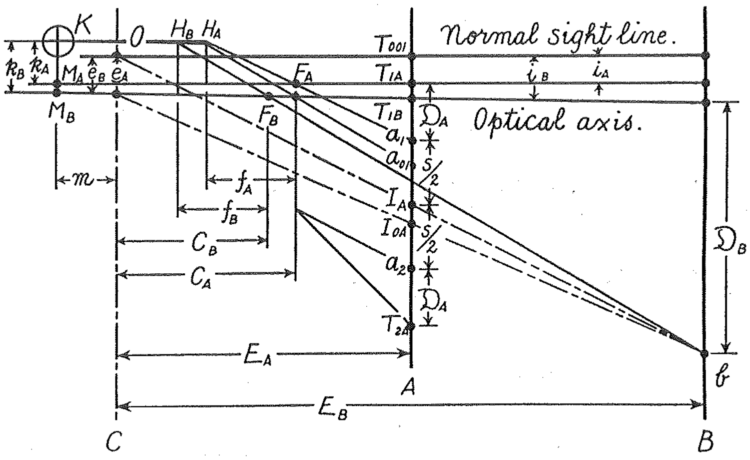


Fig. 7. Optical Relations when the Horizontal Cross-Hair is adjusted by the First Method of Integral Adjustment.

Now, from Fig. 7, when point A is sighted, the relation

$$\overline{T_{01}a_1} = \overline{T_{001}\,T_{1A}} + \overline{T_{1A}\,a_1}$$

is found, from which the deviation of point A

$$E_A\varphi_A = e_A + E_A i_A + (E_A - C_A)\frac{k_A}{f_A} \quad \ldots\ldots\ldots\ldots\ (33a)$$

is obtained, where $e_A = e_{g,\,A} - g i_A$.

Similarly, that of point B

$$E_B\varphi_B = e_B i_B + (E_B - C_B)\frac{k_B}{f_B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (33b)$$

is gotten, where $e_B = e_{g,\,B} - g i_B$.

Now, from the above two, referred to Fig. 7, the relations

$$k_A - k_B = \frac{E_A(\varphi_A - i_A) - e_A}{E_A - C_A} f_A - \frac{E_B(\varphi_B - i_B) - e_B}{E_B - C_B} f_B \quad \ldots \quad (34a)$$

and also

$$k_A - k_B = -(e_B - e_A) + m(i_B - i_A) \quad \ldots\ldots\ldots\ldots \quad (34b)$$

are gotten.

But, since, again from Fig. 7, the relation between φ_A and φ_B

$$\widehat{T_{001}Ob} = \widehat{T_{001}Oa_1} + \widehat{a_1OI_A}$$

or

$$\varphi_B = \varphi_A + \frac{s}{2E_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (35)$$

is obtained; from Eq. (34a), Eq. (34b) and Eq. (35), the angular deviation of point a_1 from the normal sight line,

$$\varphi_A = \left\{ \frac{E_B f_B}{E_B - C_B} \frac{s}{2E_A} + \frac{C_A i_A + eA}{E_A - C_A} f_A - \frac{C_B i_B + e_B}{E_B - C_B} f_B \right. \\ \left. -(e_B - e_A) - (f_B - m)i_B - (f_A - m)i_A \right\} \\ \times \frac{1}{\dfrac{E_A f_A}{E_A - C_A} - \dfrac{E_B f_B}{E_B - C_B}} \quad \ldots\ldots\ldots\ldots\ldots \quad (36)$$

and again, from (33a), referred to Fig. 7,

$$\varphi_A = \frac{E_A i_A + e_A}{E_A} + \frac{\mathfrak{D}_A}{E_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (37)$$

are derived.

Therefore, from (36) and (37), the deviation of point a_1 from the optical axis of the objective lens system

$$\mathfrak{D}_A = E_B E_A (E_A - C_A) \\ \times \frac{\dfrac{s}{2E_A} - (i_B - i_A) - \left(\dfrac{e_B}{E_B} - \dfrac{e_A}{E_A}\right) - \dfrac{e_B - e_A - m(i_B - i_A)}{f} \dfrac{E_B - C_B}{E_B}}{E_B C_A - E_A C_B - (E_B - C_B) E_A \dfrac{f_B - f_A}{f_B}} \\ \ldots\ldots \quad (38)$$

is obtained.

Now, referred to (33a) (33b) and (34a), in order that the deviation $E\varphi$ may not be influenced by the eccentricity e and the inclination i at all, the conditions

$$e = 0\,, \quad i = 0\,, \quad e_g = 0 \quad \dots\dots\dots\dots\dots\dots \quad (39)$$

must hold good, but strictly speaking, such conditions can not be expected virtually, because they really mean the absolute perfection of the parts of the instrument, correlative with the adjustment of the cross-hair.

In practice, in order that the horizontal cross-hair can be perfectly adjusted, it is necessary that the eccentricity of the optical axis of the objective lens at the front slide bearing e_g and its inclination i do not vary irrespective of the distance of the point sighted and are of small magnitude, that is to say—**the practical conditions**

$$\left.\begin{array}{l} \boldsymbol{e = \text{a small magnitude or zero,}} \\ \boldsymbol{i = \text{a small magnitude or zero,} \quad e_g = e+gi} \end{array}\right\} \dots\dots \quad (40)$$

are fulfilled, because, if they do not, the angular deviation of a collimation point varies irregularly. Accordingly the adjustment of cross-hair performed between a certain pair of two points holds no longer true at other points, so that the cross-hair can not be adjusted perfectly.

But, as a matter of fact, it was proved by the experiments performed by the present author, as reported on p. 19, p. 20, p. 25, p. 30, and p. 31 of his first paper† as check observations, that these practical conditions are fortunately satisfied. Furthermore, on the other hand, from the stand-point of Mechanical Technology and also the careful inspection of a certain first class manufactory in Tōkyō, Japan, this fact is undoubtedly admitted.

Regarding further theoretical and experimental proofs, refer to Art. 32, Art. 35, Art. 63 and Art. 65.

Therefore, again, from (33a) and (33b), the relations

$$\left.\begin{array}{l} E_A\varphi_A = e+E_A i+(E_A-C_A)\dfrac{k}{f_A}\,, \\ E_B\varphi_B = e+E_B i+(E_B-C_B)\dfrac{k}{f_B} \end{array}\right\} \dots\dots\dots\dots \quad (41)$$

are gotten, from which or directly from (38) and (40), the deviation of the collimation point A from the optical axis of the objective

† Refer to Reference (1) at the end of this work.

$$\mathfrak{D}_A = \frac{\dfrac{E_B(E_A - C_A)s}{2} + (E_B - E_A)(E_A - C_A)e}{E_B C_A - E_A C_B - (E_B - C_B) C_A \dfrac{f_B - f_B}{f_B}}, \quad \cdots\cdots\cdots \quad (42)$$

is obtained.

Now, since it is required to make $s = 0$ in actual by adjustment, **the theoretically exact correction**

$$\mathfrak{C}_A = \frac{E_B(E_A - C_A)}{E_B C_A - E_A C_B - (E_B - C_B) E_A \dfrac{f_B - f_A}{f_B}} \frac{s}{2}, \quad \cdots\cdots \quad (43)$$

is gotten from (42) and (41), where

$$k = \frac{(E_B - E_A) f_A}{E_B C_A - E_A C_B - (E_B - C_B) E_A \dfrac{f_B - f_A}{f_B}} e .$$

From this the practical formulas for correction for telescopes of miscellaneous types will be reduced in the following pages in order.

Finally, now that the correction does not depend on the optical errors e and i, the theory and formulas of correction for adjustment can be directly reduced from the case when $e = 0$ and $i = 0$, which is depicted in Fig. 8.

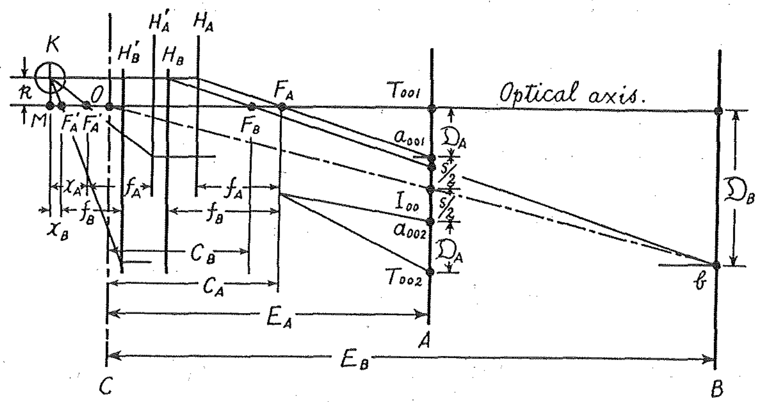


Fig. 8. Optical Relations when the Horizontal Cross-Hair is adjusted by the First Method of Integral Adjustment in case of $e = 0$ and $i = 0$.

6. The Ramsden and Huygenian Telescopes. For these, **the theoretically exact correction**

$$\mathfrak{C}_A = \frac{E_B(E_A - C_A)}{E_B C_A - E_A C_B}\frac{s}{2} \quad \dots\dots\dots\dots\dots\dots \quad (44)$$

is gotten from (9) and (43).

Therefore, from (44) and (12), neglecting terms of the higher order and satisfying such a condition as

$$E_B \gtrapprox \frac{E_A(E_A - C)}{f^2}C, \quad \dots\dots\dots\dots\dots\dots \quad (45)$$

a very handsome formula

$$\mathfrak{C}_A = \frac{E_A - C}{C}\frac{s}{2} \quad \dots\dots\dots\dots\dots\dots \quad (46)$$

is obtained, whose error of approximation does not exceed over two percent,* as far as the condition (45) is approximately satisfied.

Hereupon, neglecting terms of the higher order, the practical formulas

$$\mathfrak{C}_A = 5\,s, \quad \text{when} \quad E_A = 10\,C \quad \dots\dots\dots\dots \quad (47)$$

and

$$\mathfrak{C}_A = 10\,s, \quad \text{when} \quad E_A = 20\,C \quad \dots\dots\dots\dots \quad (48)$$

are obtained from (46). If the instrument has fairly large errors, formula (48) is far better than (47) for the reason illustrated in Art. 49.

Now, the precision of the formulas are minutely explained in Diagram 1.**

Finally, the practical examples† are shown in Column 2, 3 and 4 of Table 3.

7. The Porro's Telescope. Now, if the wire or hair diaphragm with four adjustable screws is provided for this telescope, then the horizontal cross-hair can be adjusted perfectly by **the formula**

$$\mathfrak{C}_A = \left(\frac{E_A}{f}\right)^2 \frac{s}{2} \quad \dots\dots\dots\dots\dots\dots \quad (49)$$

obtained from (13), (16) and (43).

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* Refer to Reference (2), p. 150∼151, at the end of this work.

** See Reference (2), p. 151, at the end of the present work.

† Refer also to Reference (1), p. 31, at the end of the present work.

Table 3.

Experimental Note 1.

Designation	Method 1	Method 1	Method 1	Method 2	Method 2	Method 2
Date tested	July, 1931	Aug. 1931	Oct., 1935	June, 1931	July, 1931	Oct., 1935
Transit, No.	23 *	15 *	29 †	A4 *	7 *	29 †
Size, In.	5	4	"Fuji", large	$3^1/_2$	4	"Fuji", large
Type of Telescope	Ramsden	Ramsden	Internal Focussing	Ramsden	Ramsden	Internal Focussing
Made in	U. S. A.	Japan	Japan	U. S. A.	Japan	Japan
Years of use	0	6	0	very old	6	0
Observer	T. Shingo	T. Shingo	T. Shingo	students	T. Shingo	T. Shingo
C, m.	0.347	0.2764	0.340	0.253	0.2764	0.340
E_B, m.	80	70	138.30	55	55	138.30
E_A, m.	3.817	3.588	3.40	2.53	2.764	3.40
Correction $\mathfrak{C}$	$+5s$	$+6s$	$+7.65s$	$-5r$	$-5r$	$-8.5r$
First Observation						
r or s, m.m.	+0.3	+1.2	+0.56	+ 80	− 5	− 13.5
$\mathfrak{C}$, m.m.	+1.5	+7.2	+4.27	−400	+25	+114.5
Second Observation						
r or s, m.m.	0.0	0.0	−0.11	− 10	0	− 6.2
$\mathfrak{C}$, m.m.			−0.83	+ 50		+52.5
Third Observation						
r or s, m.m.			0.00	0		0.0
Check Observation						
E_B, m., as it was		70			55	
E_D, m.		7.176			5.528	
r or s, m.m.		0.0			0.0	

* The horizontal cross-hair was not disturbed before the adjustment.

† The horizontal cross-hair was intentionally disturbed in advance before the adjustment.

Table 3.—(*Continued*)

Experimental Note 1.

Designation	Method 3	Method 3	Method 3	Method 3.	Method 3
Date tested	Oct., 1935	Oct., 1935	Jan., 1935	Oct., 1935	Oct., 1935
Transit, No.	23	23	A3	29	29
Size, In.	5	5	"Fuji", small	"Fuji", large	"Fuji", large
Type of Telescope	Ramsden	Ramsden	Internal Focussing	Internal Focussing	Internal Focussing
Made in	U. S. A.	U. S. A.	Japan	Japan	Japan
Years of use	0.2	0.2	0	0	0
Observer	T. Shingo	T. Shingo	T. Shingo	T. Shingo	T. Shingo
Method, proceeded upon	Method 1	Method 2	Method 1	Method 1	Method 2
C, m.	0.347	0.347	0.280	0.340	0.340
E_B, m.	138.30	138.30	138.30	138.30	138.30
E_A, m.	4	4	2.8	6.5	6.5
Correction	$+ 7.10s$	$- 5.59r$	$+ 8.75s$	$+18.4s$	$- 18.3r$
First Observation					
r or s, m.m.	− 0.71	+ 23.2	+ 0.14	− 0.61	− 14.6
$\mathfrak{C}$	$+10s$	$- 10r$	$+10s$	$+10s$	$- 10r$
$\mathfrak{C}$, m.m.	− 7.14	−231.7	+ 1.40	− 6.11	+146.0
Second Observation					
r or s, m.m.	+ 0.29	− 18.3	− 0.02	− 0.28	− 6.6
$\mathfrak{C}$	$+ 7.10s$	$- 5.59r$	$+ 8.75s$	$+18.4s$	$- 18.3r$
$\mathfrak{C}$, m.m.	+ 2.02	+102.1	− 0.18	− 5.10	+ 12.1
Third Observation					
r or s, m.m.	0.00	0.0	0.00	0.00	0.0

Notice: In all cases, the horizontal cross-hairs were intentionally disturbed in advance before the adjustment.

8. The Wild-Zeiss or Internal Focussing Telescope. For a telescope of the internal focussing type, the exact correction is directly obtained from (43), calculating $(f_B-f_A)/f_B$, C_A and C_B through (20) and (21), if the optical data are given, though they are not given commonly and the computation is too complicated to be performed in practice. But, from it, such a convenient formula utilized in the field as that in the Ramsden or Huygenian telescope, can not be reduced easily, because the last term in the denominator on the right hand side of (43) is not only much influenced by the change of f_1, f_2 and d or the optical data of the objective, but also the magnitude of the denominator itself is seriously controled by it, and accordingly that of the correction.

On the contrary, if $(f_B-f_A)/f_B$, C_A and C_B are required, the exact correction is easily obtained from (43), measuring f_1, $(h+\delta)$ and (d_A-d_B) and substituting f_B and d_B for f_∞ and d_∞, though f_1, f_2 and d are not given by the makers. Moreover, if (30) and (32) are put into (43) instead of (26) and (29), neglecting terms of the higher order, **the most convenient formula for correction**

$$\mathfrak{C}_A = \frac{E_A - C}{C - \frac{2}{3}\frac{d_A-d_\infty}{C}E_A}\frac{s}{2} \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots (50)$$

is obtained, in which (d_A-d_∞) may merely be measured.

Therefore, the practical formulas for correction

$$\mathfrak{C}_A = \frac{4.5\,s}{1-\frac{20}{3}\frac{d_A-d_\infty}{C}}, \quad \text{when} \quad E_A = 10\,C \;\cdots\cdots\; (51)$$

and

$$\mathfrak{C}_A = \frac{9.5\,s}{1-\frac{40}{3}\frac{d_A-d_\infty}{C}}, \quad \text{when} \quad E_A = 20\,C \;\cdots\cdots\; (52)$$

are gotten from (50), whose errors are negligible if point B is taken at a point rather far away.

Hereupon, if the instrument is tolerably erroneous, formula (52) must be adopted, for the reason which will be illustrated in Art. (49) in detail.

Example 1.——For the preceding example of the Wild-Zeiss telescope mentioned in Art. (4), if it is taken that $E_A = 10\,C = 2.85$ m. and $E_B = 137.3$ m., then the formula for correction

$$\mathfrak{C}_A = 19.73\,s \quad \text{(53)}$$

or in round numbers,

$$\mathfrak{C}_A = 20\,s \quad \text{(54)}$$

is obtained from (20) and (43).

If the horizontal cross-hair of this theodolite were truly adjustable, the correction of the highest precision would be given by (54).

Example 2.——For the telescope of the small "Fuji" transit, manufactured by Sokkisha Co., Tōkyō, Japan, if it is taken as $E_A = 10\,C = 2.8$ m. and $E_B = 137.3$ m., then

$$d_A - d_\infty = 20.5 \text{ m.m.}$$

is measured, and accordingly

$$\frac{f_\infty - f_A}{f_\infty} = 0.04637$$

and

$$C_A = 0.9921\,C$$

are computed from (26), (29), and the data tabulated in Table 2 of Art. 4.

Therefore, the exact correction

$$\mathfrak{C}_A = 8.86\,s \quad \text{(55)}$$

is obtained from (43), and the approximate correction

$$\mathfrak{C}_A = 8.79\,s \quad \text{(56)}$$

from (50) or (51), whose error comes up to only $-0.8\,\%$, as referred to (55).

Example 3.——For the telescope of a large "Fuji" transit, manufactured by Sokkisha Co., if it is taken that $E_A = 10\,C = 3.4$ m. and $E_B = 138.3$ m., then

$$d_A - d_\infty = 21.0 \text{ m.m.}$$

is directly found out, and therefore

$$\frac{f_\infty - f_A}{f_\infty} = 0.04127$$

and

$$C_A = 0.9932\,C$$

are computed from (26), (29) and the data computed as in Table 2 of Art. 4.

Hence, the exact correction

$$\mathfrak{C}_A = 8.09\,s \quad \text{(57)}$$

is calculated from (43) and the approximate correction

$$\mathfrak{C}_A = 7.65\,s \quad \text{(58)}$$

from (50) or (51), whose error is only $-5.5\,\%$, as referred to (57).

Now, the practical examples of the adjustment of the horizontal cross-hairs in the telescopes of this type are given in Table 3-continued part of Art. 6 of this chapter.

SECTION II.—THE SECOND METHOD

9. General Principle. This is the exact procedure for adjustment of the horizontal cross-hair in the Ramsden or the Huygenian telescope, which had been adopted by famous first-class authorities in their works.† This procedure is still accepted at present, but its correctness had been quite misestimated for a long time, until the correct and exact formulas were made public* by the present author under the date of Oct., 1931.

But, the formulas are now generalized so as to be applicable to all the existing telescopes.

This method is the very reverse of the First Method and is here described again and exactly proved as follows :

Hold a levelling rod on a stake A, ten or twenty times the instrumental constant C away from the instrument station C, clamp the telescope so that the line of sight is approximately level, and note the rod reading a. Without moving the telescope read a rod upon a second stake B, over 30 m. away in the straight line $\overline{CA}$, and note its reading b_1. Unclamp, plunge, turn the plates, bring the inverted telescope to the former reading a on the first stake, and tighten all clamps. If the horizontal cross-hair does not strike the former reading b_1 on the second stake but a new reading b_2, move the horizontal cross-hair, with the telescope still clamped in the position it was in when the second reading b_2 was taken, from that reading in the direction of the first reading b_1 by the amount of the correction, which is given by (64) (65) or (66), (67) or (68) (69) or (70) as is shown below. See Fig. 9.

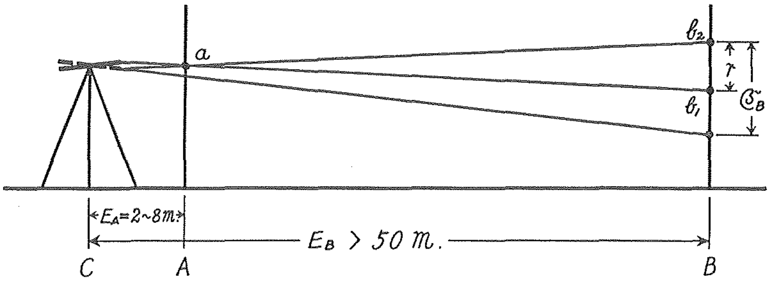
Repeat the test and the correction until the adjustment is integrated.

In this method, because in all respects it is the reverse of the first, different formulas will be obtained, merely replacing a, b, A, B and r for b, a, B, A and s in the formulas obtained in the preceding section.

Now, from (38), the deviation of the collimation point b from the optical axis of the objective lens system

† See Reference at the end of this work.

* See Reference (1) at the end of this work.



Practical correction $\mathfrak{C}_B$.

E_A	Ramsden and Huygenian telescopes.	Porro's telescope.	The telescopes of the Wild-Zeiss' type.
E_A	$\frac{E_A}{C}\,\frac{r}{2}$	$\left(\frac{E_A}{f}\right)^2\frac{r}{2}$	$\frac{E_A}{C-\frac{2}{3}\frac{d_A-d_\infty}{C}E_A}\,\frac{r}{2}$
$10C$	$5r$		$20r$ for a Wild-Zeiss' $10r$ for a Sokkisha's

Fig. 9. Illustration of the Second Method of Integral Adjustment of the Horizontal Cross-Hair in a Transit.

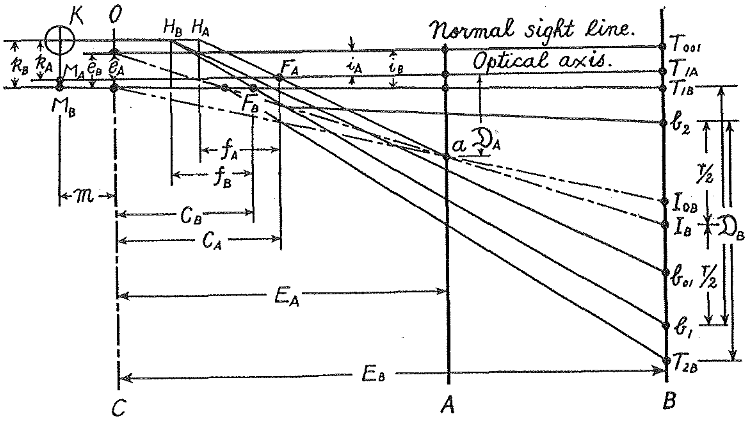


Fig. 10. Optical Relations when the Horizontal Cross-Hair is adjusted by the Second Method of Integral Adjustment.

$$\mathfrak{D}_B = E_B E_A (E_B - C_B) \times \frac{-\dfrac{r}{2E_B} - (i_B - i_A)\left(\dfrac{e_B}{E_B} - \dfrac{e_A}{E_A}\right) - \dfrac{e_B - e_A - m(i_B - i_A)}{f_A}\,\dfrac{E_A - C_A}{E_A}}{E_B C_A - E_A C_B - (E_A - C_A) E_B \dfrac{f_B - f_A}{f_A}} \quad \dots\dots\dots (59)$$

is obtained, where

$$e_A = e_{g,A} - g i_A\,, \quad e_B = e_{g,B} - g i_B\,;$$

from which, from the same ground as in Section I or directly from (39) and (40), the exact conditions for adjustability

$$e = 0\,, \qquad i = 0\,, \qquad e_g = 0\,, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (60)$$

the practical conditions for adjustability

$$\left.\begin{array}{l} \boldsymbol{e = \text{a small magnitude or zero,}} \\ \boldsymbol{i = \text{a small magnitude or zero,}} \quad \boldsymbol{e_g = e + gi} \end{array}\right\} \quad \ldots\ldots \quad (61)$$

and **the exact correction**

$$\mathfrak{C}_B = -\frac{(\mathrm{E}_B - \mathrm{C}_B)\mathrm{E}_A}{\mathrm{E}_B\mathrm{C}_A - \mathrm{E}_A\mathrm{C}_B - \mathrm{E}_B(\mathrm{E}_A - \mathrm{C}_A)\dfrac{f_B - f_A}{f_A}}\,\frac{\mathrm{r}}{2} \quad \ldots\ldots \quad (62)$$

are gotten, where

$$k = \frac{(\mathrm{E}_B - \mathrm{E}_A) f_A}{\mathrm{E}_B\mathrm{C}_A - \mathrm{E}_A\mathrm{C}_B - (\mathrm{E}_B - \mathrm{C}_B)\mathrm{E}_A\dfrac{f_B - f_A}{f_B}}\, e\,.$$

Now, since the correction (62) is independent of e and i, the theory and formulas can be simply computed from the case when $e = 0$ and $i = 0$, which is depicted in Fig. 11.

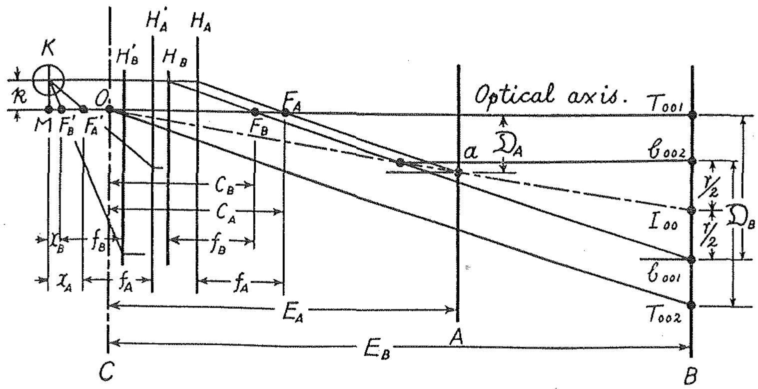


Fig. 11. Optical Relations when the Horizontal Cross-Hair is adjusted by the Second Method of Integral Adjustment in case of $e = 0$ and $i = 0$.

From the above, formulas for correction like to those in the preceding section can be readily reduced.

10. The Ramsden and the Huygenian Telescopes. For these, **the exact correction**

$$\mathfrak{C}_B = -\frac{(E_B - C_B)E_A}{E_B C_A - E_A C_B}\frac{r}{2} \quad \ldots\ldots\ldots\ldots\ldots \quad (63)$$

is obtained from (9) and (62), from which, through (12), **the practical formula**

$$\mathfrak{C}_B = -\frac{E_A}{C}\frac{r}{2}, \quad \text{when} \quad E_B \geqq \left(\frac{E_A - C}{f}\right)^2 C, \quad \ldots\ldots\ldots \quad (64)$$

is gotten.

Now, from (64), the field formulas

$$\mathfrak{C}_B = -5r, \quad \text{when} \quad E_A = 10C \quad \text{and} \quad E_B > 30\,\text{m.}, \ \ldots\ldots \quad (65)$$

and

$$\mathfrak{C}_B = -10r, \quad \text{when} \quad E_A = 20C \quad \text{and} \quad E_B > 30\,\text{m.}, \ \ldots\ldots \quad (66)$$

are obtained, the latter being much more effective than the former, for the reason to be illustrated later in Art. 49 in detail.

Respecting the precision of the formulas, refer to Diagram 1 of the author's second paper.†

Regarding the practical examples* of the adjustment, see Column 5, 6 and 7 of Table 3 in Art. 6.

11. The Porro's Telescope. For Porro's telescope, the formula for correction

$$\mathfrak{C}_B = -\left(\frac{E_A}{f}\right)^2 \frac{r}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (67)$$

is obtained from (13), (16) and (62).

12. The Wild-Zeiss Telescope. For the telescope of the Wild-Zeiss type, **the most convenient formula**

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† See Reference (2), p. 144, at the end of this work.
* See also Reference (1), p. 30, at the end of this work.

$$\mathfrak{C}_B = -\frac{\mathbf{E}_A}{\mathbf{C} - \frac{2}{3}\frac{d_A - d_\infty}{\mathbf{C}}\mathbf{E}_A}\frac{r}{2} \quad \ldots\ldots\ldots\ldots\ldots (68)$$

is gotten from (30), (32) and (62), neglecting terms of the higher order.

Therefrom, the practical corrections

$$\mathfrak{C}_B = -\frac{5}{1 - \frac{20}{3}\frac{d_A - d_\infty}{C}}, \quad \text{when} \quad E_A = 10\,C, \quad \ldots\ldots (69)$$

and

$$\mathfrak{C}_B = -\frac{5}{1 - \frac{40}{3}\frac{d_A - d_\infty}{C}}, \quad \text{when} \quad E_A = 20\,C, \quad \ldots\ldots (70)$$

are obtained, between which the latter is superior to the former for the particular reason illustrated in Art. 49.

Example 4.——For the preceding example of the Wild-Zeiss telescope mentioned in Art. 4, if it is taken that $E_A = 10\,C = 2.85$ m. and $E_B = 138.3$ m., then the formula

$$\mathfrak{C}_B = -20.23\,r \quad \ldots\ldots\ldots\ldots\ldots (71)$$

is gotten, from (20) and (62) or in practice,

$$\mathfrak{C}_B = -20\,r. \quad \ldots\ldots\ldots\ldots\ldots (72)$$

If the horizontal cross-hair of this instrument were really adjustable, the correction of the highest accuracy would be given by (72).

Example 5.——For the telescope of a small "Fuji" transit in Ex. 2 of Art. 8, if it is taken that $E_A = 10\,C = 2.8$ m. and $E_B = 137.3$ m., then

$$d_A - d_\infty = 20.5 \text{ m.m.}$$

$$\frac{f_\infty - f_A}{f_A} = 0.04862$$

and

$$C_A = 0.9921\,C$$

are obtained from Ex. 2 of Art. 8, by which the exact correction

$$\mathfrak{C}_B = -9.35\,r \quad \ldots\ldots\ldots\ldots\ldots (73)$$

is gotten from (62), and the practical correction

$$\mathfrak{C}_B = -9.76\,r \quad \ldots\ldots\ldots\ldots\ldots (74)$$

from (68) or (69), whose error amounts to no more than $-4.4\,\%$.

Example 6.——For the telescope of a large "Fuji" transit, if it be taken that $E_A = 10\,C = 3.4$ m. and $E_B = 138.3$ m., then

$$d_A - d_\infty = 21.0 \text{ m.}$$

$$\frac{f_\infty - f_A}{f_A} = 0.04304$$

and

$$C_A = 0.9932\,C$$

are gotten from Example 3 of Art. 8.

Therefore, the practical correction

$$\mathfrak{C}_B = -8.5\,r \quad \dots\dots \quad (75)$$

is obtained from (68) or (69), whose error comes up to only $+1.2\,\%$ compared to (62).

SECTION III.—THE THIRD METHOD

13. The Third or "Trial Method", Applicable to All the Telescopes of the Existing Types. Up to the present, for formulas of correction for telescopes of miscellaneous types have been reduced theoretically.

But, in order to obtain the corrections, the focal length of the objective or f must be known in Porro's telescope, as recognized from (49) and (67), and also, in the internal focussing telescope, either $(f_\infty - f_A)/f_\infty$ must be calculated from the optical data or the amount of movement of the internal lens $(d_A - d_\infty)$ measured, as is readily understood from (43), (50), (62) and (68).

For the purpose of avoiding such subtlety and trouble, the "Trial Method" is originated, which is applicable to all the telescopes of the existing types.

Now, it is expatiated as follows:

First, select the two points A and B such that

$$E_A \doteqdot 10\,C \sim 20\,C, \quad E_B \geqq 40 \text{ m.}, \quad \dots\dots \quad (76a)$$

take the reading difference of the scale at A or s according to the first method or that of the rod at B or r according to the second method between these two points, and then, denoting this observed value by s_1, adjust the horizontal cross-hair at the first procedure by the formula for correction

$$\mathfrak{C} = 10\,s_1. \quad \dots\dots \quad (76b)$$

If the process of adjustment is repeated and the second reading s_2 is gotten, then the further adjustment must be made by the formula

$$\mathfrak{C} = \frac{10}{1 - \frac{s_2}{s_1}} s, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (76c)$$

in which attention must be paid to the sign of s_2, referred to s_1, that is, it becomes + or − according to the under or over correction. Refer to Fig. 12.

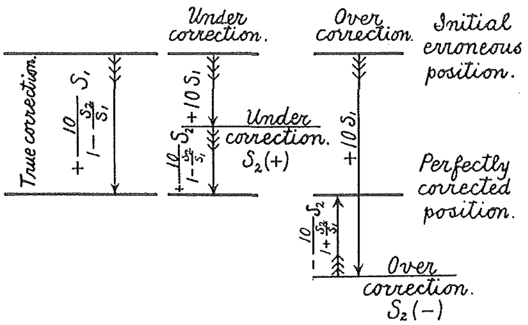


Fig. 12. Illustration of the Trial Method of Integral Adjustment of the Horizontal Cross-Hair in a Transit.

Thereupon, its fundamental principle is based upon the two facts as follows:

First, if it is taken that (76a) is fulfilled, the correction should properly be given by (76b) approximately, referred to (46), (47), (48), (51), (52), (55), (56), (57), (58), (64), (65), (66), (69), (70), (71), (72), (73), (74) and (75), admitting the error of about 100%.

Secondly, the residual error left after the first adjustment is theoretically

$$\mathfrak{C}_2 = \frac{10}{1 - \frac{s_2}{s_1}} s_2 ,$$

the theoretical reason for which is immediately understood from Fig. 12.

If E_A is taken as large as possible, a more accurate result shall be obtained, the detailed theoretical reason for which is illustrated in Art. 49.

SECTION IV.—RESIDUAL ERROR REMAINING AFTER ADJUSTMENT

14. Residual Error Remaining after Adjustment. The residual error remaining after adjustment may be negligibly small even when the minimum angular reading in the vertical circle is ten seconds, because, unlike the relation between the horizontal centering of the locus of collimation points and the standards, the vertical centering should properly be disturbed by no distorsion of the standards absolutely, if they are completely adjusted, that is to say, the vertical eccentricity of the optical axis of the objective lens from the normal sight line at the front slide bearing and its inclination or e_g and i are virtually not influenced by any defect of the standard at all.

On the contrary, since they are chiefly subjected to the dis-centering of the objective lens and the defects of the objective slide and the slide bearings, the errors e_g and i will not be remarkably increased with years of practical use, and moreover, their absolute magnitudes may probably be small in general.

The results of the check experiments of the adjustment of the horizontal cross-hair, shown in Columns 5 and 6 of Experimental Note 1 and also in those of Experimental Note 2 of the author's first paper,† should exactly prove the above reasoning.

For further explanation one should refer to Figs. 36 and 37 of Art. 49.

Section V.—Criticisms of the Old and New Methods

15. Criticisms of the Old and New Methods. Now, the procedure of the method of adjustment of the horizontal cross-hair adopted by Prof. Ira.. O. Baker and followed by first-class authorities in their works* on Surveying, in which the correction for adjustment

$$\mathfrak{C}_B = -\frac{r}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(A)}$$

was estimated from the principle of single reversion, is the very same as the second method in the present work, which differs only from the former in the point, that the formulas for correction are exactly calculated from Geometrical Optics in the latter.

Comparing formula (A) with (62), (63), (64), (65), (66), (67), (68), (69) and (71) in the second method of the present work, it can be readily comprehended that the horizontal cross-hair can not be adjusted by the former method in practice, unless several score operations of adjustment are repeated: as—even **when it is taken that $E_A = 10\,C$, the residual**

† See Reference (1), p. 30 and p. 31, at the end of this work.

* See References at the end of this paper.

error barely amounts up to 4 % for the Ramsden and the Huygenian telescopes and 21 % for the internal focussing telescope of the small "Fuji" transit manufactured by Sokkisha Co., after repetition thirty times respectively.

For the remedy of this defect, the new formulas of correction for adjustment for only the Ramsden and the Huygenian telescopes, previously made public in Japan, by moderate approximation,

$$\mathfrak{C}_B = -\frac{E_A(E_B-C)}{C(E_B-E_A)}r \quad\text{(B)}$$

and by further approximation,

$$\mathfrak{C}_B = -\frac{E_A}{C}r \quad\text{(C)}$$

may be calculated. **However, their magnitudes were respectively doubled on account of the wrong approximation in comparison with those of the true correction shown by (11) and (14) in the author's first paper† or (61) and (62) in the present paper,** that is, **the errors in them amounted up to 100%.**

Lately, even if a new process of proof of (14) in the author's first paper† or (62) in the present paper, was attempted by Assis. Prof. K. Tanaka*, assuming as the instrumental constant $C=$ a constant, from lack of the exact knowledge of Geometrical Optics, the attempt was regretably unsuccessful, because his process of proof was not only unsimplified but also (62) can be applied merely to the Ramsden and the Huygenian telescopes, but not to the telescopes of the Porro and the Wild-Zeiss types at all.

Other methods are also found in the first-class authorities' works on Surveying, but they are all un-applicable from want of the trues theoretical foundation and due to the substantial inferiority of their processes in practice.

Now, the first method of the present research is theoretically and practically superior to the second in the following respects :

1). No rod is necessary to be held at a distant point, that is—the process of adjustment is not hindered by obstacles midway in the direction of a welldefined distant point b: as a valley, a forest, a lake, etc..

† See Reference (1), p. 20 and p. 21, at the end of this paper.
* See Reference (15) and (16) at the end of this paper.

2). The adjustment is less influenced by the atmospheric conditions, because only one distant point is sighted.

3). It may not only be performed in the open air but also indoors on the rigid floor at night or in bad weather.

4). Both the vertical and horizontal cross-hairs are simultaneously adjusted by using two scales, as shown in Fig. 13, when the angle of deflexion is not measured.

5). With no other's help, the observer can accurately adjust the horizontal and vertical cross-hairs for himself.

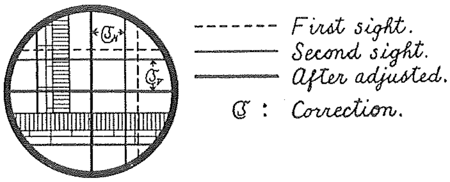


Fig. 13. Simultaneous Adjustment of the Cross-Hairs in a Transit.

Finally, the third or "Trial Method" is the most useful, because it is the most universal notwithstanding that there is no need of profound knowledge of Geometrical Optics and the data of the instrument.

CHAPTER III

INTEGRAL ADJUSTMENT OF THE VERTICAL CROSS-HAIR IN A TRANSIT

SECTION I.—GENERAL DESCRIPTION

16. General Description. The ordinary method of adjustment of the vertical cross-hair in a transit can in general be classified into two kinds according to their fundamental principles.

That of the first kind is called "**the Method of One-Quarter Adjustment**" in a broad sense†, and those of the second are the methods of Integral Adjustment of the horizontal cross-hair expounded in the preceding chapter.

In a transit adjusted by the Method of One-Quarter Adjustment of the vertical cross-hair, the collimation points are slightly dislocated out of the meridional plane or the normal sight plane of the instrument, except at the same distance as that of the points between which it was adjusted, because of the instrumental defects, viz., the inclination and the eccentricity of the optical axis of the objective lens system. The angular error due to this dislocation generally becomes large, when an angle between an infinitely distant point and an extremely near one is measured, because the locus of collimation points is practically straight, which should readily be seen from Figs. 34 and 35 of Art. 49.

The practical examples of such a case obtained from the experimental results are shown in Table 8 of Art. 28.

Now, adjustments of the second kind, namely—**the methods of Integral Adjustment of the horizontal cross-hair,** are originally conceived for the adjustment of the horizontal cross-hair in a transit, but come to be applied to that of the vertical cross-hair in a transit because of the advantage that the locus of collimation points passes through the center of rotation of the telescope*.

In a transit adjusted by one of these methods, an angular error proportional to $(\sec\theta-1)$ comes into the measured horizontal angle whose maximum value occurs at $\theta=180°$ or in the deflexion angle, where θ

† Owing to the author's research, there is no restriction on the distances of the two points, between which One-Quarter Adjustment is performed, that is, it can be performed between very near points, if the vertical cross-hair is integrally adjusted.

* See Reference (2), p. 154, and Reference (16), p. 39∼40, at the end of this work.

denotes the vertical angle. Practical examples of such cases are described in Table 8 of Art. 28.

Now, it has already been grasped that the effect of the error in collimation point, caused by the eccentricity and the inclination of the optical axis of the objective or e and i, can not be eliminated only by the adjustment of the position of the cross-hair, whatever method may be preferred.

Therefore, in order to eliminate the aforesaid defects in all the methods of adjustment and integrate the adjustment of the vertical cross-hair in a transit, the processes must be extended to those of the position and the direction of the optical axis of the objective lens system.

First of them is the adjustment of the adjustable rear slide bearing, by which the direction of the optical axis can properly be changed; second is that of the horizontal axis of the telescope in its axial direction, by which the eccentricity of the optical axis of the objective lens system can suitably be corrected; third is the combination of the preceding two and the most integral, by which the eccentricity of the optical axis of the objective lens system at the front slide bearing and its inclination can be made to disappear absolutely, namely—$e_g = 0$, and $i = 0$.

Now, because of the present construction of the instruments, the second and the third adjustments can not be performed; while, on the contrary, the first can readily be practised on all the telescopes with adjustable rear slide bearings, that is to say, the Ramsden, the Huygenian, etc.

But, it can not be performed on the telescope of the internal focussing type which is manufactured with a non-adjustable construction to obtain the highest degree of accuracy.

Now, in the manufactory, the standards and the objective slide are fiftted with **the centering tool** or "**Shin-Gané**" (心金) in Japan, so that the optical errors e and i may be made tolerably small, when a transit is newly constructed or repaired. Refer to Art. 52.

In general, the error of a precise instrument rapidly increases through use in the long run.

Therefore, no man can dogmatize that the centering tool does not get out of order by abraisions and accidents with practical use for long years.

Moreover, even if an instrument is pieced together with **the centering tool** or "**Shin-Gané**", the residual errors e and i should often yield detrimental influences on the measurement of the horizontal angle.

This is trues because the above residual errors are composed of formal errors of the objective lens system itself and of the metal parts left after centering at finishing, of which the former should be the principal parts of the residual errors and amount up to fairly large magnitudes as will be made clear by the experiments below described in Art. 26, Art. 30 and Art. 62 in detail.

Further, the optical errors in a transit grow large due to the changes of mechanical and optical conditions during practical use for a long time and especially when the standards, etc. are distorted by accidents.

Now, after several illustrative experiments will be performed, the Methods of Integral Adjustment of the vertical cross-hair will be minutely explained in order. These methods are theoretically proved from Geometrical Optics and practised by the present author in the first place, supplementarily and correlatively to the Method of One-Quarter Adjustment.

By these new methods, besides the effects of the residual errors remaining after centering*, the influences of the optical and mechanical errors grown larger during many years' use in practice and especially those caused by severe accidents in the field can be made completely to disappear on the spur of the moment.

Hereupon, the origin of the name "**Integral Adjustment**" will be explained minutely in the Conclusion at the close.

SECTION II.—LEMMAS TO INTEGRAL ADJUSTMENT OF THE VERTICAL CROSS-HAIR IN A TRANSIT

17. Deviation of a Collimation Point Referred to the Meridian of the Transit. The illustration of the optical relation in a telescope in Fig. 14 should be considend quite satisfactory for the purpose of optical analysis of the deviation of a collimation point from the meridian of the instrument.

Thereupon, for caution's sake, the notations are written down as follows :

C_p : The so-called instrumental constant when point p is sighted, or the distance of the first focal point from the center of rotation of the telescope.

---

* Refer to Art. 52 in the present work.

δ_p : The distance of the first principal point of the objective lens from the center of rotation of the telescope when point p is sighted.

δ : Do. when an infinitely distant point is sighted.

$e_{g,p}$: The eccentricity of the first principal point of the objective lens at the front slide bearing from the meridian of the transit, when point p is sighted.

e_p : The eccentricity of the optical axis of this objective lens at the center of rotation of the telescope when point p is sighted.

$e_{l,p}$: The eccentricity of the first principal point of the objective lens from the meridian when point p is sighted.

E_p : The distance of point p from the center of rotation of the telescope.

f : The focal length of the objective lens or $C-\delta$.

φ_p : The angular deviation of the collimation point p from the meridian of the transit.

g : The distance of the front slide bearing from the center of rotation of the telescope.

i_p : The inclination of the optical axis of the objective lens referred to the meridian of the transit when point p is sighted.

k_p : The departure of the cross-point of the cross-hairs from the optical axis of the objective lens.

z : Do. from the meridian of the transit.

In general, from Fig. 14, **the deviation of point p from the meridian of the transit**

$$E_p\varphi_p = e_p + E_p i_p - \frac{E_p - C_p}{f_p} k_p \quad \ldots\ldots\ldots\ldots\ldots \quad (77)$$

is exactly found out for a telescope of any type.

Therefore, for the telescope of an invariable focal length, **the formula for deviation of a collimation point p**

$$E_p\varphi_p = e_p + E_p i_p - \frac{E_p - C_p}{f} k_p \ldots\ldots\ldots\ldots\ldots \quad (78)$$

is gotten from (77).

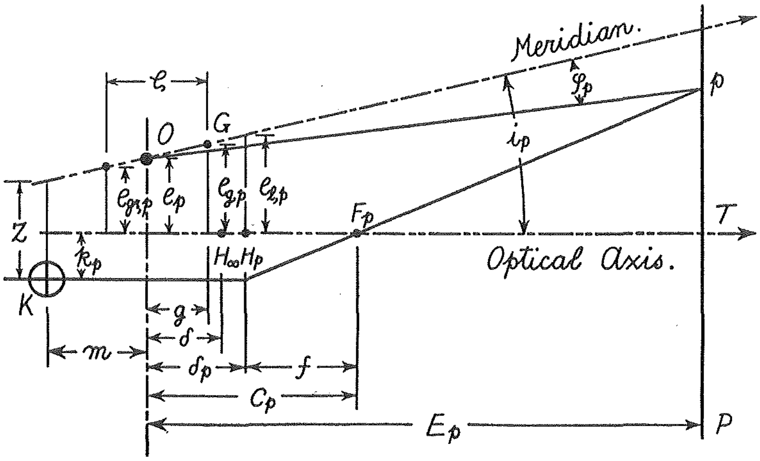


Fig. 14. Deviation of a Collimation Point.

O: Center of rotation of the telescope.
F_p: First focal point of the objective lens.
H_p: First principal point of the objective lens.
K: Cross-point of the cross-hairs.

Hereupon, since the relations

$$e_p = e_{g,p} - g i_p, \quad \dots\dots \quad (79a)$$

$$\begin{aligned} k_p &= z - e_p - m i_p \\ &= z - e_{g,p} + (C - 2\delta + g) i_p, \quad m = f - \delta, \quad f = C - \delta \quad \dots\dots \quad (79b) \end{aligned}$$

are found from Fig. 14, **the new formula for the deviation of p from the meridian for the telescope of an invariable focal length**

$$E_p \varphi_p = \frac{E_p - C_p + C - \delta}{C - \delta} e_{g,p} + \left\{ C_p - g + \frac{\delta - g}{C - \delta}(E_p - C_p) \right\} i_p - \frac{E_p - C_p}{C - \delta} z \quad \dots\dots \quad (80)$$

is gotten from (77), (79a) and (79b).

Now, from the relations among the inclination of the optical axis of the objective lens i, its eccentricity at the front slide bearing e_g, the eccentricity at the rear slide bearing e_{gr}, the interval between the two slide bearings ζ and the eccentricity of the first principal point from the meridian e_l, **the formula for the deviation of a collimation point p from the meridian of the transit**

$$E_p\varphi_p = \left\{\frac{E_p-C_p}{C-\delta}\left(1+\frac{\delta-g}{\zeta}\right)+1+\frac{C_p-g}{\zeta}\right\}e_{g,p}$$

$$-\left\{\frac{C_p-g}{\zeta}+\frac{E_p-C_p}{C-\delta}\,\frac{\delta-g}{\zeta}\right\}e_{gr,p}-\frac{E_p-C_p}{C-\delta}z$$

$$=\frac{E_p-C_p+C-\delta}{C-\delta}e_{l,p}-\frac{E_p-C_p}{C-\zeta}z \qquad (81)$$

should exactly be obtained from (80) over again, where

$$e_{g,p}-e_{gr,p}=\zeta i, \quad e_{l,p}=e_{g,p}+(C_p-C+\delta-g)i_p.$$

Hereupon, if the vertical cross-hair in a transit is adjusted by the method of One-Quarter Adjustment at the distant conjugate points B and $\bar{B}$, that is—if the deviation of those points be made to vanish, then the relation

$$e_{l,B}=z \qquad (82)$$

is gotton from (81), neglecting terms of the higher order.

In the above formulas, the relations

$$\left.\begin{aligned}\frac{\delta-g}{\zeta}&=0.32\sim0.18,\\ \frac{C-\delta}{\zeta}&=1.8\sim1.6\end{aligned}\right\} \qquad (83)$$

are generally obtained for ordinary transits in practice; refer to Table 5 of Art. 25.

Now, the variation of the angular deviation of a collimation point p from the meridian due to defects in finishing in the factory and the abraision of the cylindrical tube of the objective slide and the two slide bearings during practical use is exactly given by the formula

$$\varDelta(E_p\varphi_p) = \varDelta e_p+E_p\varDelta i_p-\frac{E_p-C_p}{f}\varDelta k_p$$

$$=\frac{E_p-C_p+C-\delta}{C-\delta}\varDelta e_{g,p}+\left\{C_p-g+\frac{E_p-C_p}{C-\delta}(\delta-g)\right\}\varDelta i_p$$

$$=\left[\frac{E_p-C_p}{C-\delta}\left\{1+\frac{\delta-g}{\zeta}\right\}+1+\frac{C_p-g}{\zeta}\right]\varDelta e_{g,p}$$

$$-\left\{\frac{C_p-g}{\zeta}-\frac{E_p-C_p}{C-\delta}\,\frac{\delta-g}{\delta}\right\}\varDelta e_{gr,p} \qquad (84)$$

This can readily gotten from (78), (80), (81) and (83), and from cases where Δ shows the variation respectively and

$$\frac{\delta-g}{\delta} = 0.32 \sim 0.18\,,$$

$$\frac{C-g}{\zeta} = 1.8 \sim 1.6\,.$$

18. Deviation of the Collimation Point Due to the Eccentricity of the First Principal Point and the Inclination of the Optical Axis of the Objective Lens Itself with Reference to its Ideal Axis. The necessity occurs fairly often to wipe the cloudy stains from the surfaces of the component lenses of the objective in the field, taking them out from the holder. The stains are generally produced by moisture in the air, the disintegration of the lens glass itself, grease, etc.

Now, by this operation, the adjustment of the cross-hair may be seriously disturbed and accordingly the deviation of the collimation point from the meridian of the transit should generally be distinctly acknowledged, because there must erroneously be the eccentricity of the first principal point and the inclination of the optical axis of the objective lens itself reference to its ideal axis.

Thereupon, in order to discuss the change of the deviation due to the above causes, the notations will be listed here as follows, referred to Fig. 15 :

(e) : The eccentricity of the first principal point of the objective lens itself projected on the ξ-axis or $(e) = \xi_H = [e] \cos \theta$.

$[e]$: The absolute eccentricity of the first principal point of the objective lens itself in the ξ-η plane.

$(E_p\varphi_p)$: The deviation of point p projected on the ξ-axis or $\xi_{\wp}$.

H : The actual position of the first principal point of the objective lens system.

H_0 : The ideal position of the first principal point of the objective lens system itself.

(i) : The inclination of the optical axis of the objective lens projected on the plane perpendicular to the ξ-η plane, including the ξ-axis, referred to the ideal optical axis, or $[i] \times \cos(\theta+\psi)$.

$[i]$: The absolute inclination of the optical axis of ths objective lens system referred to the ideal.

$\wp$: The image point corresponding to the virtual collimation point p in the field through the objective lens system.

ψ : The angular divergence between $\overline{H_0H}$ and $\overline{H\wp}$ in the ξ-η plane.

θ : The direction angle of $\overline{H_0H}$ referred to the ξ-axis.

ξ and η : The abscissa and ordinate referred to the rectangular co-ordinate axes perpendicular to the ideal optical axis of the objective lens system through the ideal position of its first principal point H_0.

For other data refer to Fig. 15.

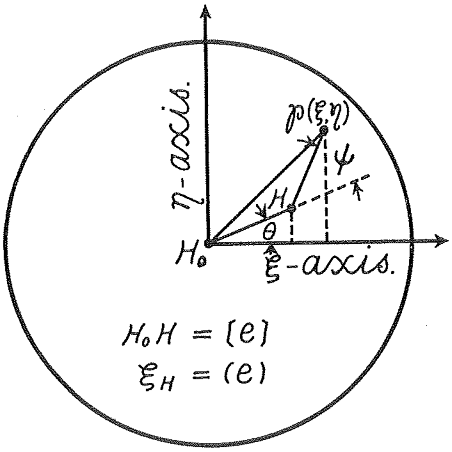


Fig. 15. Formal Errors of the Objective.

Now, from (80), **the deviation of point $\wp$ from the η-axis**

$$\xi_p = (E_p\varphi_p)$$

$$= \frac{E_p - C_p + C - \delta}{C - \delta}[e]\cos\theta + \left\{C_p - g + \frac{E_p - C_p}{C - \delta}(\delta - g)\right\}[i]\cos(\theta + \psi) \qquad \cdots\cdots\cdots (85)$$

is exactly gotten.

Therefore, since ψ is invariable, the maximum and the minimum absolute values of $\xi_\wp$ can readily be calculated from (85), because the conditions for them

$$\left.\begin{array}{l} |\xi_{\wp}| = \text{a minimum} \\ \quad\;\; = 0\,, \qquad\qquad \text{when} \quad \theta = \theta_0 \quad \text{or} \quad \theta_0+\pi\,, \\ |\xi_{\wp}| = \text{a maximum, when} \quad \theta = \theta_0+\dfrac{\pi}{2} \quad \text{or} \quad \theta_0+\dfrac{3}{2}\pi \end{array}\right\} \quad \ldots\ldots\ldots (86)$$

are found, neglecting terms of the higher order, where

$$\theta_0 = \tan^{-1}\left[\frac{[e]}{\left\{(C_p-g)\dfrac{C-\delta}{E_p-C_p}+\delta-g\right\}[i]\sin\psi}+\cot\psi\right].$$

Now, if the objective lens itself is loosened and rotated by one hundred and eighty degrees as it is in the holder and subsequently tightened again after that operation has been completed, then half the actual variation of the scale or rod reading at the collimation point p

$$(E_p\varphi_p) = \frac{E_p-C_p+C-\delta}{C-\delta}(e)+\left\{C_p-g+\frac{\delta-g}{C-\delta}(E_p-C_p)\right\}(i) \quad . \ (87)$$

is exactly obtained from (81), where $\Delta e_{l,\,p} = 2(e)$, $\Delta i_p = 2(i)$.

19. General Optical Problem for the Adjustment of the Objective Slide through the Adjustable Rear Slide Bearing.

Hereupon, the formulas will be reduced, taking i, e and e_g as invariant.

When the rear slide bearing of the objective slide is adjusted so that the direction of the optical axis of the objective lens system may be changed from $\overline{MGT}$ to $\overline{M'G'T'}$ as depicted in Fig. 17, and simultaneously the reading of a scale or a rod held at point P from p to p', through driving the objective-slide adjusting screws shown at Gr in Fig. 16, then the objective slide is transposed to a new position as it is in the front slide bearing G and the adjustable rear slide bearing Gr, as if the former were a fulcrum.

Then, the reading increment Δp, caused by the inclination correction of the optical axis of the objective lens system Δi, is calculated as follows, referred to Fig. 17 :

Now, from Fig. 17, the relation

$$\begin{aligned} \Delta p &= \overline{pp'} \\ &= \overline{T_pT_{p'}} + \overline{T_{p'}p'} - \overline{T_pp} \end{aligned}$$

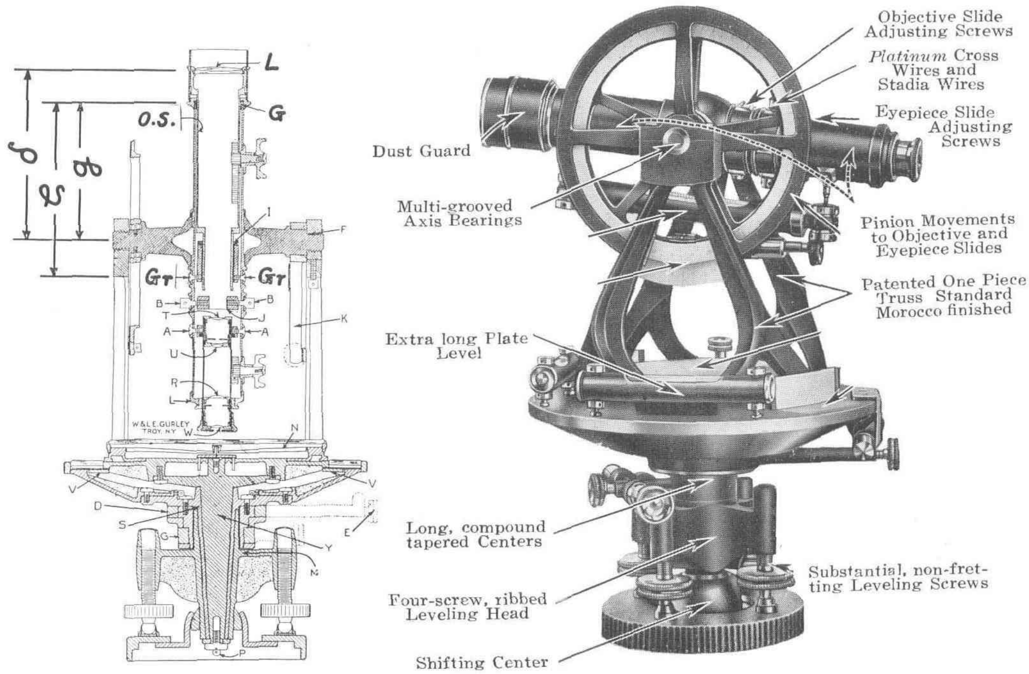


Fig. 16. Cross-Section of a Transit.

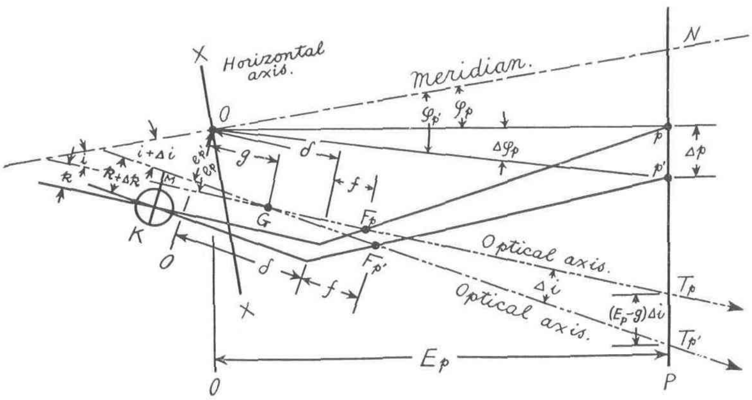


Fig. 17. Adjustment of the Inclination of the Optical Axis of the Objective Lens System.

is gotten, which, putting the notations in it, becomes

$$\varDelta p = (E_p - g)\varDelta i + \frac{k - \varDelta k}{f}(E_p - C_p) - \frac{k}{f}(E_p - C_p)\,,$$

where $$\varDelta k = (f - \delta + g)\varDelta i\,.$$

Therefore, arranging in good order, **the formula** is obtained **for adjustment of the inclination of the optical axis of the objective lens system**

$$\Delta p = \left\{C_p - g + \frac{\delta - g}{C - \delta}(E_p - C_p)\right\}\Delta i, \quad \ldots\ldots\ldots\ldots \quad (88)$$

which can exactly be obtained from (80) or (84) by its every nature, simply taking the variation of the deviation with respect only to i_p.

Further, neglecting terms of the higher order, when E_p is large,

$$\Delta p = \frac{\delta - g}{C - \delta} E_p \Delta i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (89)$$

is gotten.

Therefore, if Δi is computed according to the principle of Integral Adjustment of the vertical cross-hair described in Section VI, the correction for adjustment on the scale or the rod held at point P shonld be given by (88) and (89).

Now, the change of the angular deviation from the meridian of the instrument

$$\begin{aligned}\Delta\varphi_p &= \frac{\Delta p}{E_p} \\ &= \left\{\frac{C_p - g}{E_p} + \frac{\delta - g}{C - \delta}\frac{E_p - C_p}{E_p}\right\}\Delta i \quad \ldots\ldots\ldots\ldots \quad (90)\end{aligned}$$

and further, neglecting terms of the higher order, when E_p is large,

$$\Delta\varphi_p = \frac{\delta - g}{C - \delta}\Delta i, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (91)$$

are reduced from (88) and (89) respectively, where $f = C - \delta$.

Now, referred to the data of the transits adopted in the experiment described in Art. 25 and Art. 26, which are set down in Table 5, the formula

$$\begin{aligned}\Delta\varphi_p &= \frac{\Delta p}{E_p} \\ &= (0.23 \sim 0.13)\Delta i \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (92)\end{aligned}$$

is immediately found from (91).

20. The Distance and the Deviation of the Intersection Point of the Optical Axis of the Objective Lens System and the Straight Line Passing through a Near Point and a Distant One on the Locus of Collimation Points.

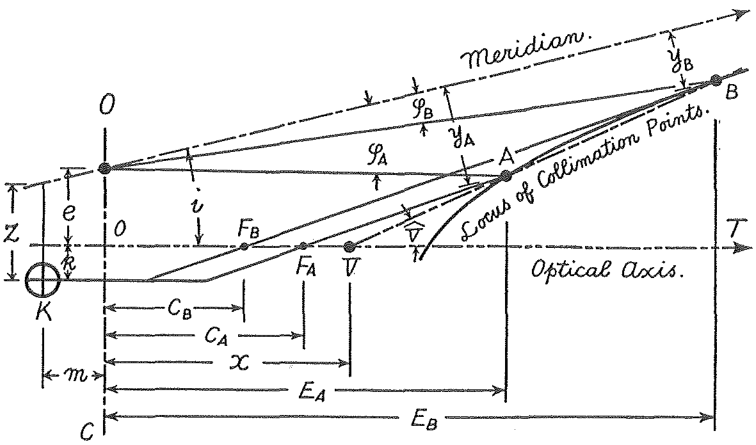


Fig. 18. Illustration of the Point V.

Now, referred to Fig. 18. denote a near point and a distant one on the locus of collimation points by A and B respectively, and the intersection point of the straight line passing through A and B and the optical axis of the objective lens system by V. Notations and Fig. 14 in Art. 17 should be referred to.

Hereupon, the distance of point V from the center of rotation of the telescope $\overline{OV} = x$ and its deviation from the meridian $e+xi$ are found out as follows :

Now, referring to Fig. 18, take the center of rotation of the telescope as the coordinate origin, the meridian itself as the abscissa, defining the forward direction as positive, and its perpendicular as the ordinate, defining the downward as positive, respectively.

Then, the equation of the straight line $\overline{BAV}$ is given by

$$\left.\begin{aligned} y-E_A\varphi_A &= \tan(\hat{V}-i)(E_A-x)\,, \\ y-E_B\varphi_B &= \tan(\hat{V}-i)(E_B-x)\,, \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad \text{(93a)}$$

where the angle $\widehat{BVT}$ is denoted by $\hat{V}$, and that of the optical axis of the objective lens system by

$$y = e + xi \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad \text{(93b)}$$

Hence, eliminating y from (93a) and (93b), the equation

$$\frac{e+xi-E_A\varphi_A}{e+xi-E_B\varphi_B} = \frac{E_A-x}{E_B-x}$$

is gotten, from which the new relation

$$\frac{E_A-C_A}{E_B-C_B}\frac{f_B}{f_A}=\frac{E_A-x}{E_B-x} \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (94)$$

is reduced, referred to (77).

Therefore, from (94), the universal formula

$$x=\frac{1-\frac{E_A-C_A}{E_B-C_B}\frac{E_B}{E_A}-\frac{f_B-f_A}{f_B}}{1-\frac{E_A-C_A}{E_B-C_B}-\frac{f_B-f_A}{f_B}}E_A$$

$$=\frac{C_A-\frac{E_A-C_A}{E_B-C_B}C_B-\frac{f_B-f_A}{f_B}E_A}{1-\frac{E_A-C_A}{E_B-C_B}-\frac{f_B-f_A}{f_B}}$$

$$=C_A+\frac{\frac{E_A-C_A}{E_B-C_B}(C_A-C_B)-\frac{f_B-f_A}{f_B}(E_A-C_A)}{1-\frac{E_A-C_A}{E_B-C_B}-\frac{f_B-f_A}{f_B}} \quad \dots\dots \quad (95)$$

is obtained.

But, referring to (12) and (21), the relation

$$\frac{E_A-C_A}{E_B-C_B}(C_A-C_B)=(C_B-C)\left\{1-\frac{E_A-C_A}{E_B-C_B}\right\} \quad \dots\dots\dots \quad (96)$$

is exactly gotten, in which, only for the telescope of the Wild-Zeiss type, terms of the much higher order are neglected.

Therefore, from (95) and (96), **the formula for the distance of the intersection point V of the optical axis of the objective lens system and the straight line passing through a near and a distant point on the locus of collimation points, measured from the center of rotation of the telescope or the abscissa of V,**

$$x=C_A+C_B-C-\frac{f_B-f_A}{f_B}\frac{E_A-C_A-C_B+C}{1-\frac{E_A-C_A}{E_B-C_B}-\frac{f_B-f_A}{f_B}} \quad \dots\dots \quad (97)$$

is universally found, where $(f_B-f_A)/f_B$ can be computed from (25), that is to say, the formula

$$\frac{f_B-f_A}{f_A}=\frac{1}{\dfrac{f_1^2}{(C-\delta-f_1)(d_A-d_B)}+1+\dfrac{d_B-d_\infty}{d_A-d_B}} \quad\cdots\cdots\cdots \quad (98)$$

or, neglecting terms of the higher order,

$$\frac{f_B-f_A}{f_B}=\frac{1}{\dfrac{f_1^2}{(C-\delta-f_1)(d_A-d_B)}+1} \quad\cdots\cdots\cdots\cdots \quad (99)$$

is obtained from (25).

Now, referring to (93b) and (97), **the deviation of point V from the meridian of the transit** or **the ordinate of V should exactly be given by the universal formula**

$$\begin{aligned} y_V &= e+x_V i \\ &= e+\left\{C_A+C_B-C-\frac{f_B-f_A}{f_B}\,\frac{E_A-C_A-C_B+C}{1-\dfrac{E_A-C_A}{E_B-C_B}-\dfrac{f_B-f_A}{f_B}}\right\}i. \quad (100) \end{aligned}$$

Therefore, **for the telescope whose focal length is invariable, that is, $f_A=f_B$, the general formulas of the distance and the deviation of point V or the coordinates of V referred to the origin O and the meridian of the instrument**

$$\left.\begin{aligned} x_V &= C_A+C_B-C \\ y_V &= e+x_V i \end{aligned}\right\} \quad\cdots\cdots\cdots\cdots\cdots\cdots \quad (101)$$

are exactly gotten from (97) and (100) respectively, where $e=e_g-gi$.

Thereupon, the deviation y has the most important significance in the present studies of the method of Integral Adjustment of the vertical cross-hair, which should readily be understood from the universal fact that $e+x_V i$ is the principal factor in various essential formulas: as in those for the deviations of collimation points, that is, (108), (109), (110), (111), (120), (121), (122) and (123), in those for the magnifiers of the optical errors, namely—(126), (127) and (128), and finally, in those for the condition of Integral Adjustment of the vertical cross-hair in a transit, or (168b).

Now, **if points B and A can simultaneously be brought on the meridian of the transit, then point V comes on that plane of itself, that is to say—$y_V=0$, and vice versa.**

21. Deviation of a Collimation Point from the Meridian of the Transit, when the Vertical Cross-Hair is adjusted by the Method of One-Quarter Adjustment. First, if the vertical cross-hair were adjusted by the method of One-Quarter Adjustment between the two unequally distant points B and $\bar{A}$ by mistake, that is, such that

$$\varphi_B = -\varphi_{\bar{A}}\,, \quad \text{when} \quad E_B \neq E_{\bar{A}}\,, \quad \ldots\ldots\ldots\ldots\ldots \quad (102)$$

the deviation of of B and $\bar{A}$ referred to the meridian of the transit

$$\left.\begin{aligned} E_B\varphi_B &= e + E_B i - \frac{E_B - C_B}{f_B}\,k\,, \\ E_{\bar{A}}\varphi_{\bar{A}} &= e + E_{\bar{A}} i - \frac{E_{\bar{A}} - C_{\bar{A}}}{f_{\bar{A}}}\,k \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (103)$$

would exactly be gotten from (78). Refer to Fig. 19.

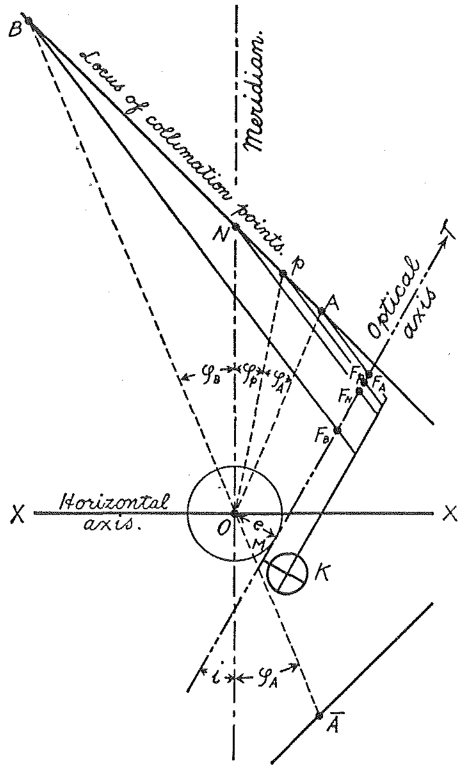


Fig. 19. Optical Relations when Point N is brought on the Meridian of the Transit.

Then, from (102) and (103), the departure of the vertical cross-hair from the optical axis of the objective lens system

$$k = \frac{(E_B + E_{\bar{A}})e + 2E_B E_{\bar{A}} i}{\frac{E_{\bar{A}} - C_{\bar{A}}}{f_{\bar{A}}} E_B + \frac{E_B - C_B}{f_B} E_{\bar{A}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (104)$$

is obtained in general.

Therefore, the deviation of a collimation point p should be exactly given by (77) and (104).

Now, if the vertical cross-hair is properly adjusted by the method of One-Quarter Adjustment between the equi-distant points N and $\bar{N}$ as usual, then the most significant relations

$$\varphi_N = \varphi_{\bar{N}} = 0\,, \quad k = \frac{e + E_N i}{E_N - C_N} f_N\,, \quad E_N = E_{\bar{N}} \quad \ldots\ldots \quad (105)$$

are found out from (102) (103) and (104), putting N and $\bar{N}$ instead of B and $\bar{A}$ respectively, because $\varphi_N = -\varphi_{\bar{N}}$ and $\varphi_N = \varphi_{\bar{N}}$. Refer to Fig. 19.

Hence, from (77) and (105), the formula for deviation of point p from the meridian of the transit for a telescope of an invariable focal length

$$E_p \varphi_p = e + C_p i - \frac{E_p - C_p}{E_N - C_N}(e + C_N i)\frac{f_N}{f_p} - i(E_p - C_p)\frac{f_N - f_p}{f_p}$$

$$\ldots\ldots\ldots \quad (106)$$

is exactly gotten, where

$$E_N = E_{\bar{N}}, \quad k = \frac{e + E_N i}{E_N - C_N} f\,, \quad e = e_g - gi\,.$$

But, exactly for the Ramsden, the Huygenian and the Porro telescopes and especially, neglecting terms of the higher order, for the telescope of the Wild-Zeiss type, the relations

$$\frac{E_p - C_p}{E_N - C_N} = \frac{C_N - C}{C_p - C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (107a)$$

and accordingly

$$C_N - C_p = \left\{\frac{E_p - C_p}{E_N - C_N} - 1\right\}(C_p - C) \quad \ldots\ldots\ldots\ldots \quad (107b)$$

are found out from (12).

Hence, transforming (106) through (107a) and (107b) into the most significant type, from which the highly satisfactory "Condition for Integral Adjustment of the vertical cross-hair in a transit", namely—

$$\varphi_p = 0 ,$$

can be immediately found out, **the new formula for the deviation of point p from the meridian of the transit**

$$E_p\varphi_p = \left\{e+(C_p+C_N-C)i\right\}\left\{1-\frac{E_p-C_p}{E_N-C_N}\right\}-(e+E_Ni)\frac{E_p-C_p}{E_N-C_N}\frac{f_N-f_p}{f_p} \qquad \text{........ (108)}$$

is exactly obtained in general, where

$$E_N = E_{\bar{N}}, \quad k = \frac{e+E_Ni}{E_N-C_N}f_N, \quad e = e_g-gi .$$

Again, from (93a) (93b) and (94), **the new formula for the same deviation of point p**

$$E_p\varphi_p = (e+xi)\left\{1-\frac{E_p-x}{E_N-x}\right\}$$

$$= (e+xi)\left\{1-\frac{E_p-C_p}{E_N-C_N}\frac{f_N}{f_p}\right\}, \quad \text{when} \quad E_N = E_{\bar{N}}, \quad \text{.. (109)}$$

is exactly gotten too, where x is given by (95) or (97), putting p and N instead of A and B respectively.

Therefore, **for the telescope with an invariable focal length, the formula of the deviation of point p from the meridian of the transit**

$$E_p\varphi_p = e+C_pi-\frac{E_p-C_p}{E_N-C_N}(e+C_Ni)$$

$$= \left\{e+(C_p+C_N-C)i\right\}\frac{E_N-C_N-E_p+C_p}{E_N-C_N}$$

$$= \left\{e+(C_p+C_N-C)i\right\}\frac{E_N-E_p}{E_N-C_N-C_p+C} \qquad \text{.......... (110)}$$

is obtained from (106) (108) and (109), where

$$E_N = E_{\bar{N}}, \quad k = \frac{e+E_Ni}{E_N-C_N}f, \quad e = e_g-gi .$$

Again, referring to (12), (110) is transformed into a new formula

$$E_p\varphi_p = \left\{e+(C_p+C_N-C)i\right\}\frac{C_p-C_N}{C_p-C}, \quad \ldots\ldots\ldots\ldots\ (111)$$

where

$$E_N = E_{\overline{N}}, \quad k = \frac{e+C_N i}{f}(C_N-C)+fi, \quad e = e_g-gi.$$

Now, if the condition

$$e+(C_J-C_N-C)i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (112)$$

is fulfilled at point J, then the formula for the residual deviation of the collimation point p

$$\begin{aligned} E_p\varphi_p &= \frac{(C_p-C_J)(C_p-C_N)}{C_p-C}i \\ &= \frac{E_J-C_J-E_p+C_p}{E_J-C_J}\frac{E_N-C_N-E_p+C_p}{E_N-C_N}(C_p-C)i \\ &= -\frac{(E_N-C_N)(C_p-C_J)+(E_p-C_p)(C_J-C_N)+(E_J-C_J)(C_N-C_p)}{(E_N-C_N)C_J-(E_J-C_J)C_J}e \\ &= \frac{E_N(C_J-C_p)+E_p(C_N-C_J)+E_J(C_p-C_N)}{E_NC_J-E_JC_N}e \quad \ldots\ldots\ldots\ (113a) \end{aligned}$$

is found out from (110) (111) and (112), referred to (12), where

$$\begin{aligned} i &= -\frac{e}{C_J+C_N-C} \\ &= -\frac{e_g}{C_J+C_N-C-g}, \\ e &= \frac{C_J+C_N-C}{C_J+C_N-C-g}e_g \\ &= -(C_J+C_N-C)i. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (113b) \end{aligned}$$

22. Deviation of a Collimation Point from the Meridian of the Transit, When the Vertical Cross-Hair is adjusted by the Method of Integral Adjustment of the Horizontal Cross-Hair. When the vertical cross-hair in a transit is adjusted by the method of Integral Adjustment of the horizontal cross-hair between a near point and a distant one—say A and B respectively, strictly speaking, any point except them is slightly dislocated, that is, it does not lie on a straight line passing through the center of rotation of the telescope. See Fig. 20.

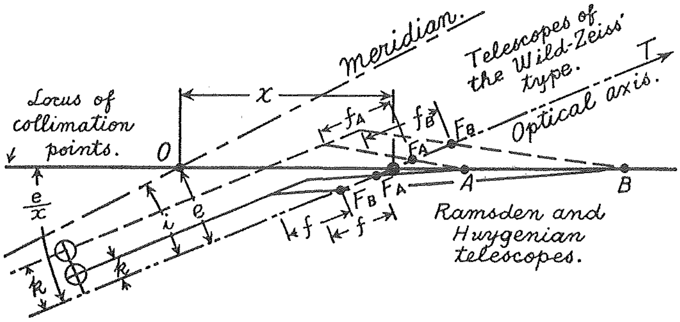


Fig. 20. Optical Relations when the Cross-Hair is adjusted by the Method of Integral Adjustment of the Horizontal Cross-Hair.

Then, from Fig. 20 and formula (77), for points A and B, the relations

$$E_A\varphi_A = E_A i + e - (E_A - C_A)\frac{k}{f_A} \quad \text{.............. (114a)}$$

$$E_B\varphi_B = E_B i + e - (E_B - C_B)\frac{k}{f_B} \quad \text{.............. (114b)}$$

and

$$\varphi_A = \varphi_B \quad \text{................................ (115)}$$

are obtained.

Hence, from (114a) (114b) and (115), the departure of the vertical cross-hair from the optical axis of the objective lens system

$$k = \frac{E_B - E_A}{\frac{E_A - C_A}{f_A}E_B - \frac{E_B - C_B}{f_B}E_A}e$$

$$= -\frac{E_B - E_A}{E_B C_A - E_A C_B + (E_B - C_B)E_A\frac{f_B - f_A}{f_B}}f_A e$$

$$= -\frac{E_B - E_A}{E_B C_A - E_A C_B + (E_A - C_A)E_B\frac{f_B - f_A}{f_A}}f_B e \quad \text{... (116)}$$

is exactly reduced.

Therefore, the deviation of a collimation point p from the meridian shall be calculated from (77) and (116).

Now, if the focal length is invariable, then, putting $f_A = f_B = f$ in (116) and (77), the formulas

$$k = -\frac{E_B - E_A}{E_B C_A - E_A C_B} f e \quad \dots\dots\dots\dots\dots\dots \text{(117)}$$

and

$$E_p \varphi_p = E_p i + e + \frac{(E_B - E_A)(E_p - C_p)}{E_B C_A - E_A C_B} e \quad \dots\dots\dots\dots \text{(118)}$$

are exactly gotten.

Hereupon, the deviation of a collimation point p from the line $\overline{OAB}$, being referred to that of A or B in Fig. 20,

$$E_p \varphi_p - E_p \varphi_A = \frac{E_B(C_A - C_p) + E_p(C_B - C_A) + E_A(C_p - C_B)}{E_B C_A - E_A C_B} e$$

$$= \frac{(E_B - C_B)(C_A - C_p) + (E_p - C_p)(C_B - C_A) + (E_A - C_A)(C_p - C_B)}{(E_B - C_B)C_A - (E_A - C_A)C_B} e$$

$$\dots\dots\dots \text{(119a)}$$

is exactly obtained from (114a) (117) and (118).

But, referred to (12), (119a) is accurately transformed into the form

$$E_p \varphi_p - E_p \varphi_A = \frac{C_A - C_p}{C_A + C_B - C} \frac{C_p - C_B}{C_p - C} e, \quad \dots\dots\dots \text{(119b)}$$

being put in order.

Now, the deviations of the collimation points A and B

$$E_A \varphi_A = E_A i + \frac{E_B - C_B - E_A + C_A}{E_B C_A - E_A C_B} E_A e$$

$$= \frac{e + (C_A + C_B - C)i}{C_A + C_B - C} E_A \quad \dots\dots\dots\dots\dots\dots \text{(120)}$$

and similarly,

$$E_B \varphi_B = E_B i + \frac{E_B - C_B - E_A + C_A}{E_B C_A - E_A C_B} E_B e$$

$$= \frac{e + (C_A + C_B - C)i}{C_A + C_B - C} E_B \quad \dots\dots\dots\dots\dots\dots \text{(121)}$$

are exactly obtained from (118) and (12).

Therefore, **the deviation of the collimation point p from the meridian of the transit**

$$E_p\varphi_p = \frac{e+(C_A+C_B-C)i}{C_A+C_B-C}E_p+\frac{E_B(C_A-C_p)+E_p(C_B-C_A)+E_A(C_p-C_B)}{E_BC_A-E_AC_B}e$$

$$= \frac{e+(C_A+C_B-C)i}{C_A+C_B-C}E_p+\frac{C_A-C_p}{C_A+C_B-C}\frac{C_p-C_B}{C_p-C}e \quad \ldots\ldots (122)$$

is obtained from (119a) (119b) and (120).

Accordingly, when the telescope is reversed, transiting about its horizontal axis as the alidade is still clamped horizontally, **the deviation of the collimation point p at fore-sight from the straight line $\overline{OAB}$,** referred to Fig. 20, becomes

$$E_p(\varphi_p+\varphi_A) = 2\frac{e+(C_A+C_B-C)i}{C_A+C_B-C}E_p$$

$$+\frac{E_B(C_A-C_p)+E_p(C_B-C_A)+E_A(C_p-C_B)}{E_BC_A-E_AC_B}e$$

$$= 2\frac{e+(C_A+C_B-C)i}{C_A+C_B-C}E_p+\frac{C_A-C_p}{C_A+C_B-C}\frac{C_p-C_B}{C_p-C}e$$

$$\ldots\ldots\ldots (123)$$

Now, if the condition

$$e+(C_A+C_B-C)i = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (124)$$

holds good for points A and B, then the formula for the residual deviation of the collimation point p from the meridian of the transit

$$E_p\varphi_p = \frac{E_B(C_A-C_p)+E_p(C_B-C_A)+E_A(C_p-C_B)}{E_BC_A-E_AC_B}e$$

$$= -\frac{(C_A-C_p)(C_p-C_B)}{C_p-C}i$$

$$= -\frac{E_p-C_p-E_A+C_A}{E_A-C_A}\frac{E_B-C_B-E_p+C_p}{E_B-C_B}(C_p-C)i$$

$$\ldots\ldots\ldots (125a)$$

is exactly gotten from (119a) (119b) (122) (123) and (124) for any position of the telescope, where

$$i = -\frac{e_g}{C_A+C_B-C-g}, \quad e = -\frac{C_A+C_B-C}{C_A+C_B-C-g}e_g. \quad \ldots\ldots (125b)$$

SECTION III.—EXPERIMENTATION UPON THE ERROR AFFECTING THE POSITION OF A COLLIMATION POINT OF A TRANSIT HORIZONTALLY

23. **General Principle of Finding the Optical Errors.** In order to research the Integral Adjustment of the vertical cross-hair in a transit, it is a necessity to know in advance the actual optical errors coming into the measured horizontal angle and subsequently their origins.

Therefore, the principle of finding them shall be explained as follows:

For the purpose of finding the eccentricity and the inclination of the optical axis of the objective lens system, that is to say, e and i, their magnified functions must be observed, so that they may easily be computed from the observed values.

Now, if the vertical cross-hair is adjusted by the method of One-Quarter Adjustment between the two equi-distant points $\overline{N}$ and N, then the deviation of the collimation point should be given exactly by (110) in general.

Thereupon, first sight at point $\overline{N}$ with the telescope normal, and then read the scale or the rod at point p still keeping the telescope clamped *in situ* when point $\overline{N}$ was sighted.

Again perform similarly as before with the telescope inverted, and then the difference of the two readings on the scale or the rod at p or t_p should theoretically be double the deviation of the collimation point p from the meridian of the transit, so that the relation

$$
\begin{aligned}
t_p &= -2E_p\varphi_p \\
&= 2\frac{E_p-C_p}{E_N-C_N}(e+C_N i)-2e-2C_p i \\
&= 2\left\{e+(C_p+C_N-C)i\right\}\frac{E_p-C_p-E_N+C_N}{E_N-C_N} \qquad \ldots\ldots\ldots\ (126)
\end{aligned}
$$

can be obtained from (110), through which the optical error $e+(C_p+C_N-C)i$ is simply magnified by $2(E_p-C_p-E_N+C_N)/(E_N-C_N)$ times.

Hereupon, it is immediately grasped that the nearer the points $\overline{N}$ and N and the more distant the point p are taken, the larger the magnitude of t_p becomes.

Now, if the vertical cross-hair is adjusted by the method of Integral Adjustment of the horizontal cross-hair between the near and the distant points A and B, the deviation of the collimation point $\overline{P}$ at fore-sight from the straight line $\overline{BAO}$ at back-sight, referred to the right figure in Fig. 23 of Art. 26 and Fig. 20 of Art. 22, should be exactly given by (123).

As soon as the adjustment has been completed, first sight at point A or B with the telescope normal, then clamping the telescope horizontally *in situ* as it was in when point A or B was sighted and reversing it, read the scale or the rod situated at point $\overline{P}$ and again perform the same as the preceding with the telescope inverted.

Then, the difference of the two readings $u_{\overline{P}}$ should be exactly double the deviation of the collimation point $\overline{P}$ from the straight line $\overline{BAO}$, (refer to the right figure of Fig. 23 of Art. 26), that is to say—the relation

$$u_{\overline{P}} = 2E_{\overline{P}}(\varphi_{\overline{P}}+\varphi_A)$$

$$= 2\frac{e+(C_A+C_B-C)\,i}{C_A+C_B-C}E_{\overline{P}} + 2\frac{C_A-C_{\overline{P}}}{C_A+C_B-C}e \quad \dots\dots\dots \quad (127)$$

is gotten from (123), neglecting terms of the higher order.

Hereupon, if the vertical cross-hair is adjusted by the same method between B and P, for which such a relation as $E_P = E_{\overline{P}}$ holds good, then the deviation of point $\overline{P}$ from the meridian of the transit

$$\frac{U}{4} = E_P\phi_P$$

$$= \frac{e+(C_P+C_B-C)\,i}{C_P+C_B-C}E_P \quad \dots\dots\dots\dots\dots \quad (128)$$

is obtained from (120) (123) or (127).

From (127) and (128), it is readily grasped that the error $e+(C_A+C_B-C)i$ or $e+(C_P+C_B-C)i$ can be magnified by $4E_P/(C_A+C_B-C)$ or $4E_P/(C_P+C_B-C)$ times, that is, for example, about 100 times when $C = 0.276$ m. and $E_P = 7$ m.

Now, the difference of the deviations of point $\overline{P}$ from the meridian when the vertical cross-hair is adjusted by the same method between B and P and B and A respectively

$$\frac{U_P}{4} - \frac{u_P}{4} = E_P\phi_P - E_P\varphi_P$$

$$= \frac{C_A-C_P}{C_A+C_B-C}\left\{\frac{E_P}{C_P+C_B-C} - \frac{1}{2}\right\}e \quad \dots\dots\dots \quad (129)$$

is computed from (128) and (127), in whose right hand member the first term of the factor in the bracket is absolutely much larger than the second since $E_P \geqq 10C$ always, so that the relative magnifying power of (128) is a little superior to that of (127), especially when the variations of e, e_g, i, etc. are researched.

Therefore, from the above demonstrations, it can be fully comprehended that the magnifying power of $e+(C_P+C_B-C)i$ of u-readings represented by (127) and (128) is much less greater than the magnifying power of t-readings represented by (126).

In the first experiment described in Art. 25 and Art. 26, the methods of magnification shown by (126) and (127) were adopted because the analytical study had not yet progressed far when the experiment was performed, and moreover such an extreme accurary as in the second and the fourth experiments, described in Art. 29, Art. 30, and Art. 32 respectively, had not been required because the magnitudes of e, e_g and i are fairly large; while, on the contrary, in the second and the fourth experiments, only the method of magnification of u-type represented by (127) and (128) was utilized from the above-described ground because the accuracy of the highest degree was required for obtaining the variations of e, e_g, i, etc.

24. The Experimentation of Finding the Optical Errors.

Now, since the subject of the present study lies in the improvement of the existing methods of adjustment of the vertical cross-hair in a transit, the actual experiments are confined to the following according to the need, namely—the first experiment, in which the eccentricity and the inclination of the optical axis of the objective lens system referred to the meridian of the transit can be gotten, the second, in which the eccentricity of the first principal point and the inclination of the optical axis of the objective lens system itself referred to its ideal axis can be found out, the third, in which the mechanical workmanship of the cylindrical slide surfaces of the objective slide is inspected, and finally, the fourth, in which the variations of the eccentricities and the inclination of the optical axis of the objective lens system and the deviation of a collimation point are considered.

Hereupon, for the first, second and fourth experiments, special test apparatuses arranged with the appliances by the author for himself, and assembled by his own hand, can be utilized, while for the third, the Zeiss Optotest is adopted.

Thus, the respective experiments are performed and the miscellaneous optical errors are found out as described in the following articles in order.

25. Apparatus for the First Experiment. The test apparatus designed for the first experiment plays the most important and efficient part for observing the optical errors. From it the eccentricities and the inclinations of optical axes of objective lens systems from the meridians of transits can be reduced. It is a special apparatus, conceived by the author for himself, as illustrated in Fig. 21 and Fig. 22. It is so arranged with appliances assembled by the author that the principle of observation of the optical errors studied in the preceding article can be practised in his laboratory for all its narrow space, because the out-door observations are not suitable for the precise experiments on account of interferences caused by sudden changes of temperature, by irregular reflexion and by partial direct exposition of the sun-light. Any necessary point in this apparatus can be adjusted at the observer's own pleasure as it is sighted through the telescope of a transit, deflecting the collimation line through five right-angled prisms numbered from 1 to 5.

Now, in Fig. 21, point C is the instrumental station of the transit which must be tested.

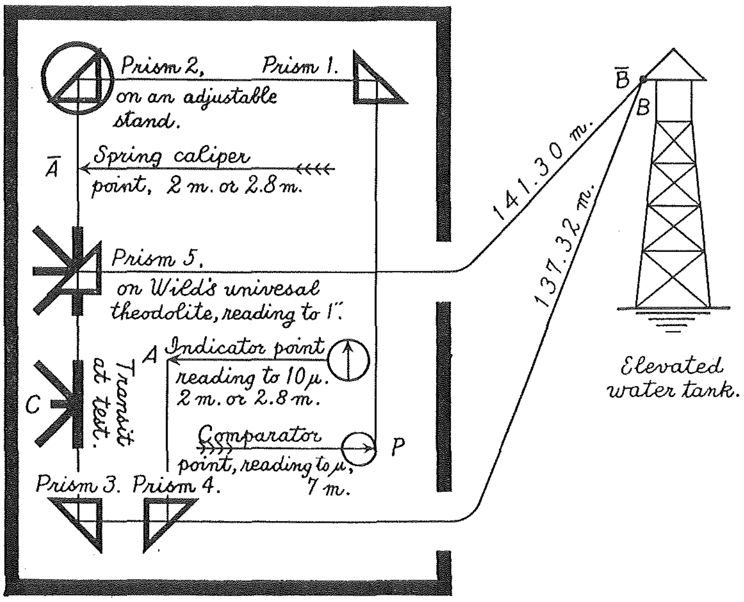


Fig. 21. Apparatus for the First Experiment.

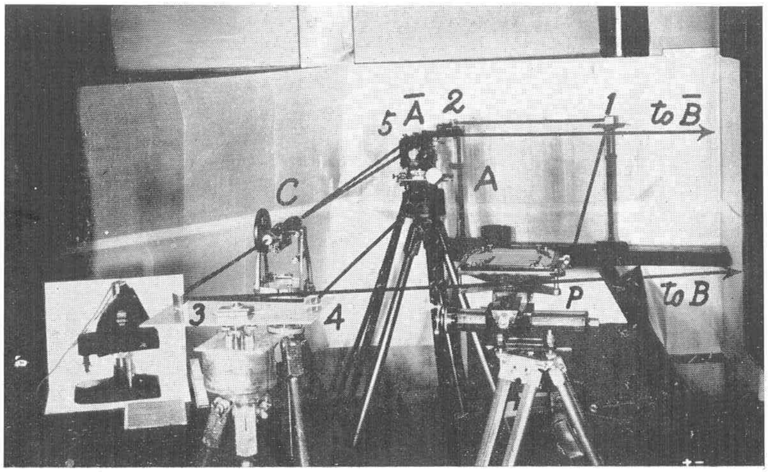


Fig. 22. Apparatus for the First Experiment.

In order to perform the One-Quarter Adjustment within an extremely short distance, two very near points $\bar{A}$ and A with an inside spring caliper and a dial indicator are established respectively. They are $E_A = 2$ m. for ordinary transits and $E_A = 2.8$ m. for a small "Fuji" transit respectively, deflecting the collimation line by a right angle through Prisms 3 and 4 arranged on its optical path for the purpose of adjustment of the indicator point at the observer's own convenience.

In addition to the above, the two points B and $\bar{B}$ are aligned in the same straight line as A and $\bar{A}$ so that the ordinary method of One-Quarter Adjustment of the vertical cross-hair can be performed between them, adjusting point $\bar{B}$ through rotating Prism 5, which is situated on a Wild's universal theodolite, reading to one second in both the vertical and the horizontal circles, so that it can be adjusted minutely in both directions by the tangent screws at the observer's own pleasure.

In this arrangement, B and $\bar{B}$ are the very same point, that is to say, the definite corner of the cover of an elevated water tank, nevertheless B differs from $\bar{B}$ on the respect that the former is sighted at B. S. deflected through Prism 3 and $E_B = 137.32$ m. away from the instrumental station, while the latter at F. S. deflected through Prism 5 and $E_{\bar{B}} = 141.30$ m. away.

Moreover, the above two combined are so arranged that the first method of adjustment of the horizontal cross-hair can also be performed between B and A, because they are $E_B = 137.32$ m. and $E_A = 2$ m. for ordinary transits or 2.8 m. for a small "Fuji" transit in the same collimation line.

Now, in order to find the inclination and the eccentricities of the optical axis of the objective lens system, besides the above, a new point P is arranged on a comparator reading to one micron, in the same straight line $\overline{BA\bar{A}}$, which is situated at the distance of $E_P = 7$ m. away from the instrumental station, so that the reading t explained in Art. 23 can be taken between A and P, as soon as the One-Quarter Adjustment of the vertical cross-hair has been completed between $\bar{A}$ and A; also the reading u, explained in Art. 23, can be taken between A and P, just after the first method of Integral Adjustment of the vertical cross-hair between B and A has been perfected.

Before point P was determined at the distance of 7 m., it was put to the test at various distances, e.g., $E_P = 107.3$ m., 53.7 m., 25 m. and 10 m. At last, point P was set at $E_P = 7$ m. as the most efficient optically for two reasons, because it could not only with difficulty be sighted owing to the fact that its collimation line was deflected many times through several right-angled prisms in order to secure a long collimation distance in the author's narrow laboratory, but also it was qualified to take a short distance, for E_P not influenced by atomospheric conditions, approximately equating to (the magnifying power of the telescope $\doteqdot 26$) $\times$ (the range of vision of man's eye $= 0.25$ m.).

26. First Experiment.—Experiment for Finding the Optical Errors Affecting the Horizontal Angle, with the Author's Own Apparatus.

Now, the method of observing the optical errors, referred to the proof in Art. 23, from which the eccentricities and the inclination of the optical axis of the objective lens system from the meridian of the instrument can be calculated, are composed of the two processes described, and referred to in Fig. 21 and Fig, 22 of Art. 25, as follows:

1. Adjust the vertical cross-hair by the method of One-Quarter Adjustment between $\bar{A}$ and A at first. As soon as it has been completed, sight at $\bar{A}$ with the telescope normal, clamp the telescope *in situ* when A was sighted, and then take the reading of the comparator point P. Again sight at $\bar{A}$ with the telescope inverted, perform the same as the preceding and then take the difference of the two readings or t_P.

Then, the relation

$$\frac{E_P - C_P}{E_A - C_A}(e + C_A i) - (e + C_P i) = \frac{t_P}{2} \quad \ldots\ldots\ldots\ldots \quad (130)$$

should be obtained from (126).

2. Adjust the vertical cross-hair by the method of Integral Adjustment of the horizontal cross-hair between B and A. As soon as it has been perfected, sight at A with the telescope normal, clamp the alidade horizontally *in situ* when A was sighted, reverse the telescope, and then take the fore-sight reading of the comparator point P. Again sight at A with the telescope inverted, repeat the preceding performance, and then take the difference of the two readings or u_P.

Then, neglecting terms of the much higher order, the relation

$$u_P = 4\frac{e+(C_A+C_B-C)i}{C_A+C_B-C}E_P + 2\frac{C_A-C_P}{C_A+C_B-C}e \quad \ldots\ldots\ldots \quad (131)$$

is gotten from (127).

Now, from (130) and (131), since

$$\begin{aligned}
&\frac{E_P-C_P}{E_A-C_A}C_A-C_P+\frac{C_A-C_P}{2E_P}\left\{\frac{E_P-C_P}{E_A-C_A}C_A-C_P\right\} \\
&\qquad\qquad -\frac{E_P-C_P}{E_A-C_A}(C_A+C_B-C)+C_A+C_B-C \\
&= C_A-C_P-\frac{E_P-C_P-E_A+C_A}{E_A-C_A}(C_B-C)+\frac{C_A-C_P}{2E_P}\left\{\frac{E_P-C_P}{E_A-C_A}C_A-C_P\right\},
\end{aligned}$$

the formula for the inclination of the optical axis of the objective lens system referred to the meridian of the transit

$$i=\frac{\left\{1+\frac{C_A-C_P}{2E_P}\right\}t_P-\frac{E_P-C_P-E_A+C_A}{E_A-C_A}\,\frac{C_A+C_B-C}{2E_P}u_P}{2\left\{C_A-C_P-\frac{E_P-C_P-E_A+C_A}{E_A-C_A}(C_B-C)-\frac{C_A-C_P}{2E_P}\left(\frac{E_P-C_P}{E_A-C_A}C_A-C_P\right)\right\}}, \quad \ldots\ldots\ldots \quad (132)$$

its eccentricity at the centre of rotation of the telescope

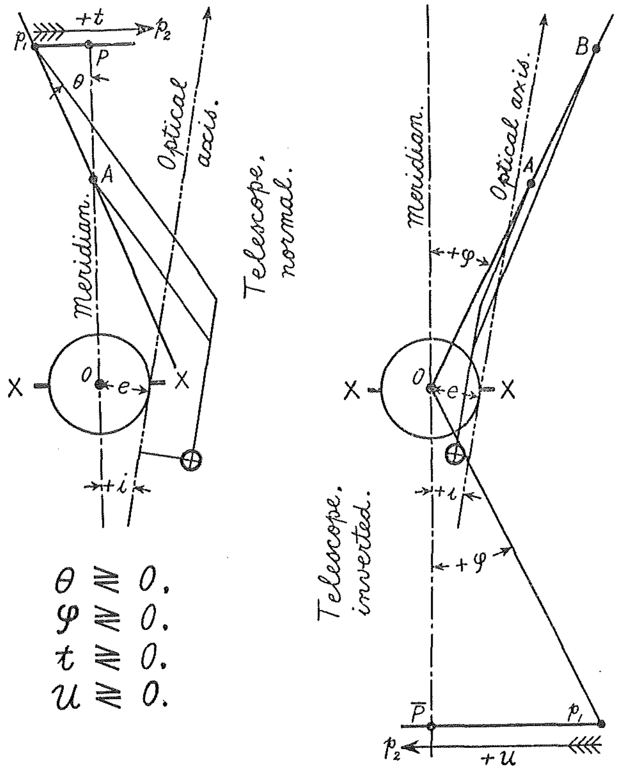
$$e=\frac{\frac{C_A+C_B-C}{2E_p}\,\frac{u_P}{2}-(C_A+C_B-C)i}{1+\frac{C_A-C_P}{2E_P}}, \quad \ldots\ldots\ldots\ldots\ldots \quad (133)$$

and also, its eccentricity at the front slide bearing of the objective slide

$$e_g = e + gi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (134)$$

are computed.

Optical Relations when the Vertical Cross-Hair is adjusted by the Ordinary Method of One-Quarter Adjustment.



Optical Relations when the Vertical Cross-Hair is adjusted by the Method of Adjustment of the Horizontal Cross-Hair.

Fig. 23.

Now, the signs of e, i, t and u are defined, with reference to Fig. 23, as follows:

The signs of t and u are positive, when the collimation line with the telescope normal strikes on the left and that with the telescope inverted, on the right.

The sign of e is positive, when it stands on the right of the meridian of the transit, with the telescope normal.

Finally, the sign of i is positive, when the optical axis of the objective lens system strikes on the right of the meridian of the instrument with the telescope normal in the direction of collimation.

Thus, it has been perfectly illustrated that the instrumental errors e, e_g and i can be found most successfully from the results obtained by the present experiment.

Thirteen representative transits were selected for the inspection. They have no defects among about thirty of the Civil Engineering course of Hokkaido Imperial University, Japan, where the present author is now engaged.

In Table 4, makers, sizes, years of use, etc. are listed.

In Table 5, are tabulated the so-called instrumental constant C, the distance of the optical center of the objective lens from the center of rotation of the telescope δ, the distance of the front slide bearing from the center of rotation of the telescope g, the interval between the two slide bearings ζ, $(\delta-g)/(C-\delta)$, $(\delta-g)C/(C-\delta)(C-g)$, $(\delta-g)/\zeta$, $(C-g)/\zeta$, etc.

In Table 6, besides the readings t and u, are set down the eccentricity of the optical axis of the objective lens system from the meridian of the transit at the center of rotation of the telescope or e, ditto at the front slide bearing or e_g, the inclination of the optical axis from the meridian or i, the correction of i for Integral Adjustment between $E_B = 137.3$ m. and $E_A = 2$ m. or Δi, and the grade of quality of transit. Hereupon, e, e_g, i and Δi are obtained from (133) (134) (132) and (169) respectively, and for the grade of quality one must refer to Table 12 of Art. 33.

Table 4.

(Jan., 1915)

Transit No.	Size, In	Made in	Date of Purchase	Years of Use	User
1	4	Japan	Dec., 1923	0.0	none
4	4	Japan	March, 1925	10.0	students
5	4	Japan	March, 1925	10.0	students
7	4	Japan	March, 1925	10.0	students
9*	4	Japan	March, 1925	10.0	students
22	5	U.S.A.	March, 1928	0.1	ass. prof.
23	5	U.S.A.	March, 1928	0.2	T. Shingo
37	4	Japan	March, 1930	3.0	students
39	4	Japan	March, 1930	3.0	students
11	$3^1/_2$	Japan	June, 1925	10.0	students
A1	$3_1/_2$	Japan	March, 1932	1.5	students
A2	$3^1/_2$	Japan	March, 1933	1.0	students
A3	small "Fuji"	Japan	March, 1934	0.0	none

* Newly repaired.

Table 5.

Transit No.	C, m.m.	δ, m.m.	g, m.m.	ζ, m.m.	$\frac{\delta-g}{\zeta}$	$\frac{C-g}{\zeta}$	$\frac{1}{C-\delta}\left\{1+\frac{\delta-g}{\zeta}\right\}$
1	315	132	103	125	0.228	1.70	0.006 71
4	276.4	122	87	110	0.318	1.72	0.008 31
5	276.4	122	87	110	0.318	1.72	0.008 31
7	276.4	122	87	110	0.318	1.72	0.008 31
9	276.4	122	87	110	0.318	1.72	0.008 31
22	347	141	112	135	0.215	1.74	0.005 90
23	347	141	112	135	0.215	1.74	0.005 90
37	325	136	100	123	0.295	1.83	0.006 84
39	320	130	98	121	0.264	1.82	0.006 66
11	254 5	109	90	105	0.181	1.57	0.008 12
A1	254.5	109	90	105	0.181	1.57	0.008 12
A2	250	110	83				
A3	280						

Table 5.—(*Continued*)

Transit No.	$\frac{1}{C-\delta}\frac{\delta-g}{\delta}$	$\frac{\delta-g}{C-\delta}$	$\frac{\delta-g}{C-\delta}\frac{C}{C-g}$	$\frac{1+\frac{\delta-g}{\zeta}}{\frac{\delta-g}{\zeta}}$
1	0.001 25	0.159	0.235	5.39
4	0.002 00	0.227	0.331	4.14
5	0.002 00	0.227	0.331	4.14
7	0.002 00	0.227	0.331	4.14
9	0.002 00	0.227	0.331	4.14
22	0.001 04	0.141	0.208	5.65
23	0.001 04	0.141	0.208	5.65
37	0.001 55	0.190	0.275	4.41
39	0.001 39	0.168	0.243	4.79
11	0.001 24	0.130	0.202	6.52
A1	0.001 24	0.130	0.202	6.52

Table 6.

Experimental Note 2.

(Jan.~Feb., 1935)

Transit No.	t, Arith Mean / Av. Error m.m.	u, Arith Mean / Av. Error m.m.	e, m.m.	e_g, m.m.	i, Radian / ° ′ ″	i, ′ ″	Grade* of Quality
1	−2.052 ±0.0245	−28.852 ±0.0381	−0.367	−0.361	+0.000 058 + 12	+ 5 06	B
4	+1.174 ±0.0123	+27.915 ±0.0378	+7.297	+5.209	−0.024 064 −1 22 44	− 4 53	D
5	+1.521 ±0.0149	+30.348 ±0.0307	+4.692	−3.384	−0.015 029 − 51 40	− 5 15	C
7	+3.777 ±0.0254	+59.710 ±0.0418	−1.028	−0.535	+0.005 662 + 19 28	−10 28	B
9	−0.836 ±0.0166	−13.967 ±0.0441	−0.355	−0.292	+0.000 721 + 2 29	+ 2 26	B
22	−0.353 ±0.0261	−9.004 ±0.0255	−3.676	−2.609	+0.009 530 − 32 46	+ 1 31	C
23	−0.794 ±0.0245	−9.207 ±0.0726	+0.209	+0.109	−0.000 888 − 3 03	+ 1 37	A
37	+1.099 ±0.0069	+16.341 ±0.0121	+1.504	+1.132	−0.003 727 − 12 49	− 2 47	C
39	−1.784 ±0.0199	−17.754 ±0.0370	+4.745	+3.321	−0.014 528 − 49 57	+ 3 08	C
11	+0.652 ±0.0625	+8.070 ±0.0401	−3.052	−1.996	+0.011 734 + 40 20	− 1 32	C
A1	+0.164 ±0.0137	+0.404 ±0.0206	−2.090	−1.384	+0.007 852 + 27 00	− 06	C
A2	+0.251 ±0.0138	+7.867 ±0.0118	+2.440	+1.693	−0.008 995 − 30 55	− 1 23	C
A3	+0.126 ±0.0146	+17.758 ±0.0151	+0.038	+0.052	+0.000 385 + 1 19		A
32†	+0.194 ±0.010						

* Refer to Art. 33.

† This is a Wild's universal theodolite, reading to one second both horizontally and vertically, and the reading t was taken between $E_B = 137.3$ m. and $E_A = 2.5$ m. .

As regards the observations of t and u, their five readings taken to one micron by the comparator are averaged and their average errors are calculated by the method of least squares, so that they may not be affected by large errors.

27. The Deviation of the Point V and the First Focal Point of the Objective Lens System of the Telescope from the Meridian of the Transit. The deviation of the intersection point V of the optical axis of the objective lens system of the telescope and the straight line passing through the two points on the locus of collimation points—say B and p, the former being adopted for One-Quarter Adjustment and the latter taken *ad libitum*, is already given by (101), that is,

$$y_V = e + (C_p + C_B - C)\,i \,. \quad \dots\dots\dots\dots\dots\dots \quad (101)$$

Hereupon, if point B is infinitely distant or $E_B = \infty$, then, from (101), that is,

$$y_V = e + C_p i \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (135)$$

is gotten, which is the exact deviation of the first focal point of the objective lens system. Refer to Fig. 14.

Table 7.

(Jan.~Feb., 1935)

Transit No.	$e+(C_A+C_B-C)i$, m.m.	$e+C_Ai$, m.m. ($E_A = 2$ m.)	$e+C_Pi$, m.m. ($E_P = 7$ m.)	$e+C_Bi$, m.m. ($E_B = 137.32$ m.)
1	−0.347	−0.347	−0.348	−0.348
4	+0.284	+0.289	+0.554	+0.641
5	+0.312	+0.315	+0.481	+0.535
7	+0.622	+0.621	+0.559	+0.538
9	−0.145	−0.145	−0.153	−0.156
22	−0.115	−0.118	−0.306	−0.367
23	−0.123	−0.123	−0.105	−0.100
37	+0.202	+0.203	+0.271	+0.292
39	−0.222	−0.218	+0.012	+0.092
11	+0.079	+0.076	−0.028	−0.063
A1	+0.005	+0.004	−0.068	−0.091
A2	+0.072	+0.074	+0.161	+0.189
A3	+0.097	+0.145	+0.146	+0.146

Notice: C_A, C_P and C_B correspond to $E_A = 2$ m., $E_P = 7$ m. and $E_B = 137.32$ m. respectively.

Further, the eccentricity of the straight line $\overline{Bp}$ at the center of rotation of the telescope is given by (135), neglecting terms of the higher order, nevertheless a detailed explanation of it will be given later.

In Table 7, the values of $e+C_p i$ are given at the distances of 2 m., 7 m. and 137.32 m.

28. Error Coming into the Measured Horizontal Angle. Next, the error coming into the measured horizontal angle should be discussed here according to the method of Adjustment.

When the vertical cross-hair is adjusted between B and $\overline{B}$ by the ordinary method of One-Quarter Adjustment, then the angular deviations of the collimation point A and P from the meridian of the transit are given by the formulas

$$\varphi_A = \left\{e+(C_A+C_B-C)i\right\}\frac{C_A-C_B}{(C_A-C)E_A} \quad \cdots\cdots\cdots\cdots \quad (136a)$$

and

$$\varphi_P = \left\{e+(C_P+C_B-C)i\right\}\frac{C_P-C_B}{(C_P-C)E_P}, \quad \cdots\cdots\cdots \quad (136b)$$

which are both obtained from (110).

Accordingly, from (136a) and (136b), the dislocations of the collimation point B in relation to A and P are given by the formulas

$$E_B\varphi_A = \left\{e+(C_A+C_B-C)i\right\}\frac{C_A-C_B}{C_A-C}\frac{E_B}{E_A} \quad \cdots\cdots\cdots\cdots \quad (137a)$$

and

$$E_B\varphi_P = \left\{e+(C_P+C_B-C)i\right\}\frac{C_P-C_B}{C_P-C}\frac{E_B}{E_P}. \quad \cdots\cdots\cdots \quad (137b)$$

Further, when the vertical cross-hair is adjusted by the method of Integral Adjustment of the horizontal cross-hair between point B and A, the error coming into the deflexion angle at the same distances as those of B and A can be gotten by the formula

$$\frac{u_A}{2E_A} = \frac{u_B}{2E_B}$$

$$= \frac{2e}{C_A+C_B-C}+2i \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (138)$$

reduced from (131), neglecting terms of the higher order.

In Table 8, are given the errors coming into the angles measured between the points of $E_B=137.32$ m. and $E_p=7$ m. or $E_A=2$ m. for the ordinary transits and $E_P=7$ m. or $E_A=2.8$ m. for the small "Fuji"

Table 8.

(Jan.~Feb., 1935)

Transit No.	Angular Error between E_B and E_A, Seconds	Deviation at E_B in Relation to E_A, m.m.	Angular Error between E_B and E_P, Seconds	Deviation at E_B in Relation to E_P, m.m.	Error in the Deflexion Angle	
					Radian	′ ″
1	−35.4	−23.5	− 9.8	− 6.5	−0.002 061	− 7 05
4	+29.0	+19.3	+15.3	+10.2	+0.001 955	+ 6 43
5	+31.8	+21.1	+13.4	+ 8.9	+0.002 142	+ 7 22
7	+63.3	+42.2	+15.6	+10.4	+0.004 271	+14 41
9	−14.8	− 9.8	− 4.3	− 2.8	−0.000 996	− 3 25
22	−11.7	− 7.8	− 8.6	− 5.7	−0.000 615	− 2 07
23	−12.6	− 8.4	− 3.0	− 2.0	−0.000 659	− 2 16
37	+20.6	+13.7	+ 7.6	+ 5.0	+0.001 156	+ 3 58
39	−22.6	−15.1	+ 0.2	+ 0.1	−0.001 300	− 4 28
11	+ 8.0	+ 5.3	− 0.7	− 0.5	+0.000 591	+ 2 02
A1	+ 0.5	+ 0.4	− 1.9	− 1.3	+0.000 039	+ 08
A2	+ 7.5	+ 5.0	+ 4.7	+ 3.1	+0.000 549	+ 1 53
A3	+ 7.1	+ 4.8	+ 2.8	+ 1.9	+0.001 270	+ 4 22

Notice: Here, $E_A = 2$ m., $E_p = 7$ m. and $E_B = 137.32$ m..

The errors shown in Column 2, 3, 4 and 5 are those occurring when the vertical cross-hair is adjusted by the method of One-Quarter Adjustment, and the error in the deflexion angle given in the last two columns is that when the vertical cross-hair is adjusted by the method of Integral Adjustment of the horizontal cross-hair.

transit A3, when they are adjusted by the method of One-Quarter Adjustment at the two points, both 137.32 m., away from the instrumental center, and also, the angular errors coming into the deflexion angles, when adjusted by the method of Integral Adjustment of the horizontal cross-hair between B and A.

The latter errors are equal to double the angular divergences of the loci of collimation points from the meridian of the instrument.

29. The Test Apparatus Adopted for the Second and the Fourth Experiments. The second test apparatus is especially designed for the most significant experiment in which the eccentricity and the inclination of the optical axis of the objective lens system with regard to the normal or ideal position as it is in the holder, can be found. It is more efficient than that selected in the first experiment from the standpoint of accuracy, because the observations of the u-reading alone, whose magnifying power of the optical errors is incomparably greater than that of the t-reading referred to the analytical proofs described in Art. 23, are performed in the present experiment. On the contrary, the observations of the u-reading and the t-reading both were in the preceding experiment.

Now, this apparatus was conceived by the present author too as in the case of the first. It is so arranged with the hand-made appliances assembled by him for himself, as illustrated in Figs. 24 and 25, that the principle of observation of the optical errors researched in Art. 23 can be realized in such a narrow space as the author's laboratory for the same grounds as described in Art. 25, because the accurate results can not be obtained from the outdoor observations. Refer to Fig. 24 and Fig. 25.

Now, points A, $\overline{A}$, D, $\overline{D}$, P and $\overline{P}$ are situated on the same comparator, among which A, D and P should be collimated at foresight and $\overline{A}$, $\overline{D}$ and $\overline{P}$ at back-sight, so that they may be adjusted at the observer's own command as they are sighted through the telescope of the transit at test, deflecting the collimation line through four right-angled prisms, numbered from 1 to 4. Among the four, only Prism 2 is fitted on a Wild's universal theodolite, reading to one second both horizontally and vertically, so that it can be finely adjusted in both directions as desired.

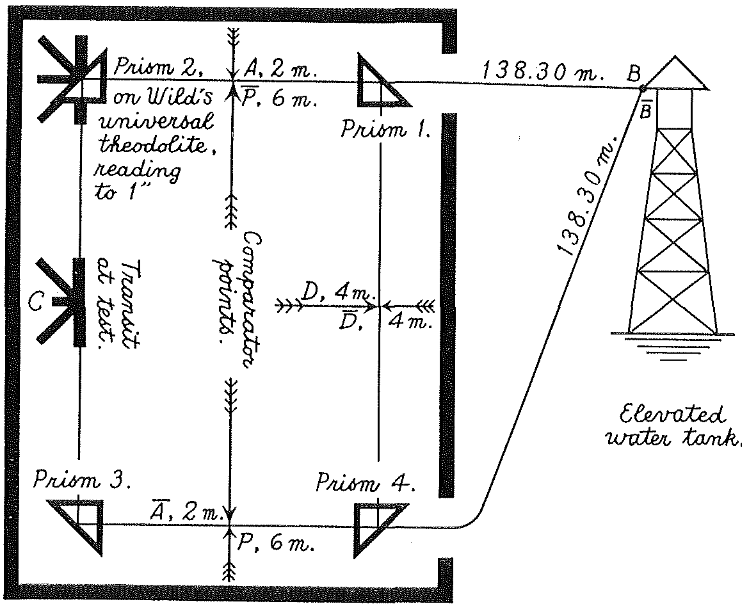


Fig. 24. Apparatus for the Second and the Fourth Experiments.

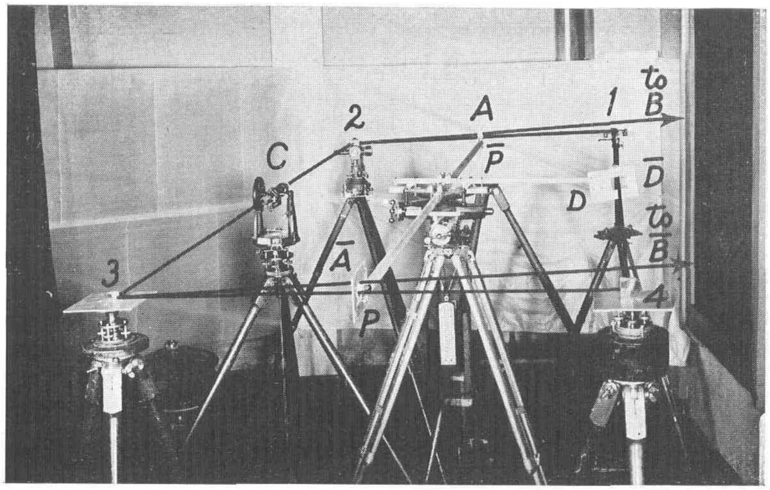


Fig. 25. Apparatus for the Second and the Fourth Experiments.

Point C is the instrumental station of the transit to be tested.

Points B and $\bar{B}$ are nothing but the same point, namely—a definite corner of the cover of an elevated water tank, the former being collimated through Prisms 2 and 1 at foresight and the latter through Prisms 3 and 4 at backsight, between which the method of One-Quarter Adjustment can be practiced.

Through this apparatus, the vertical cross-hair can be adjusted by the method of Integral Adjustment of the horizontal cross-hair between $\bar{B}$ and $\bar{A}$, $\bar{B}$ and $\bar{D}$, and also $\bar{B}$ and $\bar{P}$ through Prisms 3 and 4.

After the above adjustment is completed between $\bar{B}$ and $\bar{A}$, sight at $\bar{B}$ with the telescope inverted, reverse the telescope, clamping the alidade horizontally *in situ* when $\bar{B}$ was sighted, take the reading of the comparator point A through Prisms 1 and 2, and again, performing likewise as before with the telescope normal, take the new reading.

Thus, the difference of the two readings u_A can be obtained. Similarly, proceding among $\bar{B}$, $\bar{D}$ and D, and also $\bar{B}$, $\bar{P}$ and P, the new differences of the respective two readings, viz., u_D aud u_P can be gotten.

Regarding the sign of u, refer to the definition described above in Art. 26.

Now, as readily apprehended from Figs. 24 and 25, the nine points B, P, D, A, C, $\bar{A}$, $\bar{D}$, $\bar{P}$ and $\bar{B}$ are arranged in the same collimation line, so that B, P, D and A can be sighted from foresight and the remainder from backsight, C being the transit station at test.

Herein, their distances measured from the instrumental station are taken as $E_B = E_{\overline{B}} = 138.30$ m., $E_P = E_{\overline{P}} = 6.0$ m., $E_D = E_{\overline{D}} = 4.0$ m. and $E_A = E_{\overline{A}} = 2.0$ m. respectively, in order to get the accuracy of the highest possibility of the various quantities computed from the measurements and the simplicity of the computation, which must be referred to (128).

With regard to the arrangement in the apparatus, one should refer for the other data to Art 30.

Now, this apparatus is likewise utilized in the fourth experiment through which the variations of the eccentricity and the inclination of the optical axis of the objective lens system are found. For detailed explanation refer to Art. 32.

30. Second Experiment.—Experiment for Determining the Erroneous Inclination of the Optical Axis and the Erroneous Eccentricity of the Optical Center of the Objective Lens System Itself in Relation to the Ideal Optical Axis. It is very significant and useful to know what kinds of defects of the parts of the instrument should principally cause the eccentricity and the inclination of the optical axis of the objective lens system.

Among them, those disabilities caused by dis-centering the objective lens itself are researched exactly by the present experiment.

Originally, even a new transit, just come from the maker, may be somewhat faulty owing to the defects in centering of the standards, the horizontal axis of the telescope, the objective lens and its holder; finishing of the objective slide and the slide bearings; and especially, owing to the erroneous centering of the objective lens itself and the erroneous methods of adjustment of the objective slide. On the other hand, an instrument, used for years, may be very faulty from the distorsion of the standards, the telescope tube and the horizontal axis of the telescope, because of accidents and abraisions of the surfaces of the objective slide and the slide bearings.

A detailed discussion of such errors is fairly difficult. Such is moreover un-necessary because it is exactly concluded from the results of the experiment shown in Table 4 and Table 6 of Art. 26 and Table 10 of Art. 31 that the eccentricity of the optical axis of the objective lens system at the front slide bearing and its inclination from the meridian of the transit, viz., e_g and i, are principally caused by the lateral and rotational distorsion of the standards during practical use. Yet such defects partially the result of imperfect workmanship in the factories for they are not only extremely large in some instruments used for years but also defect of fairly large magnitudes are found even in new ones which have only adjusted by the centering tool or "Shin-Gané" (心金)† in manufactories of Japan, and thereafter may probably be tested by the present erroneous methods of adjustment of the objective slide, which are found in all the foreign celebrated works* on Surveying written by authorities of world wide-fame and in the catalogues issued from makers of the first-class. Detailed theoretical proofs and illustrations will be given in Section XI below.

In order to prove this fact practically, an experiment, from which the erroneous inclination of the optical axis and the eccentricity of the first principal point of the objective lens system in reference to the ideal axis shall be found, must be performed and explained:

Now, first, adjust the vertical cross-hair by the method of Integral Adjustment of the horizontal cross-hair between points $\overline{B}$ and $\overline{A}$.

As soon as the adjustment has been completed, reverse the telescope with the alidade still clamped horizontally *in situ* when $\overline{B}$ was sighted with the telescope inverted, and then take the reading of the comparator point A at the back.

Again perform the same as the preceding with the telescope normal.

Then the relation among the difference of the respective two readings of the comparator point or u_A, the eccentricity of the optical axis of the objective lens system at the center of rotation of the telescope or e and its inclination from the meridian of the transit or i

$$u_A = 4\frac{e+(C_A+C_B-C)i}{C_A+C_B-C}E_A \quad \text{............} \quad (139a)$$

is gotten from (128), where $e = e_g - gi$.

Subsequently, perform similarly as the preceding among points $\overline{B}$, $\overline{D}$ and D, and then the new equation

$$u_D = 4\frac{e+(C_D+C_B-C)i}{C_D+C_B-C}E_D \quad \text{.............} \quad (139b)$$

is obtained likewise as before.

Hereupon, loosen the objective lens in its holder by un-screwing, rotate it by one hundred and eighty degrees about its optical axis as it is in the holder and then tighten it by screwing again.

† Refer to Art. 52.

* See Reference at the end of this work.

Thereafter, again perform all the same as before, and then the similar equations to (139a) and (139b)

$$\bar{u}_A = 4\frac{\bar{e}+(C_A+C_B-C)\bar{i}}{C_A+C_B-C}E_A \quad \ldots\ldots\ldots\ldots \quad (140a)$$

and

$$\bar{u}_D = 4\frac{\bar{e}+(C_D+C_B-C)\bar{i}}{C_D+C_B-C}E_D \quad \ldots\ldots\ldots\ldots \quad (140b)$$

are anew obtained from (128), in which the notations $\bar{u}$, $\bar{e}$, $\bar{i}$ and $\bar{e}$ show the quantities after the objective lens was rotated by one hundred and eighty degrees in the holder, where $\bar{e} = \bar{e}_g - g\bar{i}$.

Now, from (139a), (139b), (140a) and (140b), are reduced the inclinations of the optical axis of the objective lens

$$i = \frac{\frac{C_A+C_B-C}{4}\frac{u_A}{E_A} - \frac{C_D+C_B-C}{4}\frac{u_D}{E_D}}{C_A-C_D} \quad \ldots\ldots\ldots\ldots \quad (141a)$$

and

$$\bar{i} = \frac{\frac{C_A+C_B-C}{4}\frac{\bar{u}_A}{E_A} - \frac{C_D+C_B-C}{4}\frac{\bar{u}_D}{E_D}}{C_D-C_B}, \quad \ldots\ldots\ldots\ldots \quad (141b)$$

its eccentricities at the center of the horizontal axis of the telescope

$$e = \frac{C_A+C_B-C}{4}\frac{u_A}{E_A} - (C_A+C_B-C)i \quad \ldots\ldots\ldots \quad (142a)$$

and

$$\bar{e} = \frac{C_A+C_B-C}{4}\frac{\bar{u}_A}{E_A} - (C_A+C_B-C)\bar{i}, \quad \ldots\ldots\ldots \quad (142b)$$

its eccentricities at the front slide bearing

$$e_g = e + gi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (143a)$$

and

$$\bar{e}_g = \bar{e} + g\bar{i}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (143b)$$

the erroneous inclination of the optical axis of the objective lens itself referred to its ideal axis

$$(i) = -\frac{\bar{i}-i}{2}$$

$$= -\frac{\frac{C_A+C_B-C}{4}\frac{\Delta u_A}{E_A}-\frac{C_D+C_B-C}{4}\frac{u_D}{E_D}}{C_A-C_D} \quad \ldots\ldots\ldots (144)$$

and the erroneous eccentricity of the first principal point of the objective lens system itself referred to its ideal position

$$(e) = -\frac{\bar{e}+\delta\bar{i}-e-\delta i}{2}$$

$$= -\frac{C_A+C_B-C}{8}\frac{\Delta u_A}{E_A}+(C_A+C_B-C-\delta)(i) \quad \ldots\ldots\ldots (145)$$

where $\Delta u = \bar{u} - u$.

Now, the experiment for finding (i) and (e) according to (144) and (145) was performed on three representative transits, namely—No. 4 and No. 5, in which the inclination and the eccentricity of the optical axis of the objective lens system are extraordinarily large, and No. 9, in which they are exceedingly small.

In Table 9, the results are given, from which it should readily be seen that the erroneous inclination of the optical axis and the erroneous eccentricity of the first principal point of the objective lens system referred to its ideal position, that is, (i) and (e) are so large beyond expectation that they may not be unconditionally negligible.

Table 9.

Experimental Note 3.

(July～Aug., 1935)

Transit No.	Δu_A, at $E_A = 2$ m., m.m.	Δu_D, at $E_D = 4$ m., m.m.	(i)		(e) m.m.
			Radian	′ ″	
4	−1.508	− 0.806	+0.002 555	+8 47	+0.379
5	+7.341	+13.485	−0.001 792	−6 10	− 0.380
9	+4.379	+11.599	+0.002 890	+9 56	+0.318

Other transits were not inspected, since the component lenses of the objective lens system were simply assembled with only air-spaces so that this experiment could not be performed.

31. Third Experiment. — Direct Measurement of the External Diameters of the Objective Slide and the Objective Lens. To measure directly the external diameters of the objective slide and the objective lens is not wholly meaningless for the surveying engineer, who is not aware of the mechanical workmanship in the manufactory at all.

In this experiment, in order to obtain the difference of the maximum and the minimum values, a Zeiss Optotest, reading to one micron, shown in Fig. 26, was adopted, though it was too accurate, because the workmanship and the quality of the instrument are due to the maker itself, and the measurements were performed with a hand-made back-rest and a hand-made base, which were assembled by the author with S. K. F. steel rollers of $^1/_2''$ size. But the diameters were simply measured by a scale with a vernier reading to one-twentieth m.m..

All observations were carried out at the standard temperature 20°C.

The results of the measurements are tabulated in Table 10, and the notations are illustrated in the figure therein, in which Δ shows the difference of the maximum and the minimum values.

These errors are principally due to the mechanical workmanship of the maker and theoretically analized into miscelaneous kinds from the standpoint of Mechanical Technology, that is to say — geometrical form, irregular surface, temperature effect, etc., according to Prof. Sawin's researches.

But because it is not our present subject to discuss such errors minutely, it will be sufficient to discriminate the quality of the workmanship by the second quality of fits prescribed in J. E. S., i. e., — Japanese Engineering Standard, whose results are mentioned in the last column of Table 10.

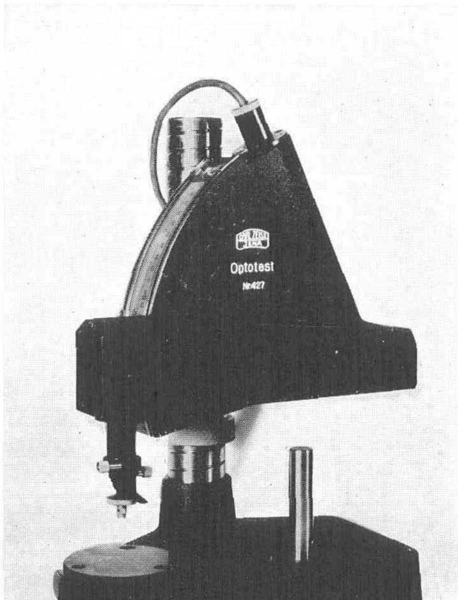
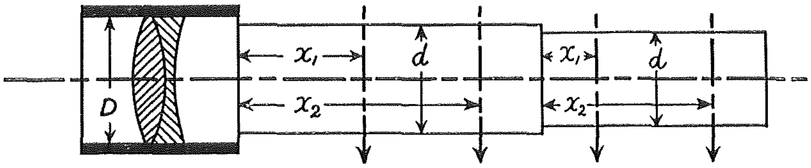


Fig. 26. Zeiss Optotest.

Table 10.

Experimental Note 4.

(Measured at 20°C, Feb., 1935)



Transit No.	D, ΔD, m.m.	W^*,	x_1, x_2, m.m.	d, Δd, m.m.	d, Δd, m.m.	d, Δd, m.m.	d, Δd, m.m.	Quality
1	31.80 0.0209	22	15 40	26.60 0.0022	26.60 0.0113	17.20 0.0174	17.20 0.0261	e_2
4	31.80 0.1237	91	7.5 30	28.45 0.0105	28.45 0.0187	17.00 0.0042	17.00 0.0056	f_2
5	32.00 0.0108	38	7.5 30	28.40 0.0048	28.40 0.0064	17.00 0.0034	17.00 0.0046	g_2
7	31.95 0.0760	53	7.5 30	28.45 0.0023	28.45 0.0094	17.00 0.0048	17.00 0.0036	g_2
9	32.00 0.0128	80	7.5 30	28.45 0.0091	28.45 0.0104	17.00 0.0023	17.00 0.0080	g_2
22			10 40	26.90 0.0098	26.90 0.0258	17.25 0.0057	17.25 0.0094	e_2
23	31.80 0.0095	25	10 40	26.90 0.0093	26.90 0.0119	17.25 0.0075	17.25 0.0081	g_2
37	31.60 0.0049	89	7 35	26.30 0.0085	26.30 0.0046	17.30 0.0120	17.30 0.0048	g_2
39	32.05 0.0275	38	7 35	28.35 0.0096	28.35 0.0280	17.00 0.0047	17.00 0.0026	e_2
11	21.50 0.0129	23	6 26	21.40 0.0000	21.40 0.0216	15.00 0.0074	15.00 0.0049	f_2
A1	21.50 0.0631	48	6 26	21.40 0.0032	21.40 0.0275	15.00 0.0072	15.00 0.0300	e_2
A2	21.45 0.0306	53	6 26	21.35 0.0076	21.35 0.0085	14.20 0.0020	14.20 0.0140	g_2

Notice: These variations of the diameters must be referred to εe_g and εe_{gr} mentioned in Table 11A of Art. 32.

* W is the angular divergence between the maximum and the minimum values of D.

Now, if there are defects due to mechanical workmanship, abraision and accidental hitches in the cylindrical tube of the objective slide and the slide bearings, then, because of the irregular variation of the eccentricities of the optical axis of the objective lens system at both the slide bearings caused by them, that is to say—the pitching and jolting motion of the objective slide when it is run out and drawn in, the irregular deviation of the collimation point, such as shown by (84), should be produced.

Hereupon, from (84), it can immediately be grasped that the finishing of the front slide bearing and the part of the objective slide, which fits into it, must be far more accurate than that of the adjustable rear slide bearing and the part of the objective slide, which fits into it, that is to say—the error in the centering of the former should virtually be made to from one-third to one-sixth of the latter; refer to (81) and Table 5 of Art. 26.

Now, it is found out that the exact relation

$$\left.\begin{aligned} \varDelta\varphi &= 311\,\varDelta e_g \quad \text{seconds, where} \quad e_g \text{ in m.m.,} \\ &= \ 85\,\varDelta e_{gr} \quad \text{seconds, where} \quad e_{gr} \text{ in m.m.,} \end{aligned}\right\} \quad \ldots\ldots \quad (167)$$

holds good among the mean variations of the deviations of collimation points and the eccentricities of the front and the rear slide bearings or $\varDelta\varphi$, $\varDelta e_g$ and $\varDelta e_{gr}$ in transits No. 4, No. 5 and No. 9, referred to the experimental results of Art. 32, which will be given in Art. 35.

Therefore, in order to minimize the above described angular deviation, the objective slide and the slide bearings must be finished according to the running fit of the highest quality.

Thereupon, it is tentatively proposed by the present author that, respecting their fits, the grade of quality g_2 is fair, f_2 tolerable and under e_2 must be rejected, according to the notations in J. E. S..

32. Fourth Experiment.—Experiment for Finding out the Mean Variations of the Angular Deviations of Collimation Points and the Eccentricities and the Inclination of the Optical Axis of the Objective Lens. If the variations in the eccentricity of the optical axis of the objective lens system at the front slide bearing and its inclination from the meridian of the instrument according to the point sighted are too large, the irregular variation of the deviation of the collimation point can not be negligible at all and moreover the adjustment of the vertical cross-hair can not be completed whatever method of adjustment may be chosen.

Therefore, though there is no method of finding their exact values absolutely, an attemps to find them out approximately will now be explained.

Since their magnitudes are in fact too small to be observed by an ordinary method, the special method of observation in which the readings represented by (128) and (127) are measured because of their great relative magnifying powers of the optical errors, should be utilized, so that the variations will be observed through the same apparatus shown in Fig. 24 and Fig. 25, in common with the second experiment described in Art. 29 and Art. 30. Accordingly the explanation should also be compared to that in Art. 29.

Now, the process in the present experiment may be described as follows :

First, adjust the vertical cross-hair by the method of Integral Adjustment of the horizontal cross-hair between $\overline{B}$ and $\overline{A}$. Just after it has been completed, sight at $\overline{B}$ with the telescope inverted, reverse it clamping the alidade horizontally *in situ* when $\overline{B}$ was sighted, and then take the reading of the compararator points A, D and P. Again do the same as before with the telescope normal ; then the differences of the respective two readings u_{AA}, u_{AD} and u_{AP} can be gotten.

Further, adjust anew the vertical cross-hair between $\overline{B}$ and $\overline{D}$ by the same method, perform likewise as before, and then the differences of the respective two readings at A, D and P, that is, u_{DA}, u_{DD} and u_{DF} can be obtained.

Further, perform similarly among point $\overline{B}$, $\overline{P}$, A, D and P over again, and then u_{PA}, u_{PD} and u_{PP} can be gotten.

Now, from (81), the relations among the above-obtained readings, the eccentricity of the optical axis of the objective lens system at the front slide bearing, ditto at the rear slide bearing and the angular deviation of the collimation point from the meridian of the transit may be written down as follows :

$$\varphi_{A,\,p} = (j_A + y_A + w_A)e_{g,\,p} - w_A e_{gr,\,p} - j_A z_A \,, \quad \cdots\cdots\cdots\cdots \quad (146)$$

$$\varphi_{D,\,p} = (j_D + y_D + w_D)e_{g,\,p} - w_D e_{gr,\,p} - j_D z_D \,, \quad \cdots\cdots\cdots\cdots \quad (147)$$

$$\varphi_{P,\,p} = (j_P + y_P + w_P)e_{g,\,p} - w_P e_{gr,\,p} - j_P z_P \,, \quad \cdots\cdots\cdots\cdots \quad (148)$$

$$\varphi_{q,\,p} = \frac{u_{q,\,p}}{2E_p} - \frac{u_{q,\,q}}{4E_q} \,, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (149a)$$

$$\left.\begin{aligned} j_p &= \frac{E_p-C_p}{E_p(C-\delta)}, \qquad y_p = \frac{1}{E_p}, \\ &\qquad\qquad\qquad (p = \text{A, D, P, B.}) \\ w_p &= \frac{C_p-g}{E_p\zeta} - \frac{E_p-C_p}{E_p(C-\delta)}\,\frac{\delta-g}{\zeta}. \end{aligned}\right\} \quad (149\text{b})$$

Hereupon, since the variations of φ, e_g and e_{gr} produced according to the point sighted are originally caused by defects of finishing, by abraision and by accidental injury of the objective slide and their slide bearings, so that their magnitudes are excedingly small, and moreover the variations of the reading difference u are due to their variations, they may substantially be considered as like the compensating errors.

Therefore, it should be safe to take it that the variations of φ, e_g and e_{gr} are approximately given by their mean square errors when the mean of five pairs of reading differences are taken respectively, because the error coming into φ, caused from the observation of u itself, can be reduced under 0.000 000 5 radian or 0.1 second, by which the solutions of the variations of e_g and e_{gr} should not be disturbed.

Respecting the errors due to the observations of u, refer to the results of those shown in Table 6 of Art. 26 above.

Therefore, from (146) (147) and (148), adding their respective sides referred to points A, D, P and B severally and transforming into the convenient forms by taking the mean, which can readily be dealt with as the observation equations, the four new relations

$$\varphi_{A,\,p} = (i_A+y_A+w_A)e_{g,\,A} - w_A\, e_{gr.\,A} - j_A z\,, \quad \ldots\ldots\ldots \quad (150\text{a})$$

$$\varphi_{D,\,p} = (j_D+y_D+w_D)e_{g,\,D} - w_D\, e_{gr.\,D} - j_D z\,, \quad \ldots\ldots\ldots \quad (150\text{b})$$

$$\varphi_{P,\,p} = (j_P+y_P+w_P)e_{g,\,P} - w_P\, e_{gr.\,P} - j_P z\,, \quad \ldots\ldots\ldots \quad (150\text{c})$$

$$\varphi_{B,\,p} = (j_B+y_B+w_B)e_{g,\,B} - w_B\, e_{gr.\,B} - j_B z \quad \ldots\ldots\ldots \quad (150\text{d})$$

can be reduced, where

$$\left.\begin{aligned} 3\,\varphi_p &= \varphi_{A.\,p}+\varphi_{D.\,p}+\varphi_{P.\,p}\,, \quad (p = \text{A, D, P.}) \\ 3\varphi_B &= \varphi_{A.\,A}+\varphi_{D.\,D}+\varphi_{P.\,P}\,, \\ 3\,z &= z_A+z_D+z_P\,. \end{aligned}\right\} \;\ldots\; (150\text{e})$$

Now, the weights of (150a) (150b) (150c) and (150d) are unity respectively, because the original form of an observation equation for the deviation of collimation point p from the meridian is

$$E_p \varphi_p = \frac{1}{3} \sum \left\{ \frac{u_{q,p}}{2} - \frac{u_{q,g}}{4} \right\}, \quad (q = \text{A, D, P, B.}) \ \dots \ (151)$$

referred to (149a) and (150e), and accordingly its weight should be

$$\omega_{E\varphi,\, p} = \frac{1}{E_p^2}. \quad \dots\dots\dots\dots\dots\dots\dots\dots \ (152)$$

Hence, considering (150a) (150b) (150c) and (150d) as the observation equations with respect to the unknown e_g, e_{gr} and z, and the weighted measurements φ_A, φ_D, φ_P and φ_B, the nornal equations

$$[(j+y+w)(j+y+w)]\,e_g - [(j+y+w)w]\,e_{gr} - [(j+y+w)j]\,z = [(j-y-w)\varphi] \quad \dots\dots\dots \ (153a)$$

$$-[w(j-y-w)]\,e_g + \quad [ww]\,e + \quad [wj]\,z = -[w\varphi] \quad \dots\dots\dots \ (153b)$$

$$-[j(j-y-w)]\,e_g + \quad [jw]\,e + \quad [jj]\,z = -[j\varphi] \quad \dots\dots\dots \ (153c)$$

can be found out by Method of Least Squares.

Then, denoting the discriminant of (135a) (135b) and (135c) by

$$\vartheta = \begin{vmatrix} [(j+y+w)(j+y+w)] & [(j+y+w)w] & [(j+y+w)j] \\ [w(j+y+w)] & [ww] & [wj] \\ [j(j+y+w)] & [jw] & [jj] \end{vmatrix}$$

$$= \begin{vmatrix} [yy] & [yw] & [yj] \\ [wy] & [ww] & [wj] \\ [jy] & [jw] & [jj] \end{vmatrix}$$

$$= \frac{1}{\zeta^2(C-\delta)^2} \begin{vmatrix} \left[\frac{1}{E}\frac{1}{E}\right] & \left[\frac{1}{E}\frac{\varDelta C}{E}\right] & \left[\frac{1}{E}\right] \\ \left[\frac{\varDelta C}{E}\frac{1}{E}\right] & \left[\frac{\varDelta C}{E}\frac{\varDelta C}{E}\right] & \left[\frac{\varDelta C}{E}\right] \\ \left[\frac{1}{E}\right] & \left[\frac{\varDelta C}{E}\right] & [1] \end{vmatrix} \quad \dots\dots\dots \ (154)$$

and solving (153a) (153b) and (153c), the mean eccentricity of the optical axis of the objective lens system at the front slide bearing becomes

$$e_g = \frac{1}{\vartheta}\begin{vmatrix} [y\varphi] & [yw] & [yj] \\ [w\varphi] & [ww] & [wj] \\ [j\varphi] & [jw] & [jj] \end{vmatrix}$$

$$= \frac{1}{\vartheta\zeta^2(C-\delta)^2}\begin{vmatrix} \left[\frac{1}{E}\varphi\right] & \left[\frac{1}{E}\frac{C-g}{E}\right] & \left[\frac{1}{E}\frac{E-C}{E}\right] \\ \left[\frac{\Delta C}{E}\varphi\right] & \left[\frac{\Delta C}{E}\frac{C-g}{E}\right] & \left[\frac{\Delta C}{E}\frac{E-C}{E}\right] \\ [\varphi] & \left[\frac{C-g}{E}\right] & \left[\frac{E-C}{E}\right] \end{vmatrix}, \quad \ldots (155)$$

ditto at the adjustable rear slide bearing becomes

$$e_{gr} = e_g + \frac{1}{\vartheta}\begin{vmatrix} [y\varphi] & [yy] & [yj] \\ [w\varphi] & [wy] & [wj] \\ [j\varphi] & [jy] & [jj] \end{vmatrix}$$

$$= e_g - \frac{1}{\vartheta\zeta(C-\delta)^2}\begin{vmatrix} \left[\frac{1}{E}\frac{1}{E}\right] & \left[\frac{1}{E}\varphi\right] & \left[\frac{1}{E}\frac{E-C}{E}\right] \\ \left[\frac{\Delta C}{E}\frac{1}{E}\right] & \left[\frac{\Delta C}{E}\varphi\right] & \left[\frac{\Delta C}{E}\frac{E-C}{E}\right] \\ \left[\frac{1}{E}\right] & [\varphi] & \left[\frac{E-C}{E}\right] \end{vmatrix} \quad \ldots\ldots (156)$$

and the departure of the cross-point of the cross-hairs from the meridian becomes

$$z = e_g - \frac{1}{\vartheta}\begin{vmatrix} [yy] & [yw] & [y\varphi] \\ [wy] & [ww] & [w\varphi] \\ [jy] & [jw] & [j\varphi] \end{vmatrix}$$

$$= e_g - \frac{1}{\vartheta\zeta^2(C-\delta)}\begin{vmatrix} \left[\frac{1}{E}\frac{1}{E}\right] & \left[\frac{1}{E}\left(\frac{\delta-g}{C-\delta}+\frac{\Delta C}{E}-\frac{\Delta C}{E}\frac{\delta-g}{C-\delta}\right)\right] & \left[\frac{1}{E}\varphi\right] \\ \left[\frac{\Delta C}{E}\frac{1}{E}\right] & \left[\frac{\Delta C}{E}\left(\frac{\delta-g}{C-\delta}+\frac{\Delta C}{E}-\frac{\Delta C}{E}\frac{\delta-g}{C-\delta}\right)\right] & \left[\frac{\Delta C}{E}\varphi\right] \\ \left[\frac{1}{E}\right] & \left[\frac{\delta-g}{C-\delta}+\frac{\Delta C}{E}-\frac{\Delta C}{E}\frac{\delta-g}{C-\delta}\right)\right] & [\varphi] \end{vmatrix},$$

$$\ldots\ldots\ldots (157)$$

where $\Delta C = C_p - C$.

Now, since the changes of e_g and e_{gr} due to the collimation point are substantially independent of z but depend on the errors in the adjustment of the vertical cross-hair and the observation, which are virtually reduced to imperceptibly small magnitudes, their weights must really be computed from only the normal equations (153a) and (153b).

Therefore, from (153a) and (153b) the weights of e_g and e_{gr}, namely—

$$\varpi_{e_g} = \frac{\Delta}{[ww]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (158a)$$

and

$$\varpi_{e_{gr}} = \frac{\Delta}{[(j+y+w)(j+y+w)]}, \quad \ldots\ldots\ldots\ldots \quad (158b)$$

where

$$\Delta = \begin{vmatrix} [(j+y+w)(j+y+w)] & [(j+y+w)w] \\ [w(j+y+w)] & [ww] \end{vmatrix}, \quad \ldots \quad (158c)$$

are exactly found, where the notations must be referred to (149b) in the present article.

Thereupon, putting the values of e_g, e_{gr} and z gotten from (155), (156) and (157) in (150a), (150b), (150c) and (150d), the residuals

$$\left.\begin{aligned} v_A &= (j_A+y_A+w_A)e_g - w_A e_{gr} - j_A z - \varphi_A, \\ v_D &= (j_D+y_D+w_D)e_g - w_D e_{gr} - j_D z - \varphi_D, \\ v_P &= (j_P+y_P+w_P)e_g - w_P e_{gr} - j_P z - \varphi_P, \\ u_B &= (j_B+y_B+w_B)e_g - w_B e_{gr} - j_B z - \varphi_B, \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (159)$$

are obtained.

Now, since φ_q is the mean of three $\varphi_{q,p}$, compare (150e), it should be free fairly from errors in observation and the mean square error of $\varphi_{g,p}$

$$\varepsilon_\varphi = \pm \sqrt{[vv]} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (160)$$

is gotten.

Therefore, from (158a), (158b), (81) and (79a), are obtained respectively the mean square error of e_g

$$\varepsilon_{e_g} = \pm \sqrt{\frac{[vv]}{\varpi_{e_g}}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (161a)$$

that of e_{gr}

$$\varepsilon_{e_{rg}} = \pm \sqrt{\frac{[vv]}{\varpi_{e_{gr}}}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (I61b)$$

that of e

$$\varepsilon_e = \pm\sqrt{\left(1+\frac{g}{\zeta}\right)^2 \varepsilon_{e_g}^2 + \left(\frac{g}{\zeta}\right)^2 \varepsilon_{e_{gr}}} \qquad \text{(162a)}$$

and that of i

$$\varepsilon_i = \pm\sqrt{\frac{\varepsilon_{e_g}^2 + \varepsilon_{e_{gr}}^2}{\zeta}}. \qquad \text{(162b)}$$

The notations must also be referred to those explained at the beginning of this paper.

Now, it may probably be safe to say that the mean square errors of φ, e_g and e_{gr} given by (160), (161a), (161b), (162a) and (162b) may approximately be taken for their actual mean variations, as it is ratiocinated in the preceding part of the present article.

Accordingly, the mean variation of the deviation of the collimation point should readily be calculated from (160).

As described above, the present experiment for finding the mean variations of φ, e_g and e_{gr} was performed using the apparatus explained in Art. 29 in common with the second experiment on three representative transits, namely—No. 4, No. 5 and No. 9. For these instruments e_g and i of the first two are extremely large and those of the latter are extraordinarily small, as shown in the experimental results in Table 6, Art. 26.

The results gotten from the present experiment are shown in Table 11A and Table 11B.

Table 11A.

Experimental Note 5A.

(July~Aug., 1935)

Transit No.	ε_φ, Radian Seconds	ε_{e_g}, m.m.	$\varepsilon_{e_{gr}}$, m.m.	ε_e, m.m.	ε_i, Radian Seconds	Mean Variation of Deviation at 100 m., m.m.
4	±0.000 003 6 ±0.8	±0.002 4	±0.008 8	±0.008 2	±0.000 082 9 ±17.1	±0.36
5	±0.000 011 3 ±2.3	±0.007 5	±0.027 4	±0.023 9	±0.000 258 4 ±53.3	±1.13
9	±0.000 002 5 ±0.5	±0.001 6	±0.006 0	±0.005 6	±0.000 056 6 ±11.7	±0.25

Notice: ε denotes the mean square error and the suffix shows the related quantity. Further, compare ε_{e_g} and $\varepsilon_{e_{gr}}$ with Δd in Table 10 shown in Art. 31.

Table 11B.

Experimental Note 5B.

(July~Aug., 1935)

Transit No.	E,* m.	$C_p - C$, † m.m.	Reading u in m.m.		
			at 2 m.	at 4 m.	at 6 m.
	2	14.72	+2.309	+ 4.482	+ 6.739
4	4	6.77	+1.972	+ 3.693	+ 5.548
	6	4.40	−0.019	− 0.368	− 0.451
	2	14.72	+1.495	+ 3.219	+ 4.847
5	4	6.77	+0.212	+ 0.438	+ 0.280
	6	4.40	+5.480	+11.396	+17.096
	2	14.72	−1.290	− 2.694	− 3.921
9	4	6.77	−1.789	− 3.649	− 5.547
	6	4.40	−0.826	− 1.690	− 2.432

Remark: It should especially be noted that the disagreement between the measurements shown in Table 6 of Art. 26 and those in Table 11B should depend on the very changes caused by the dis-jointing of instruments in the second, the third, the fourth experiments, etc., the practical exercies for Integral Adjustment shown in Art. 43, the influence of temperature, the students' use, etc.

* The distance of the near point, between which and the distant point at 138.3 m. the vertical cross-hair was adjusted by the method of Integral Adjustment of the horizontal cross-hair.

† The displaced distance of the objective lens for focussing.

SECTION IV.—CRITERIA FOR THE GRADE OF QUALITY OF A TRANSIT

33. Criterion of the Grade of Quality of a Transit. Since the transit is a precise optical instrument, it should be inspected from the standpoint of Geometrical Optics and Mechanical Technology when it is adopted; but on the contrary, it has never been found up to the present by the author from the narrow scope of his knowledge that it is actually done. Nevertheless, for the future, it must be practised.

Hereupon, in order to discriminate between grades of qualities of transits, an approximate standard criterion should be proposed for convenience sake, with reference to the experimental results shown in Arts. 26, 27, 28, 30, 31 and 32.

Now, if the transit is adjusted by the method of One-Quarter Adjustment between the two infinitely distant points N and $\overline{N}$, then the angular divergence of any collimation point p from the meridian of the instrument

$$\varphi_p = \frac{e_g+(C_p-g)i}{E_p} \frac{E_N-E_p}{E_N} \quad \ldots\ldots\ldots\ldots\ldots \quad (163)$$

is obtained from (110), neglecting terms of the higher order.

In the above formula for the angular deviation, from the standpoint of the mechanical construction of the telescope and the standards, it is neccessary and sufficient that each of them does not cause any deleterious error in a measured horizontal angle, though the errors e_g and $(C_p-g)i$ shall be partially dependent upon each other.

These antipathic errors become greater principally due to the instrumental defects and especially, the accidental impediments during practical use, although, of course by their very nature, they are partially caused by imperfect workmanship and construction in the manufactory.

Taking account of the allowed errors in triangulation survey in the prescription for River Surveying of the Department of Home Affairs and tolerating the largest error of ten seconds in an angle since it occurs only in one direction, the optical grade of quality of the transit may tentatively be classified into four classes as shown in Table 12.

Table 12.

Criterion A.

Grade	e_g, m.m.	i		E_p, m.		
		Radian	Minutes	$\varphi=5''$	$\varphi=10''$	$\varphi=20''$
Excellent, A						
	0.2	0.001	3	8	4	2
Good, B						
	1.0	0.005	17	29	17	9
Tolerable, C						
	3.5	0.0175	60	59	42	27
Too large, D						

Remark: It is taken that $C-g=200$ m.m. and the vertical cross-hair is adjusted by the method of One-Quarter Adjustment at the distance of 100 m.

E_p is the distance, at which the designated deviation φ must be produced by the given e_g or i.

Now, even if the inclination of the optical axis of the objective lens or i can be adjusted by the method of Integral Adjustment of the vertical cross-hair expounded in Section VI of this chapter so that the relation

$$e_g+(C_J+C_N-C-g)i_{\varphi=0}=0 \quad \text{or} \quad \varphi_N=\varphi_J=0$$

may practically hold good, refer to (168b), when point p is sighted, the magnitude of i should be discriminated by the above criterion.

The grade mentioned in Table 6 of Art. 26 depends on the present criterion.

34. Criterion for the Eccentricity and the Inclination of the Optical Axis of the Objective Lens System Itself. Now, even though in the second experiment, other transits than the three representative ones could not be inspected because of the air-spaced construction of their objective lens systems, the general conception of the approximate magnitude of the eccentricity and the inclination of the optical axis of the objective lens system itself or (e) and (i) should readily be obtained from the experimental results shown in Table 6 of Art. 26 and in Tables 11A and 11B of Art. 32.

Accordingly, referred to Table 12 of Art. 33, their criteria may probably be taken as shown in Table 13.

Table 13.

Criterion B.

Grade	(e), m.m.	(i) Radian	(i) Minutes
Excellent, A			
	0.1	0.001	3
Good, B			
	0.3	0.003	10
Tolerable, C			
	1.0	0.010	30
Too large, D			

In the above Table, the criterion for (i) does not seem to be too strict, but the inclination of the optical axis of the objective lens from the meridian or i can voluntarily be adjusted by the method of Integral Adjustment as expatiated and illustrated in Section VI below, so that its influence on the measurement of the horizontal angle may be minimized, that is to say—it is the very subject of the present work.

Again, if it is desired that the influence of (e) and (i), shown by (85) and (87), be made completely to disappear, it can simply be attained by finding out the aspect of the objective lens according to the theory studied in Art. 18, that is, the principle shown by (86), whose detailed process will be explained in Art. 71 below.

Further, these errors will probably be eliminated to the extent of neccessity, if, for the purpose of centering the objective lens system, the ordinary optical method* is adopted with care and skill as usual, which can exactly be performed with facility.

Now, if there were no other error but (i) and (e) in the telescope, then exactly, the deviation of the first focal point F, or approximately, that of the point V explained in Art. 20 should be given by the formula

$$y_V = (e) + f(i)\,, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (164)$$

referred to (101) and (135) respectively, when point p is sighted, where

$$i = (i)\,, \qquad e = -(C_p - f)(i) + (e)\,.$$

Hereupon, if $f = 180$ m.m., $(i) = 3/1{,}000$ and $(e) = 0.3$ m.m., then the maximum limit of the value of y_V should be given by

$$y_V = 0.9 \text{ m.m.}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (165)$$

referred to (164), which causes an angular error of $20''$ at the distance of 10 m. and is fairly harmful to the angular measurement.

Further, from the results of the second experiment shown in Table 9 of Art. 30, it can readily be seen that the erroneous inclination (i) has virtually no influence on the theory, from which the method of Integral Adjustment has been able to be ratiocinated in the present work.

Putting together these facts, and referring to Table 6 of Art. 26, it can be concluded that the inclination and the eccentricity of the optical axis of the objective lens system from the meridian of the instrument are mainly caused by accidents to the standards during practical use, because those of the new or unused transit are small, and those of the transit used for many years large; while, on the contrary, those of the transit newly repaired by the manufactory small though it has been used for a long time. That is to say, those due to the errors in centering and finishing or to the abraision of the surfaces of the objective slide and the slide

* Refer to Reference (20) at the end of the present work, or "Zeiss" Ueber den Werdegang eines photographischen Objektives, s. 18~19. which will be described in English at the end of Art. 52.

bearings can be thought to be negligibly small compared with those due to the preceding secondary defects of the standards.

As seen in Table 4 and Table 6 of Art. 26, transit No. 1 and No. 23 are unused and A3 is absolutely new but, No. 9 had been recently repaired; while, on the contrary, No. 4 and No. 5 had been used for practical exercises of students for about ten years.

From these practical examples, the above explanation can readily be understood.

Now, since, from these grounds, the objective slide is run out and drawn in with its axis practically parallel to the optical axis of the objective lens system so that the conditions for adjustability, that is, (166), which shall be proved in Art. 35 below, are usually fulfilled, it is safe to say that the cross-hair of a transit can integrally be adjusted in general.

SECTION V.—"THE CONDITIONS FOR ADJUSTABILITY"

35. "The Conditions for Adjustability." Likewise as in Art. 5 and Art. 9, the conditions for adjustability of the horizontal cross-hair in a transit is proved, those of the vertical cross-hair must also be discussed in this article in detail, before the method of Integral Adjustment shall be studied.

Since the accuracy of the mechanical workmanship of all the parts of the objective slide and the slide bearings in the manufactory should be the same in general at present, the deviation of a collimation point is far more predominatingly controled by e_g than by e_{gr}, (refer to (81) and (84)). That is to say, in order to obtain the highest degree of accuracy, the front slide bearing and the part of the objective slide, which fits into the former, must be constructed precisely and finished so that the eccentricity of the optical axis of the objective lens system at the front slide bearing and its variation will be reduced to from one-third to one-sixth of that of the adjustable rear slide bearing and the part of the objective slide, which fits into it, from (81) and (84).

Now, if the deviation of a collimation point p from the meridian of the transit varies irregularly according to the point sighted, the adjustment of the vertical cross-hair changes strikingly with the point selected for the adjustment.

Therefore, it must be a linear function of only the distance Ep to the end that can be uniquely adjusted, that is to say—referring to (78), (79a), (79b) (80), (81) and (84), **the practical "Conditions for Adjustability"**

$$\left.\begin{aligned} e_g &= \text{a small magnitude or zero,}\\ e &= \text{a small magnitude or zero,}\\ i &= \text{a small magnitude or zero,}\\ k &= \text{a small magnitude or zero,}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (166)$$

must be exactly fulfilled, so that they may not produce any deleterious error in a measured horizontal angle after the vertical cross-hair is integrally adjusted.

Hereupon, besides the above conditions, the secondary ones should properly be derived from them, through which their variations according to the collimation point must not extend over such limits that they may produce the detrimental irregular deviation of the collimation point.

Now, because there is no exact method of finding out the variation of the angular deviation of the collimation point and the eccentricities of the optical axis of the objective lens system at the front and rear slide bearings, the fourth experiment explained in Art. 32 and Art. 29 is at present the unique reliable and approximate mean of ascertaining to what limit the above conditions are virtually fulfilled, in which the mean square errors E_φ, E_{e_g} and $E_{e_{gr}}$ are considered as the mean variations of the above quantities respectively.

Although, of course, they are not accurate mean variations, their approximate tendencies and extents may probably be grasped from them.

Now an approximate criterion for the variation of the deviation of a collimation point may tentatively be given for convenience sake as in Table 14, for comparison with Table 12 of Art. 33. in which the maximum limit of "Tolerable" is taken as 2″ or 0.000 010 radian, because 3 m.m.

Table 14.

Criterion C.

Grade of Quality	ε_φ, Radian	ε_φ, Seconds
Excellent, A		
	0.000 002 5	0.5
Good, B		
	0.000 005 0	1.0
Tolerable, C		
	0.000 010 0	2.0
Too large, D		

can commonly be read at the distance of 100 m. through the ordinary telescope.

Now, the angular deviation of the collimation point p from the meridian, that is, $\Delta\varphi$ may virtually be shown by the variation of the eccentricity of the optical axis of the objective lens system at the front slide bearing and also that at the rear slide bearing or Δe_g and Δe_{gr} respectively, referred to (84), though the finishing of all the parts of the objective slide and the slide bearings are the same in practice in the manufactory.

Hence, putting together the experimental results set down in Table 11A of Art. 32, Table 5 of Art. 26, (161a) and (161b), the theoretical relation

$$\left.\begin{aligned}\Delta\varphi &= 311\,\Delta e_g \quad \text{seconds,} \quad \Delta e \quad \text{in m.m.,}\\ &= \;\;85\,\Delta e_{gr} \quad \text{seconds,} \quad \Delta e_{gr} \; \text{in m.m.,}\end{aligned}\right\} \quad \cdots\cdots\cdots \quad (167)$$

is commonly determined for transits No. 4, No. 5 and No. 9

Now, from Table 11A of Art. 32, it can be seen at once that the errors amounting up to such extents are so small that they can be positively neglected in ordinary works, except for a transit of extremely poor workmanship or design and that damaged severely, which may often be found.

Finally, it can be clearly concluded that the ordinary transit can stand this criterion in general, insofar as it has no extraordinary disability, putting together the experiments for the adjustment of the horizontal cross-hair described in Art. 5 or in p. 19, 20, 25, 30 and 31 of the author's first paper, and the new experiment described in Art. 32.

Section VI.—Methods of Integral Adjustment of the Vertical Cross-Hair in a Transit

36. General Principle of Integral Adjustment. Theoretically, whatever point may be sighted, it should always lie on the meridional plane of the transit, if the instrument is ideally perfect in construction and completely adjusted by the existing method.

But, since the transit is a manufactured product, it can not be free from mechanical errors, which can not be eliminated only by the present method of adjustment of the vertical cross-hair in the manufactory and the field, regulating the parts of the instrument, as demonstrated by the results of the experiments in the preceding articles.

Now, as soon as the vertical cross-hair has been adjusted by the method of One-Quarter Adjustment between the two distant points or very near points $\bar{N}$ and N, sight at a very near or distant point J correlative to point N with the telescope inverted, still tightly clamped *in situ* when N was sighted, draw in or run out the objective lens and then take the reading of the rod or the scale held at point J,

Again sight at N with the telescope normal, perform the same as before, and then take the difference of the first and the second readings, that is, t_J .

Hereupon, if $t_J = 0$, the instrument is to be considered perfect, that is, $e_g = 0$ and $i = 0$, or $e_g+(C_J+C_N-C-g)i = 0$ referred to (126) or (110), each of which can not actually be expected to occur except by chance.

Further, up to now, from lack of knowledge and research in Geometrical Optics, the vertical cross-hair has not only radically been misadjusted by the existing methods and a pair of centering tools or "Shin-Gané" (心金) in the manufactory in Japan, but also, in the field, it can not be adjusted by the surveying engineer at all, for the reasons adduced in Section XI, and accordingly, in general, $t_J \neq 0$ or $e_g+(C_J+C_N-C-g)i \neq 0$ from (126) or (110) because of $e_g \neq 0$ and $i \neq 0$.

Above all, it should especially be noticed that even if the centering tool or "Shin-Gané" (心金) is generally adopted for the adjustment of the objective slide as the most reliable and exact method in the manufactory at present, it is by its very nature nothing but a pair of tools by which the objective slide and the standards must mechanically be pieced together. It should not be used for the adjustment of the objective slide.

Now, since the erroneous eccentricity and the erroneous inclination of the optical axis of the objective lens system itself, the errors caused from the metal parts and those due to the centering, are entirely neglected now, among which the first should be the greatest and the most injurious as will made clear by the experiment described in Art. 30 and Art. 62, the fundamental principle of the method of adjustment by the centering tool must be radically be misunderstood,

Respecting the eccentricity and the inclination of the optical axis of the objective lens itself, refer to Table 11A of Art. 30 and Table 21 and Table 22 of Art. 62.

For that reason, an injurious error is often discovered in a new transit just come from the manufactory.

For the practical proofs, note that transit No. 1 is an unused one and No. 22 has been only used for less than one month, nevertheless, as in

Table 6 of Art. 26, $e_g = -0.36$ m.m. and $i = +5'\ 06''$ in the former while $e_g = -2.61$ m.m. and $i = -32'\ 46''$ in the latter, which can not be considered as tolerably small, especially in the latter. Further, for the values of the factor $e_g+(C_J+C_N-C-g)i$ refer to Table 7 of Art. 27.

Therefore, as described at the beginning of this chapter, in order to eliminate the influences of the mechanical errors coming into the measured angle, a new method of adjustment must be adopted, which is composed of the two correlative processes :

First, the present adjustment of the vertical cross-hair, and secondly, the new adjustment of the rear slide bearing* of the objective slide of the telescope. These adjustments must be repeated in the above order until they are completed.

Now, from (77), if the vertical cross-hair is adjusted by the method of One-Quarter Adjustment between $\bar{N}$ and N, it can plainly be seen that there are no more than two point J and N in the direction of N, whose deviations from the meridian of the instrument can be made to disappear perfectly. This is true because only the position of the vertical cross-hair and the inclination of the optical axis of the objective lens, both referred to the meridian of the transit, that is to say—only the two parameters k and i, can optionally be adjusted so that they may satisfy the condition for k at point N

$$\varphi_N = 0\,, \quad \text{when} \quad E_N = E_{\bar{N}}\,,$$

and that for i at point J

$$\varphi_J = 0$$

respectively, referred to (78).

Hence, from (110) and (111), **the general formula for the deviation of the collimation point p**

$$E_p\varphi_p = \left\{e_g+(C_p+C_N-C-g)i_0\right\}\frac{E_N-C_N-E_p+C_p}{E_N-C_N}$$

$$= \left\{e_g+(C_p+C_N-C-g)\,i_0\right\}\frac{C_p-C_N}{C_p-C} \quad \ldots\ldots\ldots \quad (168a)$$

and from it and the above second condition, **the new conditions**

$$e_g+(C_J+C_N-C-g)i_0 = 0\,, \quad \varphi_J = \varphi_N = 0 \quad \ldots\ldots\ldots \quad (168b)$$

* It is called the "Tsuri-Ring" in the factories in Japan.

are exactly gotten, that is to say—they are in fact no other than the splendid and long-desired "Conditions for Integral Adjustment of the vertical cross-hair," by which it can be realized for the first time.

Hereupon, (168a) represents the very present condition, which must generally be satisfied by an engineer in the field, performing the method of One-Quarter Adjustment between $\bar{N}$ and N.

Further, the new condition (168b) must not only be fulfilled perfectly in correlation to (168a) by the actual shop adjustment but also in the field by an engineer when a transit is newly purchased, used for years, or subjected to an accident.

Therefore, **the correction for the inclination of the optical axis of the objective lens system**

$$\Delta i = i_0 - i$$
$$= \frac{E_N - C_N}{E_J C_N - E_N C_J - (E_J - E_N) g} \frac{t_J}{2} \quad \ldots\ldots\ldots\ldots\ldots (169)$$

is obtained from (168b) and (110) or (126), where $t_J = 2E_J \varphi_J$.

Hence, **the very correction of the reading of the scale or the rod at J**

$$\Delta j = \left\{ C_J - g + \frac{\delta - g}{C - \delta}(E_J - C_J) \right\} \frac{E_N - C_N}{E_J C_N - E_N C_J - (E_J - E_N) g} \frac{t_J}{2} \quad \ldots\ldots\ldots (170)$$

can be derived from (169) and (88), where $t_J = 2E_J \varphi_J$.

Now, up to the present, e_g and i are merely taken for the invariants without reference to the point sighted for simplicity of explanations and proofs.

But, strictly speaking, both are really variable according to the point sighted, and accordingly there should properly be a superfluous deviation due to the variations of e_g and i produced when point J is sighted, referred to those when point N is sighted, and therefore it must also be nil at point J.

But, **as a matter of fact, the neccessary correction should fortunately be given by (159) and (170); because the above superfluous deviation, namely—**

$$2(e_J - e_N) + 2E_J(i_J - i_N) - 2\frac{E_J - C_J}{f}(k_J - k_N)$$

referred to (78), may approximately be taken for an invariant with respect to i at every step of adjustment, so that it should properly exert an

influence on e_g as if it were increased by the amount which causes the same superfluous deviation, and the reading t_J, in which it is certainly included, can be made to disappear by adjusting only i.

This reasoning will be proven explicitly by the illustrative experiments for Integral Adjustment described below in Art. 43 for the transit and in Art. 66 for the wye level.

From this reasoning, the present conventional explanation, that e_g and i are invariable according to the point sighted, has no influence on the correction for Integral Adjustment given absolutely by (170).

Hereupon, since similar arguments have already been found in Art. 5 and Art. 9 when the true corrections (43) and (62) were reduced from the true deviations (42) and (59) respectively, the reader should also refer to those articles.

In Table 15, the angular corrections of the optical axes of the objectives of the transits used for the present experiments are given which are computed from i given in Table 6 of Art. 26 and i_0 gotten from (168b) when it is adjusted by the method of Integral Adjustment between 2m., 10C or 20C and 137.3 m. respectively.

Table 15.

The Correction for the inclination of the optical axis.

Transit No.	Δi, When integrally adjusted between 137.3 m. and 2 m. ′ ″	When integrally adjusted between 137.3 m. and 10 C.* ′ ″	When integrally adjusted between 137.3 m. and 20 C.* ′ ″
1	+ 5 06	+ 5 20	+ 5 30
4	− 4 53	− 6 55	− 9 23
5	− 5 15	− 6 34	− 8 10
7	−10 28	−10 16	−10 00
9	+ 2 26	+ 2 33	+ 2 42
22	+ 1 31	+ 3 16	+ 4 20
23	+ 1 37	+ 1 33	+ 1 30
37	− 2 47	− 3 34	− 4 02
39	+ 3 08	+ 1 20	− 05
11	− 1 32	− 1 11	− 32
A1	− 06	+ 22	+ 1 09
A2	− 1 23	− 2 01	− 2 59

* C is the so-called instrumental constant.

Hereupon, granting that various methods of Integral Adjustment can be conceived, detailed explanations of five representative ones among them, which can be practised, will be given in order in the following articles.

Again, it is repeated that the methods of Integral Adjustment of the vertical cross-hair must be performed on a transit newly acquired on the market, occasionally on one used for years in practice and properly on one which has been subject to an accident.

37. The Bi-axial Conditions. Because **the idea of the biaxial conditions, as they are called, is often extremely mistaken,** an exact detailed explanation should be given here.

Now, the conditions are two, as follows :

First. The condition for the horizontal axis of the telescope, that is to say, that the locus of collimation points must be exactly perpendicular to that axis.

Second. The condition for the vertical axis of the transit, namely —that the locus of collimation points must pass accurately through the cenrer of rotation of the telescope.

Now, if the vertical cross-hair is adjusted by the method of One-Quarter Adjustment between the two equi-distant points N and $\overline{N}$, then, as it is called, **the "Condition for the horizontal axis of the telescope"**

$$\varphi_N = 0 = \varphi_{\overline{N}}, \quad \text{when} \quad E_N = E_{\overline{N}}, \quad \ldots\ldots\ldots\ldots \quad (105)$$

should just be fulfilled, or points N and $\overline{N}$ are exactly brought on the meridian of the instrument. Also, refer to Art. 21.

Further, if the vertical cross-hair is adjusted by the method of Integral Adjustment of the horizontal cross-hair between N and J, then **the so-called "Condition for the vertical axis of the transit"**

$$\varphi_N = \varphi_J \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (115)$$

should be precisely satisfied, or N and J would be exactly brought on to the straight line passing through the centre of rotation of the telescope O. Also, refer to Art. 22.

But, it must especially be noted here that if the vertical cross-hair were adjusted by the method of One-Quarter Adjustment between the two unequal-distant points B and $\overline{A}$, then the undesirable relation

$$\varphi_B = -\varphi_{\overline{A}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (102)$$

should not only be fulfilled, (refer to Art. 21) namely—B and $\overline{A}$ dislocated out of the meridian, but also the locus of collimation points would not pass through the center of rotation of the telescope because the condition (105) can not be satisfied at all. See Fig. 19 of Art. 21.

Now, strictly speaking, the locus of collimation points is not a straight line, which shall afterward be proved in Art. 46 and Art. 48 in detail, unless the instrument has no error, that is, $e = 0$ and $i = 0$.

Therefore, (refer to Art. 36), instead of that locus, the straight line passing through the two control points, say—N and J, must be precisely adjusted so that it may completely satisfy the above conditions, that is, the straight line passing through N and J must be brought just on the meridian of the transit.

Now, if it can properly be done, the intersection point V of the straight line $\overline{NJV}$, (see Fig. 18 of Art. 20), and the optical axis of the objective lens system should be exactly on that plane too, and vice versa.

In that case, the deviation of point V from the meridian clearly vanishes of itself or

$$e+(C_J+C_N-C)i_0 = 0 ,$$

as derived from (110) and (101), which is exactly the same as (168b).

38. The First Method of Integral Adjustment. — Conjugate to the Method of One-Quarter Adjustment Performed between Short Distances. In order to adjust the vertical cross-hair in a transit integrally, first carry out One-Quarter Adjustment between the two near equi-distant points $\overline{A}$ and A, a scale being fixed at the latter.

Just after the adjustment has been completed, draw in the objective lens, sight at a well-defined distant point B beyond A in the same direction as that of the former, and then take the reading of the scale at A, that is, a_1, with the telescope normal, with the alidade still tightly clamped horizontally *in situ* when B was sighted.

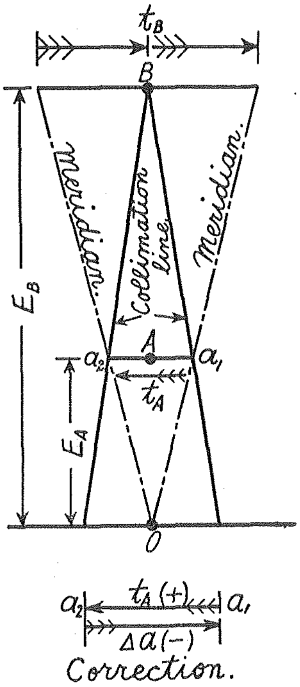


Fig. 27. First Method of Integral Adjustment.

Again, repeating as before with the telescope inverted, take the second reading a_2, and then find the difference of the two readings or t_A.

Then adjust the rear slide bearing, by means of the screws, loosening on one side and tightening on the other side, still with the telescope clamped *in situ* of the second reading, until the indication of the vertical cross-hair on the scale is transposed by the amount of correction $|\varDelta a|$, given by (175), from the second reading a_2 in the direction of the first, which reduces as follows :

Now, referring to Fig. 27, if a straight line passing through B parallel to $\overline{a_1 a_2}$ is intercepted by t_B between the two meridian lines of the transit when a_1 and a_2 are sighted respectively or the prolonged straight lines of $\overline{Oa_1}$ and $\overline{Oa_2}$, the relation

$$t_B = -\frac{E_B}{E_A} t_A \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (171)$$

is obtained.

Then, substituting A and B instead of N and J in (169) and (170) and then (171) in them, one gets exactly **the correction for the inclination of the optical axis of the objective lens system**

$$\varDelta i = -\frac{E_A - C_A}{E_B C_A - E_A C_B - (E_B - E_A)g} \frac{E_B}{E_A} \frac{t_A}{2} \quad \dots\dots\dots\dots \quad (172)$$

and **that of the reading of the scale at A corresponding to the above correction**

$$\varDelta a = -\left\{C_A - g + \frac{\delta - g}{C - \delta}(E_A - C_A)\right\} \frac{E_A - C_A}{E_B C_A - E_A C_B - (E_B - E_A)g} \frac{E_B}{E_A} \frac{t_A}{2}, \quad \dots\dots\dots \quad (173)$$

from which are reduced neglecting terms of the higher order*, **the practical formulas for correction**

$$\varDelta i = -\frac{E_A - C}{E_A} \frac{t_A}{2(C - g)} \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (174)$$

and

$$\varDelta a = -\left\{1 + \frac{\delta - g}{C - \delta} \frac{E_A - C}{C - g}\right\} \frac{E_A - C}{E_A} \frac{t_A}{2} \quad \dots\dots\dots\dots \quad (175)$$

See Fig. 27.

* Refer to the reduction of formula (14) and its accuracy in the author's second paper or Reference (2), p. 150~152. and those of (46) and (62) in the present paper.

Now, since for the transits adopted for the experiments in the present work, the relation

$$\left\{1+\frac{\delta-g}{C-\delta}\,\frac{E_A-C}{C-g}\right\}\frac{E_A-C}{E_A}\,\frac{C}{E_A}=(0.18\sim0.11),\quad \text{when}\quad E_A \leqq 20C, \tag{176}$$

is found, as seen from the observed data in Table 5 of Art. 26, the field formulas for correction

$$\left.\begin{aligned} \Delta i &= -\frac{t_A}{2(C-g)}\,, \\ \text{and}\qquad \Delta a &= -\frac{E_A}{C}\,\frac{t_A}{7}\,, \quad \text{when}\quad E_A \leqq 20C, \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots \tag{177}$$

are obtained from (174) and (175), neglecting terms of the higher order.

In the above formulas, t_A is positive, when the nomal sight strikes on the right and the inverted on the left, as shown in Fig. 27.

Further, with regard to the signs of t and i, refer to Art. 26.

For the purpose of testing this method, two practical examples of Integral Adjustment were performed on transit No. 5, of which the successful results are tabulated in Table 16 of Art. 43.

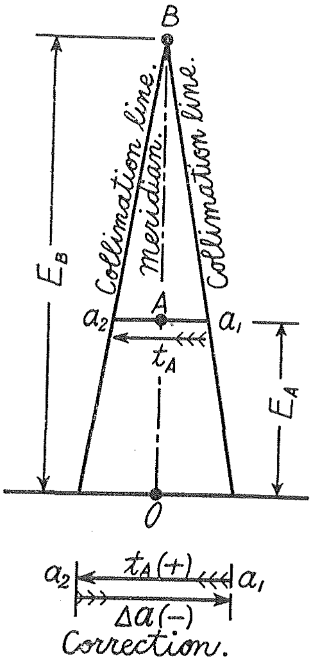


Fig. 28. Second Method of Integral Adjustment.

39. The Second Method of Integral Adjustment. — Conjugate to the Ordinary Method of One-Quarter Adjustment. As soon as the ordinary method of One-Quarter Adjustment has been completed between the two distant points $\overline{B}$ and B, take the reading a_1 of the scale fixed at the near point A with the telescope normal, keeping the alidade still clamped in the position that it was in when B was sighted. Again, performing the same as before with the telescope inverted, take the second reading a_2 of the scale at A, and then get the difference of the two readings t_A.

Then adjust the rear slide bearing, with the telescope still clamped in the position as it was in when the second reading was taken, till the reading of the scale is exactly changed by the amount of the correction $|\Delta a|$, computed from (181) reduced below, from the

second reading a_2 in the direction of the first a_1, which is gotten as follows :

Now, substituting A and B instead of J and N respectively in (169) and (170), **the correction for the inclination of the optical axis of the objective lens system**

$$\Delta i = -\frac{E_B - C_B}{E_B C_A - E_A C_B - (E_B - E_A)g}\frac{t_A}{2} \quad \cdots\cdots\cdots (178)$$

and **that of the reading of the scale at A corresponding to the above**

$$\Delta a = -\left\{C_A - g + \frac{\delta - g}{C - \delta}(E_A - C_A)\right\}\frac{E_B - C_B}{E_B C_A - E_A C_B - (E_B - E_A)g}\frac{t_A}{2} \quad \cdots\cdots\cdots (179)$$

are exactly gotten. Refer to Fig. 28.

Therefore, neglecting terms of the higher order*, **the practical formulas for correction of the inclination of the optical axis**

$$\Delta i = -\frac{t_A}{2(C - g)} \quad \cdots\cdots\cdots (180)$$

and **that of the reading**

$$\Delta a = -\left\{1 + \frac{\delta - g}{C - \delta}\frac{E_A - C}{C - g}\right\}\frac{t_A}{2} \quad \cdots\cdots\cdots (181)$$

are obtained from (178) and (179).

Further, since, from the data of the transits used in the experiments of the present paper, which are shown in Table 5 of Art. 26, the result

$$\left\{1 + \frac{\delta - g}{C - \delta}\frac{E_A - C}{C \quad g}\right\}\frac{C}{E_A} = (0.20 \sim 0.12), \quad \text{when } E_A \geqq 20\,C, \quad \cdots\cdots (182)$$

is gotten, the field formula for correction

$$\left.\begin{aligned} \Delta i &= -\frac{t_A}{2(C - g)}, \\ \text{and} \quad \Delta a &= -\frac{E_A}{C}\frac{t_A}{7} \end{aligned}\right\} \quad \cdots\cdots\cdots (183)$$

are obtained from (180) and (181), neglecting terms of the higher order.

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* Refer to the reduction (14) in the author's second paper or Reference (2), p. 142~144, and also, in the present paper, (46) and (62).

In the above formulas, t_A is positive, when the normal sight strikes on the right on the scale and the inverted on the left, as depicted in Fig. 28.

As to the signs of t and i, refer to Art. 26.

Now, similarly as in the preceding article, this method was tried for transits No. 5 and No. 23 too, with the results given in Table 16 of Art. 43.

40. The Third Method of Integral Adjustment.—Conjugate to the Ordinary Method of One-Quarter Adjustment. First adjust the vertical cross-hair by the method of One-Quarter Adjustment, as usual, between the two distant points $\overline{B}$ and B, a rod being horizontally fixed at the latter.

As soon as the adjustment has been completed, sight at a near distinct point A with the telescope normal, which is in the same direction as B, and clamping the telescope *in situ* when A was sighted, take the reading of the rod at B, that is, b_1. Again, performing likewise with the telescope inverted, take the second reading b_2, and then get the difference of the first and second readings or t_A.

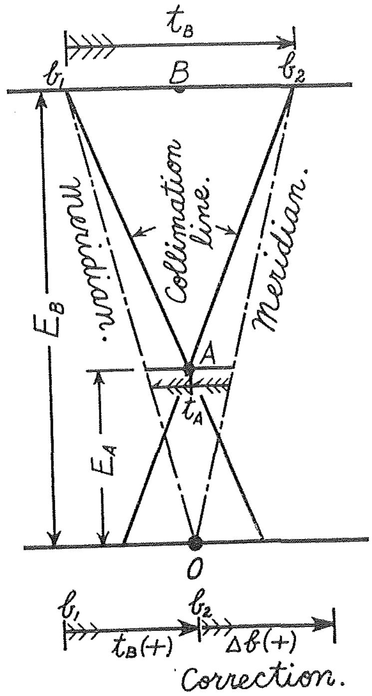

Fig. 29. Third Method of Integral Adjustment.

Then adjust the rear slide bearing, with the telescope still clamped *in situ* when the second was observed, until the vertical cross-hair is transposed on the rod by the amount of correction $|\Delta b|$, computed from (188), from the second reading b_2 in the direction opposite to b_1, which is obtained as follows:

Now, referring to Fig. 29, if the straight line passing through point A parallel to the rod at B is intercepted by t_A between the respective meridians of the instrument corresponding to the first and the second sights, the relation

$$t_A = -\frac{E_A}{E_B} t_B \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (184)$$

holds good.

Thereupon, putting B and A instead of N and J into (169) and (170) and also (184) into them, one obtains **the correction of the inclination of the optical axis of the objective lens system**

$$\Delta i = \frac{E_B - C_B}{E_B C_A - E_A C_B - (E_B - E_A) g} \frac{E_A}{E_B} \frac{t_B}{2} \quad \cdots\cdots\cdots (185)$$

and **that of the reading of the rod at B corresponding to the above correction**

$$\Delta b = \left\{ C_B - g + \frac{\delta - g}{C - \delta}(E_B - C_B) \right\} \frac{E_B - C_B}{E_B C_A - E_A C_B - (E_B - E_A) g} \frac{E_A}{E_B} \frac{t_B}{2} \quad \cdots\cdots\cdots (186)$$

Thereupon, neglecting terms of the higher order*, the practical formulas

$$\Delta i = \frac{E_A}{2E_B} \frac{t_B}{C - g} \quad \cdots\cdots\cdots (187)$$

and

$$\Delta b = \frac{\delta - g}{C - \delta} \frac{E_A}{C - g} \frac{t_B}{2} \quad \cdots\cdots\cdots (188)$$

are obtained.

Now, because the relation

$$\frac{\delta - g}{C - \delta} \frac{C}{C - g} = (0.17 \sim 0.10), \quad \text{when } E_A \leqq 20\,C, \quad \cdots\cdots\cdots (189)$$

is gotten from Table 5 of Art. 26, in which the observed constants of the transits used for the experiments of the present paper are given, the field formula

$$\Delta b = \frac{E_A}{C} \frac{t_B}{7} \quad \cdots\cdots\cdots (190)$$

is found from (188), neglecting terms of the higher order.

Hereupon, t_B is positive, when the normal sight strikes on the left and the inverted on the right as depicted in Fig. 29. Also, with regard to the signs of t and i, refer to Art. 26.

Likewise as in the preceding articles, this method was tested by adjusting transits No. 5 and No. 23, with the results shown in Table 16 of Art. 43 below.

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* Refore to the reduction of (14) in the author's second paper or Reference (2), p. 142~144, at the end of this work.

41. The Fourth Method of Integral Adjustment.—Conjugate to the Method of One-Quarter Ad ustment Performed between Short Distances. First, adjust the vertical cross-hair by the method of One-Quarter Adjustment performed between the two near equi-distant points $\bar{A}$ and A.

As soon as the adjustment has been completed, take the reading of the rod fixed horizontally at a distant station B, viz., b_1 with the telescope normal, keeping the alidade still clamped horizontally *in situ* when A was sighted.

Again, doing as before with the telescope inverted, take the new reading b_2 and then find the difference of the two readings—say t_B.

Thereupon, adjust the rear slide bearing, keeping the telescope still clamped *in situ* when the second reading b_2 was taken, until the reading of the vertical cross-hair on the rod is varied by the amount of correction $|\Delta b|$, gotten from (194), from the second reading b_2 in the direction opposite to the first b_1, which is reduced as follows:

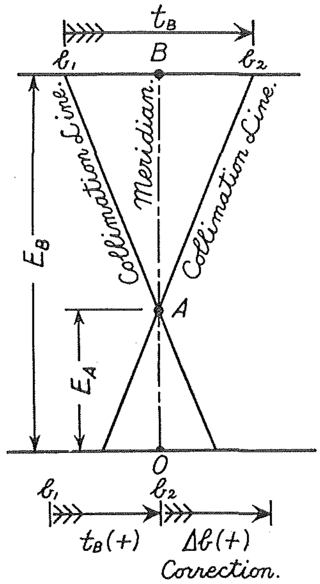

Fig. 30. Fourth Method of Integral Adjustment.

Now, putting A and B instead of N and J into (169) and (170) respectively, the correction for the inclination of the optical axis of the objective lens system

$$\Delta i=\frac{E_A-C_A}{E_BC_A-E_AC_B-(E_B-E_A)g}\,\frac{t_B}{2} \quad \cdots\cdots\cdots (191)$$

and that for the reading of the rod at B corresponding to the above

$$\Delta b=\left\{C_B-g+\frac{\delta-g}{C-\delta}(E_B-C_B)\right\}\frac{E_A-C_A}{E_BC_A-E_AC_B-(E_B-E_A)g}\,\frac{t_B}{2} \quad \cdots\cdots\cdots (192)$$

are exactly obtained. Refer to Fig. 30.

Hereupon, neglecting terms of the higher order, the practical formulas for correction

$$\Delta i = \frac{E_A - C}{E_A} \frac{\cdot t_B}{2(C-g)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (193)$$

and

$$\Delta b = \frac{\delta - g}{C - \delta} \frac{E_A - C}{C - g} \frac{t_B}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (194)$$

are gotten from (191) and (192) respectively.

Again, since, referred to the data of the transits in Table 5 of Art. 26, the relation

$$\frac{1}{2} \frac{\delta - g}{C - \delta} \frac{C}{C - g} \frac{E_A - C}{E_A} = (0.15 \sim 0.10)\,, \quad \text{when} \quad E_A \leqq 20\,C, \quad \ldots\ldots \quad (195)$$

is found, the field formulas for correction

$$\left.\begin{aligned} \Delta i &= \frac{E_A}{E_B} \frac{t_B}{2(C-g)} \\ &\text{and} \\ b &= \frac{E_A}{C} \frac{t_B}{7} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (196)$$

are reduced from (193) and (194), neglecting terms of the higher order.

In the above formulas, t_B is positive, when the normal sight strikes on the left and the inverted on the right as depicted in Fig. 30.

Also, with respect to the signs of t and i, refer to Art. 26.

Now, likewise as in the other cases, from the experimental results shown in Table 16 of Art. 43, it is distinctly proved that this method can readily be practized in the field too.

42. The Fifth Method of Integral Adjustment.—Trial Method. Similar to the third method of Integral Adjustment of the horizontal cross-hair, a trial method of Integral Adjustment of the vertical cross-hair should be given here too, in which, moreover C, g and δ are no longer necessary and, too, the exact correction is simply found so that the precision of the adjustment is the same in the preceding methods; while, on the contrary, all the corrections for the preceding must be computed from the formulas respectively.

Now, the new method is expounded as follows :

First, by one of the procedures described in the preceding articles, observe the difference of the readings of the scale at A or the rod at B, that is to say, t_{A1} or t_{B1}, and then alter the reading of the vertical cross-hair, with the telescope still clamped *in situ* when the second reading was taken, by the correction

$$\varDelta a = -t_{A1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (197a)$$

or

$$\varDelta b = +t_{B1}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (197b)$$

adjusting the rear slide bearing.

Subsequently, repeating the process, if the second difference t_{A2} or t_{B2} is obtained, then the further adjustment must be continued by the correction

$$\varDelta a = -\frac{t_A}{1-\dfrac{t_{A2}}{t_{A1}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (198a)$$

or

$$\varDelta b = +\frac{t_B}{1-\dfrac{t_{B2}}{t_{B1}}}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (198b)$$

Herein, the sign of t is positive when the first sight strikes on the right of the scale at the near point A or on the left of the rod at the distant point B, and that of the correction is the same as that of t.

Further, the signs of t_{A2} and t_{B2} are the same as those of t_{A1} and t_{B1} respectively when under correction, but the opposite when over correction.

In general, the correction must be made from the second reading in the opposite direction, in which the readings have advanced, for $\varDelta a$, but in the same for $\varDelta b$.

Now, the fundamental principle of this method consists in the two facts as follows :

First. The absolute values of $\varDelta a/t_A$, gotten from (176) and (189), and $\varDelta b/t_B$, obtained from (182) and (195), are $(0.2 \sim 0.1)\,E_A/C$ for $10C \leqq E_A \leqq 20C$, when put together, and accordingly, the mean value of the constants in the parenthesis becomes 0.15 or 1/7, which is the very coefficient of t in (177), (183), (190) and (196).

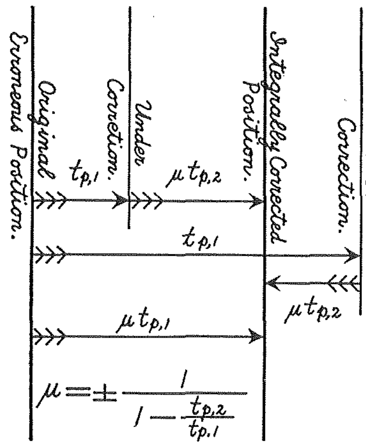

Fig. 31. Illustration of the Trial Method of Integral Adjustment.

Second. Therefore, the adjustment is made by the formula for correction

$$\Delta p_1 = \pm\, t_{r1}$$

in the first place, then the residual error left after the first adjustment is exactly given by the formula

$$\Delta p_2 = \pm \frac{t_{p2}}{1 - \dfrac{t_{p2}}{t_{p1}}}\,,$$

because the relation

$$\mu t_{p1} - t_{p1} = \mu t_{p2}$$

should hold good between t_{p1} and t_{p2} when the total actual correction is given by μt_{p1}, for the theoretical reason illustrated by Fig. 31 and where the sign of t_{p2} is the same as that of t_{p1} when under correction and the opposite when over correction.

43. Illustrative Experiments of Integral Adjustment of the Vertical Cross-Hair in a Transit. Now, it is neccessary to prove that the preceding five methods can truly be practised in the field and in the factory.

For these experimental tests, transits No. 5 and No. 23 were taken they were made in Japan and the U.S.A. respectively.

The inclinations of the optical axes of the objective lens systems of their telescopes were extraordinarily deranged on purpose in advance with the object of testing the utility of the new methods.

Now, all the test results, which were performed by the present author for himself without any preparatory practice for the first time, are minutely shown in Table 16 with absolutely no omission.

It should now be understood from Table 16 that the methods of Integral Adjustment can really be practised in the field by a civil engineer and in the manufactory, because, with all the above facts, the adjustments could successfully be completed by the repetition of the procedure two or three times at least.

Now, it should be noted here that the three experimental tests were performed in Feb., 1935, using the very apparatus, shown in Fig. 21 and Fig. 22 and used in the first experiment of Art. 25 and Art. 26, and the

Table 16.

Experimental Note 6.

Designation	First Method	First Method	Second Method	Second Method
Date, Tested	Feb., 1935	Sept., 1935	Feb., 1935	Sept., 1935
Transit, No.	5	5	5	23
Made In	Japan	Japan	Japan	U.S.A.
Size, In.	4	4	4	5
Years of Use	10.0	10.3	10.0	0.2
Observer	T. Shingo	T. Shingo	T. Shingo	T. Shingo
C, m.	0.2764	0.2764	0.2764	0.347
E_B, m.	137.32	138.30	137.32	138.30
E_A, m.	2.80	4.00	2.80	4.00
Δi, Radian	$-0.002\,370\,t$	$-0.002\,457\,t$	$-0.002\,640\,t$	$-0.002\,127\,t$
Δi, Seconds	$-\ 488.8\,t$	$-\ 506.7\,t$	$-\ 554.4\,t$	$-\ 438.8\,t$
Δa or Δb	$-\ 1.736\,t$	$-\ 2.539\,t$	$-\ 1.926\,t$	$-\ 1.594\,t$
First Adjustment				
t, m.m.	$-\ 2.16$	$+\ 1.31$	$-\ 3.46$	$+\ 1.90$
Δi, Radian	$+0.005\,112$	$-0.003\,221$	$+0.009\,138$	$-0.004\,047$
Δi, Seconds	$+\ 1045$	$-\ 664$	$+\ 1889$	$-\ 835$
Δa or Δb, m.m.	$+\ 3.74$	$-\ 3.33$	$+\ 6.69$	$-\ 3.03$
Second Adjustment				
t, m.m.	$+\ 0.15$	0.00	$+\ 0.35$	0.00
Δi, Radian	$-0.000\,363$		$-0.000\,916$	
Δi, Seconds	$-\ 75$		$-\ 189$	
Δa or Δb, m.m.	$-\ 0.27$		$-\ 0.67$	
Third Adjustment				
t, m.m.	$-\ 0.39$		0.00	
Δi, Radian	$+0.000\,932$			
Δi, Seconds	$+\ 192$			
Δa or Δb, m.m.	$+\ 0.68$			
Fourth Adjustment				
t, m.m.	0.00			

Table 16.—(*Continued*)

Experimental Note 6.

Designation	Third Method	Third Method	Fourth Method	Fourth Method
Date, Tested	Feb., 1935	Sept., 1935	Sept., 1935	Sept., 1935
Transit, No.	5	23	5	5
Made In	Japan	U.S.A.	Japan	Japan
Size, In.	4	5	4	4
Years of Use	10.0	0.2	10.3	10.3
Observer	T. Shingo	T. Shingo	T. Shingo	T. Shingo
C, m.	0.2764	0.347	0.2764	0.2764
E_B, m.	137.32	138.30	138.30	138.30
E_A, m.	2.80	4.00	4.00	4.00
Δi, Radian	$+0.000\,049\,7\,t$	$+0.000\,061\,5\,t$	$+0.000\,071\,0\,t$	$+0.000\,071\,0\,t$
Δi, Seconds	$+\ 10.25\,t$	$+\ 12.69\,t$	$+\ 14.65\,t$	$+\ 14.65\,t$
Δa or Δb	$+\ 1.495\,t$	$+\ 1.197\,t$	$+\ 2.093\,t$	$+\ 2.093\,t$
First Adjustment				
t, m.m.	+ 104.8	− 50.9	+ 42.7	− 48.3
Δi, Radian	+0.005 208	−0.003 130	+0.003 036	−0.003 430
Δi, Seconds	+1 074	− 645	+ 626	− 708
Δa or Δb, m.m.	+ 156.7	− 60.9	+ 89.5	− 101.2
Second Adjustment				
t, m.m.	0.0	+ 12.6	0.0	0.0
Δi, Radian		+0.000 776		
Δi, Seconds		+ 160		
Δa or Δb, m.m.		+ 15.1		
Third Adjustment				
t, m.m.		0.0		

Table 16.—(*Continued*)

Experimental Note 6.

Designation	Fifth or "Trial Method"			
	First Method	Second Method	Third Method	Fourth Method
Date, Tested	Oct., 1935	Oct., 1935	Oct., 1935	Sept., 1935
Transit, No.	23	23	23	5
Made In.	U.S.A.	U.S.A.	U.S.A.	Japan
Size, In.	5	5	5	4
Years of Use	0.2	0.2	0.2	10.3
Observer	T. Shingo	T. Shingo	T. Shingo	T. Shingo
C, m.	0.347	0.347	0.347	0.2764
E_B, m.	138.30	183.30	138.30	138.30
E_A, m.	3.40	3.40	3.40	4.00
Δa or Δb	$-$ 1.54 t	$-$ 1.63 t	$+$ 0.86 t	$+$ 1.45 t
First Adjustment				
t, m.m.	$+$ 2.00	$-$ 1.61	$-$ 72.2	$-$ 41.4
Δa or Δb	$-$ t	$-$ t	$+$ t	$+$ t
Δa or Δb, m.m.	$-$ 2.00	$+$ 1.61	$-$ 72.2	$-$ 41.4
Seeond Adjustment				
t, m.m.	$+$ 0.70	$-$ 1.00	$+$ 12.2	$-$ 13.1
Δa or Δb	$-$ 1.54 t	$-$ 1.63 t	$+$ 0.86 t	$+$ 1.45 t
Δa or Δb, m.m.	$-$ 1.07	$+$ 1.63	$+$ 10.4	$-$ 19.0
Third Adjustment				
t, m.m.	$-$ 0.17	$-$ 0.06	0.0	0.0
Δa or Δb	$-$ 1.54 t	$-$ 1.63 t		
Δa or Δb, m.m.	$+$ 0.26	$+$ 0.10		
Fourth Adjustment				
t, m.m.	0.00	0.00		

other tests in Sept. and Oct., 1935, with that shown in Fig. 32, that is, the apparatus of Figs. 24 and 25 slightly reconstructed, which had been used in the second experiment of Art. 29 and Art. 30.

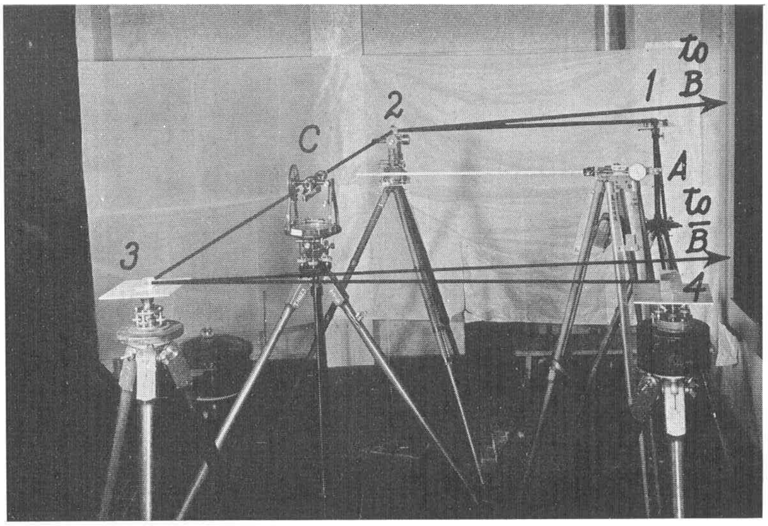

Fig. 32. Apparatus for Test Experiments of Integral Adjustment of the Vertical Cross-hairs.

In making the Integral Adjustment, the following facts must especially be noted :

1. Oil the levelling screws, the tangent screws and the clamp screws, etc., which should be driven. Especially, the screws which sustain the adjustable rear slide bearing must fully be oiled.

2. All the clamps must be tight so that they may not slip.

3. All the tangent screws must be passably forced, so that no minute variation in the direction of collimation through the telescope may be caused at all.

4. The One-Quarter Adjustment must be performed carefully with accuracy of the highest possibility.

5. It must be taken as $E_A \doteqdot 20C$ for the transit of poor finishing, used for years, or subjected to the accident for the grounds which will be explained below in Art. 49 in detail.

SECTION VII.—STUDY OF THE METHOD OF INTEGRAL ADJUSTMENT OF THE VERTICAL CROSS-HAIR BY THAT OF THE HORIZONTAL CROSS-HAIR

44. General Principle. When the vertical cross-hair is adjusted by the method of Integral Adjustment of the horizontal cross-hair between the distant point B and the near point A, the deviation of B and A from the meridian of the transit is given by $u/4$ gotten from (128), in reference to (120) and (121).

Therefore, if the condition

$$e_g + (C_A + C_B - C - g)i = 0 \quad \dots\dots\dots\dots \quad (199)$$

is fulfilled, then $u = 0$, in reference to (128).

This is the very fundamental principle of the method of Integral Adjustment of the vertical cross-hair, performed by that of the horizontal cross-hair.

Further, the content of the condition (199) is all the same as that of (168b), that is to say, it plainly proves the uniqueness of Integral Adjustment.

Therefore, in order to make $u = 0$ at A and B, the correction for the inclination of the optical axis of the objective lens system

$$\Delta i = -\frac{C_A + C_B - C}{C_A + C_B - C - g}\frac{u_p}{4E_p}, \quad (p = A, B) \quad \dots\dots\dots \quad (200)$$

and that of the reading of the scale at A or the rod at B

$$\Delta p = -\left\{1 + \frac{\delta - g}{C - \delta}\frac{E_p - C_p}{C_p - g}\right\}\frac{C_A + C_B - C}{C_A + C_B - C - g}\frac{C_p - g}{E_p}\frac{u_p}{4}, \quad (p = a,\ b \text{ or } A,\ B) \quad \dots\dots \quad (201)$$

are exactly gotten from (120) and (199), and (88) respectively.

Respecting the signs of u, e, e_g and i, refer to Art. 26.

Hereupon, neglecting terms of the higher order, the practical formulas for correction

$$\Delta i = -\frac{C}{C - g}\frac{u_p}{4E_p}, \quad (p = A, B) \quad \dots\dots\dots\dots \quad (202)$$

and

$$\Delta p = -\left\{1 + \frac{\delta - g}{C - \delta}\frac{E_p - C}{C - g}\right\}\frac{C}{E_p}\frac{u_p}{4}, \quad (p = a, b \text{ or } A, B) \quad \dots\dots \quad (203)$$

are reduced from (200) and (201).

45. Experiments of Integral Adjustment of the Vertical Cross-Hair by the Method of Integral Adjustment of the Horizontal Cross-Hair. There are only two methods of this kind by its very nature.

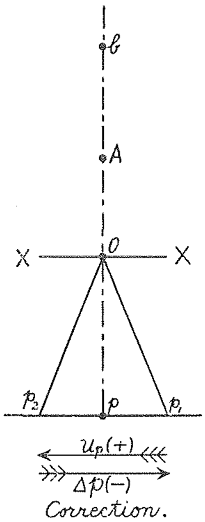

Fig. 33.

The first of them is that in which the vertical cross-hair is firstly adjusted by the first method of Integral Adjustment of the horizontal cross-hair and thereafter made so that $u_A = 0$ at A, and the second is that in which it is adjusted by the second method at first and subsequently made so that $u_B = 0$ at B.

Hereupon, in order to know whether the vertical cross-hair can be adjusted practically by those methods or not, several experiments were performed in which the reading u was observed at point P through the apparatus adopted in the first experiment described in Arts. 25 and 26, where P, A and B were taken as $E_P = 7$ m., $E_A = 2$ m. and $E_B = 137.3$ m. respectively, because the errors due to this approximation are exactly within those of observation and accordingly, referred to (123), the experiments for the adjustability of the vertical cross-hair by the methods of this kind are hardly subjected to any influence of it. Refer to Fig. 33.

Now, the experiments for Integral Adjustment of this kind were performed on several transits, among which the results with transits No. 7, 9 and 23 are representative and accordingly set down in Table 17.

Table 17.

Experimental Note 7.

(January, 1935)

Transit No.	Observation No.	u_P, m.m.	$e+xi$, m.m.	Correction Δi, Seconds	Correction Δp, m.m.
7		u	$+0.010\,41u$	$-10.51u$	$-0.086\,64u$
	1	+59.710	+0.621	−627.4	−5.173
	2	− 6.025	−0.063	+ 93.1	+0.522
	3	+ 5.671	+0.059	− 59.6	−0.491
	4	+ 1.664	+0.017	− 17.5	−0.144
	5	+ 3.050	+0.032	− 32.0	−0.264
	6	+ 2.083	+0.022	− 21.9	−0.181
9		u	$+0.010\,41u$	$-10.51u$	$-0.086\,64u$
	1	−13.967	−0.145	+146.8	+1.210
	2	+ 3.736	+0.039	− 39.3	−0.324
	3	+ 1.923	+0.020	− 20.2	−0.167
23		u	$+0.012\,28u$	$-10.52u$	$-0.075\,90u$
	1	− 9.207	−0.123	+ 96.9	+0.699
	2	− 1.804	−0.022	+ 19.0	+0.137
	3	+ 1.403	+0.017	− 14.8	−0.107
	4	+ 1.057	+0.013	− 11.1	−0.080
	5	+ 2.143	+0.026	− 22.6	−0.163

Notice: $E_A = 2$ m., $E_P = 7$ m. and $E_B = 137.32$ m.

Of these instruments, transit No. 23 is a Gurley's five inch, made in the U.S.A., and the other two are manufactured in Japan.

From the results mentioned in Table 17, it is readily comprehended that it is generally impossible to reduce the value of u under 1 m.m.

The reason why the adjustment can not be integrated due to this residual error, consits in the facts that first, the errors due to the observation of the control points for the objective slide adjustment sometimes amount up to over 0.1 m.m. at point P at the distance of 7 m., which is known from the experimental results; secondly, although the screws, which sustain the slide bearing ring, are carefully driven by a screw-driver with a very small force when the rear slide bearing is adjusted, the minute rotational motion of the tripod about the vertical axis of the instrument, and the imperceptible yielding of the springs of the tangent screws are produced by that force; and thirdly, the above errors are magnified by about 12~14 times. In regard to this one should refer to (203). Table 4 and εp in Table 17.

For this reason, the methods of this kind are merely suitable for the experiment of determining the optical and mechanical errors in the transit and other like instruments, because of their incomparable power of magnifying the errors.

SECTION VIII.—RESIDUAL DEVIATIONS REMAINING AFTER THE VERTICAL CROSS-HAIR WAS ADJUSTED BY THE METHOD OF INTEGRAL ADJUSTMENT

46. General Problem. In truth, the locus of collimation points generally becomes an hyperbola of the higher order irrespective of the adjustment of the collimation, as long as the eccentricity and the inclination of the optical axis of the objective lens system are found.

But, although, for simplicity of the explanations, it has been described as a straight line up to now, this explanatory approximation does not disturb the deduction of the exact theory of "Integral Adjustment" in the present work at all, because only absolutely exact calculations are carried out.

Now, the deviation of point p is generally given by (78), which is newly written down, with reference to (12), as follows:

$$E_p\varphi_p = E_p\left(i-\frac{k}{f}\right)+e+C\frac{k}{f}+\frac{fk}{E_p-C_p}. \quad \ldots\ldots \quad (204a)$$

The right hand side is the sum of the ordinate of the hyperbola of the higher order, considering the meridian of the transit as the abscissa, referred to Fig. 34 and Fig. 35,

$$y_1=\frac{fk}{E_p-C_p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (204b)$$

and that of the straight line

$$y_2 = E_p\left(i - \frac{k}{f}\right) + e + C\frac{k}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (204c)$$

which is the very asymptote of the curve of the locus of collimation points represented by (204a).

The deviation of the ideal collimation points should be directly proportional to only E_p, because their locus must be a straight line.

Therefore, the irregularity of the deviation should be caused by the remainder

$$(E_p\varphi_p) = e + C\frac{k}{f} + \frac{fk}{E_p - C_p}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (205)$$

which is approximately a simple hyperbola, because the variation of C_p is small compared with E_p, with reference to (12).

47. Residual Deviation Remaining after the Vertical Cross-Hair has been adjusted by the Method of Integral Adjustment. If the vertical cross-hair is adjusted between the two equi-distant points B and $\bar{B}$ by the ordinary method of One-Quarter Adjustment, the deviation of a collimation point p from the meridian of the transit

$$E_p\varphi_p = e + C_p i - \frac{E_p - C_p}{E_B - C_B}(e + C_B i) \quad \ldots\ldots\ldots\ldots \quad (206)$$

is obtained, putting B instead of N into (110), in which the irregularity of the deviation is caused by the superfluous deviation

$$\Delta(E_p\varphi_p) = \frac{e + E_B i}{E_B - C_B}(C_p - C) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (207)$$

or, neglecting terms of the higher order,

$$\Delta(E_p\varphi_p) = (C_p - C)i\,. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (208)$$

Further, if the vertical cross-hair is integrally adjusted between a near point A and a distant one B, so that the condition

$$e_g + (C_A + C_B - C - g)i_0 = 0$$

is satisfied, with reference to (168b), the residual deviation of point p from the meridian of the transit

$$E_p\varphi_p = -\frac{C_A-C_p}{C_p-C}(C_p-C_B)i_0$$

$$= -\frac{E_p-C_p-E_A+C_A}{E_A-C_A}\frac{E_B-C_B-E_p+C_p}{E_B-C_B}(C_p-C)i_0$$

$$= -\frac{E_B(C_A-C_p)+E_p(C_B-C_A)+E_A(C_p-C_B)}{E_BC_A-E_AC_B}(C_A+C_B-C)i_0 \quad \text{........ (209)}$$

and neglecting terms of the higher order,

$$E_p\,\varphi_p = -(C_A-C_p)i_0 \quad \text{........................ (210)}$$

are obtained from (125), where

$$i_0 = -\frac{e_g}{C_A+C_B-C-g}$$

$$= -\frac{e}{C_A+C_B-C}$$

with reference to (168b).

48. Residual Deviation Left after the Vertical Cross-Hair has been adjusted by the Method of Integral Adjustment of the Horizontal Cross-Hair. Now, if the vertical cross-hair is adjusted by the method of Integral Adjustment of the horizontal cross-hair between a near and a distant point—say A and B respectively, then the deviation of the collimation point p from the meridian of the transit

$$E_p\varphi_p = \frac{e+(C_A+C_B-C)i}{C_A+C_B-C}E_p + \frac{C_A-C_p}{C_A+C_B-C}\,\frac{C_p-C_B}{C_p-C}e$$

$$= \frac{e+(C_A+C_B-C)i}{C_A+C_B-C}E_p + \frac{E_B(C_A-C_p)+E_p(C_B-C_A)+E_A(C_p-C_B)}{E_BC_A-E_AC_B}e \quad \text{........ (211)}$$

and the deviation of the collimation point p sighted by the deflexion angle method from the original line of adjustment or $\overline{OAB}$ referred to Fig. 20 of Art. 22

$$E_p\varphi_p = 2\frac{e+(C_A+C_B-C)i}{C_A+C_B-C}E_p + \frac{C_A-C_p}{C_A+C_B-C}\,\frac{C_p-C_B}{C_p-C}e$$

$$= 2\frac{e+(C_A+C_B-C)i}{C_AC_B-C}E_p + \frac{E_B(C_A-C_p)+E_p(C_B-C_A)+E_A(C_p-C_B)}{E_BC_A-E_AC_B}e \quad \text{........ (212)}$$

are obtained from (122) and (123), whose unnecessary variations are given by

$$\Delta(E_p\varphi_p) = \frac{C_A-C_p}{C_A+C_B-C}\,\frac{C_p-C_B}{C_p-C}e$$

$$= \frac{E_B(C_A-C_p)+E_p(C_B-C_A)+E_A(C_p-C_B)}{E_BC_A-E_AC_B}e \quad \ldots\ldots \quad (213)$$

respectively.

But, if the vertical cross-hair were completely adjusted by the method of Integral Adjustment of the horizontal cross-hair, so that the condition

$$e_0+(C_A+C_B-C)i_0 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (214a)$$

were fulfilled, with reference to (199), where $e_0 = e_g-gi_0$, the above deviations should simultaneously become

$$E_p\varphi_p = -\frac{C_p-C_B}{C_p-C}(C_A-C_p)i_0$$

$$= -\frac{E_B(C_A-C_p)+E_p(C_B-C_A)+E_A(C_p-C_B)}{E_BC_A-E_AC_B}(C_A+C_B-C)i_0$$

$$\ldots\ldots\ldots \quad (214b)$$

referred to (125), where

$$i_0 = -\frac{e_g}{C_A+C_B-C-g}$$

$$= -\frac{e_0}{C_A+C_B-C}\,.$$

These are the truest condition and the truest residual deviation respectively, shown by (209) and (210) in the preceding article.

49. To Make the Residual Deviation under a Certain Allowable Limit after the Vertical Cross-Hair has been adjusted integrally. After the vertical cross-hair has been adjusted integrally, the residual deviation must not exceed a certain limit, otherwise all the labour for adjustment should not only go for nothing but also undesirable error would still remain in the measurement.

Now, the residual deviation after integral adjustment

$$\varphi_p = -\frac{C_A-C_p}{E_p}i_0$$

is gotten from (210).

Hereupon, φ_p becomes a maximum when

$$\frac{d\varphi_p}{dE_p} = \frac{C_A-C_p}{E_p^2} - \frac{C_p-C}{E_p(E_p-C)}$$
$$= 0 ,$$

from which

$$C_A-C = 2(C_p-C) , \quad 2(E_A-C) = E_p-C \quad \text{..........} \quad (215)$$

is obtained, with reference to (12), neglecting terms of the higher order.

Therefore, the maximum angular deviation

$$\varphi_{p,\,\text{max.}} = -\frac{C_A-C}{E_A}\frac{i_0}{4}$$
$$= \frac{C_A-C}{C_A}\frac{e}{4E_A}$$
$$= \frac{C_A-C}{C_A-g}\frac{e_g}{4E_A} \quad \text{........................} \quad (216)$$

is gotten from (210) and (215).

Accordingly, a distance E_A must be taken such that $\varphi_{p,\,\text{max.}}$ does not exceed the allowable limit, but such procedure as to obtain i_0, e_0 or e_g and to determine the suitable distance E_A for point A can not be carried out in practice because of the difficulty.

Figures 34 to 39 will exactly illustrate the important principle correlative with this fact which relieves one of this trouble.

In an ordinary transit, which has the eccentricity of the optical axis of the objective lens system at the front slide bearing e_g less than 1 m.m. or its inclination i_0 less than 0.005 or 15′, the vertical cross-hair can be adjusted perfectly by the method of Integral Adjustment of the vertical cross-hair, if it is taken that $E_A = 10\,C$.

But, when there is an error $e_g > 3.5$ m.m. or $i_0 > 0.0175$ or 60′ in the telescope now, it can not be perfectly adjusted, even if it be taken that $E_A = 10C$; while, on the contrary, it can be only if it be taken that $E_A \geqq 20C$.

The transit with an error between these limits should be safe to be adjusted by taking as $E_A = 20C$.

Now, transit No. 4, which has an error $e_g = +5.209$ m.m. or $i_0 = -0.0255 = -1°\,27'\,37''$, and No. 7, which has $e_g = -0.535$ m.m. or $i_0 = +0.00262 = +9'\,00''$, are preferred as the best example to illustrate the above description and the further demonstration.

In Fig. 34, when the vertical cross-hair of transit No. 4 is adjusted merely by the ordinary method of One-Quarter Adjustment between $\overline{B}$ and B, where $E_{\overline{B}} = E_B = 137.32$ m., and subsequently by the method of Integral Adjustment of the vertical cross-hair among $\overline{B}$, B and A in three ways, that is to say — $E_{\overline{B}} = E_B = 137.32$ m. and $E_A = 2$ m. in the first adjustment, $E_{\overline{B}} = E_B = 137.32$ m. and $E_A = 10\,C = 2.764$ m. in the second, and $E_{\overline{B}} = E_B = 137.32$ m. and $E_A = 20\,C = 5.527$ m. in the third; the loci of the collimation points or their deviations from the meridian are shown, named Curve 1, 2, 3 and 4 respectively, by taking the meridian as the abscissa and the deviation as the ordinate.

Similarly, in Fig. 35, when the vertical cross-hair is adjusted by the same methods, the same things are shown respectively.

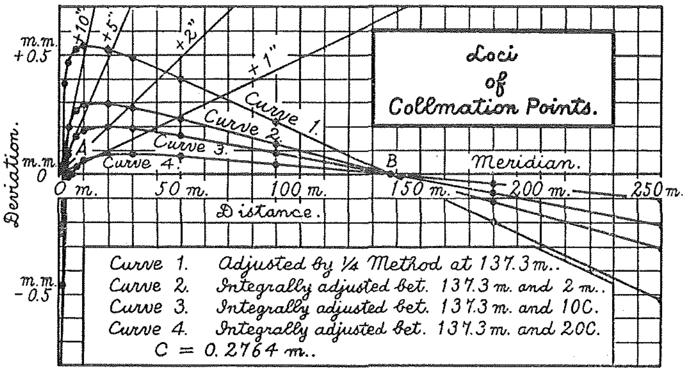

Fig. 34. Illustration of Integral Adjustment of the Vertical Cross-Hair of Transit No. 4.

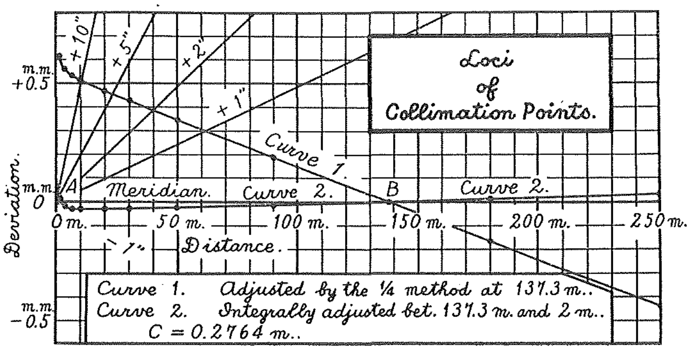

Fig. 35. Illustration of Integral Adjustment of the Vertical Cross-Hair of Transit No. 7.

Now, from Figs. 34 and 35, it can immediately be apprehended that in transit No. 4 with the large optical errors, the derogatory deviation, still remaining after One-Quarter Adjustment, can be eliminated to the extent of only about one-half of the original when adjusted by the method of Integral Adjustment of the vertical cross-hair between $E_B = 137.32$ m. and $E_A = 10C = 2.764$ m. Nevertheless it can be entirely removed when adjusted between $E_B = 137.32$ m. and $E_A = 20C = 5.527$ m. On the contrary, transit No. 7 with the small optical errors can be perfectly adjusted by the same method performed between $E_B = 137.32$ m. and $E_A = 10C = 2.764$ m..

In Figs. 36 and 37, the loci of collimation points of transits No. 4 and No. 7 respectively, when the vertical cross-hairs are adjusted by the method of the horizontal cross-hair between $E_B = 137.32$ m. and E_A

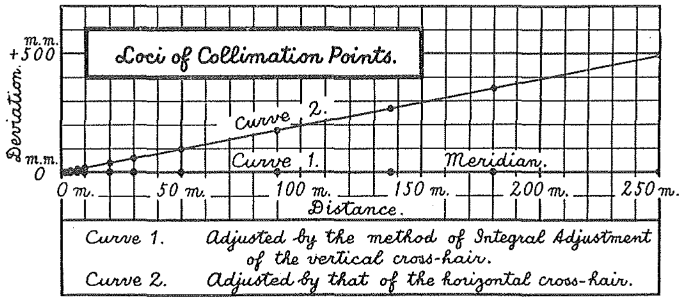

Fig. 36. Illustration of Integral Adjustment of the Vertical Cross-Hair of Transit No. 4.

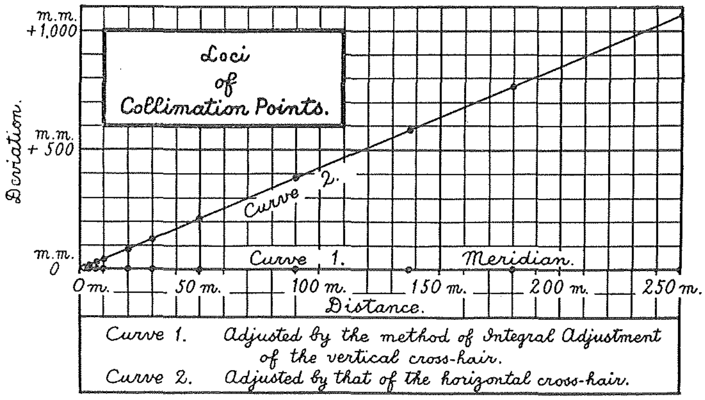

Fig. 37. Illustration of Integral Adjustment of the Vertical Cross-Hair of Transit No. 7.

$= 2$ m., are illustrated by Curve 2, together with those loci when they are adjusted by the method of Integral Adjustment of the vertical cross-hair between the same points or Curve 1, from which the angular errors coming into the deflexion angles are readily grasped.

In Figs. 38 and 39, the variations of the deviations of collimation points of transits No. 4 and No. 7, when the vertical cross-hairs are adjusted by the method of Integral Adjustment of the horizontal cross-hair and also that of the vertical cross-hair between $E_B = 137.32$ m. and $E_A = 2$ m. respectively, that is, Curve 2 and Curve 1 respectively, are shown, from which it can readily be seen that there is no difference

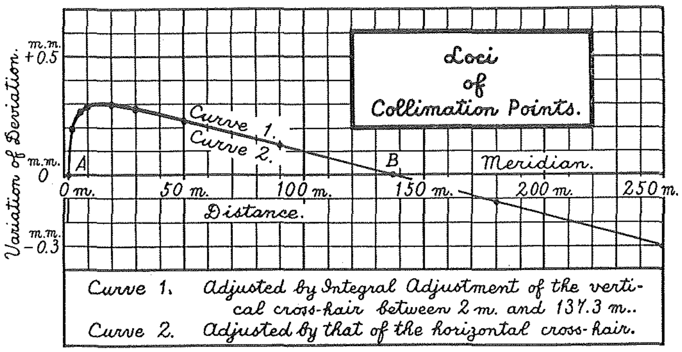

Fig. 38. Variations of Deviations of Collimation Points in Transit No. 4 after Integral Adjustment.

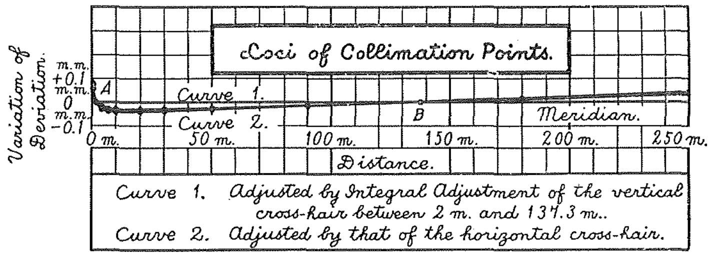

Fig. 39. Variations of Deviations of Collimation Points in Transit No. 7 after Integral Adjustment.

between the superfluous variations of collimation points when the vertical cross-hair is adjusted by the above two methods.

For the purpose of illustration in the above examples, the angular deviations of collimation points are calculated from e_g and i_0 of transits No. 4 and No. 7, but in practice, their maximum values can be found through direct observations after the adjustment from the fact formulated by (215).

Thereupon, the suitable distance of the near point A can simply be found in the following way:

First, adjust the vertical cross-hair by the ordinary method of One-Quarter Adjustment between the two distant points $\bar{B}$ and B, thereafter observe the deviations of several collimation points near the nearest control point by the method of t-reading shown by (126) described in Art. 23, at which the angular deviation must not exceed a certain limit, and then plotting these values in Fig. 40, as depicted in Fig. 34, Curve 1 can be obtained.

Now, if the vertical cross-hair is adjusted by the method of Integral Adjustment of the vertical cross-hair among the distant conjugate points $\bar{B}$ and B and a near point A, then the deviations of point p and D near to A

$$(E_p\varphi_p)_A = -(C_A - C_p)i_0 \quad \cdots\cdots (217a)$$

and

$$(E_D\varphi_D)_A = -(C_A - C_D)i_0 \quad \cdots\cdots (217b)$$

are obtained from (210), as long as the distances of point A, D and p are negligibly small compared with that of B.

Further, if the vertical cross-hair is adjusted by the same method among the distant conjugate points $\bar{B}$ and B and a different near point D, then the deviations of point p and A near to D

$$(E_p\varphi_p)_D = -(C_D - C_p)i_0 \quad \cdots\cdots (218a)$$

and

$$(E_A\varphi_A)_D = -(C_D - C_A)i_0 \quad \cdots\cdots (218b)$$

are gotten from (210) again.

Therefore, from (217a) (217b) (218a) and (218b), the relations

$$(E_p\varphi_p)_D = (E_p\varphi_p)_A - (E_D\varphi_D)_A\,, \quad \cdots\cdots (219a)$$

and

$$(E_p\varphi_p)_A = (E_p\varphi_p)_D - (E_A\varphi_A)_D \quad \cdots\cdots (219b)$$

are obtained. These exactly clarify that the deviation of point p referred to the meridian $\overline{BD}$ is equal to the algebraical difference of the deviations of point p and the control point D, both referred to the other meridian $\overline{BA}$, or vice versa, as long as the distances of A, D and p from the instrumental center are negligibly small compared with that of B. See Fig. 40.

Accordingly, if a straight line is drawn through B and A situated on Curve 1, that is, the locus of collimation points when adjusted by only the method of One-Quarter Adjustment in Fig. 40, which is the same as that in Fig. 34, it becomes a new meridian when the vertical cross-hair is perfectly adjusted by the method of Integral Adjustment of the vertical cross-hair between B and A.

Therefore, draw a straight line through B and A on the locus of collimation points or Curve 1, so that the angular deviation of the control point may not surpass the allowable limit, referred to (215), and then it should give the right distance E_A.

Now, according to the new distance E_A, adjust the vertical cross-hair between B and the new point A by the method of Integral Adjustment. Refer to Fig. 40.

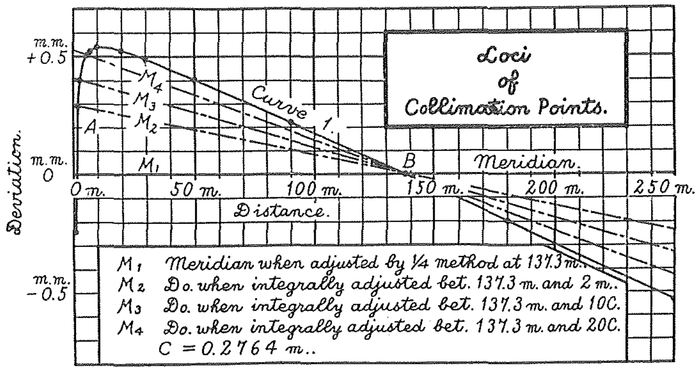

Fig. 40. Method for Determining E_A.

Also, in Fig. 40, the remainders, when Integral Adjustment is performed between 2 m., $10\,C = 2.764$ m. or $20\,C = 5.527$ m., and 137.32 m., are shown according to this method.

If, in the special work, such as the meridian determination in the field, the control points and the points for Integral Adjustment are made to coincide with each other, then the residual error should be only that due to the mis-centering of the transit.

50. Eccentricity of the Locus of Collimation Points. Now, from the curves of loci of collimation points illustrated in Figs. 34 and 35, it can immediately be seen that the magnitudes of the eccentricities of those loci can not be invariant generally, because the tangents to these curves can not have definite intercepts on the horizontal axes of the telescopes or ordinates in the figures respectively, except at infinity, and this fact does not depend upon the degree of adjustment of the vertical cross-hairs at all, but upon the existence of the instrumental errors e_g and i.

Analytically, **the eccentricity of the locus of collimation points at point p** is immediately obtained by the formula

$$\epsilon = E_p\varphi_p - E_p\frac{d}{dE_p}(E_p\varphi_p)\,, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (220)$$

with reference to Fig. 40.

Now, from (12) and (110), neglecting terms of the higher order, this equation is reduced to the form

$$\epsilon = e + (2C_p - C)i\,, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (221)$$

from which it is readily understood that the eccentricity varies according to the transposition of point p, as illustrated in the preceding figures but it has a limiting value

$$\epsilon_\infty = e + Ci \quad \ldots\ldots\ldots\ldots \quad (222)$$

for the infinitely distant point.

Even if the relation

$$\epsilon_\infty = 0$$

holds good, the residual eccentricity

$$\epsilon = 2(C_p - C)i \quad \ldots\ldots\ldots\ldots \quad (223)$$

should amount up to a respectable magnitude, referred to (222), when i is very large.

Now, the eccentricities of the transits employed in the experiments of the present research at points 2 m., 7 m,, 137.32 m. and infinity are shown in Table 18.

Table 18.

Transit No.	$\epsilon = e + 2(C_p - C)i$, m.m.			
	At 2 m.	At 7 m.	At 137.32 m.	At infinity
1	−0.346	−0.348	−0.348	−0.348
4	−0.066	+0.464	+0.636	+0.650
5	+0.093	+0.425	+0.532	+0.539
7	+0.705	+0.580	+0.539	+0.537
9	−0.134	−0.150	−0.156	−0.156
22	+0.134	−0.242	−0.364	−0.369
23	−0.147	−0.111	−0.100	−0.099
37	+0.113	+0.249	+0.291	+0.293
39	−0.532	−0.072	+0.088	+0.095
11	+0.218	+0.010	−0.061	−0.066
A1	+0.100	−0.043	−0.090	−0.092
A2	−0.043	+0.132	+0.187	+0.191

Finally, it is asserted that, strictly speaking, because there is no definite value of the eccentricity of the locus of collimation points except at infinity, that at infinity should properly be taken if neccessary.

SECTION IX.—RELATIVE MERITS OF THE METHODS OF INTEGRAL ADJUSTMENT OF THE VERTICAL CROSS-HAIR IN A TRANSIT

51. Relative Merits of the Methods of Integral Adjustment of the Vertical Cross-Hair in a Transit. Now, the value of the method of Integral Adjustment of the vertical cross-hair is approximately indicated by their ordinal number judging from the theoretical and practical points of view, and their merits are described in detail as follows :

The First Method.—Up to the present, it has been especially forbidden to adjust the vertical cross-hair between the near points for the purpose of avoiding errors coming into measured horizontal angles, caused by the instrumental errors ; yet, on the contrary, in this method, the optical errors are not only magnified slightly more than in the former but also rods or marks and accordingly, rodmen or markers are no longer neccessary at the distant points.

However, the greatest merit of this method consists in the respect that since, notwithstanding obstacles existing on the way to the distant point B, such as, a river, a lake, a valley, etc., the adjustment can immediately be performed, any far distant point can be taken for B *ad libitum* : as—the distant triangular mark.

Therefore, the vertical cross-hair should be adjusted most ideally. Hence, this mothod is especially suited for the transit, which is used for the observation of the meridian, the azymuth of a straight line in practical astronomy and a large triangulation survey.

For these reasons, the first method is the best compared with the others.

The Second Method.—In this method, the rods or the marks at the distant points $\overline{B}$ and B and also a scale or a mark at the near point A are indispensable.

However, this is the easiest method to adjust integrally not only the vertical cross-hair in a transit but also the horizontal cross-hair in a wye level.

The Third Method.—This procedure itself is none other than that found in two works* of first-class authorities in the U. S. A., but the corrections* for adjustment given in them are fundamentally mis-esti-

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* Refer to Reference at the end of this paper, and also Art. 55 and Art. 56 respectively.

mated severally; while, on the contrary, those shown by (186) and (188) of Art. 40 of the present work really are the exact solutions gotten from Geometrical Optics by the present author.

Now, in this method, the rods or the marks used for the first half of this adjustment, viz., the one-quarter adjustment of the vertical cross-hair, can be utilized again for the latter half or that of the objective slide, as it is called, and the adjustment should be made more easily than by the other methods, because of customary use and skill in the method of One-Quarter Adjustment.

As a matter of fact, the field formula for correction (188) has no perceptible error due to the neglect of the higher order, as long as point B is taken so far away.

The Fourth Method.—This is the reverse of the second and the rods or the marks at the distant points and a scale or a mark at the near point are necessary too.

The field formula for correction (194) has no error practically.

Further, like the second, this can be successfully applied to the Integral Adjustment of the horizontal cross-hair of a wye level.

The Fifth Method.—Because this is not only universal, including the above four, but also the principle is the simplest and the most efficient, it may generally be practised not only in the field but also in the manufactory.

Hence, it may be stated briefly that the most excellent method is the first, the second and the third rank after it in order, both the second and the fourth have special features applicable to Integral Adjustment of the horizontal cross-hair of a wye level, while the fifth is matchless efficient, because it is the most universal notwithstanding the facts that no profound knowledge of Geometrical Optics and no datum of the instrument are absolutely necessary.

SECTION X.—A BRIEF OUTLINE OF THE METHOD OF MANUFACTURING AND CONSTRUCTING THE PARTS OF A TRANSIT, CLOSELY RELATED TO THE CROSS-HAIRS

52. A Brief Outline of the Method of Manufacturing and Constructing the Parts of a Transit, closely Related to the Cross-Hairs. The following descriptions are extracts from the present author's note on the study by inspection and observation of a certain first-class manufactory of surveying instruments Tōkyō, Japan.

At present, the objective slide of a telescope is worked by a lathe till finishing, simply measuring its diameter by a vernier caliper reading to 0.05 m.m. during turning.

The bearing surfaces of the slide bearings of the objective slide are generally made with a kind of white metal at present, which is finished by a lathe to the accuracy of the same grade as the objective slide.

Next, the horizontal axis of the telescope is turned by a lathe, and the hole in it, into which the telescope tube is closely fitted, is turned by it too, tightly fastening the horizontal axis with bolts and nuts on the triangular notched rest to the chuck, after the horizontal axis is roughly worked in advance.

Then, after that hole is perfectly finished, the parts of the horizontal axis, which closely rest on the bearings of the standards are finished.

Subsequently, after both the horizontal axis of the telescope and the standards have been assembled, the heights of both legs are inspected, by measuring with a dial indicator of the division of 0.01 m.m. on a surface plate, and if their heights are not equal, they are adjusted by filing their bases.

Hereupon, the **centering tool A**, shown in Fig. 41, is closely fitted into the objective slide, which is previously inserted into the front slide bearing of the telescope tube of the above-explained assembled set and the adjustable rear slide bearing, as they are all set on the same surface plate, and then the telescope tube is simultaneously made prependicular to the surface of that plate too by touching the projecting arms of the **centering tool B,** shown in Fig. 41, to the outside of that tube in the plane of its rotation about the horizontal axis.

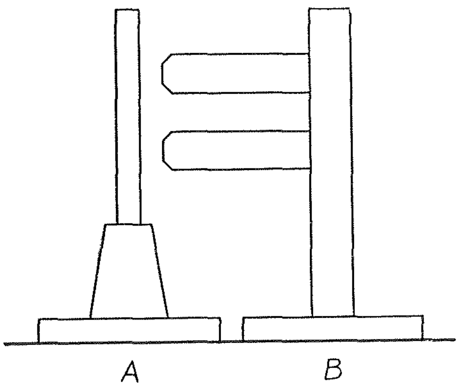


Fig. 41. Centering Tool or "Shin-Gané".

Next, in the position when the axis of the telescope tube is made vertically downward by means of the **centering tool B,** the adjustable rear slide bearing* is firmly fixed by tightly screwing the four screws around the telescope tube by a screw-driver.

* It is called "Tsuri-ring" in the manufactory of Japan.

Then the whole newly assembled set, as it was on the surface plate, is brought on the horizontal plate of the alidade, and precisely centered by fitting the **centering tool A** into the small hole at the center of that plate, and then the standards are firmly fixed to the same plate by means of screws.

Fig. 42. One Piece Truss Standard.

Thus, at present, the **centering tool** is not only a unique one in any manufactory, by which the objective slide, the telescope tube, the horizontal axis of the telescope, the standards and the horizontal plate can mechanically be centered when they are fitted together, but also the above-explained is the very unique and comparatively reliable method of adjustment of the objective slide which can be practised in only the manufactory.

However, there probably is no maker or no surveyor in the world now, who knows the accuracy of the result gotten by this present method of objective slide adjustment.

With respect to the efficiency of the centering tool, refer to Art, 57, Art. 58 and Art. 36.

Regarding the erroneous eccentricity and the inclination of the optical axis of the objective lens system itself, refer to Art. 30 and Art. 62.

The **centering tool** shown in Fig. 41 is that, which is commonly called "**Shin-Gané**" (心金) in the manufactories of Japan.

Further, in the telescope of the Wild-Zeiss type, the internal surface of the telescope tube itself is nothing but a slide of the internal focussing lens and its case itself an objective slide, that is to say, they are made in one piece, which construction is adopted in order to obtain the highest degree of accuracy.

Fig. 43.
Tube of the Internal Focussing Telescope.

For reference, the section of the telescope tube of a "Fuji" transit, manufactured by Sokkisha Co., Tōkyō, Japan, is shown in Fig. 43.

At the factory, the cross-hair of a transit is generally adjusted on a rigid standard in a limited space by the ordinary method of One-Quarter Adjustment, sighting at artificial near points through the test telescopes, by which they are seen as they were in the distance because they are focussed at infinite distances respectively.

In conclusion, the methods of centering the component lenses of the objective lens system should be described as follows; from "Zeiss' Photographic Lenses and how they are made", p. 18~19:

"When the lenses have acquired perfectly polished surfaces and when by the tests applied to them they have proved to strictly conform to the calculation the next task is to 'center' them. This means to so grind the peripfery of the lens that the optical axis is at the center of the cylindrical boundary of the lens. The optical 'axis', it should be noted, is the line joining the centers of the spheres which form the free lens surfaces."

For testing this adjustment there are two methods, an optical and a mechanical one, and, as before, the optical method is the more accurate of the two. It is based in principle upon the reflecting properties of polished lens surfaces. The lens which is to be centered is cemented by one of its surfaces to the chuck of a lathe and a flame, or point of light set up at some little distance from it. When the lens if made to rotate the two images of the flame formed by reflection at the two spherical surfaces will be seen to rotate about one another if the optical axis does not coincide with the axis of rotation of the lens. If both are coincident the reflected image appears to stand still. The lens is trued on the chuck, that is to say, it is brought into such a position that the reflected image appears to stand still, by displacing and tilting the lens over the cementing medium. When this has been accomplished the outer boundary of the lens is ground down to a cylinder on the lathe.

Where centered lenses require to be cemented together so as to form a single compound lens, this is done by means of Canada balsam.

Cementing is done applying a drop of Canada balsam to the carefully cleaned and warmed lenses. They are then placed upon one another, and by gentle pressure air and excess of balsam expelled. When the balsam has cooled the lenses will be firmly united, but they require once more trued up on the lathe, in order that the optical axis of two lenses may accurately coincide. The finished combination is then lacquered black at the cylindrical boundary, to obviate reflection of light at the translucent rim. It is now ready to be set in the lens mount.

The mounting is mostly done on the lathe with treadle motion. The degree of precision with which this operation is performed is the same as that applied in the processes of grinding and polishing, for it will be remembered that the distances between the componet lenses of the objective enter into the calculation and must therefore be adhered to within the smallest fraction. Extreme care must also be exercised to avoid any excessive pressure and still more so any one-sided pressure being exerted by the mount upon the lens, as this would give rise to internal strains in the glass, or it might even cause otherwise perfect lens surfaces to be distorted in a more or less pronounced degree."

SECTION XI.—CRITICISMS OF THE OLD METHODS OF COMPLETE ADJUSTMENT OF THE VERTICAL CROSS-HAIR IN A TRANSIT

53. **General Description.** The reason why the methods of Integral Adjustment of the vertical cross-hair in a transit have been studied by the present author for a long time, lies in the fact that all the existing so-called methods of Objective slide Adjustment expatiated in works on surveying published by first-class authorities up to present are radically mistaken from insufficiency of knowledge in Geometrical Optics and deliberation over the truth of the matter.

Really, as far as it relates to Geometrical Optics, the calculation must be made with the highest possibility of precision, because the neglect of terms of an extremely small magnitude should always cause a fatal error in it.

Once, it was experienced by the present author that in the special design of the field lenses used for the photo-elastic apparatus, whose diameters amount up to 200 m.m. exactly and which are the greatest in the world now, only precise results to three figures were given even by the ten-figured table of trigonometrical functions.

Further, it is generally known that in order to complete the design and the computation of a new photographic objective, it takes even an authority on the subject a few years.

This means that even a discursive approximation of a very small quantity should sometimes bring an astonishingly much larger error into the solution, if it were not thoroughly investigated by analytical computation.

Now, in all the first class works on surveying, the method of complete adjustment studied in the present paper is commonly called "the **method of Objective Slide Adjustment.**"

Following concise criticisms of the representative methods should exactly be given with analytical proofs in the following articles in order.

54. Old Method A. This method is found in two works of foreign first-class authorities. It is described respectively as follows:

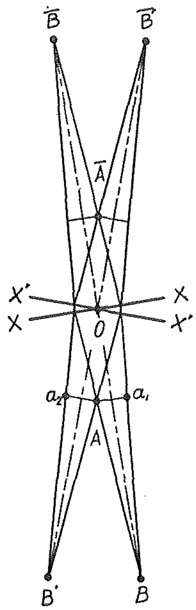


Fig. 44.
Illustration of the Adjustment by Old Method *A*.

In the first work: "Provision of some kind should be made for making the objective glass move, when focussed, in a right perpendicular to the horizontal axis. This is sometimes accomplished by making the telescope work through a ring which can be adjusted like the ring carrying the cross-wires. One way of adjusting the slide is as follows: After adjusting the line of sight by" the ordinary method of One-Quarter Adjustment, "using long sights, repeat the operation, using very short sights, the first requiring the telescope drawn well in, the second requiring the telescope run out. If the slide is out of adjustment the line of sight in the second test will not be in adjustment. In this case move the adjusting screws of the objective slide so as to apparently increase the error by one-quarter the error found.

Most makers make this adjustment permanently. In the higher grades of instrument the whole slide except the middle third is made to fit the inside of the telescope tube, a condition requiring considerable skill."

In the second: "To make the Objective Slide move Parallel to the Line of Sight. If the tube holding the objective is adjustable it must be placed so that the direction of the line of sight will not be disturbed when the telescope is focussed. The adjustment may be made as follows. Adjust the line of sight" by the ordinary method of One-Quarter Adjustment, "using very distant points. This will require the objective to be drawn in nearly as far as it will go and hence the position of the objective will be changed but little by any subsequent adjustment of the tube. Next repeat the test for the adjustment of the line of sight by using two points which are very near the instrument. In sighting on these points the objective must be run out and any error in its adjustment will change the direction of the line of sight so that it is no longer perpendicular to the horizontal axis of the instrument. In case the instrument fails to stand this test the objective slide does not move parallel to the line of sight. The adjustment is made by moving the adjustment screws of the objective slide so as to apparently increase the error making, by estimation, one-quarter the correction required. The adjustment of the line of sight should be again tested on two distant points and the cross-hairs moved in case the second adjustment appears to be disturbed the first."

"Shop Adjustments.—The adjustment of the objective slide and other adjustments such as centering the eyepiece tube and centering the circles are usually made by the instrument maker."

Now, the description of the process should again be repeated for the explanatory proof as follows:

After adjusting the vertical cross-hair by the ordinary method of One-Quarter Adjustment between the two distant points $\bar{B}$ and B, sight at the very near point $\bar{A}$ with the telescope normal, reverse the telescope, keeping the alidade still clamped tightly in the position when $\bar{A}$ was sighted, and then take the reading of the scale at point A—say a_1, whose distance is the same as that of A, that is, $E_{\bar{A}} = E_A$.

Again, performing likewise as before with the telescope inverted now, take the second reading of the scale at A—say a_2 and then the difference of the two readings is —say $\bar{t}_A$. Refer to Fig. 44.

Hereupon, the correction given by the above process is expressed by the formula

$$\varDelta a' = +\frac{\bar{t}_A}{4} \quad \dots\dots\dots\dots \quad (224)$$

with attention to the signs.

Now, from the above explanation and Fig. 44, it should readily be seen that the reading difference $\bar{t}_A$ is just equal to four times the diviation of collimation point A or $\bar{A}$ from the corresponding meridian.

Hereupon, comparing the description and Fig. 44 in the present article with that and Fig. 28 in Art. 39, where the second method of Integral Adjustment is fully expatiated, it can immediately be understood that the reading $\bar{t}_A$ in the present method is just equal to twice the reading t_A in the second and that both the processes and the magnitudes of correction are exactly the same.

Therefore, the correction for reading

$$\varDelta a = -\left\{1 + \frac{\delta - g}{C - \delta}\,\frac{E_A - C}{C - g}\right\}\frac{\bar{t}_A}{4} \quad \dots\dots\dots\dots \quad (225)$$

is found from (181).

Therefore, as in the second method of Integral Adjustment, the objective slide must be adjusted by screwing, so that the reading of the scale is changed exactly by the amount of the correction $|\varDelta a|$, from the second reading a_2 in the direction of the first a_1, with reference to (225).

Now, since an extremely large disagreement is found between the estimated correction in the present Old Method A or (224) and the new (225) analytically reduced above, a practical example must be shown here.

For example, for a Gurley's five-inch transit, the true correction for the reading of the scale

$$\varDelta a = -\left\{1 + 0.21\frac{E_A - C}{C}\right\}\frac{\bar{t}_A}{4} \quad \ldots\ldots\ldots\ldots\ (226)$$

is exactly reduced, putting into (225) the data gotten from Table 5 of Art. 26.

Hence, if the objective slide is adjusted by Old Method A according to (224), **the residual error should, at every adjustment, be increased by 35% of the original when adjusted at $E_A = 10C$ and 20% when adjusted at $E_A = 20C$, compare (226), in stead of decreasing it.**

From these, ***it is but too true that the correction for adjustment in* Old Method A has been radically mistaken for a long time.**

Moreover, it should readily be seen that this method is far inferior to the second method of Integral Adjustment described in Art. 39 because of subtlety and less accuracy.

At the beginning of the above-quoted instructions in the first work of the authority, "To make the objective glass move, when focussed, in a right perpendicular to the horizontal axis", and in that of the second, "To make the objective slide move parallel to the line of sight" are required.

Now, *it is definitely to be seen that, in a strict sense, the fundamental* idea for adjustment in the latter can not be practised because the locus of collimation points can not be a straight line but a hyperbola in general, as proved in Art. 46 and Art. 49 and illustrated in Fig. 34 to Fig. 40.

Therefore, if, taking it in a favourable sense, the normal collimation line were considered as a straight line perpendicular to the horizontal axis of the telescope through the center of rotation of the telescope, the idea for adjustment in the latter should be consistent with that in the former.

For the idea of adjustment in both the above, the inclination of the optical axis of the objective lens system from the meridian of the transit should be adjusted so that the eccentricity of the first principal point of the objective lens system from the meridian, e.g., e_l may be exactly invariable or that inclination i may just be equal to that of the objective lens itself (i), for which one must refer to Art. 30.

Therefore, the eccentricity of the first principal point of the objective lens system from the meridian

$$\begin{aligned} e_l &= e_{g,'p}+(C_p-C+\delta-g)\,(i) \\ &= a \text{ constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (227) \end{aligned}$$

is found from (81).

Now, since the vertical cross-hair is adjusted by the method of One-Quarter Adjustment, the deviation of the collimation point p

$$\begin{aligned} E_p\varphi_p &= e_l \\ &= a \text{ constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (228) \end{aligned}$$

is gotten from (81) and (227).

Thereupon, because the residual deviation of the collimation point p

$$\varphi_p = \frac{e_l}{E_p}$$

should generally increase to an injurious magnitude in the measurement of the horizontal angle, the idea for adjustment is mis-explained too.

The adjustment of the objective slide must be performed so as to make φ_p or $E_p\varphi_p$ virtually disappear at a pair of two points on the locus of collimation points A and B, that is to say—$e+(C_A+C_B-C)\,i = 0$, refer to (110), which is exactly the second condition for Integral Adjustment (168 b). Again in other words, when point A is sighted, the deviation of point V from the meridian must be made to disappear, refer to (101) and Fig. 18 of Art. 20, if the vertical cross-hair has been adjusted between $\overline{B}$ and B by the ordinary method of One-Quarter Adjustment.

55. Old Method B. This method is found in the work of a first-class foreign authority too. The procedure itself is fairly superior to the preceding except for the correction and that which is written down in it as follows:

"Sight to some well-defined point as far off as it can be distinctly seen. Then, having the plates firmly clamped, move out the object-glass slide, and fix a point in the line of sight as close to the instrument as can be distinctly seen. Then turn the limb half-way around horizontally, reverse the telescope, and again sight to the near point, by clamping the plates and bringing the vertical cross-hair on the point by means of the tangent screw. Then draw in the object-glass slide until the distant object is distinctly seen. If the vertical cross-hair bisects it, no adjustment is necessary. If not, correct one half of the apparent error by means of the screws attached to the rear slide bearing" (Gr in Fig. 16 of Art. 19.) "This may disturb the second adjustment. Try that over again, and again perform the operation of centering the object-glass.

This adjustment is always performed by the maker, and its screws are covered by a short tube."

Now, since this procedure itself is exactly the same as that of the third method of Integral Adjustment described in Art. 40 but the magnitude of the correction and the direction of the adjustment are different, all the explanation and the proofs should be referred to it.

Accordingly, from the above description, referred to the notations in Art. 40, the correction at B is given by the formula

$$\varDelta b' = -\frac{t_B}{2}, \quad \ldots\ldots\ldots\ldots \quad (229)$$

taking care for the sign, where t_B is the departure of the second sight point from the first at B.

Now, because in the third method of Integral Adjustment expatiated in Art. 40, the correction

$$\varDelta b = \frac{\delta - g}{C - \delta}\frac{E_A}{C - g}\frac{t_B}{2}$$

is exactly given by (188), the above estimated correction (229) should be erroneous exceedingly in its magnitude and its direction of adjustment.

For example, for a Gurley's five-inch transit,

$$\varDelta b = 0.104\frac{E_A}{C}t_B \quad \ldots\ldots\ldots\ldots \quad (230)$$

is exactly gotten, putting the practical data given in Table 5 of Art. 26 into (188).

Therefore, **the error should, at every adjustment, be increased by 48% of the original when the slide is adjusted by the present Old Method B through (229) at $E_A = 10C$ and by 24% when adjusted at $E_A = 20C$ so far from decreasing it, compare (230).**

For that reason, **the correction for adjustment in Old Method B has been fundamentally mis-estimated for a long time** too, but the process itself is superior as it is graded third among the others.

56. Old Method C. This procedure, described in the work of a certain maker of the first-class in a foreign country, is far superior to any hitherto mentioned but the correction is still extremely erroneous.

Now, it is stated in it as follows :

"In case of accident or injury, it may be neccessary to adjust the objective slide, and this should be done as follows. First make sure that the vertical wire is as nearly plumb as it is possible to make it. Having set up and leveled the instrument, the line of collimation being adjusted for objects from three hundred to five hundred feet distant, clamp the plates, and fix the vertical cross wire upon an object as distant as may be distinctly seen. Without disturbing the instrument, move out the objective so as to bring the vertical wire upon an object as near as the range of the telescope will allow. Having this object clearly in mind, loosen the upper clamp, turn the instrument half way around, reverse the telescope, clamp the instrument, and with the tangent screw bring the vertical wire again upon the near object ; then draw in the objective until the distant object first sighted upon is brought into distinct vision. If the vertical wire strikes the same line as at first, the slide is correct for both near and remote objects, and, being itself straight, is correct for all the distances.

But if there is an error, proceed as follows : With a screw driver turn the two screws attached to the rear slide bearing" (Gr shown in Fig. 16 of Art. 19), "on the opposite sides of the telescope, loosening one and tightening the other, so as to apparently increase the error, making, by estimation, one half the correction required. Then go over the usual adjustment of the line of collimation, and, having completed it, repeat the operation above described, first sighting upon the distant object, then upon near one in line, then reversing, making corrections, etc., until the adjustment is complete.

This adjustment is a distinctive feature of Gurley transits and furnishes the only way in which the line of collimation can be correct for all distances."

Hereupon, it can immediately be seen that this process itself is wholy the same as that of the third method of Integral Adjustment excepting the correction for adjustment.

Thereupon, from the above description, according to the notations in Art. 40, the correction at B should be given by the formula

$$\Delta b'' = +\frac{t_B}{2}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (231)$$

attending to the signs.

Now, since, in the third method of Integral Adjustment described in Art. 40, the correction

$$\Delta b = \frac{\delta-g}{C-\delta}\,\frac{E_A}{C-g}\,\frac{t_B}{2}$$

is accurately given by (188), the preceding correction (231) must be much mis-estimated.

For example, for a Gurley's five-inch transit, the correction

$$\Delta b = 0.104\frac{E_A}{C}t_B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (232)$$

is accurately obtained from (188), putting the observed data given in Table 5 of Art. 26 into it.

Hence, referring to (232), **the error should, at every adjustment, be decreased by only 36% of the original when the objective slide is adjusted by the present Old Method C according to (231) at $E_A = 15$ ft. $=4.574$ m. and also by only 24% if adjusted at $E_A= 20C$ by the same.**

Therefore, **in order to decrease the error to a point under 5% by Old Method C, the operations for adjustment must be repeated seven times at least when adjusted at $E_A = 15$ ft. $= 4.574$ m., and also eleven times at least if adjusted at $E_A=20C$.**

For that reason, **this method can not yet be utilized in practice** but the procedure itself is fairly superior to the preceding old methods so that it is graded third among methods of Integral Adjustment.

57. The Centering Tool for Fitting together and Centering the Objective Slide. Because of nature, the centering tool has been conveniently used up to the present, when the objective slide, the telescope tube, the horizontal axis of the telescope, the standards and the horizontal plate are pieced together when the transit is newly manufactured or fitted together after repair.

It is called the "Shin-Ganè" (心金) in the manufactory in Japan, and is sketched in Fig. 41 of Art. 52.

Therefore, respecting the process of use of the centering tool, refer to Art. 52.

Now, it should principally be used in the factory as the most reliable method, because, for the adjustment of the objective slide, the preceding old methods are not reliable at all as proved in Art. 54, Art. 55 and Art. 56 in detail; while, on the contrary, a quite tolerable accuracy is commonly obtained by the centering tool.

Despite this fact, the experimental results gotten in Art. 30 and Art. 62 make it clear that the eccentricity and the inclination of the optical axis of the objective lens system itself amount up to fairly large magnitudes in general, which can not at all adjusted by only the centering tool.

In addition, the eccentricities and the inclinations of the optical axis of the objective lens system caused by the clearance between the objective lens itself and its holder, the error in centering and the inclination of the optical axis of the objective slide, and the clearance between the inner surface of the objective slide and the outer of the centering tool can absolutely be eliminated by no centering tool.

Now, the passably large magnitude of these injurious residual errors, remaining after the adjustment of the objective slide by only the centering tool, can often be found in a new instrument just come from the manufactory: as — $e = -0.37$ m.m. and $i = +5' \, 06''$ in transit No. 1 and $e = -3.68$ m.m. and $i = -32' \, 46''$ in No. 22 as in Table 4 and Table 6 in Art. 26, and similarly, $t_P/2 = +0.18$ m.m. and $t_B/2 = -2.5$ m.m. in level No. 23, it being that $E_P = 10$ m. and $E_B = 138.3$ m., as in Table 20 of Art. 61.

58. Summary to Criticisms.* Originally, it should be right that first the centering tool is used for fitting together the parts above the

* During the present author's studies, two new methods of adjustment were made public by Assistant Prof. Kichirō Tanaka of Civil Engineering, the Kyūshū Imperial University, Fukuoka, Japan.

His first method, described in Reference (16) at the end of the present paper, is nothing but a method of One-Quarter Adjustment performed between a distant point B and a near point $\overline{A}$, the rod at the former being read.

Nevertheless, it does not satisfy both the conditions of the horizontal and the vertical axes, namely —— (105) and (115) as in Art. 37 and Fig. 19 of Art. 21 in the present work. Moreover, it is proved experimentally that it is inferior to the ordinary method of One-Quarter Adjustment.

Similarly, by his second method explained in Reference (17) at the end of the present paper, the biaxial conditions expounded in Art. 37 in the present paper can not be fulfilled from the same reasons, the deviations of collimation points can not be eliminated at all, and, what is more, the process for adjustment is more lengthy than any of those of Integral Adjustment studied in the present paper.

hotizontal plate, and after they have been wholly put together, the instrument is satisfactorily readjusted by the method of objective slide adjustment.

Notwithstanding the above fact, it can be synthetically apprehended from the preceding discussions that there is no reliable theoretically calculated method of adjustment of the objective slide for both the shop and the field, up to the present.

Hereupon, the centering tool itself can be utilized in only the shop but not in the field by a surveyor at all.

Nevertheless, its accuracy is known by almost no one at present, and it can not clearly be dogmatized either that the accuracy of the centering tool itself is invariable, because it may possibly be subjected to internal strains of the metal, abraisions and accidents during the use for a long time before anyone knows.

For that reason, the methods of Integral Adjustment of the vertial cross-hair are conceived so that the instrument may not only be perfectly adjusted by them in the factory after being assembled by the centering tool but also by a surveyor in the field when the transit has been used practically for year or subjected to accidents.

CHAPTER IV

INTEGRAL ADJUSTMENT OF THE HORIZONTAL CROSS-HAIR IN A WYE LEVEL

59. General Description. From the theoretical point of view, the ordinary method of One-Half adjustment of the horizontal cross-hair in a wye level is substantially the same as that of One-Quarter Adjustment of the vertical cross-hair in a transit. This is true because the level plane determined by a wye level, which is perpendicular to the vertical axis of rotation of the telescope and on which the locus of collimation points must lie when levelled up, exactly corresponds by its very nature to the meridian of the transit, which is perpendicular to the horizontal axis of rotation of the telescope and in which the locus of collimation points must be included.

Nevertheless, as a matter of fact, the error is magnified by just twice in the former, while, on the contrary, by four times in the latter.

Now, after the horizontal cross-hair of a wye level has been perfectly adjusted by the ordinary method of One-Half Adjustment at a certain distant point B, if the adjustment is tested at any other near point p, reading the scale there with the telescope normal in the first place and subsequently with the telescope inverted by rotating it one hundred and eighty degrees as in the wyes, then the difference of the two readings—say t_p, whose value should exactly be given by (126) or (110), will not generally vanish, as already made clear in case of the transit.

Therefore, the vertical deviation of the collimation point p from the level plane when levelled up is given by the formula

$$\begin{aligned} E_p\varphi_p &= \frac{t_p}{2} \\ &= \left\{e+(C_p+C_B-C)i\right\}\frac{E_p-C_p-E_B+C_B}{E_B-C_B}, \quad \ldots\ldots\ldots \quad (233) \end{aligned}$$

compare (126).

Therefore, on the contrary, if the horizontal cross-hair is adjusted at point p, then the deviation of the collimation point B from the level plane when levelled up is given by the formula

$$E_B \varphi_B = \left\{e+(C_B+C_p-C)i\right\} \frac{E_B-E_p}{E_p}$$

$$= -\frac{t_p}{2} \frac{E_B}{E_p}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (234)$$

compare (233), neglecting terms of the higher order.

Now, the experiments for finding these values were performed on twenty five wye levels in the Civil Engineering Division of the Faculty of Engineering, Hokkaidō Imperial University, where the present author is now engaged, with the results shown below in Table 20 of Art. 61. Among them, the greatest errors $E_A \varphi_A = +0.49$ m.m., $E_D \varphi_D = -0.51$ m.m. and $E_P \varphi_P = +0.47$ m.m. were found when adjusted at B, and $E_B \varphi_B = -23$ m.m., $+14$ m.m. and -7 m.m. when adjusted at A, D and P respectively, taking $E_B = 138.3$ m., $E_A = 3$ m., $E_D = 5$ m. and $E_P = 10$ m. Such errors are injurious to even the ordinary survey.

For example, if, after the horizontal cross-hair of level No. 1 has been adjusted by the ordinary method of One-Half Adjustment at the range of 20 m. away from the instrument, a slope of 5 km. length and a grade 0.06 is levelled with it, always taking the readings of the rod held at the distance of 80 m. for the sight of the down-grade and at the distance of 20 m. for that of the up-grade, then an error of 1.4 m.m. always comes into the elevation difference at every levelling and the total should properly accumulate up to 70 m.m. when ascent or descent.

Now, even if the horizontal cross-hair is adjusted at the range of 80 m., the total error should be 23 m.m.

For all that, since this error always comes in with the same sign without reference to the ascent or the descent, there is absolutely no method of finding this error in the field and accordingly, making it disappear in the measurement of levelling.

These antipathic errors are principally produced during practical use for years, because of the alteration of the optical and mechanical conditions due to the change of the aspect of the objective lens caused by the removal of it from its holder for the purpore of clearning it and refitting it after cleaning, the changes due to the conditions and the abraision of the metal parts, accidental causes, etc.

Nevertheless, these are tolerably found in even a new instrument, compare Table 20 of Art. 61, as shown by the experimental results for the transit in Art. 26, Art. 27, Art. 28 and Art. 31.

Now, the negative basic origin of these errors is incomplete workmanship in the factory. The positive principal cause is the erroneous adjustment in both the factory and the field by the surveying engineer. Between them the improvement of the former closely correlates to the economical problem; while, on the contrary, the perfection of the latter increases no economical charge, and it has very successfully been unravelled by the present work, so that it can generally be practised in both the factory and the field by the surveyor.

From the above described proof, it is readily seen that the new methods of complete adjustment of the horizontal cross-hair in a wye level are no other than those of Integral Adjustment of the vertical cross-hair in a transit described in Section VI of Chapter III in detail. There the condition

$$t = 0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (235)$$

is virtually satisfied at another control point but B, by experiments as shown in Table 26 of Art. 66.

Now, the experiments for finding out the errors and the adjustment will particularly be shown in the following articles in order.

60. Apparatus Used in Experiments for Finding out the Optical Errors in a Wye Level. Now, in order to determine the magnitudes of the existing optical errors in a wye level before the experiment for the adjustment of the horizontal cross-hair is performed, a special test apparatus is suitably designed as illustrated in Fig. 45 and Fig. 46. It bears a striking resemblance to that used for determining the opical errors connected with the vertical cross-hair in a transit in Art. 29 and Art. 30, but differs in the respect that the errors relating to the horizontal cross-hair in a wye level had to be researched in the former. See Figs. 45 and 46.

Point C is the station of the instrument for the test, while point A is an indicator point, reading to 0.01 m.m. the range of reading being 10 m.m., situated at 3 m. away from the instrumental center.

Points D and P are nothing but the same indicator point reading to 0.01 m.m., which are arranged so that the former is the very point sighted at the distance of 5 m. away from the instrumental center through Prism 2 and Prism 3 and the latter that at the distance of 10 m. away through Prism 1 in addition to the above two.

Since the range of reading of this indicator is only 6 m.m., it is fitted on the vernier caliper reading to 0.02 m.m., whose range of reading is 340 m.m., so that more reading can be taken.

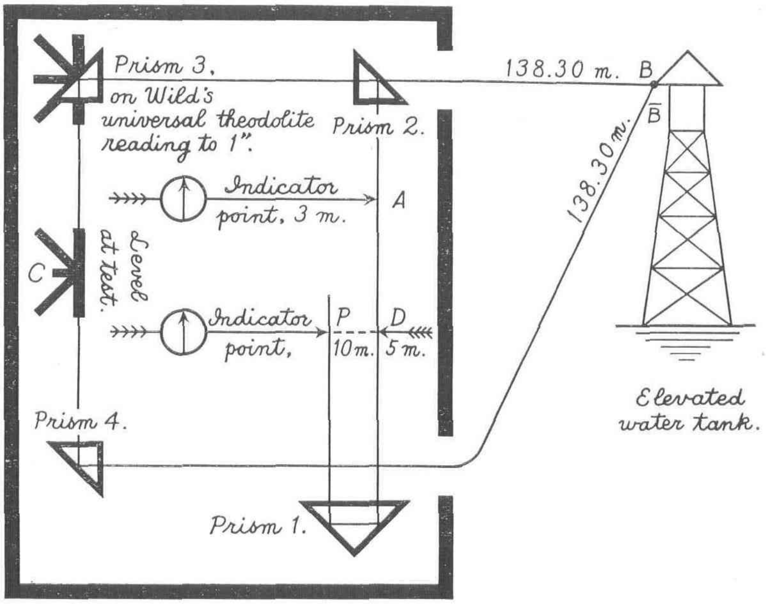


Fig. 45. Apparatus for Test and Integral Adjustment of the Horizontal Cross-Hair in a Wye Level.

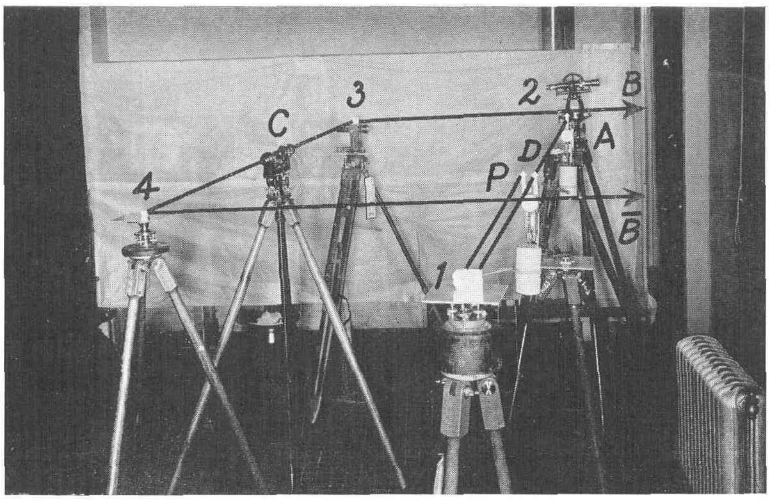


Fig. 46. Apparatus for Test and Integral Adjustment of the Horizontal Cross-Hair in a Wye Level.

Points B and $\bar{B}$ are nothing but the same corner of an elevated water tank at the distance of 138.3 m. away from the instrumental center, that is to say, the former is that sighted through Prism 3 at foresight and the latter through Prism 4 at backsight.

Hereupon, since Prism 3 is fixed on a Wild's universal theodolite reading to one second in both the vertical and the horizontal angles, points B, A, D and P can be adjusted vertically and horizontally by the observer as desired, when sighted through the telescope.

In addition of the above, of course, as shown in Fig. 46, Points A, D and P themselves are arranged so that they can also be individually adjusted by the observer as desired when sighted through the telescope.

Thus, in the above apparatus, the five points B, P, d, A and $\bar{B}$ are arranged in a straight line, so that the first four are sighted at fore sight and only the last at back sight.

Therefore, through this apparatus, after the adjustment of the horizontal cross-hair in a wye level has been completed by the ordinary method of One-Half Adjustment at B, P, D, etc., the test reading t shown by (126) can readily be found out at any other point as described in Art. 59.

Then, after it has been adjusted between B and A, D or P by the method of Integral Adjustment of the horizontal cross-hair in a transit described in Chapter II, using the levelling screws, rotate the telescope about the vertical axis of the instrument by 180°, sight at point $\bar{B}$ at back sight, again rotate it about its vertical axis by 180°, and then take the readings of the indicator points at A, D and P respectively.

Subsequently, perform the same process with the telescope inverted, rotating it by 180° as it is in the wyes, and also take the second readings.

Then, the differences of the first and the second readings shuld be the very readings of the u's shown in (127).

Through the same apparatus, the experiments for the adjustment can be performed too.

Now, the experiments performed through this apparatus will be explained in particular in the following articles in order.

61. First Experiment.—Test for the Adjustment of the Horizontal Cross-Hair in a Wye Level. Through the apparatus explained in Art. 60 according to the principle described in Art. 59, the twenty-five wye levels in the Civil Engineering Division of the Faculty of Engineering, Hokkaidō Imperial University, Sappro, Japan, where the present author is now engaged, were tested. The experiment is expatiated as follows, with reference to Fig. 45 and Fig. 46 of Art. 60 :

Now, first, after the horizontal cross-hair has been adjusted at point B by the ordinary method of One-Half Adjustment, take the the indicator reading at point A with the telescope normal and subsequently with it inverted, rotating it by 180° as it is in the wyes, and then obtain the difference of the two readings t_A.

Similarly, perform between B and D, and also B and P, and so obtain t_D and t_P.

Hereupon, the distances of A, D, P and B from the instrumental station are 3 m., 5 m., 10 m. and 138.3 m. respectively.

Now, the one-half of t is the very deviation of the collimation point from the level plane referred to in (233).

In Table 19, the sizes, the countries of production, the date of purchase, the years of use, etc. are shown.

Table 19.

(Sept., 1935)

Transit No.	Size, In	Made in	Date of Purchase	Years of Use	User
1	18	U.S.A.	Nov., 1923	0.1	ass. prof.
2	15	Japan	Dec., 1923	0.0	none
3	15	Japan	Mar., 1925	10.0	students
4	15	Japan	Mar., 1925	10.0	students
5	15	Japan	Mar., 1925	10.0	students
6	15	Japan	Mar., 1925	10.0	students
7	15	Japan	Mar., 1925	10.0	students
8	15	Japan	Mar., 1925	10.0	students
9	15	Japan	Mar., 1925	10.0	students
10	15	Japan	Mar., 1925	10.0	students
11	15	Japan	June, 1925	9.0	students
12	15	Japan	June, 1925	9.0	students
13	15	Japan	June, 1925	9.0	students
14	15	Japan	Dec., 1925	9.0	students
15	15	Japan	Dec., 1925	9.0	students
16	15	Japan	Mar., 1926	9.0	students

Table 19.—(*Continued*)

Transit No.	Size, In	Made in	Date of Purchase	Years of Use	User
17	15	Japan	Mar., 1926	9.0	students
18	15	Japan	Mar., 1926	9.0	students
19	15	Japan	Mar., 1926	9.0	students
20	15	Japan	Mar., 1926	9.0	students
23	15	U.S.A.	Mar., 1929	0.0	none
24	18	U.S.A.	Mar., 1929	0.0	none
30†	15	Japan	Mar., 1930	0.0	none
26*	15	Japan	Feb., 1930	0.0	none
31*	15	Japan	Mar., 1930	0.0	none

† The light metal instrument.
* With the telescope of the internal focussing type.

In the second, fifth and the eighth columns of Table 20, the values of $t/2$ are given.

Now, if the horizontal cross-hair is adjusted by the same method at A, D or P, the error in the height of B should be given by (234), whose values are shown in the third, sixth and nineth columns in the same table.

Further, the erroneous angular divergences due to these erroneous deviations are shown in the fourth, seventh and tenth columns.

Now, it can readily be nnderstood from the results shown in Table 20, that No. 1, No. 7, No. 17 and No. 20 are extraordinarily erroneous, compared with the others, because, referred to (233) and (234), these erroneous deviations are caused by e and i, which change according to the point sighted in direct proportion to the inclination of the optical axis of the objective lens system i.

Finally, because the angular error of three seconds or 0.000 015 radian is exactly perceptible by the ordinary telescope-level graduated to thirty seconds, an erroneous angular deviation over three seconds should properly be unsatisfactory in practice.

Table 20.

Experimental Note 8.

(Sept., 1935)

Level No.	Error in Elevation of A when adjusted at B, m.m.	Error in Elevation of B when adjusted at A, m.m.	Erroneous Angular Divergence between A and B, sec.	Error in Elevation of D when adjusted at B, m.m.	Error in Elevation of B when adjusted at D, m.m.	Erroneous Angular Divergence between D and B, sec.
1	+0.493	−22.8	33.9	+0.439	−12.2	18.1
2	+0.098	− 4.5	6.7	+0.007	− 1.9	0.3
3	−0.044	+ 2.0	3.0	−0.051	+ 1.4	2.1
4	0.000	0.0	0.0	−0.016	+ 0.4	0.7
5	+0.062	− 2.9	4.3	+0.077	− 2.1	3.2
6	−0.051	+ 2.4	3.5	−0.061	+ 1.7	2.5
7	+0.069	− 3.2	4.7	−0.507	+14.0	20.9
8	−0.142	+ 6.6	9.8	−0.192	+ 5.3	7.9
9	+0.013	− 6.0	0.9	+0.002	− 0.1	0.1
10	0.000	0.0	0.0	+0.048	− 1.3	2.0
11	−0.025	+ 1.2	1.7	−0.044	+ 1.2	1.8
12	+0.072	− 3.3	5.0	+0.054	− 1.5	2.2
13	+0.107	− 4.9	7.4	+0.096	− 2.7	4.0
14	−0.048	+ 2.2	3.3	−0.119	+ 3.3	4.9
15	−0.085	+ 3.9	5.8	−0.095	+ 2.6	3.9
16	+0.024	− 1.1	1.7	−0.080	+ 2.2	3.3
17	+0.213	− 9.8	14.7	+0.164	− 4.5	6.8
18	+0.058	− 2.7	4.0	+0.079	− 2.2	3.3
19	−0.047	+ 2.2	3.2	−0.115	+ 3.2	4.7
20	+0.179	− 8.3	12.3	+0.228	− 6.3	9.4
23	+0.099	− 4.6	6.8	+0.086	− 2.4	3.5
24	−0.062	+ 2.9	4.3	−0.021	+ 0.6	0.9
30†	+0.025	− 1.2	1.7	−0.056	+ 1.5	2.3
26*	−0.002	+ 0.1	0.1	−0.001	+ 0.0	0.0
31*	−0.065	+ 3.0	4.5	−0.111	+ 3.1	4.6

Notice: The points A, D, P and B are 3 m., 5 m., 10 m. and 138.3 m. away from the instrumental center respectively.

† The light metal instrument.
* With the telescope of the internal focussing type.

Table 20.—(*Continued*)

Experimental Note 8.

(Sept., 1935)

Level No.	Error in Elevation of P when adjusted at B, m.m.	Error in Elevation of B when adjusted at P, m.m.	Erroneous Angular Divergence between P and B, sec.
1	+0.468	−6.5	9.7
2	+0.013	−0.2	0.3
3	−0.049	+0.7	1.0
4	−0.035	+0.5	0.7
5	+0.077	−1.1	1.6
6	−0.008	+0.1	0.2
7	+0.004	−0.1	0.1
8	+0.025	−0.3	0.5
9	−0.017	+0.2	0.4
10	+0.012	−0.2	0.2
11	−0.078	+1.1	1.6
12	+0.047	−0.7	1.0
13	+0.070	−1.0	1.4
14	−0.120	+1.7	2.5
15	−0.009	−0.1	0.2
16	−0.086	+1.2	1.8
17	+0.055	−0.8	1.1
18	+0.150	−2.1	3.1
19	−0.173	+2.4	3.6
20	+0.243	−3.4	5.0
23	+0.183	−2.5	3.8
24	+0.031	−0.4	0.6
30†	+0.061	−0.8	1.3
26*	−0.015	+0.2	0.3
31*	−0.074	+1.0	1.5

Notice: The points A, D, P and B are 3 m., 5 m., 10 m. and 138.3 m. away from the instrumental center respectively.

† The light metal instrument.

* With the telescope of the internal focussing type.

62. **Second Experiment.—Determination of the Error Due to the Eccentricity and the Inclination of the Optical Axis of the Objective Lens System Referred to its Ideal Axis.** The optical error in the objective lens system itself in a wye level has also never been observed up to now. Accordingly in order to ascertain whether it is detrimental or negligible, the present experiment consists in making clear the magnitude of it. It will be shown that even the adjustment of the horizontal cross-hair is extremely disturbed due to the change of the mechanical and optical conditions according to the change of the aspect of the objective lens, as soon as it is taken out of the holder for cleaning the cloud on its surfaces and again is tightly fitted.

The process of the experiment may be described in detail as follows:

First, take the objective lens out of the holder, graduate its circumference every thirty degrees, again fit and tighten it in the holder, checking not only the counter-marks put by the maker on the periferies of the several component lenses but also the zero-degree mark and that on the holder, and then fit the holder to the telescope tube, also checking the counter-marks on them put by the observer in advance.

Subsequently, adjust the horizontal cross-hair by the ordinary method of One-Half Adjustment at point B, 138,3 m. away from the instrumental center.

Now, since all the experiment was performed through the same apparatus as depicted in Fig. 45 and shown in Fig. 46 of Art. 60, for the detailed explanation one should refer to Art. 60.

After the adjustment has been completed, take the readings of the indicator points A, D and P, situated at 3 m., 5 m. and 10 m. away from the instrumental center respectively, with the telescope normal and also inverted, and then obtain the deviations at A, D and P, namely—$t_A/2$, $t_D/2$ and $t_P/2$, taking one-half the differences of the respective readings, as described in Art. 61.

Again loosen the objective lens in the holder or take it out check anew the thirty-degree mark on the former and that on the latter this time, changing the aspect of the former correlate to the latter by thirty degrees, perform the same and get the new deviations.

Repeat the process likewise for the sixty-degree mark and so on.

These experiments were performed on two representative wye levels, that is, No. 7 and No. 20, with the results shown in the second, fifth and eighth columns of Table 21 and Table 22 respectively. The values in the tables are each the exact means of five readings.

Table 21.

Experimental Note 9.

(Sept., 1935)

Angle of Rotational Aspect of the Objective Lens, Degrees	Error in Elevation of A when adjusted at B, m.m.	Error in Elevation of B when adjusted at A, m.m.	Erroneous Angular Divergence between A and B, sec.	Error in Elevation of D when adjusted at B, m.m.	Error in Elevation of B when adjusted at D, m.m.	Erroneous Angular Divergence between D and B, sec.
0	−0.355	+16.4	24.4	−0.322	+ 8.9	13.3
30	−0.387	+17.9	26.6	−0.366	+10.1	15.1
60	−0.330	+15.2	22.7	−0.303	+ 8.4	12.5
90	−0.193	+ 8.9	13.3	−0.186	+ 5.1	7.7
120	−0.151	+ 7.0	10.4	−0.154	+ 4.3	6.4
150	−0.110	+ 5.1	7.6	−0.075	+ 2.1	3.1
180	−0.044	+ 2.0	3.0	−0.026	+ 0.7	1.1
210	−0.034	+ 1.6	2.3	−0.040	+ 1.1	1.7
240	−0.383	+17.7	26.3	−0.380	+10.5	15.7
270	−0.317	+14.6	21.8	−0.297	+ 8.2	12.3
300	−0.243	+11.2	16.7	−0.228	+ 6.3	9.4
330	−0.190	+ 8.8	13.1	−0.175	+ 4.8	7.2

Note: Points A, D, P and B are 3 m., 5 m., 10 m. and 138.3 m. away from the instrument, respectively.

The values are the means of the differences of five readings respectively.

The maximum and the minimum values of the difference of the external diameter of the objective lens itself and the internal diameter of the holder are 0.26 m.m. and 0.20 m.m. respectively.

Further, with regard to the exact theory of the deviation of the collimation point caused by the eccentricity of the first principal point of the objective lens system and the inclination of its optical axis referred to its formal axis, one may refer to Art. 18.

As in Table 20 of Art. 61, the errors in the elevations of point B, when adjusted at A, D and P respectively, are shown in the third, the sixth and the ninth columns of the same respective tables, and the

Table 21.—(*Continued*)

Experimental Note 9.

(Sept., 1935)

Angle of Rotational Aspect of the Objective Lens, Degrees.	Error in Elevation of P when adjusted at B, m.m.	Error in Elevation of B when adjusted at P, m.m.	Erroneous Angular Divergence between P and B, sec.
0	−0.303	+4.2	6.3
30	−0.356	+4.9	7.4
60	−0.270	+3.7	5.6
90	−0.171	+2.4	3.5
120	−0.151	+2.1	3.1
150	−0.038	+0.5	0.8
180	+0.066	−0.9	1.4
210	−0.015	+0.2	0.3
240	−0.263	+3.6	5.4
270	−0.255	+3.5	5.3
300	−0.166	+2.3	3.4
330	−0.165	+2.3	3.4

Note: Points P and B are 10 m. and 138.3 m. away from the instrumental center respectively.

The values are the means of the differences five readings respectively.

The maximum and the minimum values of the difference of the external diameter of the objective lens itself and the internal diameter of the holder are 0.26 m.m. and 0.20 m.m. respectively.

erroneous angular divergences between B and A, D and P in the fourth, the seventh and the tenth respectively.

Notwithstanding the fact that the horizontal cross-hairs were completely adjusted by the ordinary method of One-Half Adjustment at point B, 138.3 m. away from the instrument, the experimental results of levels No. 7 and No. 20 are so unsatisfactory that almost the entire aspect of the objective lenses may be unsatisfactory or rejectable even at point P, 10 m. away from the instrument, because the erroneous angular divergence of three seconds or 0.000 015 radian is perceptible by an ordinary telescope level graduated to thirty seconds.

Table 22.

Experimental Note 10.

(Sept., 1935)

Angle of Rotational Aspect of the Objective Lens, Degrees	Error in Elevation of A when adjusted at B, m.m.	Error in Elevation of B when adjusted at A, m.m.	Erroneous Angular Divergence between A and B, sec.	Error in Elevation of D when adjusted at B, sec.	Error in Elevation of B when adjusted at D, m.m.	Erroneous Angular Divergence between D and B, sec.
0	−0.284	+13.1	19.5	−0.308	+ 8.5	12.7
30	−0.088	+ 4.1	6.1	−0.100	+ 2.8	4.1
60	+0.122	− 5.6	8.4	+0.105	− 2.9	4.3
90	+0.207	− 9.6	14.2	+0.165	− 4.6	6.8
120	+0.255	−11.8	17.5	+0.219	− 6.1	9.0
150	+0.187	− 8.7	12.9	+0.147	− 4.1	6.1
180	+0.049	− 2.3	3.4	−0.006	+ 0.2	0.2
210	−0.108	+ 5.0	7.4	−0.156	+ 4.3	6.4
240	−0.229	+10.6	15.8	−0.280	+ 7.8	11.6
270	−0.319	+14.7	21.9	−0.336	+ 9.3	13.9
300	−0.337	+15.5	23.2	−0.382	+10.6	15.8
330	−0.333	+15.4	22.9	−0.350	+ 9.7	14.4

Note: Points A, D, P and B are 3 m., 5 m., 10 m. and 138.3 m. away from the instrumental center respectively.

The objective is composed of two air-spaced lenses.

The values are the means of the differences of five reading severally.

The mean difference of the external diameter of the objective lens itself and the internal diameter of the holder is just 0.10 m.m.

63. Third Experiment.—Determination of the Mean Variations of the Angular Deviations of Collimation Points and the Eccentricities and the Inclination of the Optical Axis of the Objective Lens System.

Since a similar experiment as already performed on transits No. 4, No. 5 and No. 9, has been descrived in Art. 32, all respecting the present problem must be referred to it.

Now this experiment was likewise performed through the apparatus depicted in Fig. 45 and also shown schematically in Fig. 46 of Art. 60, so for particular explanations one must refer to it. The procedure may be described as follows:

First, adjust the horizontal cross-hair by the method of Integral Adjustment of the horizontal cross-hair in a transit, studied in Chapter

Table 22.—(*Continued*)

Experimental Note 10.

(Sept., 1935)

Angle of Rotational Aspect of the Objective Lens, Degrees	Error in Elevation of P when adjusted at B, m.m.	Error in Elevation of B when adjusted at P, m.m.	Erroneous Angular Divergence between P and B, sec.
0	−0.299	+4.1	6.2
30	−0.118	+1.6	2.4
60	+0.083	−1.1	1.7
90	+0.131	−1.8	2.7
120	+0.192	−2.7	4.0
150	+0.147	−2.0	3.0
180	−0.006	+0.1	0.1
210	−0.155	+2.1	3.2
240	−0.246	+3.4	5.1
270	−0.333	+4.6	6.9
300	−0.363	+5.0	7.3
330	−0.385	+5.3	8.0

Note: Points P and B are 10 m. and 138.3 m. away from the instrumental center respectively.

The objective is composed of two air-spaced lenses.

The values are the means of the differences of the respective five readings.

The mean difference of the external diameter of the objective lens itself and the internal diameter of the holder is just 0.10 m.m.

II, between point B and A, using the levelling screws. Subsequently sight at point $\overline{B}$ at the back with the telescope normal, rotating it about its vertical axis and using the levelling screws, rotate the telescope about the vertical axis by nearly 180°, and then read the indicator points at A, D and P.

Next, again carry out the same procedure with the telescope inverted, and then take the differences of the respective readings, that is, u's shown by (127) and (128).

Repeat likewise after the horizontal cross-hair has by the same method been perfectly adjusted between B and D and then B and P severally.

Thus, for level No. 7 and No. 20, the readings u's given in Table 23A are gotten, which are the means of the differences of the respective six readings, so that the error in ε_7 may not surpass the upper limit 0.″1 or 0.000 005 radian.

Table 23A.

Experimental Note 11A.

(Sept., 1935)

Level No.	Horizontal Cross-Hair adjusted between 138.3 m. and E_q, m.	C_p, m.m.	Displacement of the Objective for Focussing C_p-C, m.m.	u_p, m.m.		
				At $E_A=3$ m.	At $E_D=5$ m.	At $E_P=10$m.
7	3	513.6	33.9	+8.288	+14.066	+27.747
	5	498.3	18.6	+7.682	+12.974	+25.667
	10	488.6	8.9	+0.529	+ 0.402	0.000
20	3	513.6	33.9	−0.729	− 1.168	− 2.535
	5	498.3	18.6	−1.681	− 2.876	− 5.931
	10	488.6	8.9	−0.890	− 1.374	− 2.826

Table 23B.

Experimental Note 11B.

(Sept., 1935)

Level No.	C, m.m.	f, m.m.	δ, m.m.	g, m.m.	$\delta-g$, m.m.	ζ, m.m.
7	479.5	290.5	189.0	154	35	154
20	479.9	290.4	189.5	154	35.5	154
Mean	479.7	290.5	189.2	154	35.2	154

Table 23C.

Experimental Note 11C.

(Sept., 1935)

Level No.	e_g, m.m.	e_{gr}, m.m.	e, m.m.	z, m.m.	i,	
					Radian	′ ″
7	−1.519	−2.242	−1.354	−1.485	+0.004 696	+16 08
20	−0.150	−0.192	−0.140	+0.036	+0.000 273	+ 56

Table 23D.

Experimental Note 11D.

(Sept., 1935)

Level No.	ε_φ, Radian Seconds	ε_{e_g}, m.m.	$\varepsilon_{e_{gr}}$, m.m.	ε_e, m.m.	ε_i, Radian Seconds
7	±0.000 017 7 ±3.6	±0.011 1	±0.044 2	±0.049 5	± 0.000 296 0 ±61.0
20	±0.000 002 6 ±0.5	±0.001 7	±0.006 6	±0.007 4	± 0.000 044 1 ± 9.1

In Table 23*B*, the data of the instruments are shown.

In Table 23*C* and Table 23*D*, e_g, e_{gr}, e, z, i, ε_φ, ε_{e_g}, $\varepsilon_{e_{gr}}$, ε_e and ε_i are given. They are computed through the formulas from (149 a) to (162 b) in Art. 32, and where ε's denote the mean square errors of the suffixed quantities.

As is ratiocinated in Art. 32, it should be safe to say that the mean variations of φ, e_g, e_{gr}, e and i are approximately given by their mean square errors respectively.

The reason why the eccentricities and the inclination of the optical axis of the objective lens system of No. 7 are extremely large, lies in the fact that the objective lens was fitted in the holder at the aspect of the maximum deviation of the collimation point from the level plane according to the principle meant by (86) in Art. 18, whose virtual process is described as follows:

First, put counter-marks at the highest or the lowest central point of the face of the objective lens, its holder and the telescope tube, sight at a distant well-defined point, pursue that point horizontally with the vertical cross-hair by means of the tangent screw, rotating the holder as it is fitted in the telescope tube which itself is clamped in the wyes, find out the relative position of the maximum horizontal deviation due to the aspect of the objective lens, mark that aspect at the highest or the lowest central point of the face of the objective lens itself, and then anew mark on its extremities with the aspect difference of ninety degrees in reference to the former.

Subsequently loosen the objective lens itself in its holder, check the new mark on the face of the objective lens and the old counter-mark on the holder, again tighten it in the holder, fit it into the telescope tube, and then check the counter-marks on them.

Then, from (86), this is the very aspect, in which the absolute vertical deviation of the collimation point due to the formal errors of the objective lens itself is the maximum and the horizontal is the minimum or zero.

Now, since level No. 7 was chosen by chance out of the twenty-five instruments, referred to in the experiment described in Art. 61 without any close inspection, there should be many instruments with the same errors, and moreover, ones with larger errors.

64. Grade of Quality of a Wye Level. As in the case of a transit described in Art. 33, a wye level must also be inspected from the standpoint of Geometrical Optics and Mechanical Technology when it is purchased.

Accordingly, the grade of quality of a wye level should be studied, as below.

Since the telescope level of an ordinary wye level is graduated every thirty seconds, it can be read by the eye-measurement to three seconds.

Further, a rod held at the range of 100 m. can accurately be read to 3 m.m. through the telescope of an ordinary wye level, which is equal to six seconds in the angular divergence.

Table 24.

Criterion D.

Grade	e_g, m.m.	i, Radian	i, Minutes	E, m.
Excellent, A				
	0.2	0.000 6	2	12
Good, B				
	1.0	0.003 0	10	41
Tolerable, C				
	3.5	0.010 0	35	71
Too large, D				

Remark: It is taken that $C-g = 350$ m.m. and the horizontal cross-hair is adjusted by the ordinary method of One-Half Adjustment.

E means the distance at which the angular error $3''$ should be produced.

From the above it should properly be understood that the erroneous deviation of the collimation point must not exceed three seconds or 0.000 015 radian at least.

Thereupon, putting together this and the assumption that $C-g=350$ m. m. and that horizontal cross-hair is adjusted by the ordinary method of One-Half Adjustment at the range of 100 m. away from the instrumental station, the tentative criterion shown in Table 24, similar to Table 12 of Art. 33, may be gotten, in which E shows the distance at which the disignated error $3''$ should be produced. Further, refer to Art. 33.

65. The Conditions for Adjustability. Since the similar conditions have already been studied in Art. 5, Art. 9 and Art. 35 in detail, and the principle for them is all the same, one must refer to them.

Now, in a wye level, the eccentricities of the optical axis of the objective lens system, viz., e_g, e_{gr} and e, and the inclination of the optical axis i must likewise be small, that is to say, referring to the fromulas from (78) to (84), there must hold good **"the Conditions for Adjustability"**

$$\left.\begin{aligned} e_p &= \textbf{a small magnitude or zero,} \\ e_g &= \textbf{a small magnitude or zero,} \\ e &= \textbf{a small magnitude or zero,} \\ i &= \textbf{a small magnitude or zero,} \\ k &= \textbf{a small magnitude or zero,} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (236)$$

so that they may not cause any deleterious error in the height of the collimation point or that computed from it.

Beside the above, as secondary conditions, the variations of e_g, e_{gr}, e and i according to the collimation point must be under certain allowances, low enough not to cause any injurious irregular deviation of the collimation point.

Now, taking into consideration the experiments respecting the above errors in Art. 32 and Art. 63, the Conditions for Adjustability in Art. 35, and the description and Criterion D of Art. 64, the tentative criterion may be given as follows:

Table 25

Criterion E.

Grade	ε_{φ},	
	Radian	Seconds
Excellent, . . . A		
	$\frac{1}{600,000}$	$\frac{1}{3}$
Good, ... B		
	$\frac{1}{400,000}$	$\frac{1}{2}$
Tolerable, . . . C		
	$\frac{1}{200,000}$	1
Too large, . . . D		

66. The Method of Integral Adjustment of the Horizontal Cross-Hair in a Wye Level. From the study of the deviation of the collimation point from the level plane described in Art. 59, it is made clear by (233) and (235) that the superfluous deviation, remaining after the ordinary method of One-Half Adjustment of the horizontal cross-hair at a distant point, must be made to disappear at another control point too, which is likewise produced by the defects of workmanship and centering of the metal and optical parts by its very nature as in a transit.

Here, **the conditions (233) and (235) for the wye level are exactly the same as (168a) and (168b) for the transit**, that is to say—it proves satisfactorily that **the same principle of Integral Adjustment holds accurately true to them at the same time.**

Now, from the above argument, it can readily be grasped that **the methods of Integral Adjustment of the vertical cross-hair in a transit studied in Section VI of Chapter III or from Art. 36 to Art. 43 can directly be applied to the adjustment of the horizontal cross-hair in a wye level.**

Therefore, according to the need, for all the theory and the explanation one should properly refer to those articles.

Among the five methods of Integral Adjustment of the vertical cross-hair in a transit, only **the second, fourth and fifth can favourably be practised in the field for a wye level.**

Now, the second method is slightly superior to the fourth and both require the measurement of C, δ and g; while, on the contrary, the fifth or the trial method needs none of them absolutely so that it should be the most competent and universal.

Hereupon, one should refer to Fig. 47 for the notations.

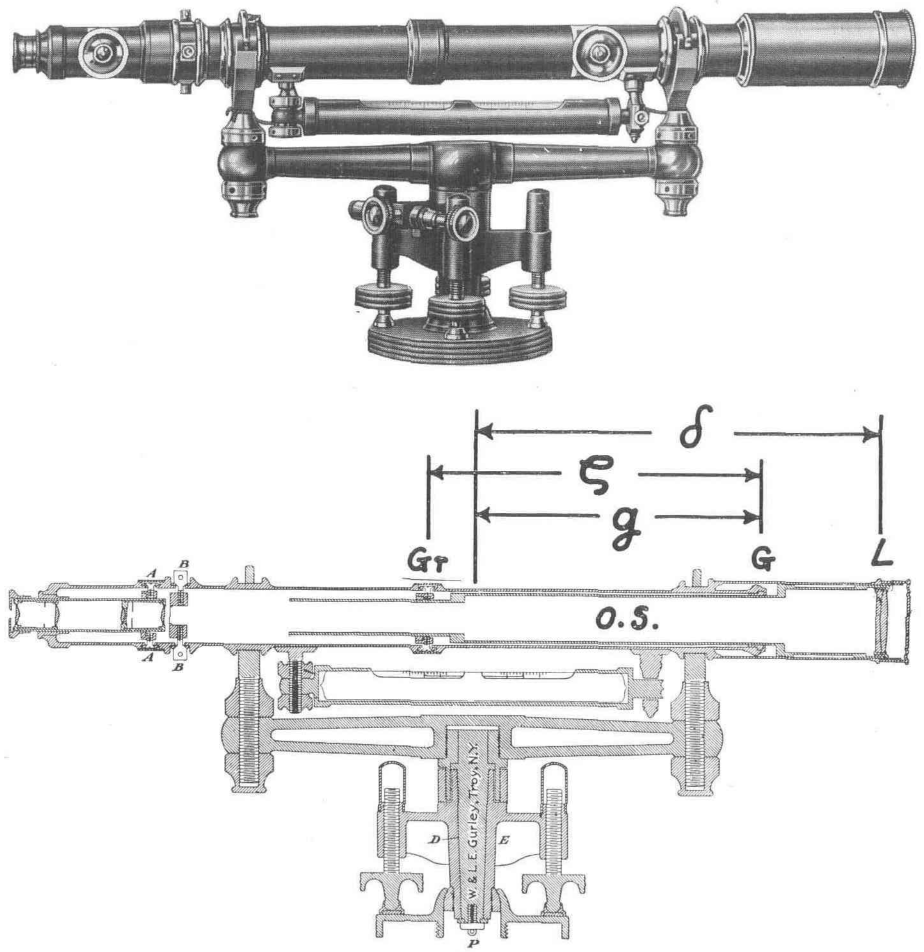


Fig. 47. Sectional View of a Wye Level.

Now, the illustrative experiments for the methods of Integral Adjustment were successfully performed on levels No. 1, No. 7 and No. 20 by the present author for himself without any preliminary practice excepting those for the transit, with the results given in Table 26.

The adjustment had been extremely disturbed in advance as in the case for the transit before the experiments.

For other data one should refer to Art. 43.

Table 26.

Experimental Note 12.

(Sept. ~ Oct., 1935)

Designation	Second Method	Second Method	Second Method
Date tested	30 Sept.	1 Oct.	2 Oct.
Level, No.	20	1	7
Made In	Japan	U.S.A.	Japan
Size, In.	15	18	15
Years of Use	9.0	0.1	10.0
Observer	T. Shingo	T. Shingo	T. Shingo
E_B, m.	138.3	138.3	138.3
E_A, m.	10.0	10.0	10.0
Δi, Radian	$-0.001\,53\,t$	$-0.001\,17\,t$	$-0.001\,53\,t$
Δi, Seconds	$-\ 316.3\,t$	$-\ 240.3\,t$	$-\ 316.3\,t$
Δa or Δb	$-\ 2.44\,t$	$-\ 2.35\,t$	$-\ 2.58\,t$
First Adjustment			
t, m.m.	$-\ 1.15$	$-\ 0.83$	$-\ 0.92$
Δi, Radian	$+0.001\,76$	$+0.000\,97$	$+0.001\,41$
Δi, Seconds	$+\ 364$	$+\ 199$	$+\ 291$
Δa or Δb, m.m.	$+\ 2.81$	$+\ 1.96$	$+\ 2.38$
Second Adjustment			
t, m.m.	0.00	0.00	$-\ 0.18$
Δi, Radian			$+0.000\,28$
Δi, Seconds			$+\ 57$
Δa or Δb, m.m.			$+\ 0.45$
Third Adjustment			
t, m.m.			0.00

Table 26.—(*Continued*)

Experimental Note 12.

(Sept.~Oct., 1935)

Designation	Fourth Method	Fourth Method	Fourth Method
Date tested	30 Sept.	1 Oct.	3 Oct.
Level, No.	20	1	7
Made in	Japan	U.S.A.	Japan
Size, In.	15	18	15
Years of Use	9.0	0.1	10.0
Observer	T. Shingo	T. Shingo	T. Shingo
E_B, m.	138.3	138.3	138.3
E_A, m.	10.0	10.0	10.0
Δi, Radian	$+0.000\,111\,t$	$+0.000\,084\,t$	$+0.000\,111\,t$
Δi, Seconds	$+\ 22.86\,t$	$+\ 17.38\,t$	$+\ 22.86\,t$
Δa or Δb	$+\ 1.94\,t$	$+\ 1.85\,t$	$+\ 2.08\,t$
First Adjustment			
t, m.m.	+ 8.6	+ 7.3	+ 15.9
Δi, Radian	+0.000 95	+0.000 62	+0.001 76
Δi, Seconds	+ 197	+ 127	+ 364
Δa or Δb, m.m.	+ 16.7	+ 13.5	+ 33.1
Second Adjustment			
t, m.m.	0.0	+ 0.9	− 1.9
Δi, Radian		+0.000 08	−0.000 21
Δi, Seconds		+ 16	− 43
Δa or Δb, m.m.		+ 1.7	− 3.9
Third Adjustment			
t, m.m.		0.0	0.0

Table 26.—(*Continued*)

Experimental Note 12.

Designation	Fifth or Trial Method			
	Second Method	Second Method	Fourth Method	Fourth Method
Date tested	16 Oct.	16 Oct.	16 Oct.	16 Oct.
Level, No.	1	20	1	20
Made in	U.S.A.	Japan	U.S.A.	Japan
Size, In.	18	15	18	15
Years of Use	0.1	9.0	0.1	9.0
Observer	T. Shingo	T. Shingo	T. Shingo	T. Shingo
E_B, m.	138.3	138.3	138.3	138.3
E_A, m.	6.5	6.5	6.5	6.5
Δa or Δb	$-$ 1.92t	$-$ 1.95t	$+$ 1.0t	$+$ 1.0t
First Adjustment				
t, m.m.	$-$ 1.47	$+$ 1.20	$-$ 16.5	$+$ 14.1
Δa or Δb	$-$ t	$-$ t	$+$ t	$+$ t
Δa or Δb, m.m.	$+$ 1.47	$-$ 1.20	$-$ 16.5	$+$ 14.1
Second Adjustment				
t, m.m.	$-$ 0.71	$+$ 0.59	0.0	$-$ 0.8
Δa or Δb	$-$ 1.92t	$-$ 1.95t		$+$ 1.0t
Δa or Δb, m.m.	$+$ 1.35	$-$ 1.14		$-$ 0.8
Third Adjustment				
t, m.m.	0.00	$+$ 0.13		0.0
Δa or Δb		$-$ 1.95t		
Δa or Δb, m.m.		$-$ 0.25		
Fourth Adjustment				
t, m.m.		0.00		

67. Residual Errors. Since, according to the descriptions and the proofs shown in Art. 59 and Art. 66, the adjustment of the horizontal cross-hair in a wye level can be completed by the same principle of Integral Adjustment of the vertical cross-hair in a transit, the theory of residual errors remaining after Integral Adjustment should also be comparable to that of the latter.

For that reason, reference must be made to Section VIII of Chapter III or from Art. 46 to Art. 50.

68. Criticism of the Old Method of Adjustment.* So far as known to the present author, the only procedure is found in the work of a certain first-class maker in a foreign country, which is described as follows:

"To Adjust the Objective Slide. The adjustment of the objective slide is a distinctive feature of Gurley instruments and is always made by us so permanently as to need no attention at the hands of the engineer, unless in case of derangement by accident.

In making this adjustment, it is necessary to remove the level tube in order that the screw immediately above it may be accessible.

To adjust the objective slide, select an object as distant as may be distinctly observed, and upon it adjust the line of collimation," by the ordinary method of One-Half Adjustment, "making the intersection of the wires to rotate in the wyes without passing either above or below the point or line selected. In this position the slide will be drawn in nearly as far as the telescope tube will allow.

With the pinion head then move out the slide until an object, distant about ten or fifteen feet, is brought clearly into view. Again rotating the telescope in the wyes, observe whether the wyes will reverse upon this second object.

Should this be the case, it is assumed that, as the line of collimation is in adjustment for these two distances, it will be for all intermediate ones, since the bearings of the slide are true and their surfaces parallel with each other.

* Also, by Asst. Prof. K. Tanaka of Civil Engineering of the Faculty of Engineering, Kyūshū Imperial University, Fukuoka, Japan, a new method was proposed on *p* 119~122 of his third paper or Reference (17) at the end of this work. But it can not be practised in the field from the theoretical and practical standpoint, because the formula for correction is too complicated to be calculated in the field, the correction is severely subjected to the errors in observations, the same complicated process must always be repeated every adjustment, and moreover the correction of the decimal fraction of one milimeter to the reading of the rod at a distant point is scarecely to be discerned so that no check can be done absolutely, among which the last is the final disqualification.

If, however, either or both wires fail to reverse upon the second point, by estimation, remove half the error by the screws at" Gr, † "at right angles with the wire to be corrected, remembering that, on account of the inverting power of the eyepiece, the slide must be moved in the direction which apparently increases the error. When both wires have been thus treated, the line of collimation is again adjusted on the near object, and the telescope again brought upon the most distant point. The tube is again rotated, the reversion of the wires upon the object once more tested, and the correction, if neccessary, made in the same manner."

Now, according to the above description, the correction for the complete adjustment of the horizontal cross-hair should be given by the formula

$$\varDelta a'' = -\frac{t_A}{2}, \qquad \text{(237)}$$

where t_A is the actual reading difference at the near point and $\varDelta a''$ the correction.

But, since the above procedure is the same as that of the second method of Integral Adjustment of the vertical cross-hair in a transit described in Art. 39, the correction

$$\varDelta a = -\left\{1+\frac{\delta-g}{C-\delta}\frac{E_A-C}{C-g}\right\}\frac{t_A}{2}$$

must exactly be given as in (181).

Therefore, from the above, the preceding correction (237) must be extrémely erroneous.

For example, since for a Gurley's eighteen-inch wye level, the correction

$$\varDelta a = -\left\{1+0.227\frac{E_A-C}{C}\right\}\frac{t_A}{2} \qquad \text{(238)}$$

is accurately computed from (181), putting the observed values of C, δ and g in it, **the error in (237) should mount up to +76% when adjusted at E_A=10C=5.8 m., and +88% when adjusted at E_A=20C=11.6 m.**

Hence, **in order to decrease the error t_A under 5% by the Old Method or (237), the process for adjustment should properly be repeated ten times at E_A = 5.8 m., and twenty-four times at E_A = 11.6 m.**

This is the reason why the Old Method can not be practised.

† See Fig. 47, Art. 66 of this paper.

69. A Temporary Method for Complete Adjustment. Now, an emergency method for perfect adjustment is described as follows:

First adjust the horizontal cross-hair by the ordinary method of One-Half Adjustment at a distant point B.

Subsequently take the reading of the scale held at a near point A with the telescope normal and again with the telescope inverted, simply rotating it about its spindle by one hundred and eighty degrees as it is in the wyes, with the instrument still clamped *in situ* when the first reading was taken. Then, again adjust the horizontal cross-hair at B by the amount of half the difference of the two readings taken at A moving it in the direction from the first to the second, with the telescope still clamped in the position it was in when the second reading was taken at A.

Now, this theoretical procedure is founded on the facts as follows:

When the horizontal cross-hair is adjusted by the ordinary method of One-Half Adjustment at a distant point B, its deviation from the level plane should properly vanish, that is to say—the relation

$$o = e + E_B\, i - \frac{k}{f}(E_B - C_B) \quad \dots\dots\dots\dots\dots \quad (239a)$$

holds good, with reference to (78).

Subsequently, when the readings of the scale at the near point A are taken with the telescope normal and inverted respectively, the relation

$$\frac{t_A}{2} = e + E_A\, i - \frac{k}{f}(E_A - C_A) \quad \dots\dots\dots\dots \quad (239b)$$

holds good, with reference to (78), t_A being the difference of the two readings at A.

Now, in order to make the collimation points A and B deviate equally from the level plane, the horizontal cross-hair must again be adjusted so that the new relation

$$E_A \varphi_A = E_B \varphi_B\,, \quad -\frac{\Delta k}{f}(E_B - C_B) = -\frac{t_A}{2} - \frac{\Delta k}{f}(E_A - C_A)$$

or
$$\frac{\Delta k}{f} = \frac{1}{2}\,\frac{t_A}{E_B - C_B - E_A + C_A} \quad \dots\dots\dots\dots\dots\dots\dots \quad (240)$$

may hold good, in which Δk is the correction for k.

Therefore, the new correction for adjustment of the reading of the horizontal cross-hair on the rod held at B

$$\Delta b = -\frac{\Delta k}{f}(E_B - C_B)$$

$$= -\frac{E_B - C_B}{E_B - C_B - E_A + C_A}\,\frac{t_A}{2} \quad \dots\dots\dots\dots\dots\dots \quad (241)$$

is gotten from (239b) and (240), from which the practical correction

$$\Delta b = -\frac{t_B}{2} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (242)$$

is obtained, neglecting terms of the higher order.

Hence, the horizontal cross-hair should properly be adjusted to the new position, at which the deviation of the collimation point B is equal to that of A.

Now, in consequence of this adjustment, the new relation between k and i

$$\frac{k}{f} = \frac{E_B - E_A}{E_B - C_B - E_A + C_A}\,i \quad \dots\dots\dots\dots\dots\dots \quad (243)$$

and the residual deviation of a collimation point p from the level plane

$$\Delta(E_p\varphi_p) = e + \frac{(C_A - C_B)E_p + C_p(E_B - E_A)}{E_B - C_B - E_A + C_A}\,i$$

$$= e + \left\{\frac{(C_A - C)(C_B - C)}{C_p - C} + C_p\right\} i \quad \dots\dots\dots \quad (244)$$

or, neglecting terms of the higher order,

$$\Delta(E_p\varphi_p) = e + C_p i \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (245)$$

are gotten from (78), using (12).

Now, this method is rather simple, compared with that of Integral Adjustment, and the residual deviation is nearly equal to that of Integral Adjustment, compare (209).

But, since the influence of the optical errors can not radically be eliminated by this method, the complicated process must always be repeated at every adjustment and moreover no check for adjustment can be made absolutely.

CHAPTER V

ADDENDA REFERRED TO THE INTEGRAL ADJUSTMENT OF THE CROSS-HAIRS IN A TRANSIT AND A WYE LEVEL

70. General Description. Secondary problems, which were found in the first place during the studies of the adjustment of the cross-hairs in a transit and a wye level and concern it both directly and indirectly, should be treated in this chapter *en bloc* so that they may not cause complications.

They are described in detail in the following several articles.

71. Method of Fitting the Objective Lens in its Holder in a Transit and a Wye Level. It has already been demonstrated by the experiments described in Art. 30 and Art. 62 that the deviation of the collimation point from the meridian of the transit or the level plane of the wye level due to the errer in the centering of the objective lens often amounts up to such a large magnitude that it can not possibly be neglected.

Therefore, for the purpose of elimination of this error, detailed study was made as described in Art. 18. The method for elimination has been conceived according to the principle found. The processes may be described as follows:

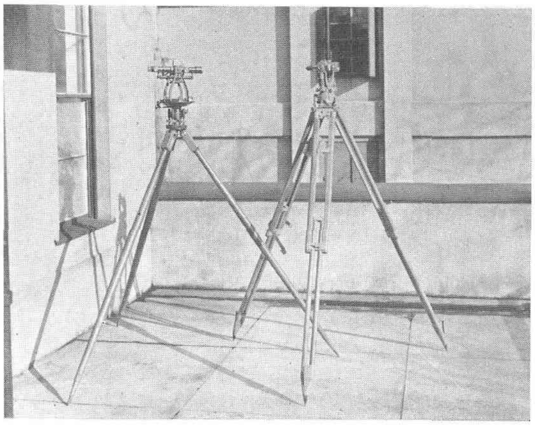
First put counter-marks on the holder of the objective lens and the telescope tube, sight at a well-defined point, pursue it horizontally with the vertical cross-hair in a wye level or vertically with the horizontal cross-hair in a transit by the tangent screw respectively, rotating only the holder as it is fitted in the telescope tube which itself is clamped in the wyes in a wye level or both vertically and horizontally clamped in a transit, find out the relative position of the maximum horizontal deviation in a wye level or the maximum vertical deviation due to the aspect of the objective lens, and then mark that aspect on the extremity of the face of the objective lens, aligning with the old counter-mark on the telescope tube.

Hereupon loosen the objective lens in its holder, check the new mark on the face of the objective lens and the old counter-mark on the holder, again tighten in the holder, and then fit it into the telescope tube, checking the counter-marks on them.

Then, referring to (86), this is the very position, at which the absolute horizontal deviation of the collimation point in a wye level and the absolute vertical deviation of the collimation point in a transit should properly be at the maximum : while, on the contrary, the vertical deviation in the former and the horizontal in the latter are the minimum or zero.

Now, this procedure must be practised in the field by a surveyor when the objective lens is taken out of its holder for some purpose, as well as in the factory when the instrument is newly assembled or after being repaired, before the ordinary adjustments of the cross-hair are performed.

72. Superfine Tripods for a Transit and a Level. The tripod of type *A* shown in Fig. 48 is the same as those used for a transit and a wye level in general, but so unfirm and unsteady that the observation may sometimes be disturbed or interfered with.



Type A Type E

Fig. 48. Comparison of Tripods.

Besides, many tripods of this type with extremely warped legs are found.

Since the disturbances due to the former phenomena were extraordinarily experienced during the experiments described in Art. 45 and the latter during those in Art. 26 and Art. 61, systematic and comparative experiments were performed on transits No. 23 and No. 29 with tripods of type *A* and on No. 32 with one of type *E* shown in Fig. 48.

Now, in these experiments, the heights of the respective instruments were kept at 1.53 m., the test load of 500 grams was horizontally loaded at the center of each leg of the tripods in the four directions, namely—the outward, the inward, the clockwise and the counter-clockwise, and the influences on the horizontal and the vertical angles were observed, taking the readings of the respective circles, first without loading when point *B*, 138.3 m. away from the instrumental center, was exactly sighted and then with the above load of 500 grams. The results are set down in Table 27.

Table 27.

Experimental Note 13.

(Oct., 1935)

Transit No.	Made in or Maker / Length of the Tripod, m.	Reading of the Vertical Circle, sec. / Reading of the Horizontal Circle, sec.	Height of Instrument, m. / Load, kg.	Influence*: Direction of Load	Leg of Tripod	Vertical Angle, sec.	Horizontal Angle, sec.
23	U.S.A.	60	1.53	Outward	a	p	0
					b	−60	0
					c	p	0
				Inward	a	p	+20
					b	p	0
					c	−60	−20
	1.46	20	0.5	Clockwise	a	0	−60
					b	0	−80
					c	p	−80
				Counter-clockwise	a	p	+40
					b	0	+80
					c	−60	+60
29	Japan	60	1.53	Outward	a	p	0
					b	+60	0
					c	p	0
				Inward	a	+60	0
					b	p	−20
					c	p	0
	1.36	20	0.5	Clockwise	a	p	−60
					b	+60	−80
					c	+60	−70
				Counter-clockwise	a	p	+50
					b	p	+60
					c	p	+70
32	Wild, Switzerland	1	1.53	Outward	a	+4	0
					b	+10	0
					c	+3	−7
				Inward	a	+2	0
					b	+4	+3
					c	−7	+10
	1.46	1	0.5	Clockwise	a	−6	−11
					b	0	−7
					c	+2	−8
				Counter-clockwise	a	+5	+9
					b	+1	+11
					c	−4	+14

*

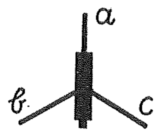


Fig. 49.

The legs of the tripod a, b and c are spaced at 120° from each other, always keeping at $H.\ I. = 1.53$ m., and b is pointed just in the same direction as that of the telescope, refer to Fig. 49.

p denotes perceptible.

Now, from these results, it should properly be grasped that a tripod of type E is far superior to that of type A, notwithstanding the fact that the former seems to be more delicate and slender than the latter.

Transit No. 23 was made in U. S. A., No. 29 in Japan, and No. 32 is a theodolite manufactured by Wild, Switzerland.

Further, the vertical and the horizontal angles can be read to $1'$ and $20''$ with No. 23 and No. 29 respectively; while, on the contrary, both $1''$ to with No. 32.

Now, upon consideration, the above results prove distinctly that the ordinary tripod of type A must immediately be improved or altered to that which is less influenced by any force put on the instrument and the tripod.

In conclusion, from the standpoint of mechanics the merit of the tripod of type E principally consists in the superior construction of the metal joints at the tops of the legs.

73. Superfine Standard of a Transit. It was cleared up by the experiment described in Art. 26, that the eccentricity and the inclination of the optical axis of the objective lens sometimes mount up to large magnitudes in a transit. Refer to Table 6 in Art. 26.

Now, there are two principal origins of these errors, namely—the first, the formal errors of the objective lens itself and the second, the distorsion of the standards due to an abrupt accident and the change of the mechanical condition from long years of service.

Among them, the former can virtually be eliminated by the method explained in Art. 71.

Furthermore, the latter should properly be diminished by improvement in the construction of the standards.

For this purpose, the one-piece truss standard shown in Fig. 42 of Art. 52 is the most excellent and suitable from the standpoint of mechanics and mechanical technology.

For this reason, this construction is adopted for precise transits: as—Fuji transit manufactured by Sokkisha Co., Japan, and Precise transit by Gurley Co., U. S. A., as well as for most theodolites.

Nevertheless, it is regretable that it is not yet used for most ordinary engineer's transits.

74. Finishing of Slide Bearings of the Objective Slides of the Ramsden and Huygenian Telescopes. From the studies of the deviation of the collimation point, it was cleared up from (81) (83) and (84), that the error in the finishing of the rear slide bearing can be admitted up to from three to six times that of the front slide bearing according to the instrument, as already described in Art. 31 and 35.

Again, the same fact is distinctly illustrated by the experiments described in Art. 31 in a transit and Art. 63 in a wye level, that is to say—in the former, for transits No, 4, No. 5 and No. 9,

$$\varepsilon_{e_{gr}} = 3.65\,\varepsilon_{e_g} \qquad \text{(246a)}$$

and in the latter, for levels No. 7 and No. 20,

$$\varepsilon_{e_{gr}} = 3.97\,\varepsilon_{e_g} \qquad \text{(246b)}$$

are actually gotten from (161a) (161b) and the experimental results.

Now, these relations exactly show that the front slide bearing must be finished with over three times the precision of the rear.

75. The Telescope of the Ramsden or the Huygenian Type, in Which the Condition $\delta = g$ holds Good. If a telescope of the Ramsden or the Huygenian type can be constructed so that the condition

$$\delta = g \qquad \text{(247)}$$

may exactly hold good, that is to say, the first principal point of the objective lens may just come on the plane of the front slide bearing when an infinitely distant point is sighted, then the formulas for the correction for Integral Adjustment of the vertical cross-hair in a transit and the horizontal cross-hair in a wye level, namely— (175), (181), (188) and (194), are extremely simplified neglecting terms of the higher order as follows:

For the first Method, $$\Delta a = -\frac{t_A}{2}. \qquad \text{(248)}$$

For the Second Method, $$\Delta a = -\frac{t_A}{2}. \qquad \text{(249)}$$

For the Third Method, $$\Delta b = +\frac{E_A}{E_B}\frac{t_B}{2}. \qquad \text{(250)}$$

For the Fourth Method, $$\Delta b = +\frac{E_A}{E_B}\frac{t_B}{2}. \qquad \text{(251)}$$

Hence, the correction for adjustment of the old method for a wye level described in Art. 67 is completely consistent with that of the second or that given by (249) at last.

Further, deviation of a collimation point p

$$E_p \varphi_p = \left\{ \frac{E_p - C_p}{C - \delta} + 1 + \frac{C_p - \delta}{\zeta} \right\} e_g - \frac{C_p - \delta}{\zeta} e - \frac{E_p - C_p}{C - \delta} z \quad . \quad (252)$$

is obtained from (81) and (247), where $e_g - e_{gr} = \zeta i$. From (252) it is cleared up that a part of the influence of e_g and almost all that of e_{gr} are eliminated.

Now, the relation shown by (247) is the very condition, at which the deviation of collimation points should properly be the minimum for all values of E, e_g and e_{gr}.

Therefore, referring to (81), it may be dogmatized that the telescope must be constructed at least so that the value $\delta - g$ may be diminished to the smallest possible, even if condition (247) can not be realized.

CONCLUSION

Up to the present, from lack of knowledge of Geometrical Optics and Mechanical Technology, civil engineers and lecturers on Surveying have too much trusted to the maker concerning the precision of a transit and a way level, so that they have not only been adopted but also inspected during practical use with absolutely no rational test.

But, now, it should properly be acknowledged from the experiments described in the present paper that the optical errors in some transits or wye levels are too large to be neglected in such an ordinary survey as when a distant and a near point are correlatively sighted in angle measurement or in levelling, much more in a precise survey.

It is quite natural that these errors were found even in a new instrument, because all existing methods for adjustment of the cross-hairs have been absolutely and fundamentally mistaken.

Besides, they are extraordinarily increased during practical use for long years due to the change of the mechanical condition, especially when the objective lens is taken out of the holder for the purposes of cleaning it, etc. and subsequently tightened in the latter again or when the instrument is subjected to an accident.

Respecting these points, refer to Art. 26, Art. 30, Art. 31, Art. 32, Art. 33, Art. 61, Art. 62 and Art. 63.

Therefore, in the present paper, to eliminate these, errors Integral Adjustment has been exactly studied from the standpoint of Gemetrical Optics, from which excellent methods of adjustment have been evolved for a transit and a wye level.

Firstly, a new procedure and correct new formulas for the old method of adjustment of the horizontal cross-hair in the Ramsden, the Huygen and the Porro telescopes have already been made public in the present author's first and second papers or Reference (1) and (2).

These have been generalized in Chapter II of the present paper so that they may be applied to the telescope of the Wild-Zeiss type too, which is the very first attempt up to the present.

In addition, the trial method, found newly, is the most efficient and universal.

As to these matters, refer to Art. 15; the error of the old formula A was over 90% and those of (B) and (C) were over 100%.

Secondly, in Chapter III, five methods for Integral Adjustment of the vertical cross-hair in a transit are proposed, which are correct new ones of the so called "Objective Slide Adjustment", and among which the fifth or trial method is the most efficient and universal.

The errors in the formulas for correction by old methods A and B generally mount up to over 120% and that of old method C to over nearly 64% at the very least respectively, compare Art. 54, Art. 55, Art. 56, Art. 39 and Art. 40 severally.

Thirdly, since the theory and the principle of Integral Adjustment of the horizontal cross-hair in a wye level, studied in Chapter IV, are wholly the same as those of the vertical cross-hair in a transit, the same methods should directly be applicable, but from the standpoint of construction, the second, the fourth and the fifth can only be practised with a wye level.

The error of the formula for correction in the old method of the so-called "Objective Slide Adjustment" described in Art. 68 generally comes up to over 80%.

The above descriptions have been experimentally confirmed by practical examples for Integral Adjustment performed with unrivalled success, with the results shown in Table 1 of Art. 6, Table 16 of Art. 43 and Table 26 of Art. 66 respectively.

The name "**Integral Adjustment**" is founded on the fact that the word "**Integral**" originally comes out of Latin "**Integralis**" and means "**necessary to the completeness of the whole**", "**made up of component parts which together constitute an unity**" and "**having no part or element lacking**".

Consequently, in the same sense, **the substance of the fundamental principle of "Integral Adjustment" of the vertical cross-hair in a transit and the horizontal cross-hair in a wye level originally constitutes in making**

$$e_g+(C_J+C_N-C-g)i=0\,,\quad \varphi_N=\varphi_J=0$$

in general to complete the adjustment, in addition to the condition for the ordinary One-Quarter Adjustment in a transit and One-Half Adjustment in a wye level

$$\varphi_N=\varphi_{\bar{N}}=0\,,$$

as already proved by (168a) and (168b) in Art. 36 and (233) in Art. 59 respectively. They are the very conditions that the two points

J and N must simultaneously lie on the meridional plane of the transit or the level plane of a wye level.

Accordingly, even if these conditions do not hold true in Integral Adjustment of the horizontal cross-hair in a transit, since the similar condition for complete adjustment

$$\varphi_A = \varphi_B$$

holds good, compare (115), all adjustment described in the present paper have come to be named "Integral Adjustment."

Now, if e_g is pretty large, the residual errors should properly be diminished to an imperceptibly small magnitude according to the principle studied in Art. 49.

Further, if e_g is too large, the instrument must be repaired or rejected.

On this point, Table 12 of Art. 33 and Table 24 of Art. 64 may probably afford convenient criteria for judging the grades of quality of a transit and a wye level.

In addition to the above, the formal errors of the objective lens itself correlative with the holder and the telescope tube, which are acknowledged to be fairly large, can absolutely be adjusted by no mechanical centering tool, though it is thought to be the most reliable tool in the factory at present. Refer to Art. 30, Art. 60, Art. 52 and Art. 57.

Up to the publication of the present paper, there has probably been none who thoroughly knew the accuracy of the result of adjustment by using the centering tool, nor either that by the old methods of Objective Slide Adjustment.

But it becomes possible easily to eliminate the influence of the formal errors of the objective lens itself by finding out the relative aspect of the maximum deviation of a collimation point by the method described in Art. 71.

Notwithstanding this fact, these formal errors must be carefully diminished as much as possible at the time of the centering in the manufactory.

For this purpose, a tentative criterion for the formal errors is shown in Art. 34 for convenience sake.

Now, in the Ramsden or the Huygenian telescope, the front slide bearing and the part of the objective slide, which fits into it, must be finished three times at the very least more accurately than the rear slide bearing and part of the objective slide, which fits into it, that is to say—the irregular variation of the eccentricity of the optical axis of the objective lens system at the front slide bearing must be made under one-third that at the rear. Refer to Art. 74.

Next, the mean variations of the deviations of collimation points should simultaneously not exceed certain limits according to the grade of quality of the instrument. Tentative criteria are conveniently shown in Table 14 of Art. 35 for a transit and Table 24 of Art. 65 for a wye level.

Now, if the criterion can not at all be satisfied according to the object of surveying, then the instrument should properly be rejected, regardless of the adjustment of the cross-hair.

Further, if the telescope is constructed in such a manner that the first principal point of the objective lens comes on the plane of the front slide bearing when an infinitely distant point is sighted, that is to say—when the condition

$$\delta = g$$

is fulfilled, the deviation and its angular variation are very much diminished and moreover the formulas of correction for Integral Adjustment remarkably simplified, refer to Art. 75.

During the experiments reported in the present work, it was very frequently experienced by the author that such an ordinary tripod as type A shown in Fig. 48 was extremely unfirm and unsteady due to the defect of the construction of its top.

Accordingly, the tripod must generally be constructed in such a manner as type E of Fig. 48. Refer to Art. 72.

Finally, since, in such an ordinary transit as shown in Fig. 16 of Art. 19, the eccentricity and the inclination of the optical axis of the objective lens system are principally produced by the distorsion due to accident from lack of solidity, it is recommended here that the instrument should properly be constructed with the one-piece truss standard, as shown in Fig. 42 of Art. 52.

In conclusion, the advice to all theoretical and practical specialists is ventured, that **the horizontal cross-hair in a transit must always be adjusted by the methods of Integral Adjustment in both the field and the manufactory; also the vertical cross-hair in a transit and the horizontal cross-hair in a wye level must always be adjusted by the methods**

of Integral Adjustment in the manufactory and in the field by the civil engineer, when it is found that the adjustment gets out of order after inspection when the instrument is newly adopted, when it has been used for years and especially when it is subjected to an accident during practical use.

If it is found by inspection that the disturbance is too large to be neglected, it must be rejected or repaired.

Now, at a glance, **the formulas of correction for Integral Adjustment appear to be somewhat complex, but they can easily be applied even in the field, if they are written down on the lid of the case of the instrument in the manufactory or by the surveyor in advance, or still more, when the trial methods are used.**

This paper has come to be too voluminous to read up in the field, because it has been brought to completion from the standpoint of the exact theory, for reference in similar studies in the future, but it can be strikingly abridged for practical use as stated in the Introduction.

Again, it is earnestly hoped that the descriptions and the experimental results may be satisfactorily understood and completely applied in practice by all makers.

Futher, it is hoped that **the standards for a transit and a level may rapidly be prescribed by civil engineers and that the instruments may always be adopted after inspection according to them.**

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