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Some Problems in Soil Mechanics.

By

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The present paper deals with a part of the results of the authors' study on the following three problems in soil mechanics :

- (1) Experimental investigations on angle of natural slope of dry sand at certain depth.
- (2) Sand pressure experiments for partial yielding of wall in certain depth.
- (3) Sand pressure experiments under special consideration of horizontal displacement and frictional condition of wall surface.

I. Experimental Investigations on Angle of Natural Slope of Dry Sand at Certain Depth.

In various investigations concerned with soil mechanics, the angle of the natural slope of sand as well as the surface of rupture of the same seem to play a very important rôle in the design of revetment walls.

On this matter, the senior author has already made exact experimental investigations under several conditions and in different ways.*

* These Memoirs, see the following reports : F. Takabeya; Experimental Investigations on the Internal Granular Movements of Sand. Memoirs Vol. 2, No. 6. Experimental Investigations on Surface of Rupture in Sand. Memoirs Vol. 3, No. 2.

In the present note the experimental results on the angle of natural slope of sand under the ground surface will be reported.

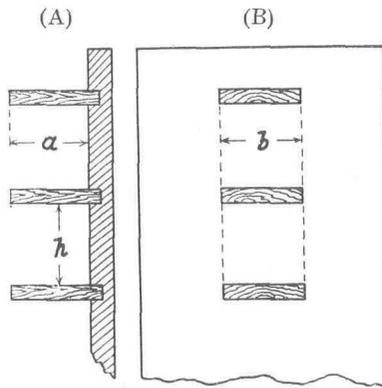


Fig. 1.

The angle between the surface of soil or sand and horizontal plane is called, in this note, the angle of repose or angle of natural slope. These are, in the case of dry sand, easily observed and measured.

For a dry sand, this natural slope is shown as a straight line and the amount of the angle is considered always to be constant, concerning with a definite species

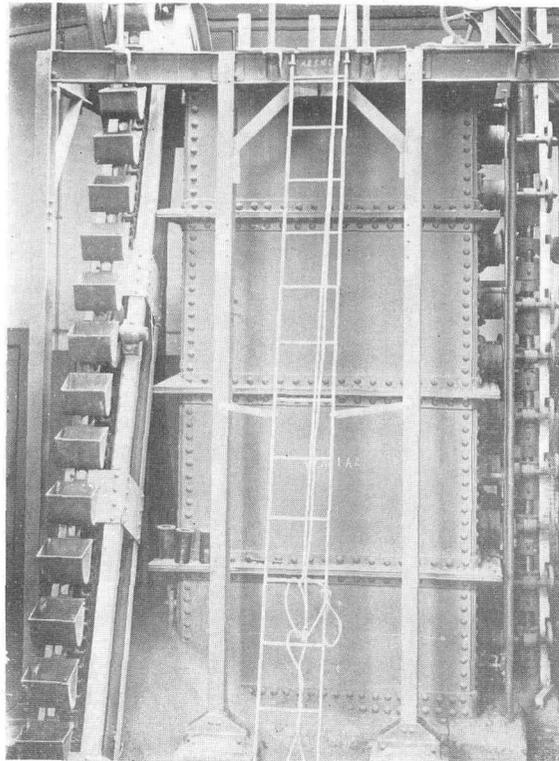


Fig. 2.

of sand grains. However, under the ground surface, i.e. at a certain depth of the sand, the question arises whether the angle keeps constant or whether it varies in quite different ways.

The angle of natural slope of that kind has been measured experimentally as follows :

The apparatus employed was as shown in Figs. 1 and 2.

The apparatus was erected which is shown in Fig. 1 in the steel tank shown in Fig. 2, having a glass side and measuring 1.8 m wide and long, 3.6 m deep. Then, dry sand was filled up loosely in the tank.

In this way, it was possible to observe the angle of natural slope of sand at any depth as shown in Fig. 3.

In the experiments fine beach sand was used dried and sifted to a grain size of 0.86 mm dia. mixed with an unspecified proportion of finer ones. The specific weight was 2.56, the weight of one cubic *m* 1300 kg, and the angle of natural slope $34^{\circ} 20'$.

The sand tank above mentioned had too little capacity to observe the variation of the angle of natural slope in question, i.e. the dimensions of the tank were too small to determine the relation between the amount of depth and the aspect of the natural slope at the given depth. A screw testing machine was sufficiently powerful to use in an experiment of this kind, so the surface of the sand mass was pressed by the testing machine substitution for the immense mass of filling sand, in order to obtain an equivalent height for very much larger depth than the tank could furnish.

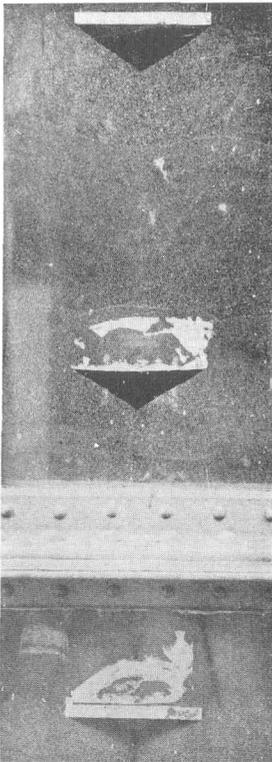


Fig. 3.

This method of experiments is as follows :

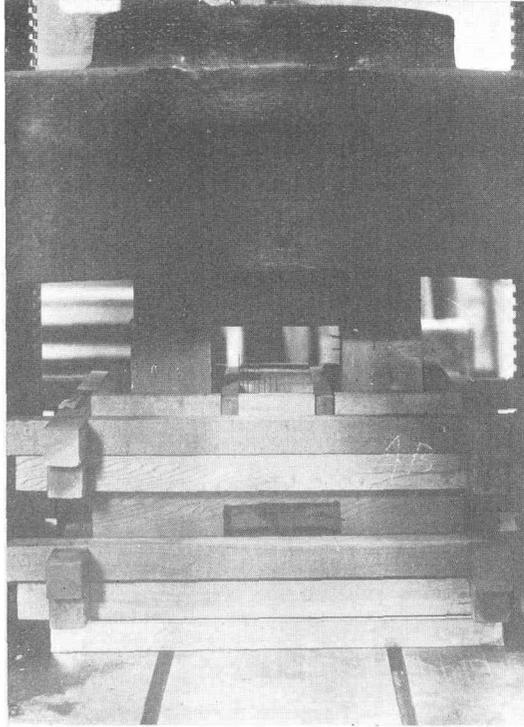


Fig. 4.

To observe the natural slope of sand, a wooden box was made, strong enough to resist the maximum loading of 10 tons, an equivalent depth of 50 m in sand. This box has been constructed with a glass plate on one side, that the measurement may be made by the side

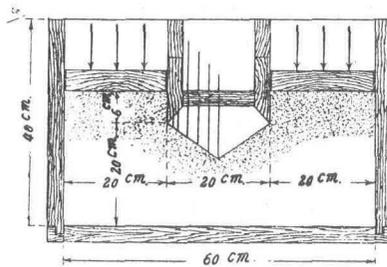


Fig. 5.

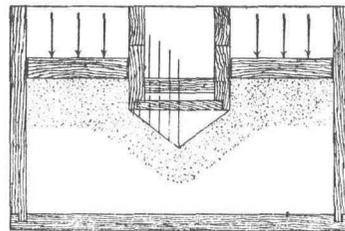


Fig. 6.

of the box ; and to avoid the frictional effect of the glass plate upon the sand mass, whose effect may not be negligible, the angle of natural slope was measured at a certain distance from the glass plate and at the same time at the central part of the box.

Next, to make the relative movement of the sand mass easier to observe, the sand was filled in the box in horizontal layers and the

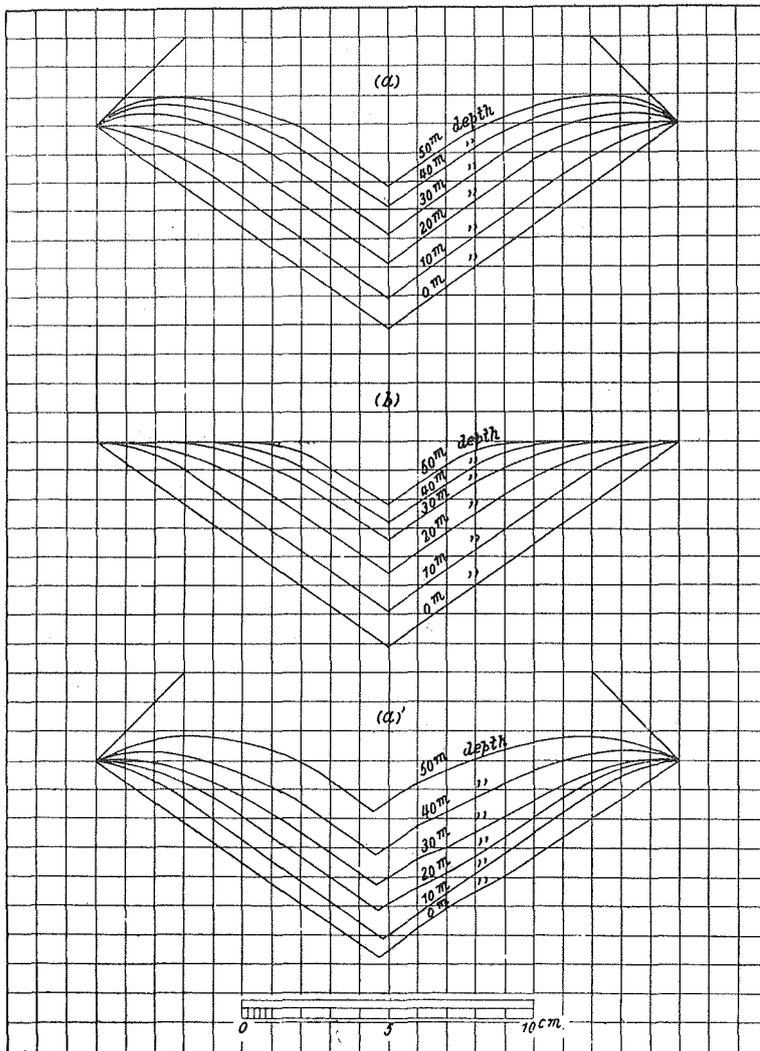


Fig. 7.

different horizontal surfaces were marked by a number of lines of black coloured sand. After pouring in and loading the sand mass was wet and fixed with plain water, applying the cementing action of water, and then having cut through the central part of the sand mass, the observer could note the relative movement of the sand grains. The experiment of this kind applying the cementing action of water has already been reported in *Memoirs Vol. 3, No. 2.** Fig. 4 shows the wooden box and the testing machine used; from the central window of the glass plate there peeps the plane of natural slope to be measured and at the top of the box one sees also eight steel pins used to determine the relative position of the points which lie in the line of natural slope.

Figs. 7 and 8 show the results of the experiments. Fig. 7 (a) shows the bulging curve obtained by the use of the apparatus of inclined edges shown in Fig. 5. Fig. 7 (b) shows the similar curve obtained with the apparatus of flat edge with base plate, shown in Fig. 6. Fig. 7 (a)' is obtained with the former apparatus (Fig. 5), filling the sand more compactly compared with the above two cases.

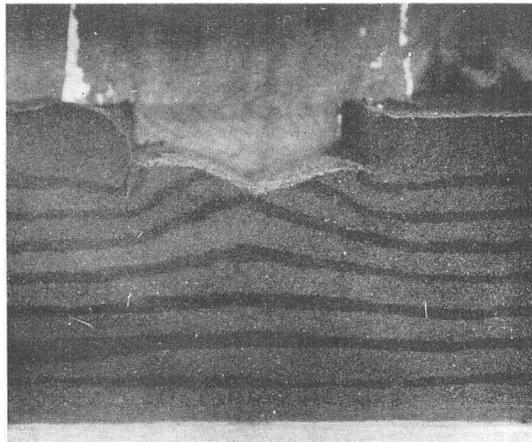


Fig. 8.

* F. Takabeya: *Experimental Investigations on Surface of Rupture in Sand.* *Memoirs Vol. 3, No. 2, P. 76.*

F. Takabeya: *Internal Granular Movement of Sand Treated as Three Dimensional Problems.* *Memoirs Vol. 3, No. 2, P. 81.*

The sand was always packed in horizontal layers; the different horizontal surfaces were marked by a number of lines of black coloured sand.

In every case, natural slopes were measured under the pressures 2 tons, 4 tons, 6 tons, 8 tons and 10 tons, their equivalent depths being 10 m, 20 m, 30 m, 40 m and 50 m in these cases. The screw testing machine was operated at the rate of 2.5 mm/minute and pressure was increased. The machine was stopped during 10 minutes for the loadings of the five different kinds mentioned above.

Fig. 9 shows the relation between pressure and settlement of the surface of sand mass. The horizontal parts in the curves show the pressure decrease due to the gradual proceeding of the mutual compensation of unbalanced frictional resistances during the time of stopping of the testing machine, i.e. 10 minutes; but, the pressure decrease seemed to be very gradually and feeble after the duration of about 10 minutes.

Fig. 8 shows the relative movement of the sand masses when the pressure was increased to 10 tons from 0 tons, i.e. when the

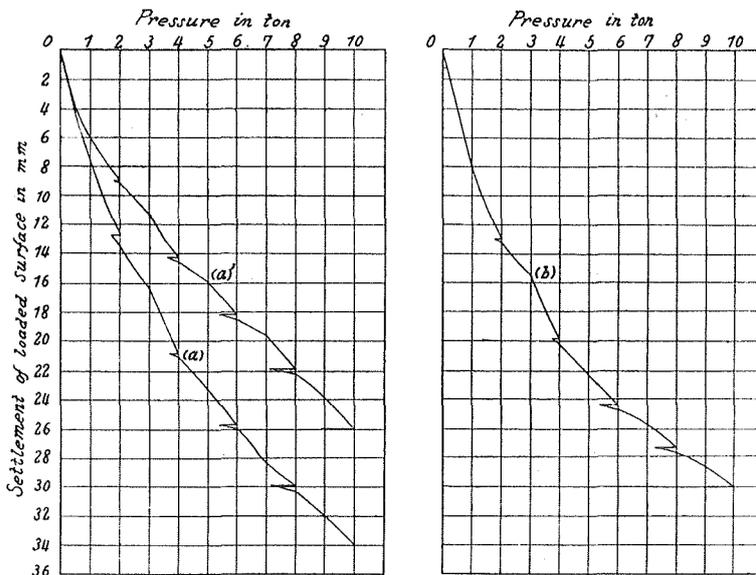


Fig. 9.

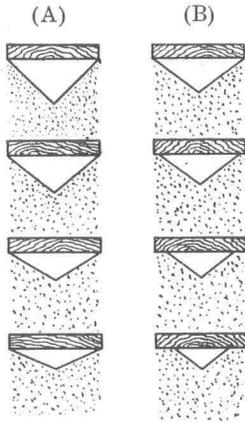


Fig. 10.

depth of sand was increased to 50 m from 0 m in equivalent height.

By these experiments it may be concluded that the natural slope of dry sand under the ground surface takes the aspects shown in Fig. 10 (B), always keeping a constant inclination to the horizontal plane, except near the ends of the cantilever arm and that it does not change its inclination with the depth as is shown in Fig. 10 (A).

For the mere upper portion of the slope, a curved surface is observed, caused by a

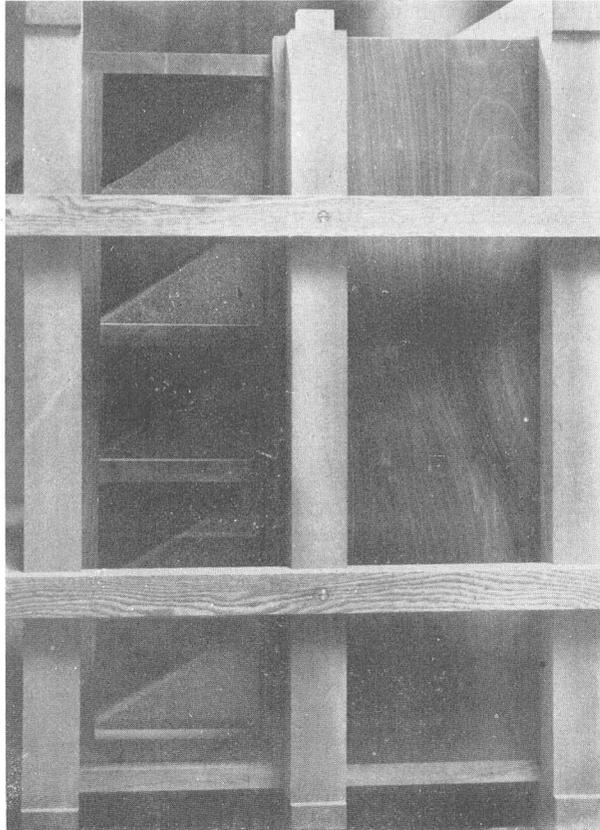


Fig. 11.

bulging of the sand mass pressed by the weight of the sand. These characteristics of the dry sand are obvious in the case of the wall with cantilever partitions as shown in Fig. 11.

(January 20, 1933)

II. Sand Pressure Experiments for Partial Yielding of Wall in Certain Depth.

Experimental investigations of soil mechanics, especially on earth pressure measurement have been extensively made by many investigators. Almost all of those experiments seem to have been to determine the resultant force of earth pressure, laying less importance upon the yielding or displacement of the wall; while the yielding of the contact surface on the pressure gives a fatal effect upon the amount of the pressure to be measured.

In this paper the authors describe the experimental results on the relations amongst the horizontal component of the sand pressure against a wall, which covers a rectangular opening in the lower part of the side wall of a tank, the yielding of this wall and the height of the back filling. In addition to these, the effect of yielding of the wall upon the structure of the sand mass in the tank is described.

Experiment and Results.

To measure the horizontal component of the sand pressure in a certain depth, use was made of a steel sand tank, measuring 1.8 m wide and long, 3.6 m deep, which has a rectangular opening (0.35 m long and 1.1 m wide) in the lower part of the side wall, the opening being covered by a sliding wall which is exposed to the pressure to be measured. (see Fig. 12)

In Fig. 12 may be seen the sliding wall *A* which stands on a pair of rollers *R*. Connection is made between the sliding wall and the weight box *W* by a steel wire. The weight box contains shots and the weight can be gradually reduced by letting the shots fall down one by one from slit *B* of the weight box.

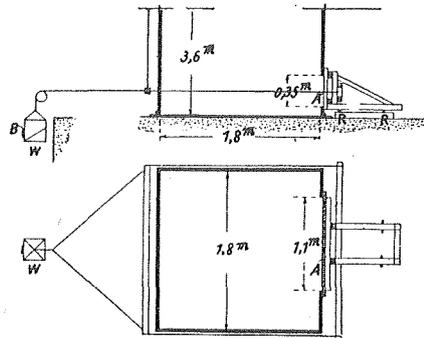


Fig. 12.

The sand tank was first filled with dry sand and the weight was then reduced gradually to the moment when the sliding wall began to yield in a horizontal direction. The observer noted the movements of the sliding wall and read the readings up to 0.1 mm by using the indicator of the similar mechanism shown in Fig. 13.

The material of the back filling was fine dry beach sand, the maximum size of grains 0.86 mm dia., the specific weight of grains 2.56, the weight of one cubic meter 1300 kg and the angle of repose of the sand $34^{\circ}20'$.

The surface of back filling was horizontal and it was poured in horizontal layers. The heights of the back filling were $0.75 h_0$, h_0 , $1.5 h_0$, $2 h_0$, $3 h_0$ and $4 h_0$, where h_0 denote the height of the opening, i.e. 0.35 m.

Next, another smaller wooden box was introduced, in order to investigate the transformations of the structure of the back filling mass of sand, which were produced by the yielding of the sliding wall, the chief part of the transformation being the change of surface of rupture in the back filling sand. The box had an opening in the lower part of its lateral side and was able to yield gradually at intensional moments, giving a horizontal movement to a board connected to the opening (Fig. 13). One of the side walls of this box was a glass plate in order to facilitate observation of the internal granular movements.

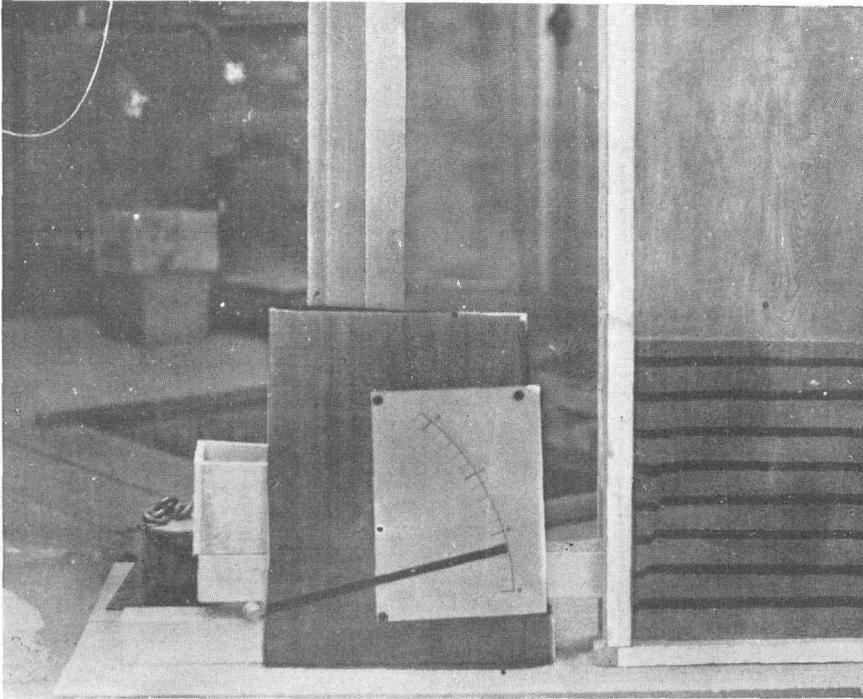


Fig. 13.

Sand was filled loosely in the box always in horizontal layers and the different horizontal surfaces were marked by a number of horizontal lines of black sand; thus the relative movement of the granular masses was made easier to observe.

Let h_0 be the height of the opening, i.e. the height of the sliding wall of the sand tank which yields in a horizontal direction, h the height of the back filling, $E_{(h)}$ the horizontal component of the pressure on the sliding wall which covers the opening, and S the distance that the sliding wall is displaced, or yields, horizontally.

Fig. 14 represents the relations between $E_{(h)}$ and S for $h = 2h_0$, $h = 3h_0$, and $h = 4h_0$; and Fig. 15 represents the same relations when $h = 0.75h_0$, $h = h_0$, $h = 1.5h_0$ and $h = 1.75h_0$.

It is noticeable that every curve consists of two different phases; in the first phase the sand pressure decreases proportionally with the distance that the wall yields; this relation can be represented by a

straight line. In the second phase, the yielding of the wall is remarkably sensible, it proceeds in a greater proportion than the sand pressure decreases and the yielding of the wall occurs by a few jerks; the zigzag lines show this part of this phase. In the advanced state of the second phase, a large yielding of the wall occurs with a single jerk and the equilibrium of the sand mass is thereby completely broken. C. Terzaghi treats similar results.*

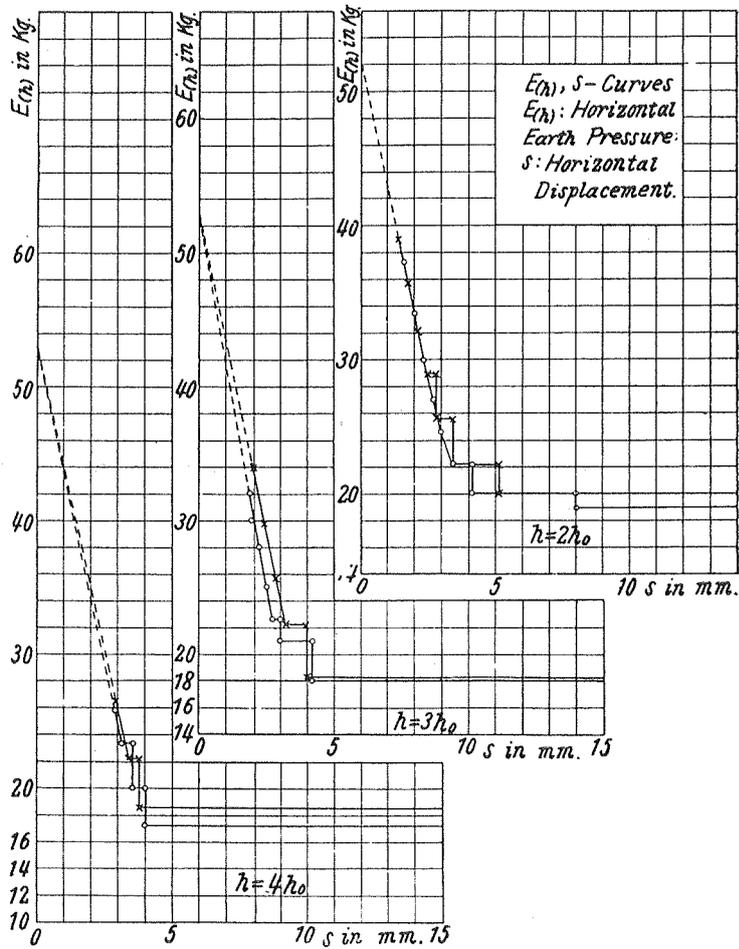


Fig. 14.

* Old Earth Pressure Theories and New Test Results. Eng. News-Record, Vol. 85, No. 14. (1920)

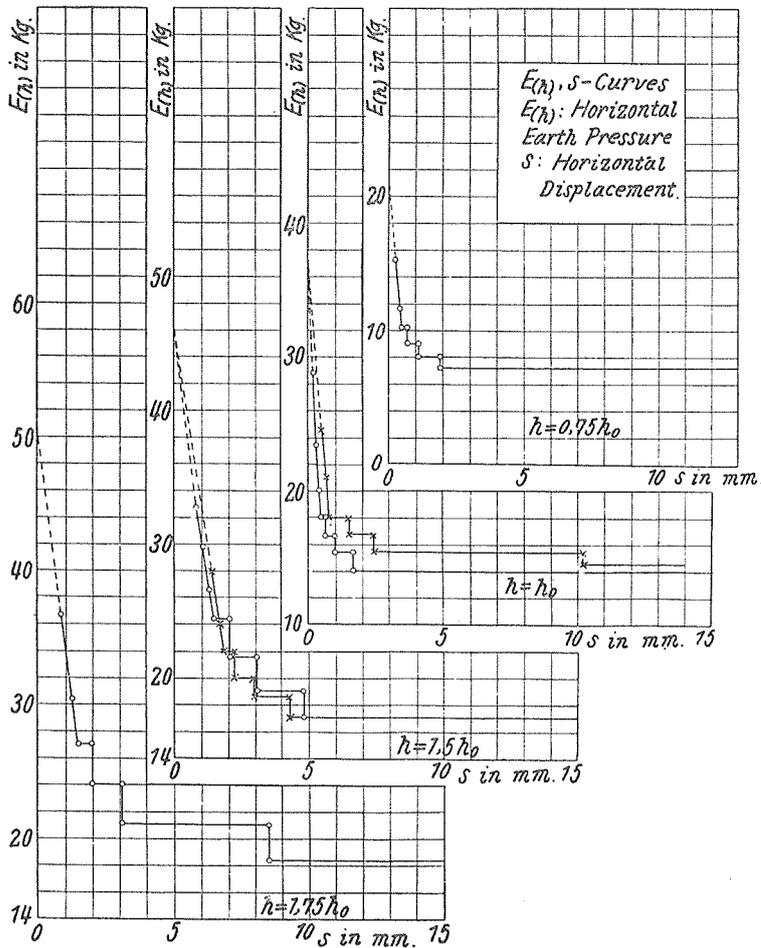


Fig. 15.

Photographs No. 1, Pl. I to No. 10, Pl. II represent the effect of horizontal yielding of the wall upon the structures of the sand mass of the back filling. They are of great importance to explain the curves in Figs. 14 and 15. If we consider, however, the effect of the frictional resistance between the glass plate and the sand as well as the arch action of the sand mass, the figures of the surface of rupture appearing in these photographs may be somewhat different from the true figures. Upon these investigations reference may again be made here to Memoirs Vol. 3, No. 2. Thus, these photo-

graphs, No. 1, Pl. I to No. 10, Pl. II explain the singular points in the curves as below :

In the first phase, the plane of rupture does not appear, along which the sand mass slides, though the corresponding movements of the grains to the points which may be traced in the curves of Figs. 14 and 15 appear as internal movements of the granular mass. They are confined to a region between the side wall and a little outside of the surface of rupture as shown in the photograph. The sand grains, which take their places nearer to the wall surface of the tank, show a greater movement than the further ones.

For the beginning of the second phase, one may observe in the photographs an early transformation to the plane of rupture in the lower part of the back filling. With the yielding of the wall, the plane of rupture propagates upwards to the upper layers, and the sand mass which slides along the plane of rupture increases with the yielding. The slip loosens the structure of a narrow zone of sand along the plane of rupture and the sand mass begins to contain a layer of less compact material, gaining the plane of least resistance. Then the sand pressure increases and at last the large yielding of the wall occurs with a jerk, allowing the equilibrium of the sand mass to break down completely.

Fig. 16 represents the relation between $E_{(h)}$ and h . Curve α gives the estimated point of intersection of the straight line of the first phase and the axis of $E_{(h)}$. In other words, this point may be considered as the one which represents the pressure $E_{(h)}$ at the moment when the wall begins to yield.

Curve β gives the pressure $E_{(h)}$ at the beginning point of the second phase. Concerning with this transition point the first phase to the second one, the experiments gave somewhat uncertain results and therefrom the corresponding pressure $E_{(h)}$ is only the approximate one. Curve γ gives the value $E_{(h)}$ at the moment when the equilibrium has been completely broken.

In the case of $h < h_0$, these curves seem to become parabolic and to coincide with

$$E_{(h)} = \frac{1}{2}Kwh^2,$$

where w is the weight of unit volume of sand, and K is a constant.

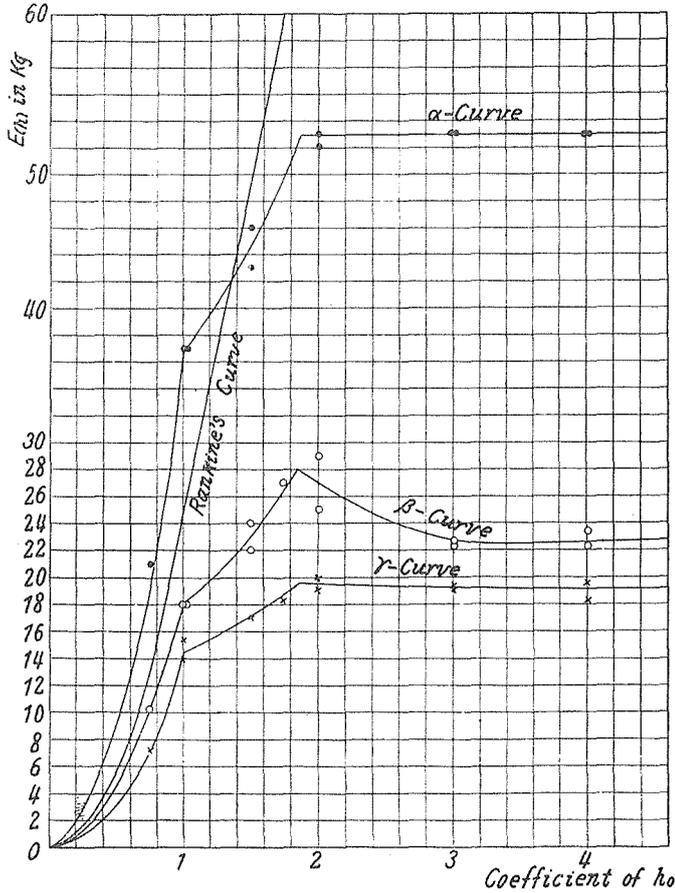


Fig. 16.

In the case of $h > h_0$, the curves do not coincide with

$$E_{(h)} = \frac{1}{2}Kw\{h^2 - (h-h_0)^2\},$$

where K is a constant and values the same in the case of $h < h_0$. Inclination of these curves is very flat, compared with the curve of $\frac{1}{2}Kw\{h^2 - (h-h_0)^2\}$.

In the case of $h > 2h_0$, the curves become approximately horizontal; in other words, the pressures are constant for any height of back filling.

It is worthy of notice that measurements made in the present experiments show no sand pressure in the case where the side wall stands completely at rest, i.e., measurements show no sand pressure where no yielding of the side wall is allowed; but the experimental results show the pressure at the moment when a certain partial yielding of the side wall in the lower part of the tank wall has been allowed. In the case of this yielding, a part of the pressure seems to have been transmitted to the adjacent portion in the wall and the measured pressure shows a much smaller value than that at the moment when the wall begins to yield, which is to be estimated from the experimental data.

Sectional Conclusions.

The general conclusions to be drawn from the results of the present investigations may be summarized as follows:

- (1) The horizontal component of the sand pressure against a wall which covers a rectangular opening in the lower part of a side wall of the sand tank, decreases with the yielding of the wall. During the decrease of the pressure, there are noticed two different phases in the curves of the relation between the pressure and the amount of the yielding or the horizontal displacement. For the first phase, the pressure decreases proportionally with the distance that the wall yields and the plane of rupture does not appear during the yielding of the wall in this phase, although one may observe the faint motion of the granular grains. For the second phase, the plane of rupture appears and the amount of the yielding is remarkable by jerks.
- (2) The relation between the sand pressure and the height of the back filling may be stated as follows:

In the case of $h < h_0$, the pressures increase proportionally with the square of h . In the case of $2h_0 > h > h_0$, the increase of the pressure is approximately proportional to the value of h . A part of the pressure which is exerted in the case of no yielding seems to be transmitted to the adjacent part of the side wall when the wall begins to yield. The rate of the increasing of the pressure is very small.

In the case of $h > 2h_0$, the pressure seems to be approximately constant for any height of back filling.

(May 15, 1933)

III. Sand Pressure Experiments under Special Considerations of Horizontal Displacement and Frictional Condition of Wall Surface.

Employing the same sand tank as described in the preceding section, sand pressure measurements were repeatedly carried out with the horizontal movement ranging between 0.5 mm and 1.0 cm, under the supposition that the sand pressure on the wall surface is dependent in a great measure on the finish of the wall surface, i.e. of the wall surfaces made of glass, of planed pine board and of rough hewn pine board.

The granular materials used in these experiments and the method of filling up were entirely the same as described in the preceding section.

Experimental Results and Sectional Resumés.

(1) *K-S*-Diagram and Coefficient of Earth Pressure at Rest.

In general, the earth pressure can be expressed by the equation :

$$E_{(h)} = \frac{1}{2} Kwh^2 \dots\dots\dots (1)$$

where $E_{(h)}$ represents the horizontal component of the pressure exerted by the sand against a unit length of the wall, w the weight of a unit volume of the sand, h the height of the retaining wall and K the constant which may be determined by the boundary conditions. In the special case, i.e. in the case $K = 1$, equation (1) gives the horizontal component of the corresponding pressure of a liquid.

The horizontal pressure of the sand was measured by means of the apparatus and granular materials which were exactly the same as described in the preceding sections and from these experiments the values of K were obtained. It is worthy of notice, as frequently mentioned before, that the sand pressures measured have relation to the horizontal yielding of the wall. This consideration did not appear in the earth pressure theories of Coulomb, Rankine and their successors. The present writers, however, have paid attention to Terzaghi's experiments which deal with the relation between the sand pressure and the yielding of the wall.

Hence, in addition to the consideration of the horizontal displacement of the wall surface, the present sand pressure experiments were carried out taking into consideration of the roughness of the wall surface, i.e. of surfaces made glass, of planed pine board and of rough hewn pine board.

The results of the experiments are summarized and represented in Figs. 17, 18 and 19, which show the relations between S and K , for three different conditions of wall-surface roughness and S denotes in the figures the amount of the horizontal yielding of the wall. That is, Fig. 17 is for the most rough surface made of rough hewn pine board; Fig. 18 for the smooth surface of planed pine board and Fig. 19 for the most smooth surface of glass. Fig. 20 represents these resumé's.

Every series of experiments was repeated at least ten times, but in these diagrams the experimental results of four times are shown for the sake of simplicity. More clearly, the curve in Fig. 21 may serve as an example of the result of the experiment, separated from the another ones.

Wall surfaces made of rough hewn pine board

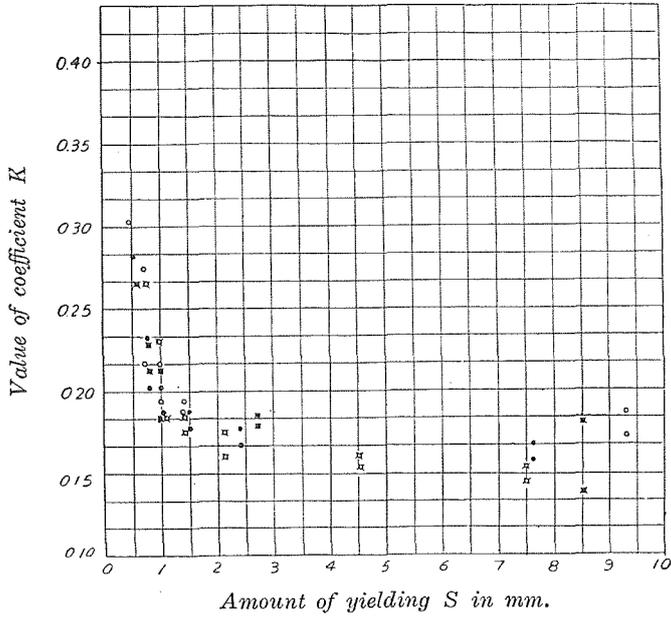


Fig. 17.

Wall surfaces made of planed pine board

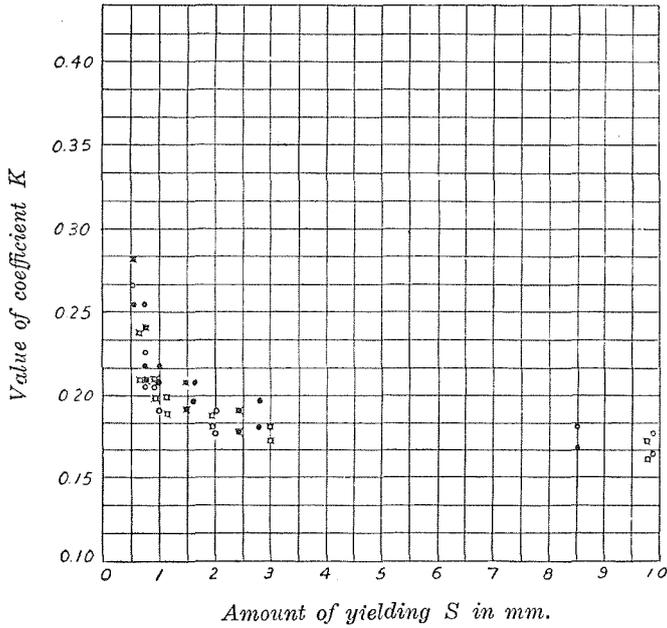


Fig. 18.

Wall surfaces made of glass

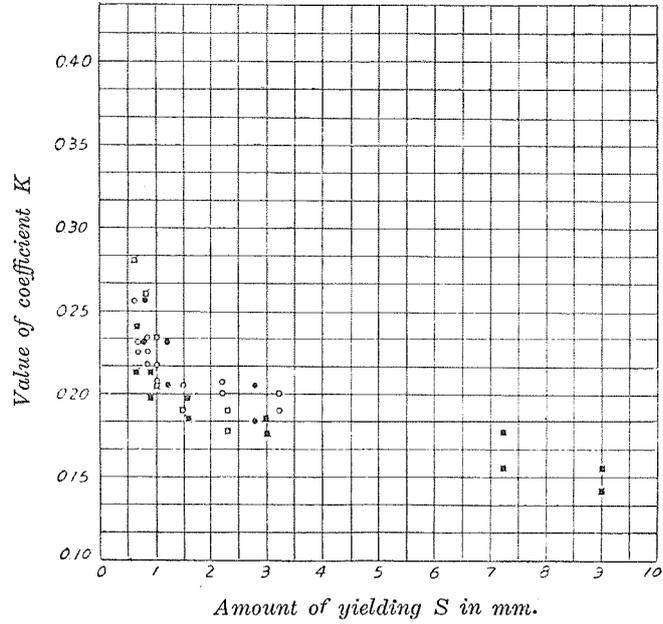


Fig. 19.

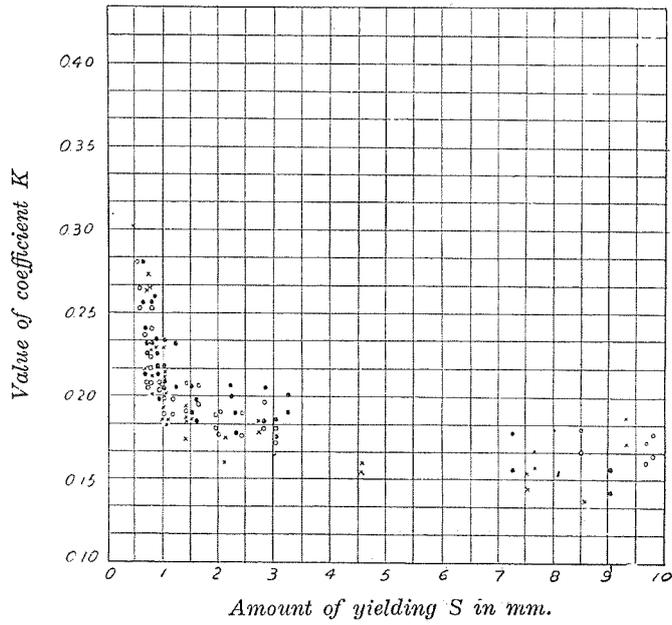


Fig. 20.

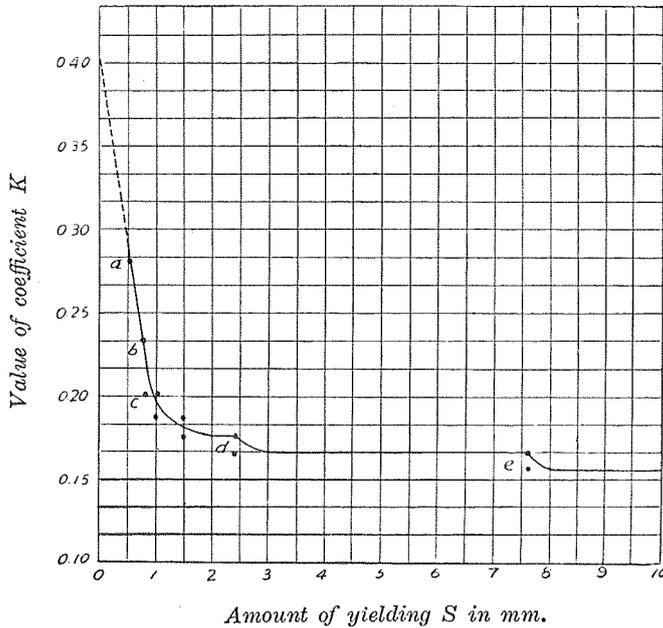


Fig. 21.

Attention is called to the characteristics in the shape of curve which have been already mentioned in the preceding sections, however, they may be mentioned again here.

As long as the amount of the yielding of the wall is small, the relation between S and K can be represented by a straight line ab (Fig. 21), in other words, the sand pressure decreases proportionally with the amount of the yielding of the wall like the relation between stress and strain in elastic solid body. However, with the increase of the yielding of the wall this relation deviates gradually from the straight line and the yielding becomes remarkably sensible and proceeds in a greater proportion than the decrease in sand pressure and the yielding of the wall is given by a few jerks. Curve cd in Fig. 21 shows this part. In the advanced state of the yielding, i.e. at the point d , a large yielding of the wall happens by a simple jerk and the equilibrium of the sand mass is completely broken.

The investigations upon the transformation of the structure of the sand mass which is produced by the yielding of the wall, concisely explain these phenomena. Photographs No. 1, Pl. I to No. 10, Pl. II show the effect of the partial yielding, in horizontal direction, of the side wall upon the structure of the sand mass of the back filling. Photographs No. 11 to No. 14, Pl. III show the same, but of the total yielding of the side wall. By these photographs we know that the plane of rupture appears at the point d (Fig. 21) and the wedge shaped sand prism begins to slide down at this point. By the outbreak of the plane of rupture, the narrow zone of sand along the plane of rupture becomes the plane of least resistance and a large yielding of the wall happens with a simple jerk, allowing the equilibrium of sand mass to break down completely. At the slight yielding of the wall, one observes only the internal movements of the granular mass which are confined to a region above the surface of rupture or near by as shown in Photograph No. 11, Pl. III; and this observation gives an idea that the sand pressure seems to have a linear relation to the yielding of the wall as long as the amount of the yielding is small.

Next, in the more advanced state of the yielding the plane of rupture is far from the wall and the sliding down of the sand prism is stopped unless the weight decreases in the weight box. When the amount of the yielding of the wall reaches to about 20 mm, a second plane of rupture appears and a large yielding happens again with a jerk, which, however, could not be marked in Fig. 21.

Now, if the straight line ab (Fig. 21) is prolonged, then it cuts the axis of ordinates at the point where K has the value 0.42 approximately. This value gives the sand pressure against a wall of absolutely no yielding and the writers call the value of K , in this case, the coefficient of the earth pressure at rest. This coincides with the value which Terzaghi's experiments gave.

(2) Comparison between Experimental Results and Those Computed from Old Earth Theories.

In the Rankine Theory, K is given by the formula :

$$K = \frac{1 - \sin \varphi}{1 + \sin \varphi} \dots\dots\dots (2)$$

where φ is the angle of repose of the sand ; in our case it takes the value :

$$\varphi = 34^{\circ}20' .$$

Substituting this value in equ. (2), one obtains

$$K = 0.278 .$$

In Coulomb Theory, the constant K is given by the formula :

$$K = \frac{\cos^2 \varphi}{\left[1 + \sqrt{\frac{\sin (\varphi + \delta') \sin \varphi}{\cos \delta'}} \right]^2} \dots\dots\dots (3)$$

where δ' is the angle between the normal to the surface of the wall and the direction of the earth pressure.

As the angle δ' we take the angle at which the sand grains are about to move along the surface of the wall inclined to the horizontal. However, for the rough hewn pine board, δ' is greater than φ . So, in this case, we take

$$\delta' = \varphi .$$

φ is the same as mentioned in equ. (2). Then, the values of K are computed as follows :

a) For the wall surface made of rough hewn pine board, substituting $\varphi = 34^{\circ}20'$ and $\delta' = 34^{\circ}20'$ in equ. (3), one obtains :

$$K = 0.211 .$$

b) For the wall surface made of planed pine board, substituting $\varphi = 34^{\circ}20'$ and $\delta' = 31^{\circ}20'$ in equ. (3), one obtains :

$$K = 0.216 .$$

c) For the wall surface made of glass, substituting $\varphi = 34^{\circ}20'$ and $\delta' = 25^{\circ}40'$ in equ. (3), one obtains :

$$K = 0.226 .$$

Let α be the angle which the plane of rupture makes with the horizontal. Then, adapting the angle α to Coulomb Theory, the next formula is obtained under the assumption that the directions of the forces are as shown in Fig. 22.

$$E = \frac{1}{2}wh^2 \frac{\cot \alpha \sin (\alpha - \delta)}{\cos (\alpha - \delta - \delta')} \dots\dots\dots (4)$$

where E is the resultant force of the earth pressure.

Therefore, its horizontal component $E_{(h)}$ is given by the formula :

$$E_{(h)} = \frac{1}{2}wh^2 \frac{\cot \alpha \sin (\alpha - \delta) \cos \delta'}{\cos (\alpha - \delta - \delta')} \dots\dots\dots (5)$$

Hence,

$$K = \frac{\cot \alpha \sin (\alpha - \delta) \cos \delta'}{\cos (\alpha - \delta - \delta')} \dots\dots\dots (6)$$

The values of the angle α can be determined from experiments ; these values were approximately equal, in the present case, for three different conditions of roughness on the wall surface :

$$\alpha = 63^{\circ}30' .$$

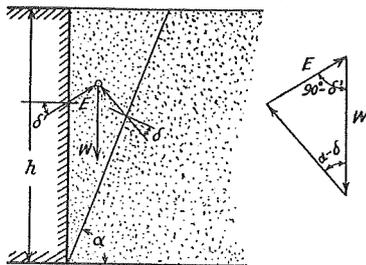


Fig. 22.

δ' is the same as mentioned in equ. (3). For the angle of the internal friction of the sand mass i.e. for δ , we assume that this is identical to the angle of repose of the sand. Then the values of K for the three different wall surfaces are computed by substituting the corresponding values of φ , δ and δ' in equ. (6) as follows :

- a) For the wall surface made of rough hewn pine board :

$$K = 0.201 .$$

- b) For the wall surface made of planed pine board :

$$K = 0.208 .$$

- c) For the wall surface made of glass :

$$K = 0.219 .$$

Comparing the values of K obtained by equ. (3) with the corresponding ones obtained by equ. (6), we know that these corresponding values are approximately identical, in other words, the plane of rupture which is assumed in Coulomb Theory coincides nearly with the plane of actual rupture.

The values of K which have been obtained by the experiments are the function of the amount of yielding of the wall. The values of K at the moment when the plane of rupture appears, i.e. at the point d , are as follows :

- a) For the wall surface made of rough hewn pine board :

$$K = 0.170 .$$

- b) For the wall surface made of planed pine board :

$$K = 0.176 .$$

- c) For the wall surface made of glass :

$$K = 0.180 .$$

The values of K at the beginning of the second phase, i.e. at the moment when the yielding of the wall becomes sensible are as follows :

a) For the wall surface made of rough hewn pine board :

$$K = 0.20 .$$

b) For the wall surface made of planed pine board :

$$K = 0.21 .$$

c) For the wall surface made of glass :

$$K = 0.22 .$$

These values of K which are obtained by the formulae as well as by the experiments are tabulated in Table I.

Table I.
Values of K .

Material of the Wall Surface	K_R	K_C	$K_{C'}$	K_{Ec}	K_{Ed}
Rough hewn pine board.	0.278	0.211	0.201	0.20	0.170
Planed pine board.	0.278	0.216	0.208	0.21	0.176
Glass.	0.278	0.226	0.219	0.22	0.180

In Table I the many K 's denote as below :

K_R : K computed from Rankine's formula (2),

K_C : K computed from Coulomb's formula (3),

$K_{C'}$: K computed from the formula (6) which contains a term of the angle of the actual plane of rupture,

K_{Ec} : K obtained by the experiments and at the moment when the yielding of the wall becomes sensible, i.e. at point c in Fig. 21,

K_{Ed} : K obtained by the experiments and at the moment when the plane of rupture appears, i.e. at point d in Fig. 21.

Sectional Conclusions.

The conclusions to be drawn from the results of the present investigation may be stated as follows :

(1) THE EFFECTS OF THE HORIZONTAL YIELDING OF THE
WALL ON THE SAND PRESSURE.

Any yielding of the wall causes a decrease of the horizontal component of the sand pressure. Two different phases are noticed in the curve of this relation between the pressure and the amount of the yielding of the wall as follows :

- a) During the first phase, i.e. during the slight yielding of the wall, the pressure decreases proportionally with the amount of the yielding of the wall.
- b) During the second phase, i.e. in the advanced state of the yielding, the yielding of the wall is remarkably sensible and it is given by a few jerks.

At last the plane of rupture appears and a large yielding of the wall occurs with a single jerk.

(2) THE EFFECTS OF THE ROUGHNESS OF THE WALL
SURFACE ON THE SAND PRESSURE.

The effects of the roughness of the wall surface on the horizontal component of the sand pressure are shown in Fig. 20. At a definite point in the yielding of the wall, the wall surface which is made of material with smaller roughness induces greater pressure than that of rougher ones.

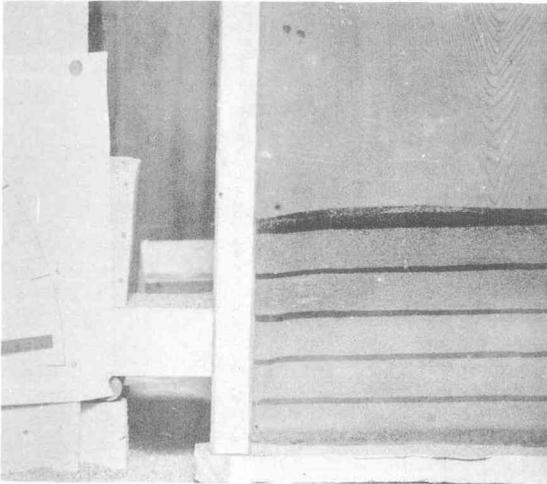
The sand pressure computed from Rankine's or Coulomb's formula is identical to the actual sand pressure which acts when the slight amount of the yielding of the wall takes place and the

sand pressure on a perfectly unyielding wall seems to take a greater value than the Rankine's or Coulomb's one.

These conclusions are, however, for sands which are completely dried up, sieved to the size of a definite range as mentioned previously and filled in the vessel in horizontal layers.

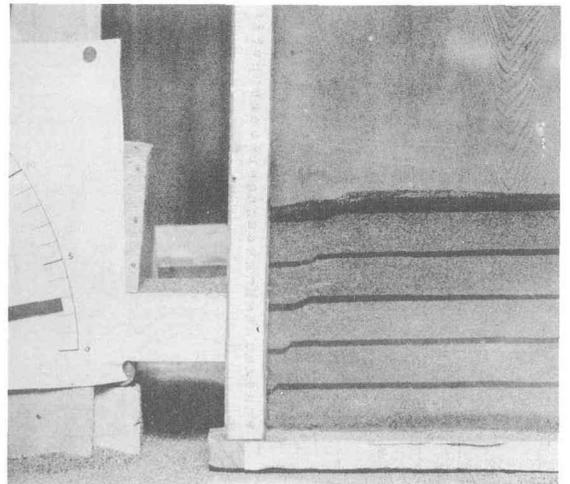
(December 10, 1932)

No. 1



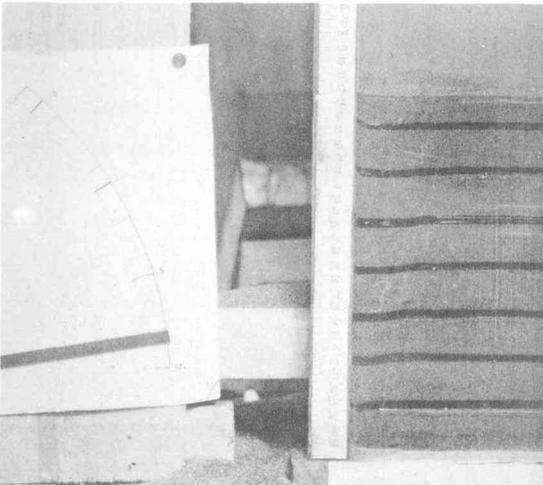
Partial Yielding: $s = 2.0 \text{ mm}$, $h = h_0$

No. 2



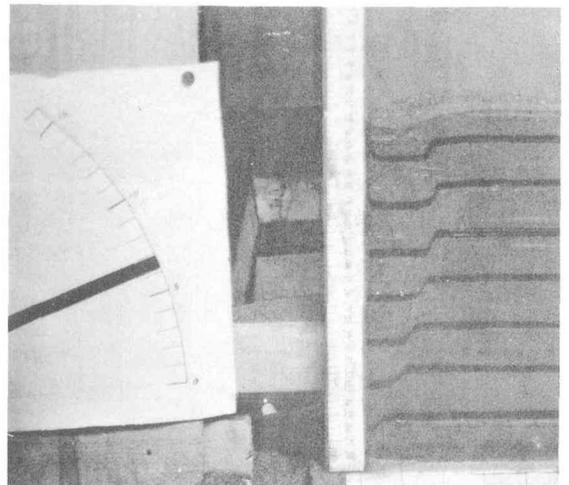
Partial Yielding: $s = 3.0 \text{ mm}$, $h = h_0$

No. 3



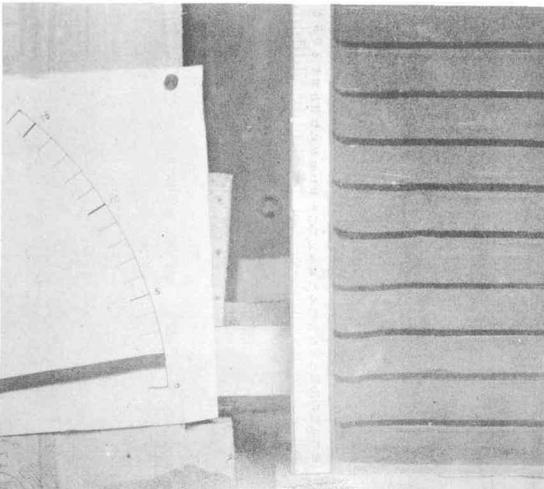
Partial Yielding: $s = 2.0 \text{ mm}$, $h = 1.5 h_0$

No. 4



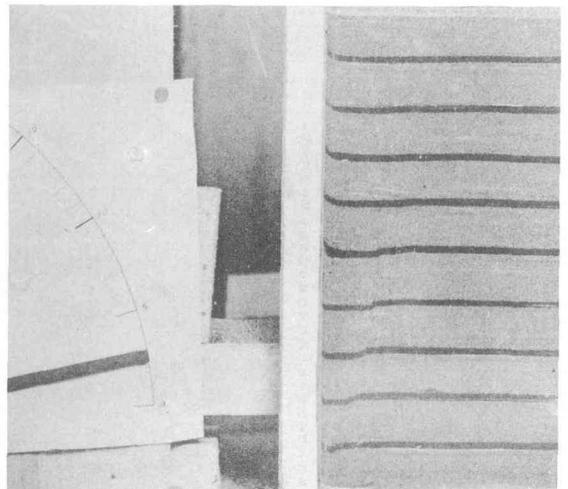
Partial Yielding: $s = 7.0 \text{ mm}$, $h = 1.5 h_0$

No. 5



Partial Yielding: $s = 2.0 \text{ mm}$, $h = 2 h_0$

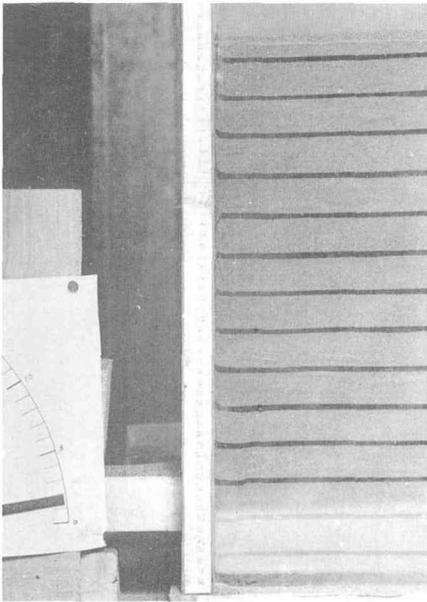
No. 6



Partial Yielding: $s = 3.0 \text{ mm}$, $h = 2 h_0$

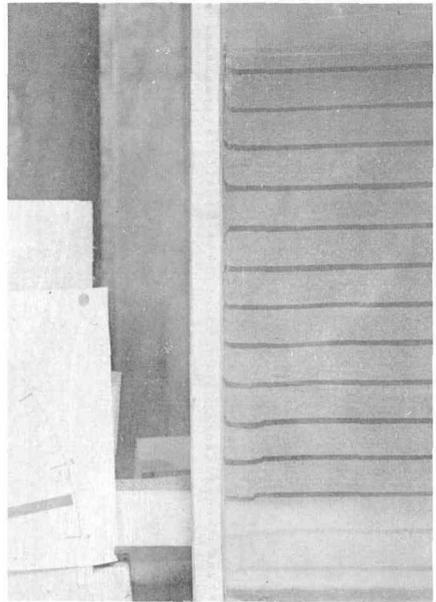


No. 7



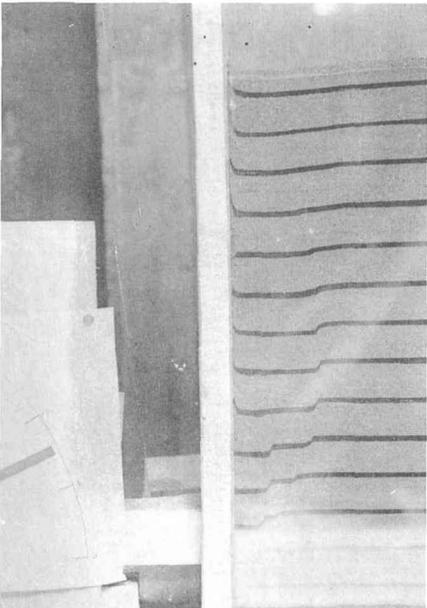
Partial Yielding: $s = 2.0 \text{ mm}$, $h = 3 h_0$

No. 8



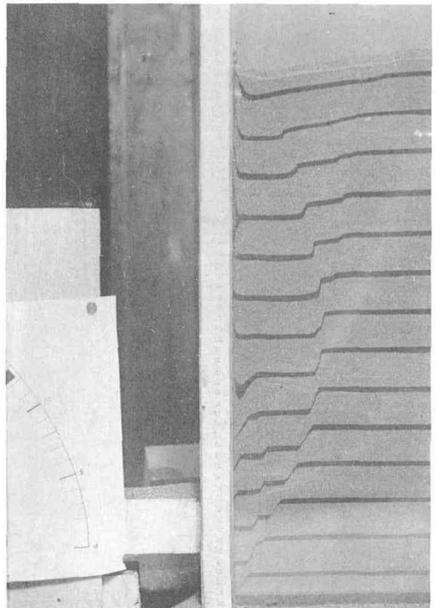
Partial Yielding: $s = 3.0 \text{ mm}$, $h = 3 h_0$

No. 9



Partial Yielding: $s = 8.0 \text{ mm}$, $h = 3 h_0$

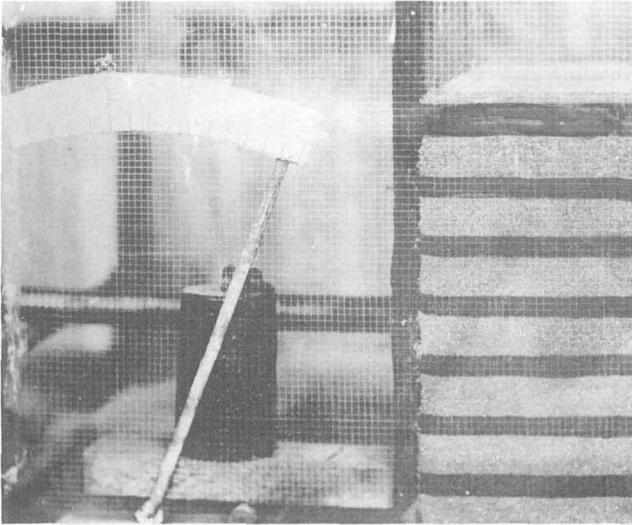
No. 10



Partial Yielding: $s = 13.0 \text{ mm}$, $h = 3 h_0$

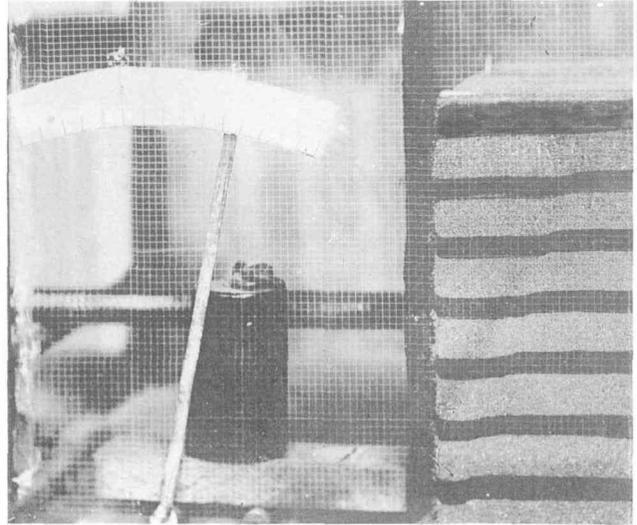


No. 11



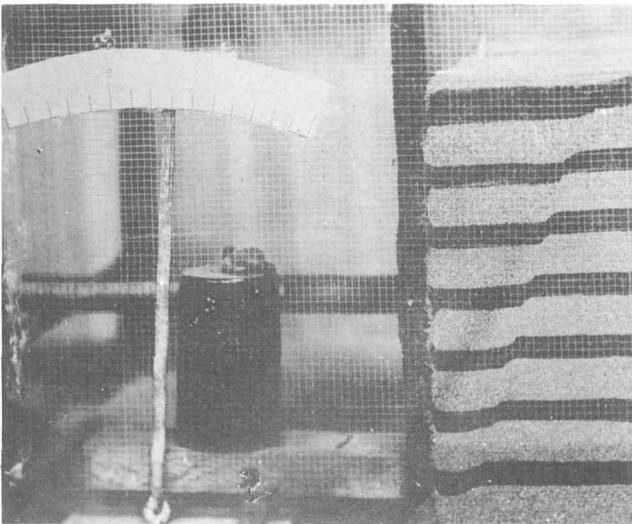
Total Yielding: 0.5 mm

No. 12



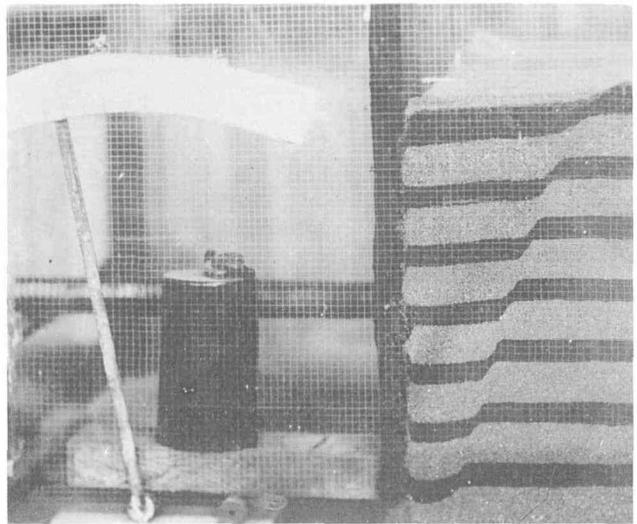
Total Yielding: 4.0 mm

No. 13



Total Yielding: 7.0 mm

No. 14



Total Yielding: 12.0 mm

