Application of Photo-Elastic Effect to Measurement of Pressure.

By

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Problems of photo-elastic effect have been extensively studied both experimentally and mathematically since twenty years ago. Those experimental investigations deal with finding the stress distribution in some beams, plates, frames and other such specimens of several transparent isotropic materials under stressed state of various different conditions. The application of these photo-elastic characteristics to the measurement of unknown pressures seems to have escaped the notice of investigators up to this time, so far as the present authors are aware.

As we sometimes find, there are some cases where the displacement of the contact surface on the pressure gauge gives a fatal effect upon the investigation of the pressure to be measured and it is therefore necessary to take such a gauge type as will allow no displacement on the pressure contact. For example, it is experimentally very difficult to measure the expansion force which some steel test pieces exert under a given temperature rise. In the analogous difficulty of such measurement, we have had some bitter experiences in the measurement of soil pressure, especially in a horizontal direction. Caused by the displacement of the contact surface, the gauge shows no true indication but yields fatal errors.

The application of piezo-electric phenomena may be one of the best fitted devices for the investigation of this kind and that of photo-elastic phenomena may be another one. The former, since first investigated by Curie, have often been used for the measurement of pressure in comparatively difficult conditions of boundary, while the photo-elastic phenomena have never been applied, to the slight knowledge of the authors, to the purpose, i.e., as a measure of the pressure.

It is well known that a transparent isotropic material becomes doubly refractive under some stressed condition, and when it is brought between crossed Nicol's prisms with or without quarter wave plates, it shows certain regularly colored or black lines. Experimental investigation upon the number of these lines may be very useful as a measure of direct forces and the position of these lines, which shift under variable loadings, may also be applied as a measure of force to the study of unkown reaction or pressure. In the case, which we have been chiefly concerned with, the changing position of the colored or black lines bore a very important relation to the unknown external forces.

Such being the case, in our opinion, a direct compression at a certain point of a test piece, made of Phenolite, Bakelite, Xylonite, Celluloid, glass and some other transparent materials must have a certain relation to the number as well as to the position of the lines which are produced.

Therefore after several experiments, we have calibrated this position of the lines for some trapezoidal and beam pieces which seem to have been the most convenient and best forms for our investigations, and as verification of the results we have applied these elastic characteristics to the measurement of unknown reactions of a continuous beam with three supports. In the following note, there are described, this experimentation and also the reading positions of the lines produced.

Experiments and Results.

1. **Source of Light.** As a light source, we used Nagaoka's quicksilver-quartz lamp, type "A", manufactured by the Institute of Physical and Chemical Research, Tokyo, without filter for Series K, and then with filter No. 77A, Eastman, for Series L and M.

2. **Axes of Nicols.** As we are satisfied with any symmetrical figures of the photo-elastic lines, the direction of the axes of Nicols and also of the quarter wave plates are not necessarily required to be in normal positions, though the plates are not necessarily used in our experiments.

The axes of the crossed or parallel Nicols must be therefore set in a symmetrical position with the line of force, i.e., they must be inclined at zero and ninety degrees or both at forty five degrees with vertical, when loaded vertically, and the axes of the quarter wave plates, when they are used, are crossed or parallel to one another and inclined always forty five degrees to those of the Nicols. The positions of the parallel Nicols and the parallel quarter wave plates above mentioned are of no importance in the stress analysis commonly used, because such are not usually employed. Above all, the crossed and parallel Nicols are very effective, while the zero and ninety degrees inclined ones without quarter wave plates are valueless, because of the darkness of the figures, which is probably due to stresses accumulated in the direction of the load.

As the results of our experiments, the following two positions seem to be conveniently applicable to the determination of the number and position of the produced lines in order to investigate some unknown amount of direct compression:

a). Crossed or parallel Nicols inclined at zero and ninety degrees, with quarter wave plates.

b). Crossed or parallel Nicols both inclined at forty five degrees, without quarter wave plates.

In the first experiment, we preferred crossed Nicols inclined at zero and ninety degrees and also parallel axes of the quarter wave plates; in the second, crossed Nicols both inclined at forty five degrees and in the third, parallel Nicols, and crossed ones, both inclined at forty five degrees and in the fourth, as an example of beam piece, parallel Nicols both inclined at forty five degrees.

3. Measuring Pieces. The forms of the measuring pieces are to be symmetrical about the line of the force and easily to be manufactured. These requirements have made us pay particular attention to some rectangular and trapezoidal sections of the specimens; our experiments have been done upon these two forms after having arrived at the conclusion that the trapezoidal and beam sections are very effective for our present purposes. The material of the measuring pieces is Phenolite.

4. Trapezoidal Specimens (Type A).

The dimensions of the trapezoidal specimen, Type A, are as follows:

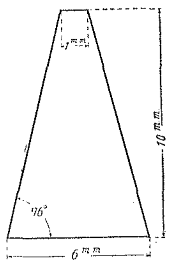


Fig. 1.

Height: 10 mm,
Upper face: 1 mm,
Base breadth: 6 mm,
Thickness: 5 mm,
Base angle: 76 degrees.

We measured the center height of a line from the base upon the photographic plate, and plotted the relation among the load L, the center height from the base H and the number of the lines n.

Now, when we let an unknown load come down upon the measuring piece, it displays certain lines, hereupon we read the center height and its number, and from the H-n-L diagram previously calibrated, we are able to find the magnitude of the unknown load.

These are shown in the photographs and diagrams of Experiments 1 and 2.

In Experiment 1, the axes of the crossed Nicols are inclined at zero and ninety degrees to vertical, with parallel quarter wave plates, while in Experiment 2, they are inclined both at forty five degrees without quarter wave plates.

All the Experiments 1 and 2 are carried out upon Series K, the measuring pieces being used as the supports of a continuous beam with three supports, whose bearing forces or reactions at the supports are required to be found experimentally.

The loading details of the experiments are shown as in the sketch below (Fig. 2).

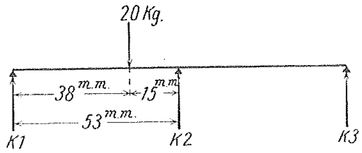
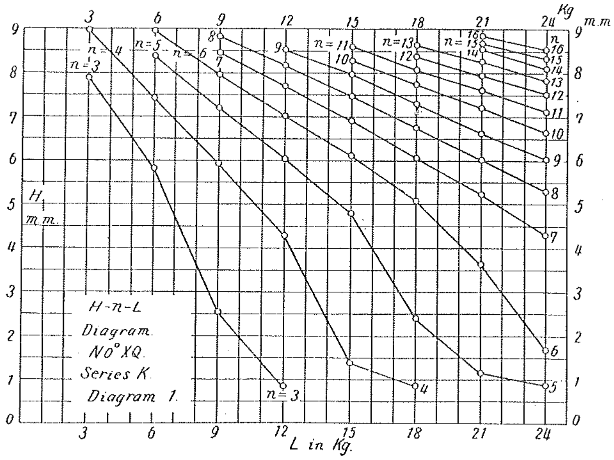


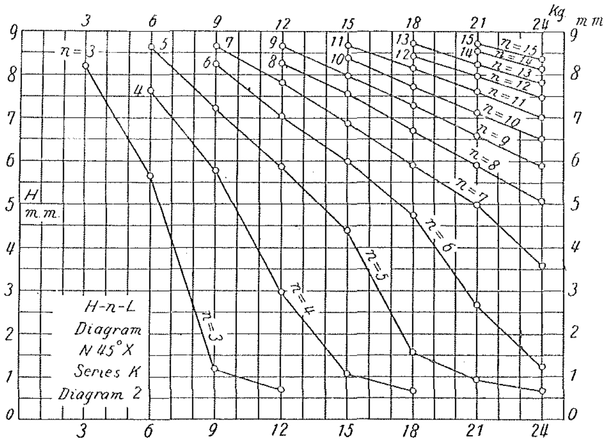
Fig. 2.

The method of taking the readings is described in article 6 From the photographs of Experiment 1 and Diagram 1, the unknown reactions are to be estimated as follows:

Measuring Piece	Estimated Load	Calculated Load	Error
K 1	5.32 kg	5.66 kg	—5.8%
K 2	13.43	14.34	—6.4
Total Sum	18.75	20.0	—6.3



For Experiment 2, by the same principles and method, we have estimated the unknown reactions as follows (Refer to Diagram 2):



Measuring Piece	Estimated Load	Calculated Load	Error
K 1	6.04 kg	5.65 kg	6.9%
K 2	13.80	14.35	−3.8
Total Sum	19.84	20.0	−0.8

From the diagrams, we can ascertain that the applicable height of lines is of from 2 mm to 8.5 mm, especially the lines from 5 mm to 8 mm furnish good results, because the H-n-L lines in those parts are nearly straight.

If the height of a piece be increased to 20 mm, the number of horizontal lines increases, and if the thickness be increased to 8 mm, then the bearing power of the load seems to be increased by about 60% and it shows a greater number of lines for greater loadings, from which the better results are to be gained.

The experiment using a continuous beam carrying a single load adopted here, seems to have been unsuitable for the measurement of unknown reactions, because the elasticity of the beam specimen has caused the beam not to be a continuous one, but to be a simply supported beam, exerting no reaction K3 at the right end support in Fig. 2. The insufficiency, however, may have been met by the following experiments.

5. Trapezoidal Specimens (Type B).

The dimensions of the trapezoidal specimen, Type B, are as follows:

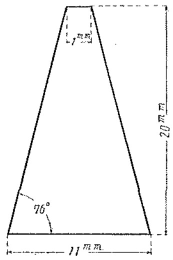


Fig. 3.

Height: 20 mm,
Upper face: 1 mm,
Base breadth: 11 mm,
Thickness: 6 mm,
Base angle: 76 degrees.

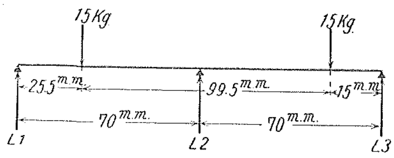
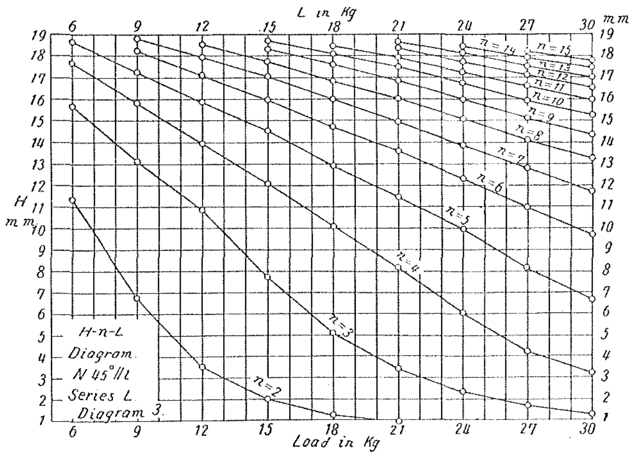


Fig. 4.

The method and process used in the experiment are the same as described in the above example.

In Experiment 3, the axes of the parallel Nicols are inclined forty five degrees to both horizon and vertical, having no quarter wave plates, while in Experiment 3a, the axes of the crossed Nicols are inclined at forty five degrees without quarter wave plates.

The loading details of the experiments are depicted in the sketch above (Fig. 4).



From the Photograph of Experiment 3 and Diagram 3, the unknown reactions are to be estimated as follows:

Measuring Pieces	Estimated Load		Calculated Load	Error	
	Ordinarily	Specially		Ordinarily	Specially
L 1	8.87 kg	7.97 kg	7.58 kg	17%	5%
L 2	12.47	12.47	12.59	-1	-1
L 3	9.88	9.88	9.83	0.5	0.5
Total Sum	31.22	30.32	30.00	4	1

(About the special estimation, refer to article 6, c).

These results are evidently more accurate than those of Experiments 1 and 2, which is due not only to better finishing of the test pieces, but also to the larger dimensions, about which we have already spoken in the end of Experiment 2.

As we can now show from the experimental results that there exists approximately a linear relation among the height of lines H, number n, and load L, we may therefore be able to plot necessary points controlled by calibration and then to draw diagrams connecting them.

The explanation how to choose correct and proper lines, has not yet been mentioned in the foregoing notes; the next section may solve it accurately for the trapezoidal measuring pieces.

6. Method of Selection of a Line and the Position, Whose Height is to be Measured in Trapezoidal Pieces. The way to select a line or to investigate the position of the line, whose height is to be measured in the trapezoidal pieces, is as follows:

a). When there is neither eccentric loading nor irregular bearing of base, any line which we may choose can be measured in the center vertical.

b). When there is irregular bearing of base and no eccentric loadings, lines near base are abnormal. Measure upper lines in the center vertical, which are uninfluenced by loading and base conditions.

c). When there exist eccentric loadings and no irregular bearing of base and further when lines near base are very much inclined, measure lines in the vertical of loading.

d). When there exist eccentric loadings and also irregular bearings of base, measure middle lines in the center vertical which are not influenced by above conditions.

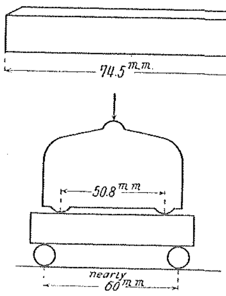
To the investigation, the following caution may be somewhat important:

1). Irregular and indistinct lines in calibration and also in estimation are to be avoided if possible.

2). If we know previously the approximate magnitude of a load, which is to be estimated, better results may be gained by more precise calibration, especially near the value.

3). In the preceding experiments of the trapezoidal specimens, the first and second lines have disappeared at the load 24 kg and 30 kg, while this may probably be avoidable if the base of the specimen is widened to about three times.

7. **Beam Specimens.** This is Series M. For the experiments, the dimensions of the beam piece are as follows (Fig. 5):



Figs. 5 and 6.

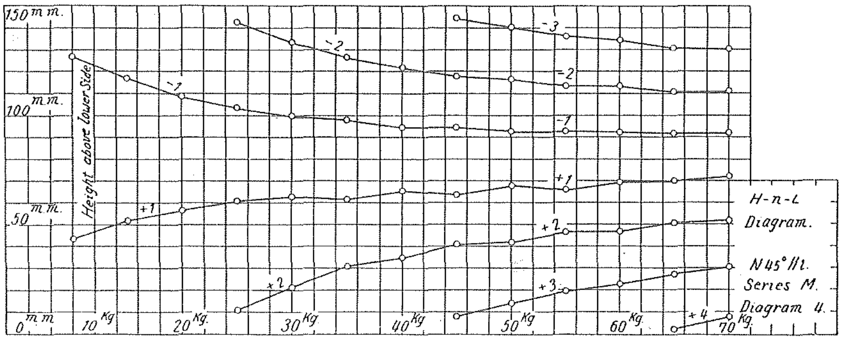
Length: 74.5 mm,
Depth: 15.0 mm,
Thickness: 7.0 mm.

This has been used as shown in Fig. 6 and Photographs of Experiment 4 are serviceable here.

We measured the height of lines above the lower face in the center vertical between the supports, and plotted H-n-L diagram or the relation among the center load L, the center height H above lower face and number of the lines n.

Now, when we let a certain unknown load come down upon the beam piece, it displays certain lines, hereupon we measure their center heights and numbers, and from the precalibrated H-n-L diagram, we can find the magnitude of the unknown load. This experiment is shown by Photographs of Experiment 4 and Diagram 4.

The axes of the parallel Nicols are inclined in this case at forty five degrees, without quarter wave plates, and this was applied to a



simple beam, carrying two loads as shown in Fig. 7. For this example, refer to Photographs of Experiment 4.

In this experiment, the spans of the loading tools were kept always constant and unchanged, while the spans of the beams were variable, on account of the free supporting method and imperfect improvised manufacture by the junior author. The lines near the

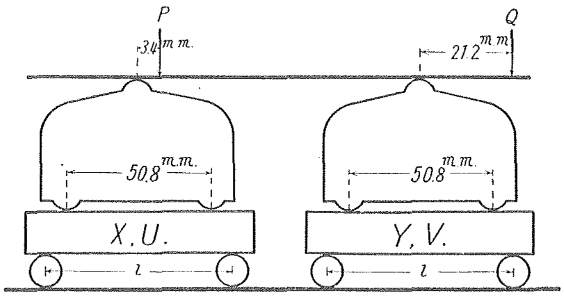


Fig. 7.

lower face of the beam were accordingly quite variable; on the other hand, the lines near the top were kept almost constant and therefore only those were adopted in our considerations.

From Photographs of Experiment 4 and Diagram 4, the required unknowns are estimated as follows:

First Experiment: P 38.6 kg and Q 22.4 kg.

Measuring Piece	Estimated Reaction	Calculated Reaction	Error
X	29.1 kg	28.5 kg	2%
Y	28.9	32.5	11
Total Sum	58.0	61.0	5

Second Experiment: P 57.6 kg and Q 33.4 kg.

Measuring Piece	Estimated Reaction	Calculated Reaction	Error
U	44.5 kg	42.5 kg	5%
V	35.4	48.5	27
Total Sum	79.9	91.0	12

The errors in these experiments may be due to the imperfect finishing of attachments as well as to unequal degeneration of the material, in spite of careful manufacturing and proper annealing.

Conclusions.

Several kinds of stress intensities have been, up to date, measured by many ways such as by a small test piece fitted in a compensator, Soleil-Babinet's compensator, Babinet's compensator in special case, a precalibrated color chart and a series of mica plates etc.

To the best of the authors' knowledge, however, no form of force has ever been measured universally by photo-elastic method, which we have proposed in the foregoing notes.

In the preceding argument, we have chosen trapezoidal specimens and beam pieces as an example of measuring compression and reactions at supports. To the measurement of some tension as well as other kind of stresses, to which the displacement of the contact position on the pressure gauge results fatally, the analogous principle of measuring process may be applied.

Caused by the inaccurate hand-made attachments and test pieces, our experiments were nothing but an example to show the principle of our process; however we can affirm that more complete accessories and precise attachments will give more accurate and much better results.

Our experiments have been carried out in thirteen Series A to M, but some were unsuccessful in trials and tests, and others have been abandoned for lack of proper facilities principally because the working-up and finishing of test pieces i.e. sawing, filing, grinding, annealing and polishing, were very particular work and took much time.

For the present object, eighty-nine photographic plates have been used for photographing over two hundred figures.

For all the matters concerned with the experiments, T. Shingo is responsible.

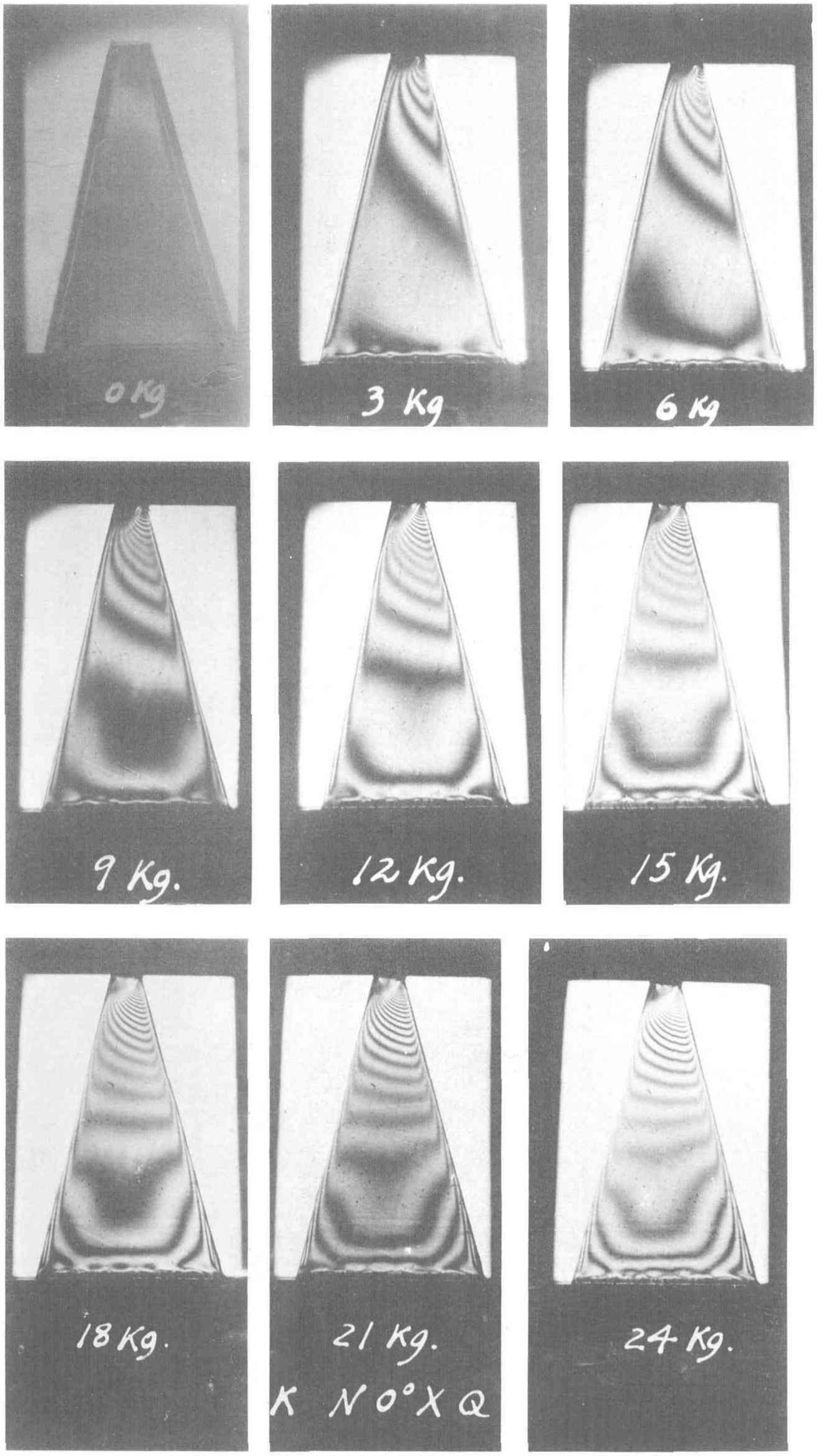
Now, we are partially satisfied with the results of our labour, constructing all the optical systems and further making attachments and test pieces by ourselves, but it is hoped that more valuable and better researches will be worked out so as to relegate our studies into comparative insignificance.

The authors gratefully acknowledge indebtedness to Profs. Ikeda and Kujime. The former has kindly granted our request to allow the junior author to work in the workshops, letting his workmen make a part of our apparatus; the latter has also willingly allowed some accessories to be made in his workshop.

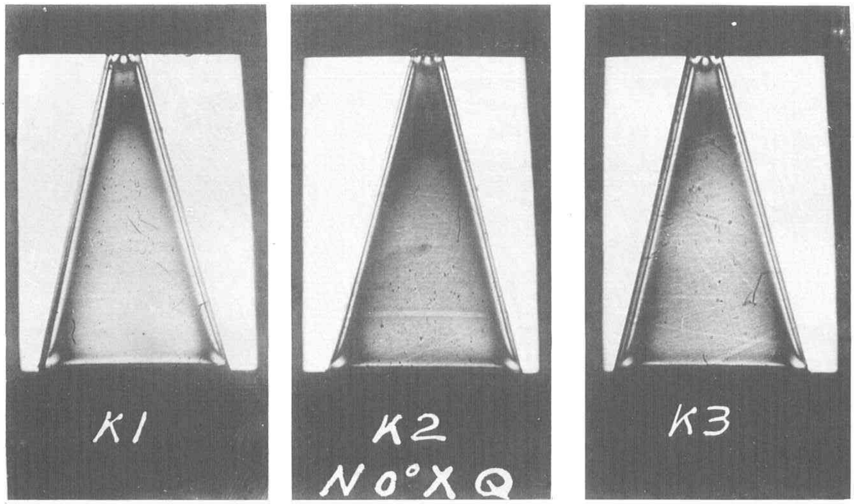
Sapporo, September, 1931.

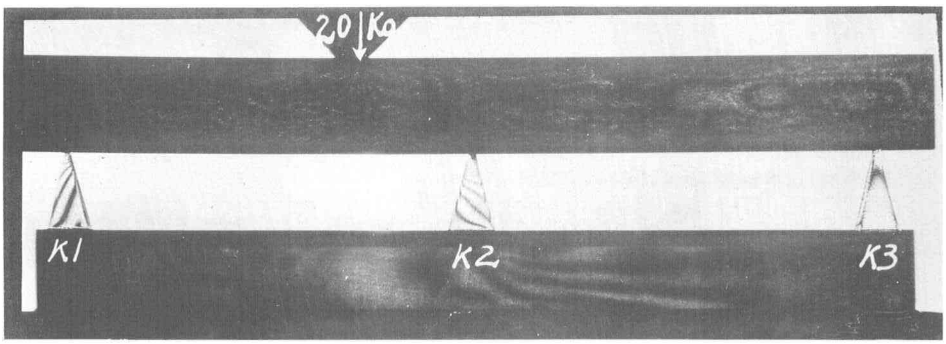
EXPERIMENT 1. (Series K.)

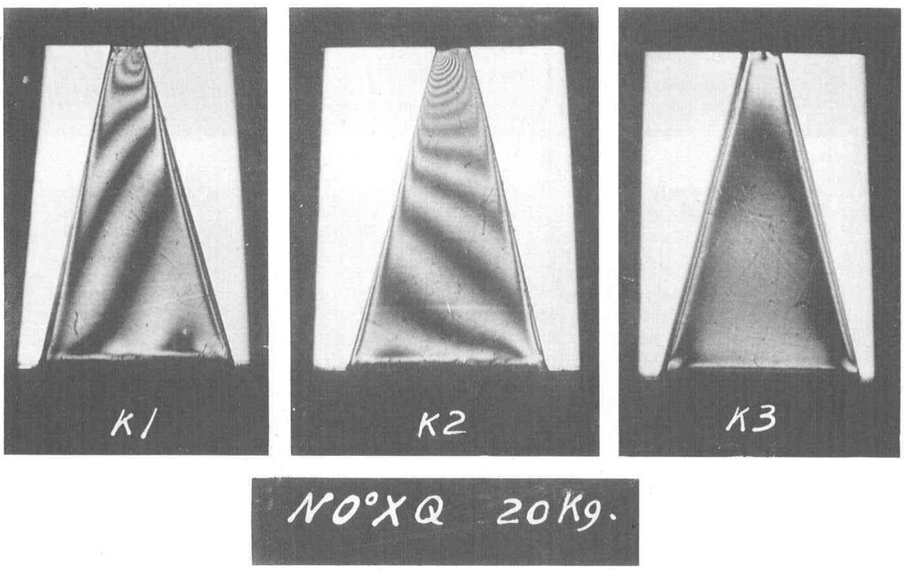
A. Photographs for Calibration.



B. Photographs of Experiment 1.

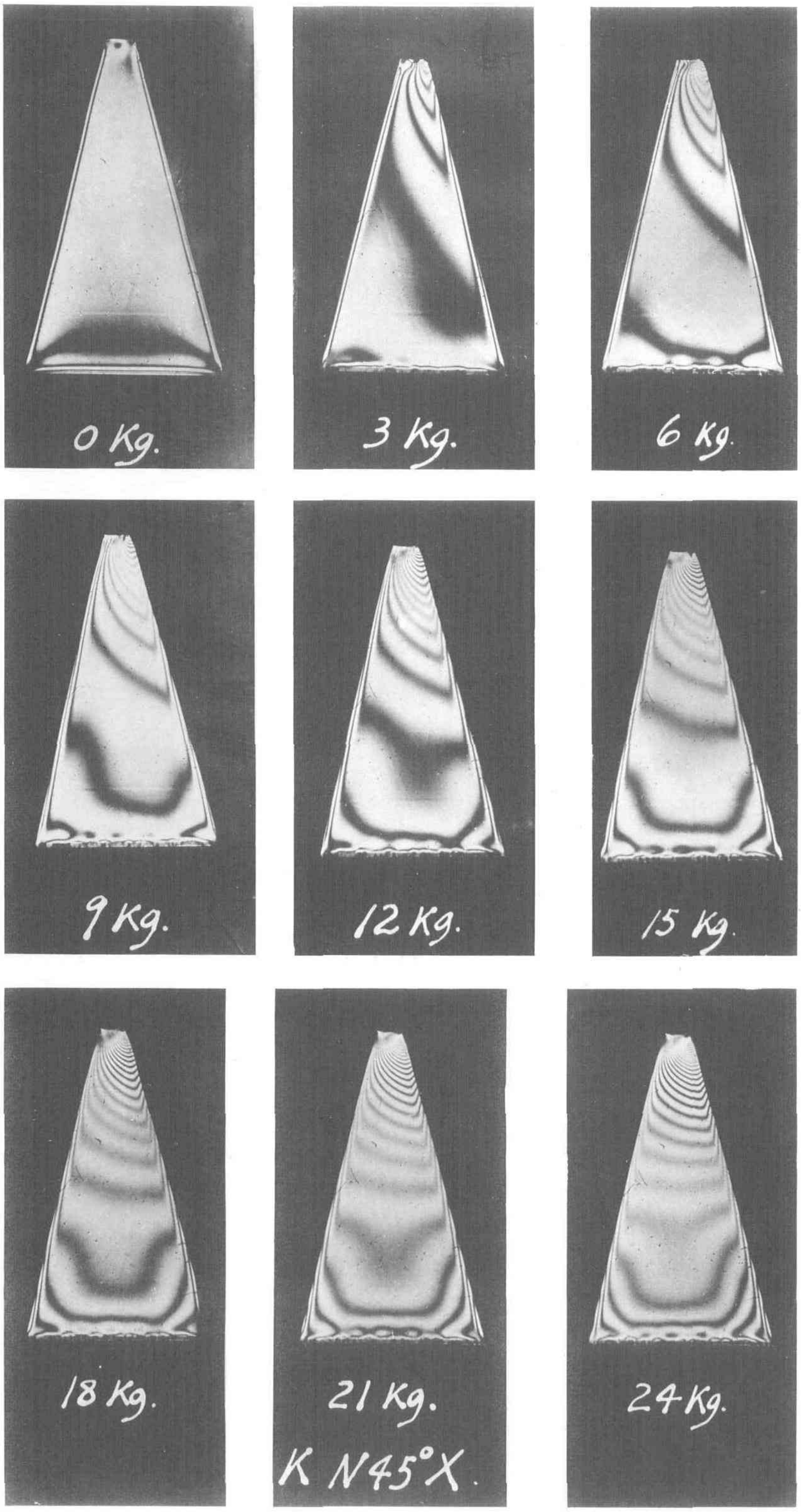




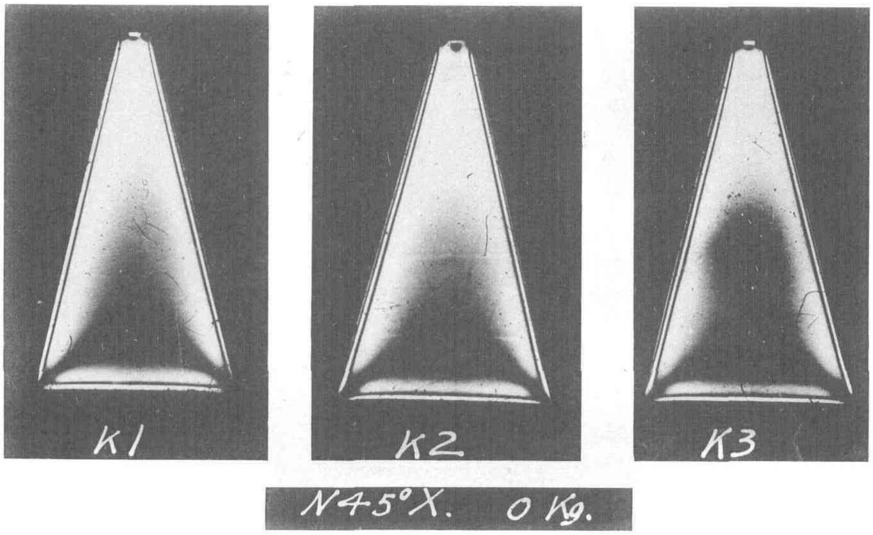


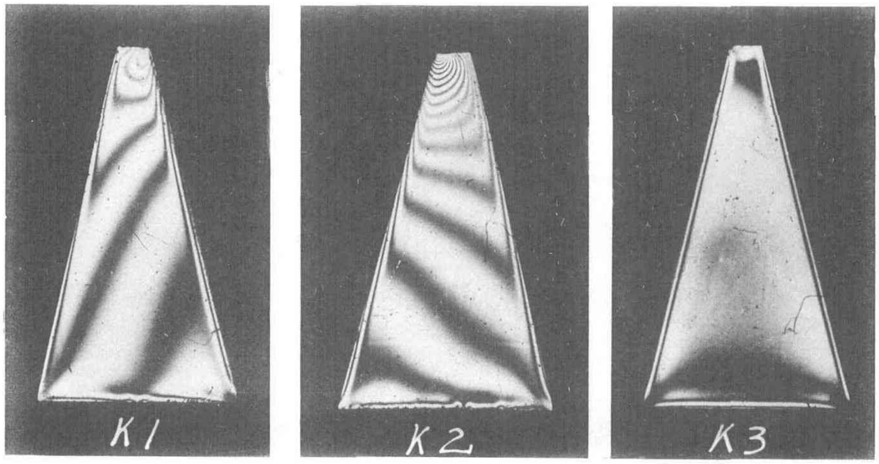
Experiment 2. (Series K.)

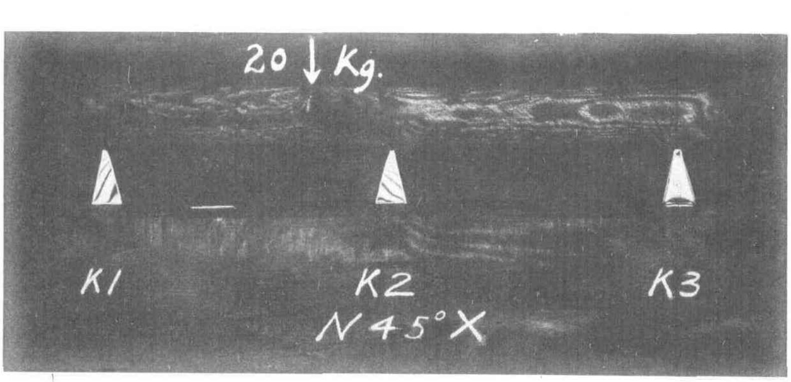
A. Photographs for Calibration.



B. Photographs of Experiment 2.

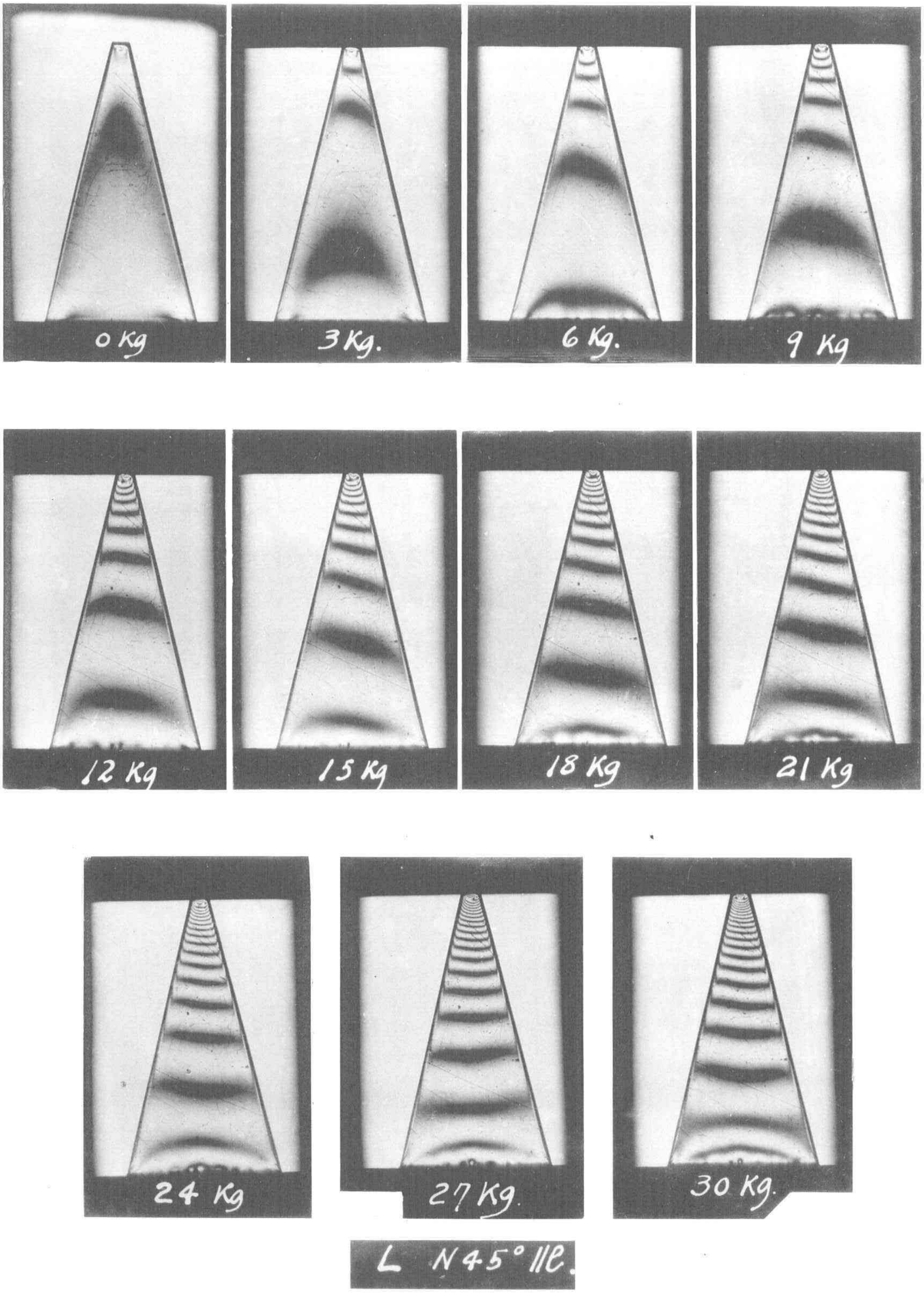




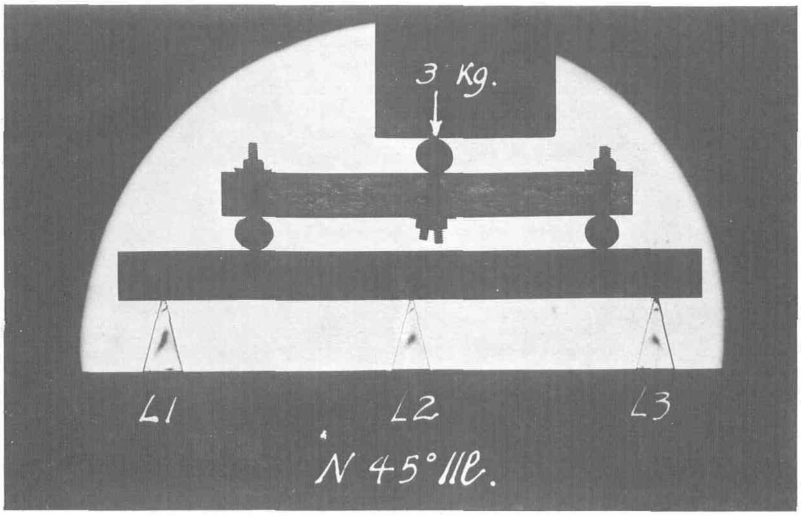


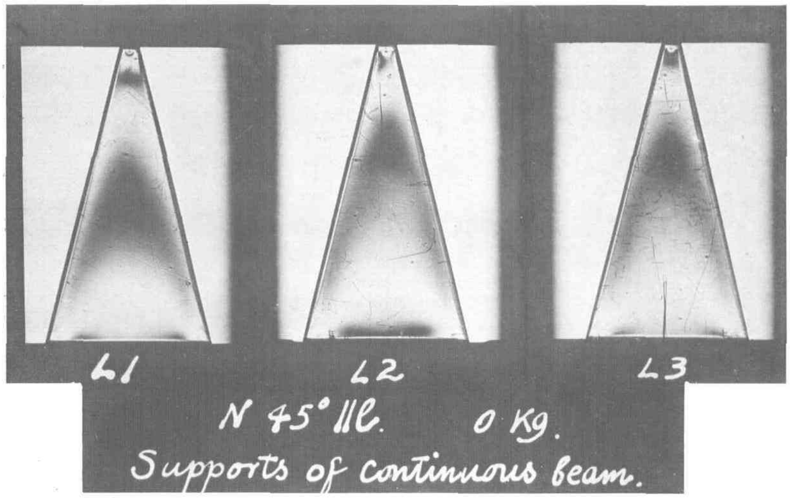
EXPERIMENT 3. (Series L.)

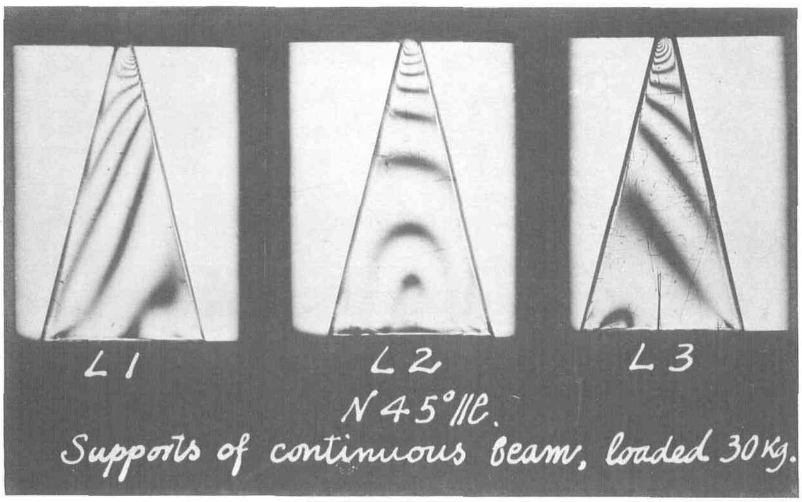
A. Photographs for Calibration.



B. Photographs of Experiment 3.

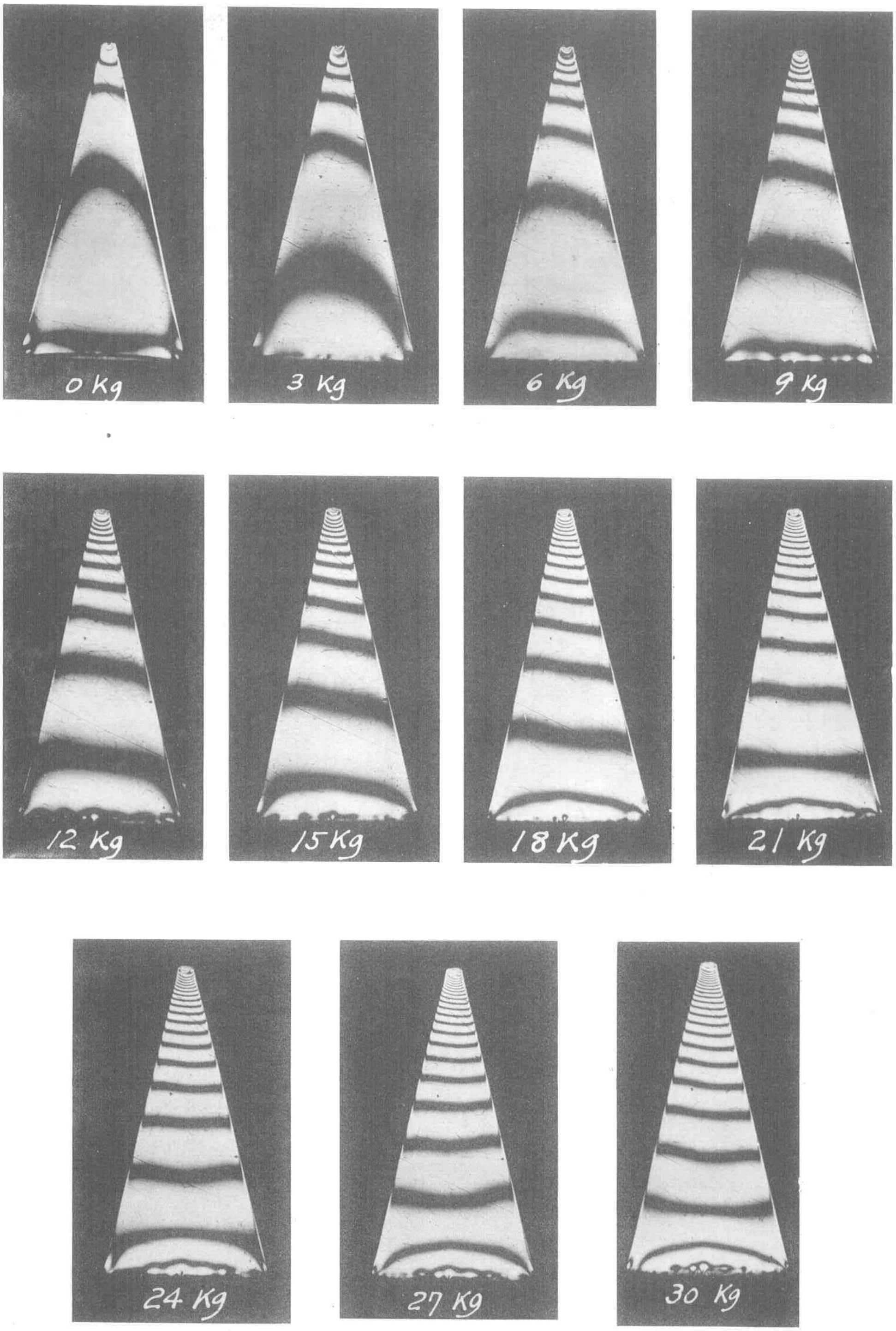






EXPERIMENT 3a. (Series L.)

A. Photographs for Calibration.



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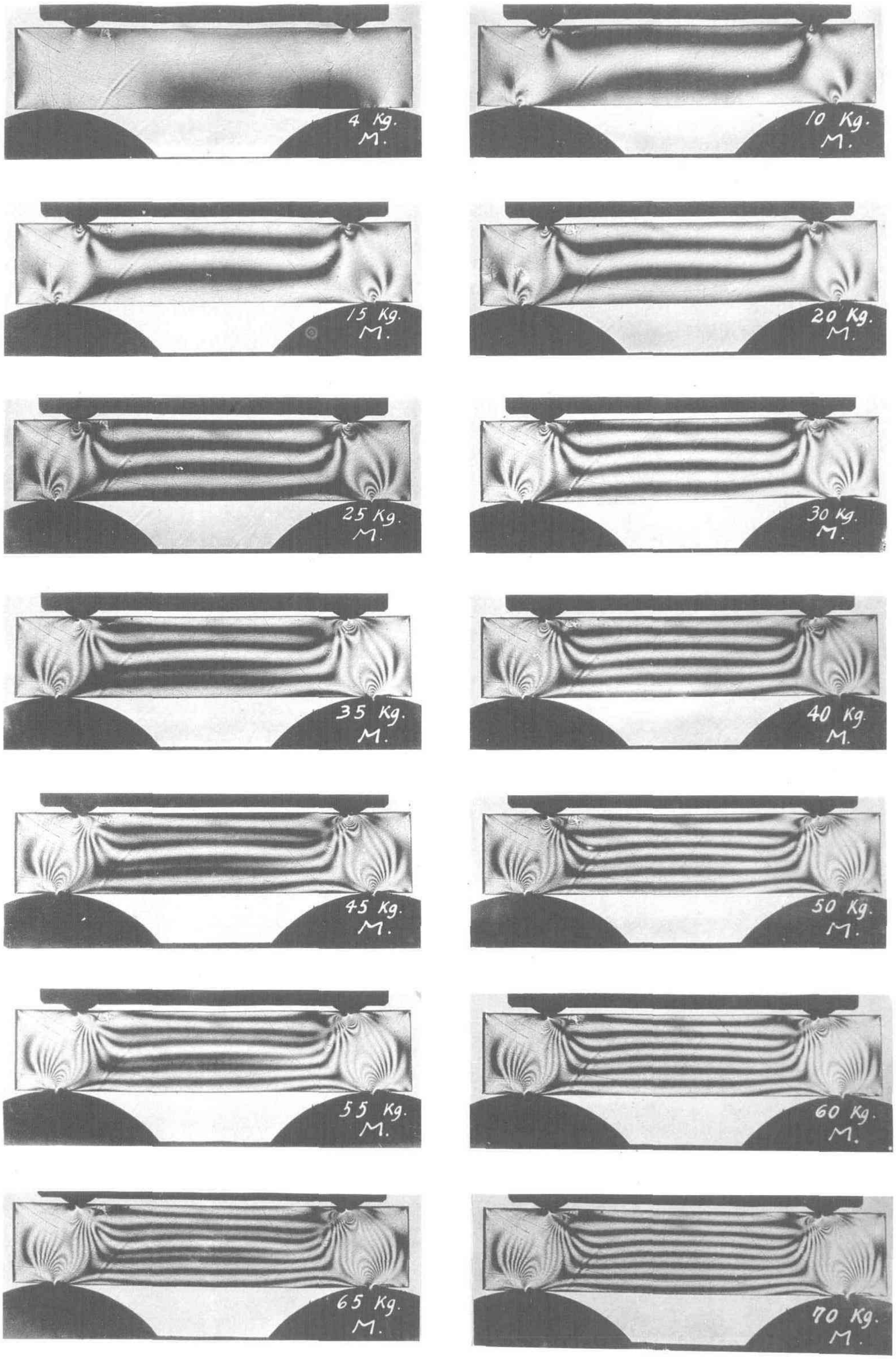
EXPERIMENT 4. (Series M.)

A. Photographs for Calibration.



B. Photographs of Experiment 4.