Conformal Representation and Electric Discharge Figure.

By

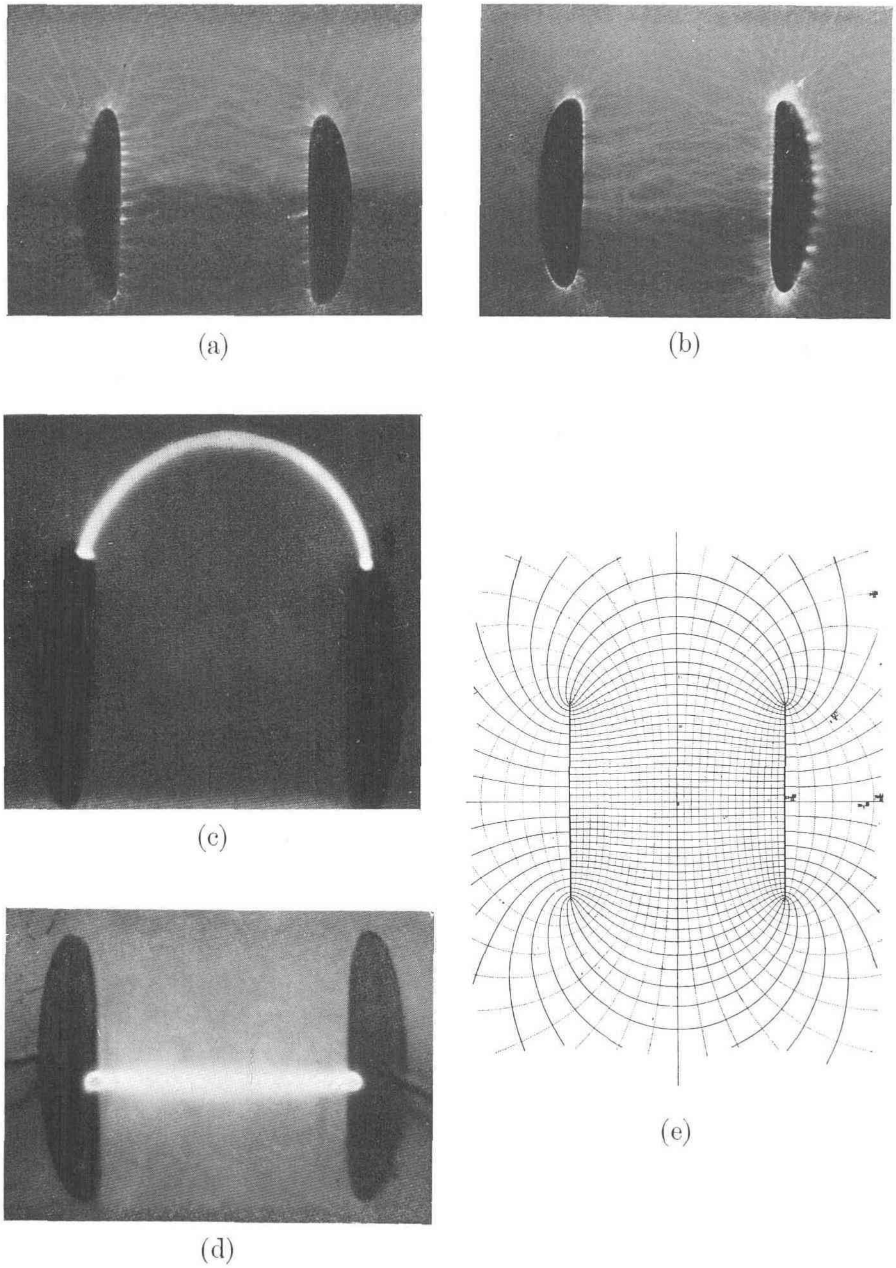
Yosirô Ikeda.

In this article are shown some photographs of electric discharge taken in such favourable cases where each electric discharge figure indicates the direction of the electric field. As a conformal representation with ordinary (mathematically not so complicated) boundary can be obtained easily by our method, we can compare the photograph of the electric discharge with the conformal representation having the same boundary, and can see at once if the electric discharge occurs along the direction of the electric field.

If a glass plate is put on a metal plate connected to earth and if a terminal of high tension is put on a glass plate, then brilliant sparks will appear on the glass plate. Instead of the glass plate a photographic plate is used and high tension is applied to the terminal on the plate for a very short duration of time. Then Lichtenberg figure is obtained. It does not indicate the direction of the electric field generally, but if all the apparatus is put in an evacuated vessel, the Lichtenberg figure in such a low pressure of air indicates nearly the direction of the electric field. Now let two terminals be used whose boundaries coincide with equipotential lines of the conformal representation shown in Fig 1, (e), and let the high tension of alternating current be applied to the terminals. The Lichtenberg figures in such a low pressure of air are shown in Fig. 1, (a) and (b). Next the glass plate is used again instead of a photographic plate and an electric arc is produced on the glass plate by making the pressure of air suitable. The photographs of the arc are shown in Fig. 1, (c) and (d). It can be seen that the arc is formed along the direction of the electric field.

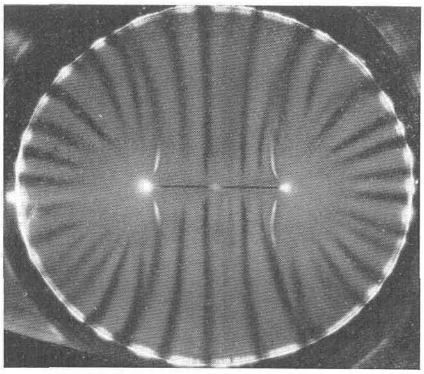
Next a high tension and high frequency current is applied to the terminals by inserting a small series spark gap in the high tension circuit. Then very beautiful streamers appear on the glass plate. The figure of the streamer is most stable, if the pressure of air in the vessel is a few mm. in Hg. This figure has been investigated fully by Dr. T. Itoh.[1]

(1) T. Itoh. Radial Electric Discharge Figure on Dielectric Plate at Low Pressure of Gas. Vol. 1, No. 5 of this memoirs.

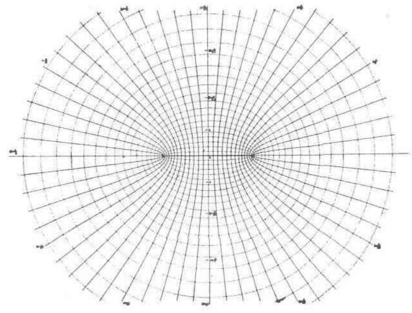


(a) (b) (c) (d) (e)

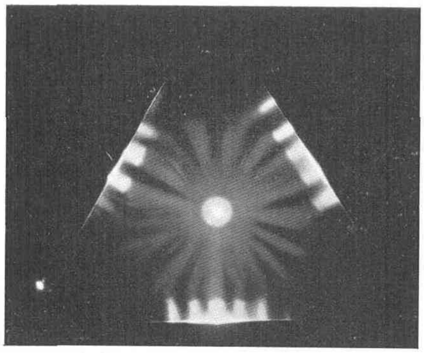
Fig. 1.



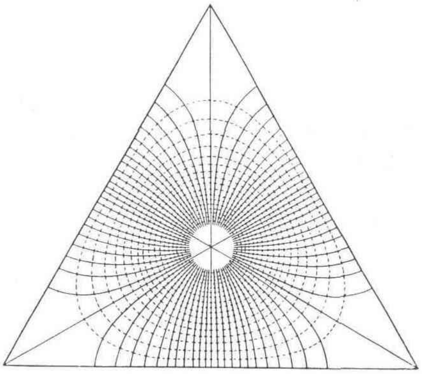
(a)



(a′)

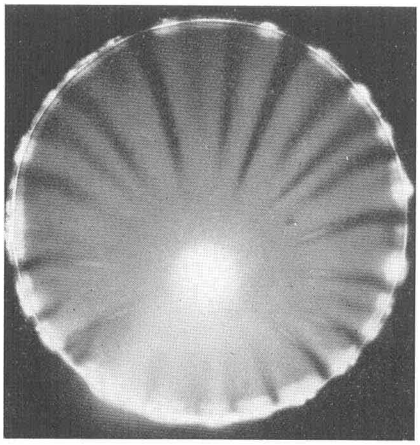


(b)

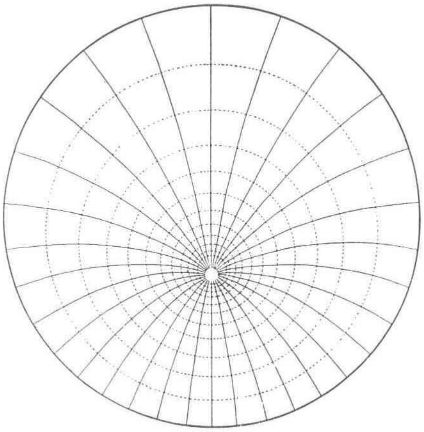


(b′)

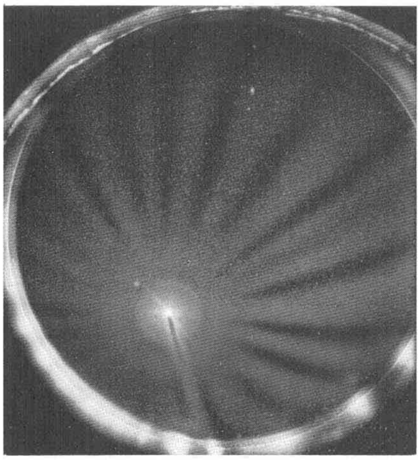
Fig. 2.



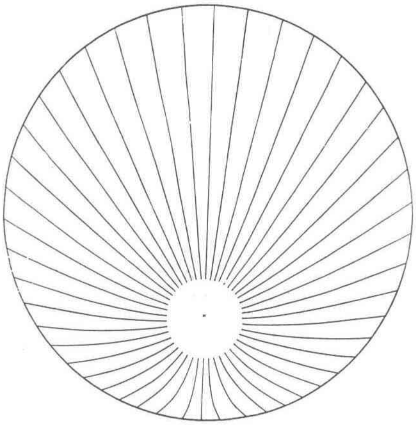
(c)



(c′)



(d)

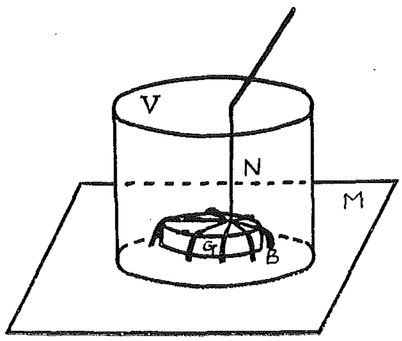


(d′)

Fig. 2.

The reason why the figure appears is not yet perfectly explained, but it is certain that the direction of the streamer coincides with that of the electric field. In order to make the problem two dimensional, the edge of the glass plate is covered with metal connected to earth. The photographs taken in such a case are shown in Fig. 2, (a) (b) and (c), The conformal representations which correspond to the photographs are shown in Fig. 2, (a′) (b′) and (c′). Fig. 2, (a) is the photograph taken when the one terminal is connected to the elliptic boundary of metal and the other terminal to a metal wire laid between the foci of the ellipse. Fig. 2, (b) is the photograph taken when the one terminal is connected to the triangular boundary of metal and the other terminal to a point at its center. Fig. 2, (c) is the photograph taken when the one terminal is connected to the circular boundary of metal and the other terminal to a point eccentrical on the circular glass plate. It can be found at sight that the discharge figures indicate the direction of the electric field.

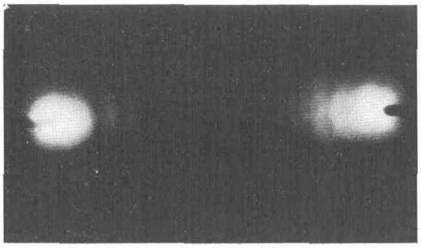
Next let the metal which covers the edge of the glass be taken away and the metal plate under the glass plate be connected to earth. The figure of the streamer changes into another type. The streamer is not perpendicular to the edge of the glass, but perpendicular to the metal plate. Then there appears such a figure as is shown in Fig. 3.



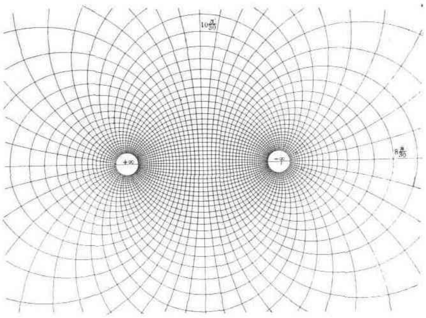
V : Vessel in which the pressure of air is a few mm. in Hg.
N : High tension terminal.
G : Glass plate.
B : Brightest column of the streamer.
M : Metal plate connected to earth.

Fig. 3.

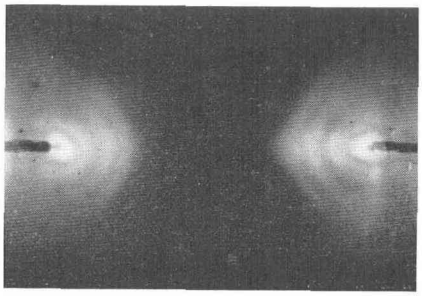
Though the brightest streamer appears along the shortest route between the terminal N and the metal plate M, the streamers exist all over the glass plate. As this phenomenon is three dimensional, the direction of the electric field is not shown by the conformal representation exactly, but it can be shown roughly if one assumes that there exist two poles at N and B. Further by assuming the intensity to be 1 at N and 1/3 at B, the conformal representation is obtained as shown in Fig. 2, (d′). If this is compared with the photograph shown in Fig. 2, (d), the assumption may



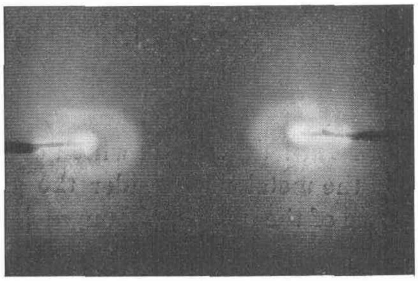
(a)



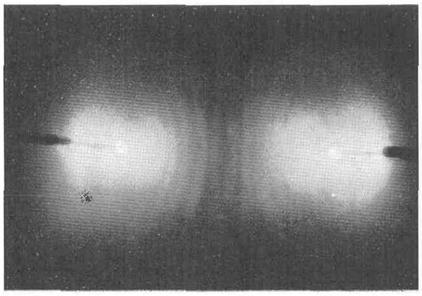
(a′)



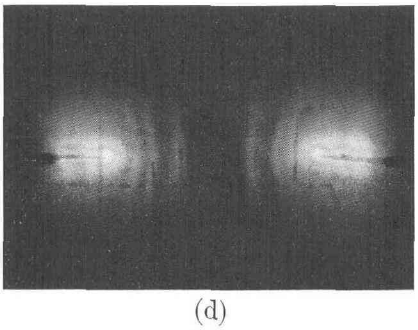
(c)



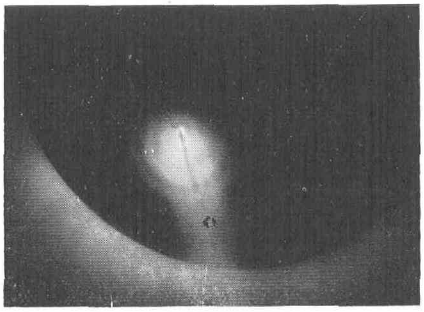
(b)



(e)



(d)



(f)

Fig. 4.

be justfied. Thus it can be concluded that the discharge figures investigated by Dr. T. Itoh indicate the direction of the electric field.

It is very remarkable that the equi-potential lines are obtained with the same apparatus as above if the electric potential and the pressure of air are varied. The photographs taken in such cases are shown in Fig. 4, (a) (b) (c) (d) (e) and (f). The last figure indicates the equi-potential line shown in Fig. 2, (d) and the others indicate nearly the equi-potential lines of the conformal representation by the function $Z = \log\frac{z-a}{z+a}$ as shown in Fig. 4, (a′).

Lastly we will explain how to have the conformal representations. Fig. 1, (e) is the conformal representation given by the function

$$Z = (\sigma^2 k^2 - 1)F(z, k) + E(z, k)$$

where F and E are the first and second kind of elliptic integral and

$$\sigma^2 k^2 = \frac{E'}{K'}\ , \quad E' = \int_0^1 \sqrt{\frac{1-k'^2x^2}{1-x^2}}\,dx\ , \quad K' = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-k'^2x^2)}}$$

Fig. 2, (a) is the conformal representation given by the function

$$Z = \cos^{-1} z$$

Fig. 2, (b) is the conformal representation directly composed from the one given by the function

$$Z = f(u)$$

where

$$Z = \int \frac{dz}{\sqrt[3]{(1-z)^2 z^2}}\ , \quad u = \log z$$

Fig. 2, (c′) is a part of the conformal representation by the function

$$Z = \log\frac{z-a}{z+a}$$

which is shown in Fig. 4, (a′)

These representations have been published in the "Scientific Papers of the Institute of Physical and Chemical Research. No. 561."

Fig. 2, (d′) is the conformal representation given by the function

$$Z = \log z + \frac{3}{4}\log(z-a)$$

The conformal representations by means of the functions

$$Z_1 = \log z \qquad Z_2 = \frac{3}{4}\log(z-a)$$

are given by the concentric circles and radii whose centers are at $z=0$ and $z=a$. According to method II in the preceding article we can obtain Fig. 2. (d′).