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Title	Parallel stream disturbed by barriers and gate
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Citation	Memoirs of the Faculty of Engineering, Hokkaido Imperial University, 4, 23-27
Issue Date	1938
Doc URL	http://hdl.handle.net/2115/37718
Type	bulletin (article)
File Information	4_23-28.pdf

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Parallel Stream Disturbed by Barriers and Gate.

By

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In this article are drawn equi-potential and stream lines in the several cases where the parallel streams are disturbed by one, two or three barriers perpendicular to the stream and by a gate opened at an angle of 45°. Though it is evident that such an ideal stream can not exist in reality, it is not worthless to have the representation of the exact potential stream.

For the first example, it is proposed to draw the streamlines in the domain, which has the following boundary.

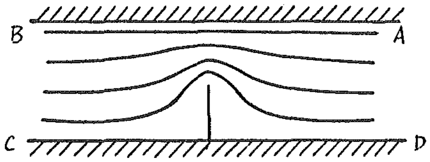
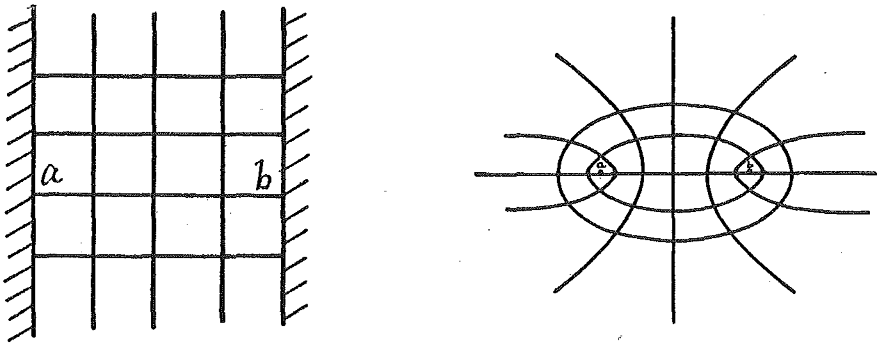


Fig. 1.

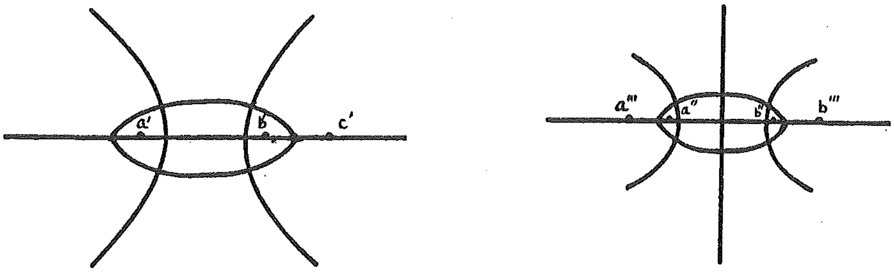
As has been seen in the preceding article, we have the correspondence between the two figures shown in Fig. 2 (a) and (b).



(a)

(b)

Fig. 2.



(a) (b)

Fig. 3.

Now one draws similar figures to those in Fig. 2 (b) a little smaller in size as shown in Fig. 3 (a) and (b). Let Fig. 3 (a) be put on Fi. 2 (b), so that c′ overlaps on b and a′ on a. By method II the curves in Fig. 3 (a) are transformed conformally in the domain shown in Fig. 2 (a), then the streamlines in Fig. 1 are obtained. If the curves in Fig. 1 thus obtained be drawn, symmetrically to *AB*, Pl. XI is obtained.

Again let Fig. 3 (b) be put on Fig. 2 (b), so that a′′′ overlaps on a and b′′′ on b then in the same way as above Pl. XII is obtained.

Lastly we draw the stream and equi-potential lines in the domain with the following boundary.

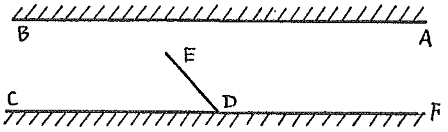


Fig. 4.

Now consider the integral

$$Z = -\int \frac{a-z}{z\sqrt[4]{(1-z)^3(c-z)}}\,dz \qquad 0<1<a<c$$

$$= -\int \left(\frac{a}{z}+\frac{a-1}{1-z}\right)\sqrt[4]{\frac{1-z}{c-z}}\,dz\ .$$

By changing the variable z into t, so that

$$t^4 = \frac{1-z}{c-z}\ ,$$

it follows

$$Z=4a\int^{\sqrt[4]{\frac{1-z}{c-z}}}\frac{dt}{1-ct^4}-4\int^{\sqrt[4]{\frac{1-z}{c-z}}}\frac{dt}{1-t^4}\ ,$$

or

$$Z=\frac{4a}{\sqrt[4]{c}}\int^{\sqrt[4]{\frac{c-cz}{c-z}}}\frac{dt}{1-t^4}-4\int^{\sqrt[4]{\frac{1-z}{c-z}}}\frac{dt}{1-t^4}\ .$$

As the integral becomes

$$\int\frac{dt}{1-t^4}=\frac{1}{4}\left\{\cosh^{-1}\frac{1+t^2}{1-t^2}+\cos^{-1}\frac{1-t^2}{1+t^2}\right\}\ ,$$

it follows

$$Z=\cos^{-1}\frac{(\sqrt{c-z}-\sqrt{1-z})^2}{c-1}+\cosh^{-1}\frac{(\sqrt{c-z}+\sqrt{1-z})^2}{c-1}$$

$$-\frac{a}{\sqrt[4]{c}}\left\{\cos^{-1}\frac{(\sqrt{c-z}-\sqrt{c-cz})^2}{(c-1)z}+\cosh^{-1}\frac{(\sqrt{c-z}+\sqrt{c-cz})^2}{(c-1)z}\right\}.$$

If it be put that

$$\zeta_1=\frac{(\sqrt{c-z}-\sqrt{1-z})^2}{c-1}\ ,\quad \zeta_2=\frac{(\sqrt{c-z}-\sqrt{c-cz})^2}{(c-1)z}\ ,$$

$$Z_1=\cos^{-1}\zeta_1+\cosh^{-1}\frac{1}{\zeta_1}\ ,\quad Z_2=\cos^{-1}\zeta_2+\cosh^{-1}\frac{1}{\zeta_2}\ ,$$

then

$$Z=Z_1-\frac{a}{\sqrt[4]{c}}Z_2\ .$$

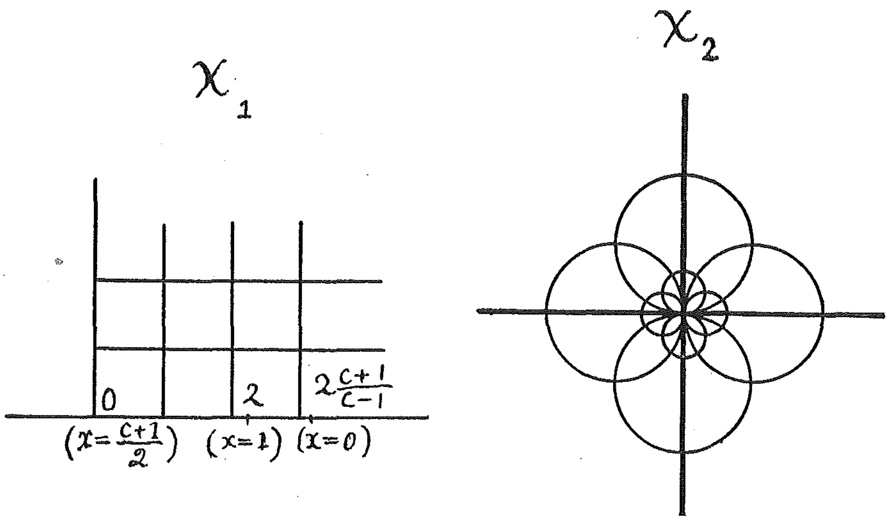
This is the sum of the representations shown in the preceding article. Now put

$$\zeta_1 + \frac{1}{\zeta_1} = \frac{2(c+1)}{c-1} - \frac{4}{c-1} z = \chi_1 \ ,$$

$$\zeta_2 + \frac{1}{\zeta_2} = \frac{4c}{c-1} \frac{1}{z} = \frac{2(c+1)}{c-1} = \chi_2 \ .$$

Thus the loci of $\Re(\chi_1) =$ const. and $\Im(\chi_1) =$ const. in Z_1-plane and that of $\Re(\chi_2) =$ const. and $\Im(\chi_2) =$ const. in Z_2-plane are represented by the similar curves shown in Pl. VIII in the preceding article.

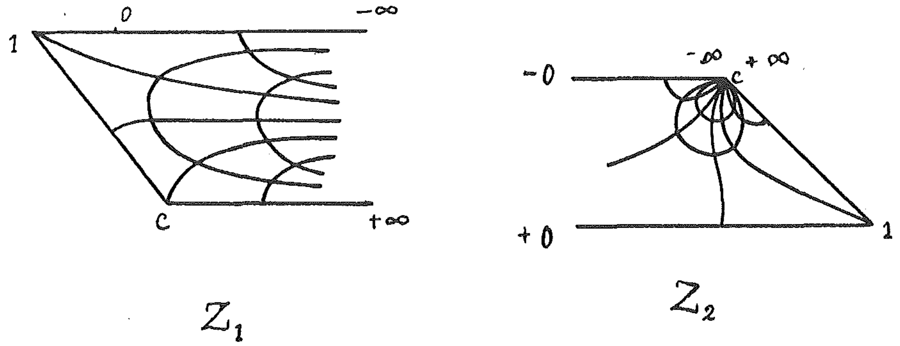
The correspondences between χ_1 and z, and between χ_2 and z are shown in Fig. 5 (a) and (b);



(a) (b)

Fig. 5.

By superimposing these figures on Fig. 10 of the preceding article and by applying method II, one can transform these curves of Fig. 5 (a) and (b) conformally in the same domain with Fig. 11 (c) of the preceding article. Thus is obtained Fig. 6 (a) and (b) respectively.



(a) (b)

Fig. 6.

In order to have the representation in the domain shown in Fig. 3, one must put

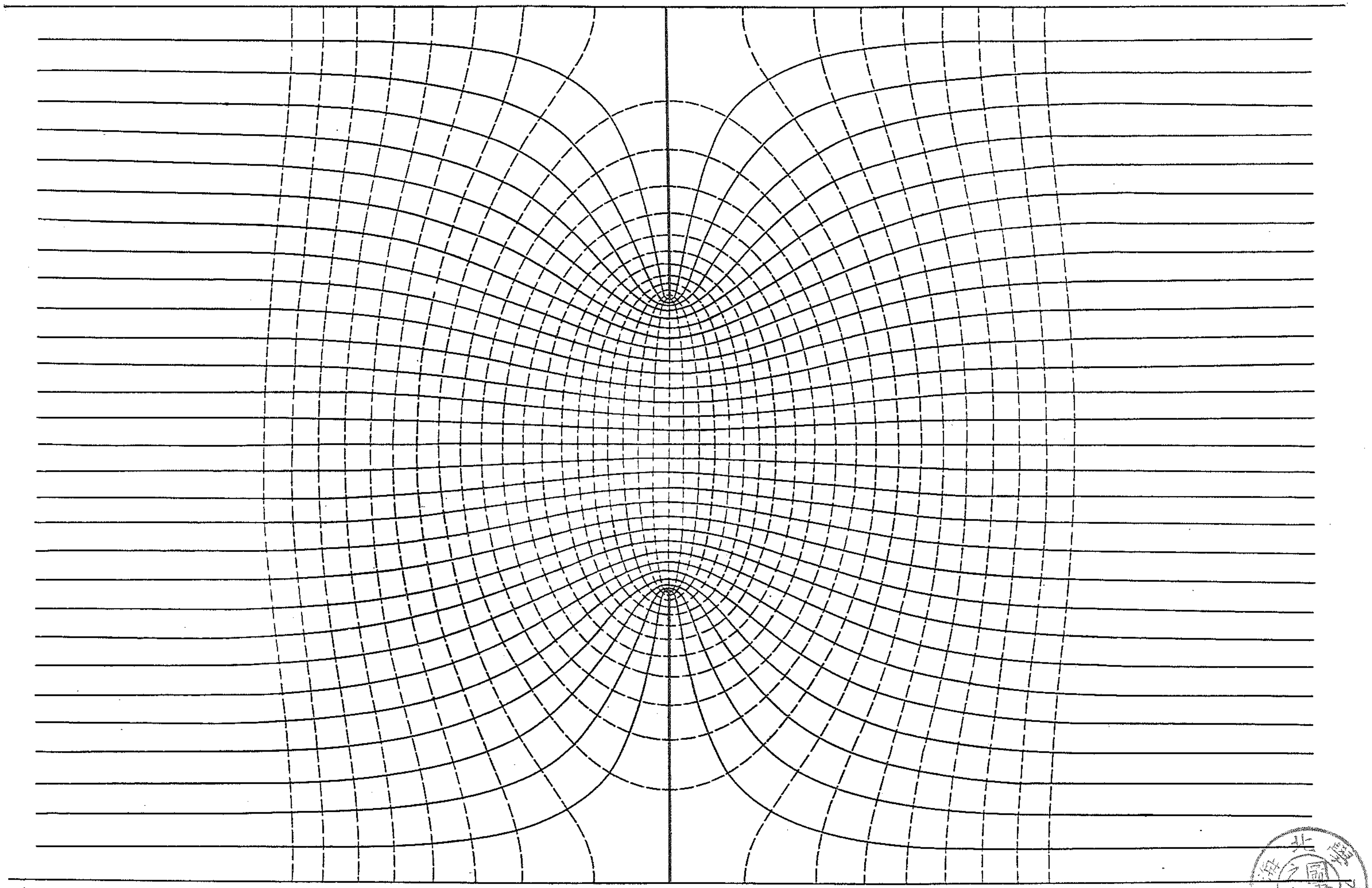
$$\frac{a}{\sqrt[4]{c}} = 1$$

for the distance between the points which correspond to $z = +\infty$ and $z = -\infty$ must be equal to that between the points which correspond to $z = +0$ and $z = -0$. Moreover it is assumed that the length DE is equal to the distance between the parallel boundary, for the boundary line DE is considered as a gate. Thus it follows

$$a = 6.5 \qquad c = a^4 = 1784.96$$

If these numerical values be used, the points which correspond to $z = c$ and $z = \pm\infty$ approach each other and the calculation becomes very easy. Fig 6. (a) and (b) are exactly drawn in Pl. VIII and XIII.

If measurement is made of the values of Z_1 and Z_2 which correspond to the same value of z from Fig. 6 (a) and (b) and the value $Z_1 - Z_2$ in the domain shown in Fig. 4 is plotted, then according to method IV the conformal representation shown in Pl. XIV can be drawn. In order to have the streamlines which begin from the point $z = 0$ and ends in $z = \infty$, one puts again $u = \log z$ and transforms the loci of real and imaginary parts of $u =$ constant conformally in the domain given in Pl. XIV by method III. Thus Pl. XV is obtained. If the curves of Pl. XV be drawn symmetrically to the boundary line AB in Fig. 3, Pl. XVI is obtained.



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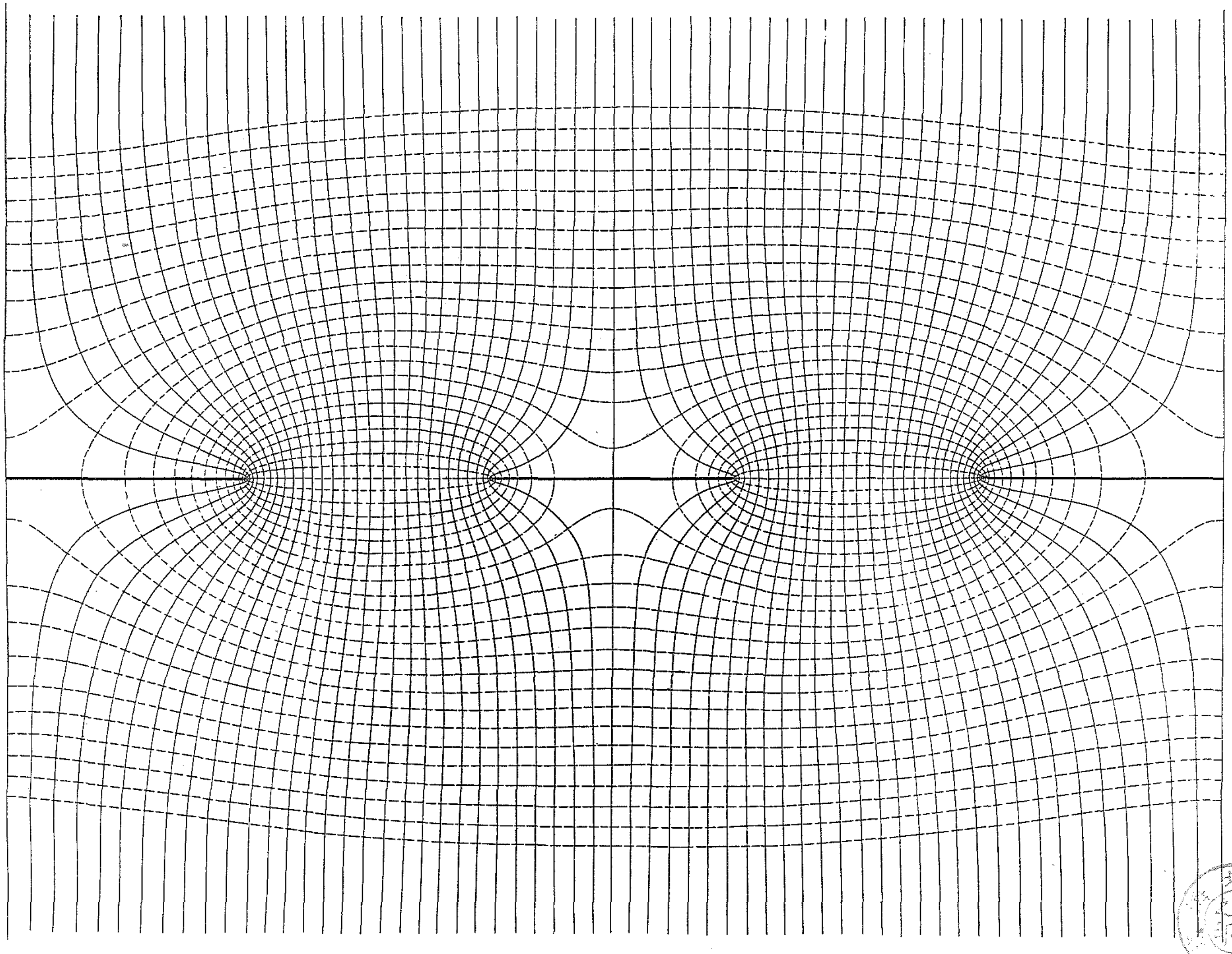

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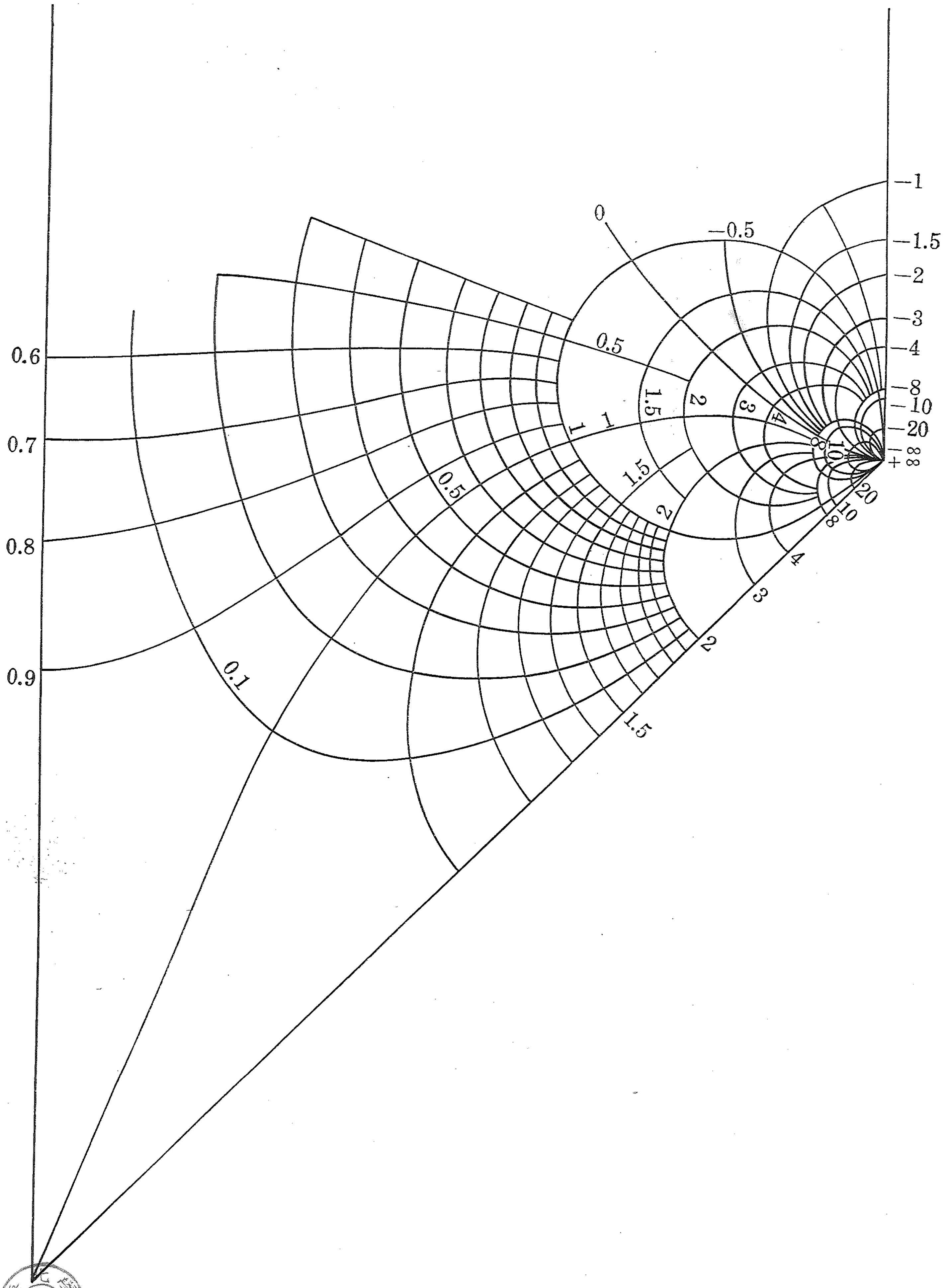

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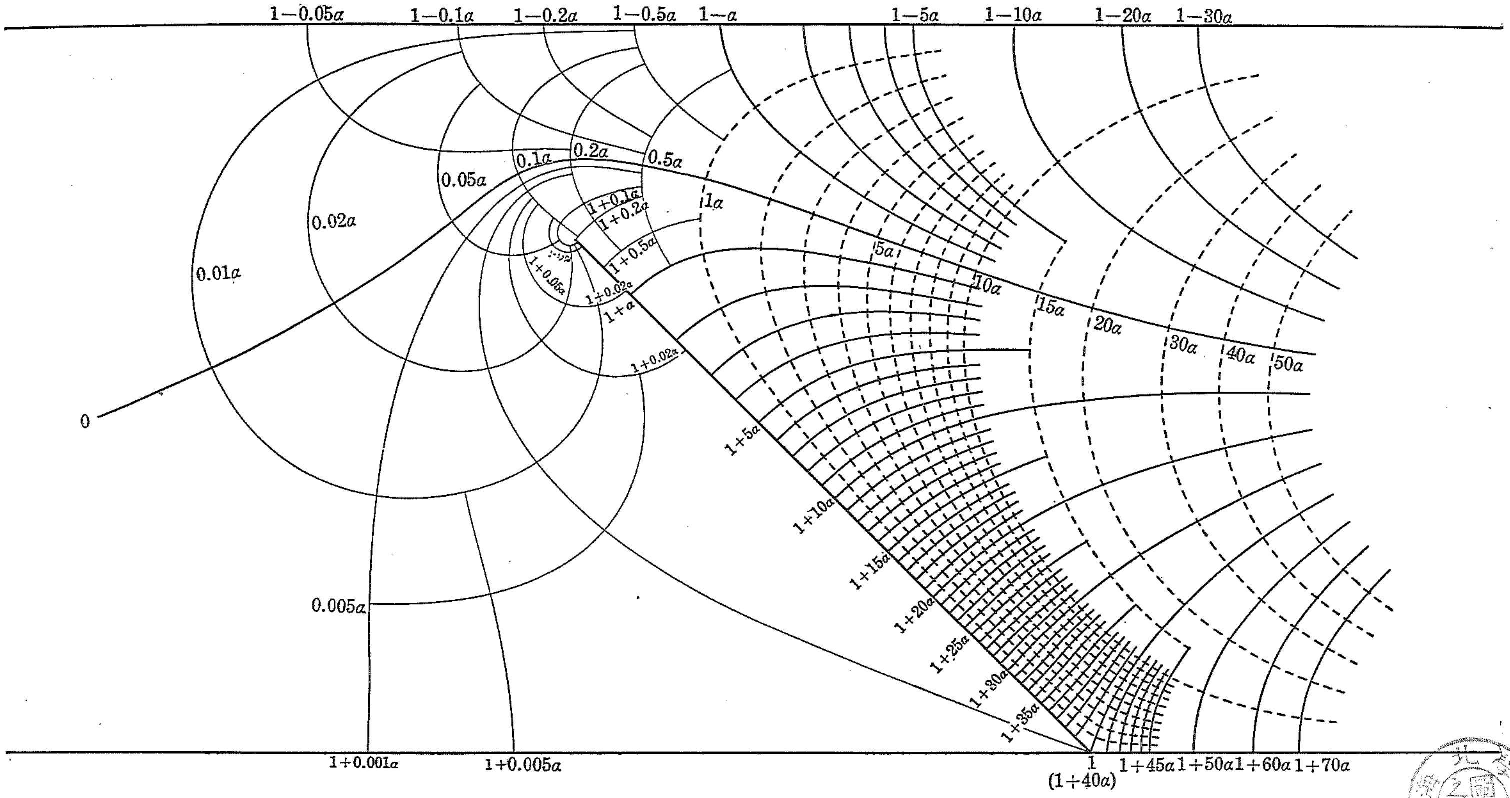

α = 44.6

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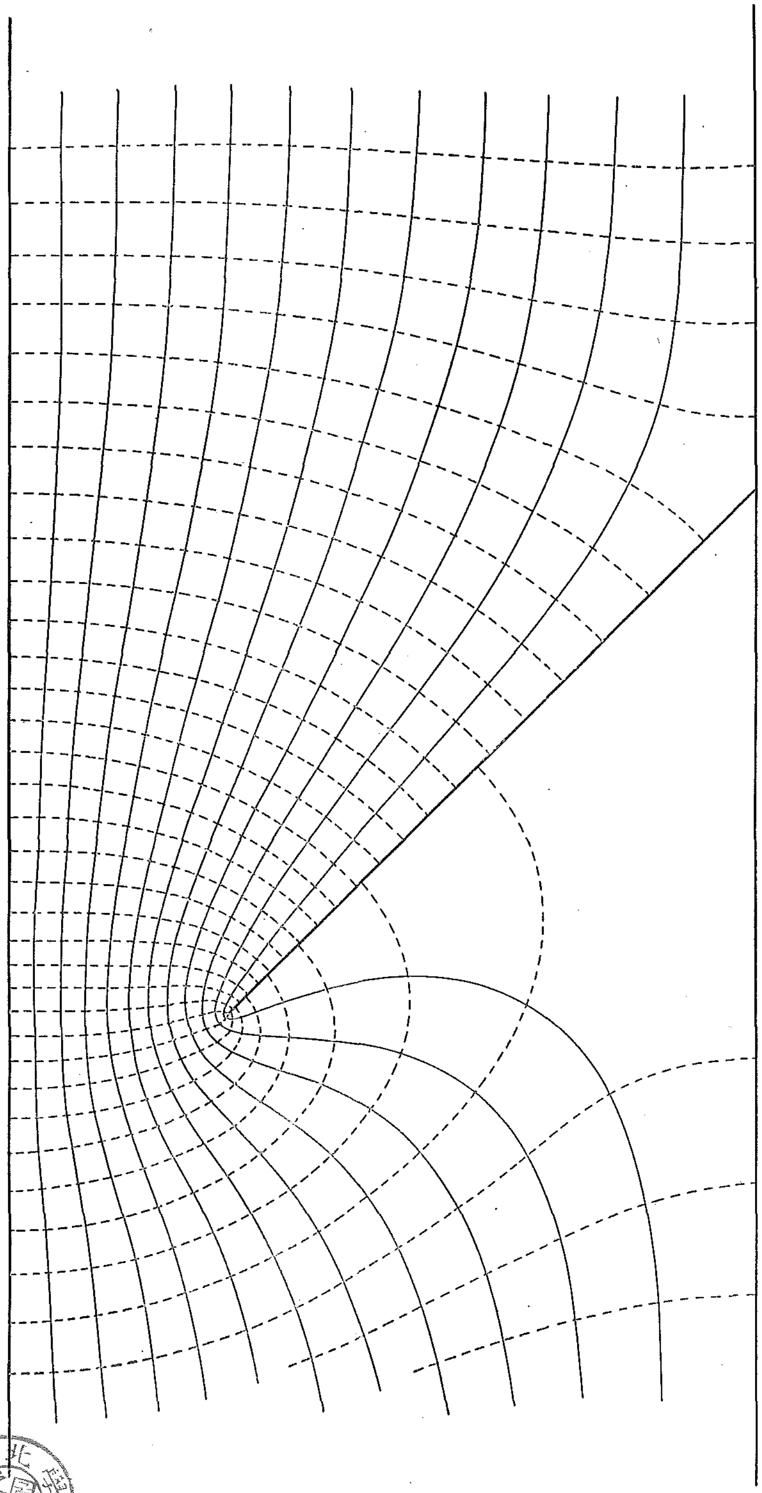

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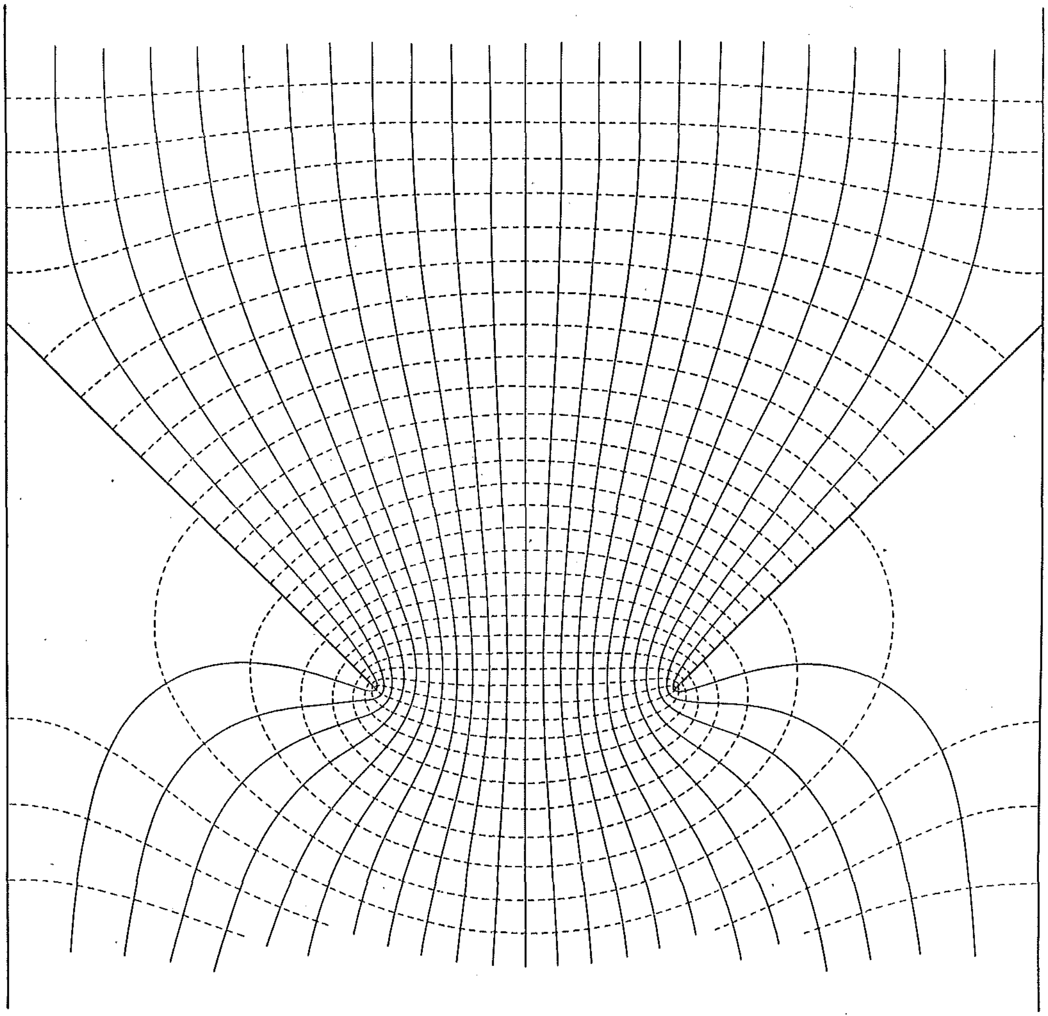

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