On Wave Propagation on the Surface of a Sand Mass.*

By

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Introduction.

Determinations of the propagation velocity of a shock and periodic vibration through a granular or powder mass were carried out by the present author several years ago.[1] The measurement of the velocity of propagation of the disturbance produced on the sand surface by mechanical shock at the beach of Iwanai and Isikari in Hokkaido are also precisely shown in this volume by Prof. Y. IKEDA[2] and Mr. M. ARAMATA.

According to their results, the velocity of the propagation of a shock wave has the same value in either case when the disturbance is propagated on the surface of or through a sand mass. This definite value of the velocity is estimated as about 150 m/sec. Strictly inspecting the osillograms taken by them, one can easily find several greater or smaller ones other than this definite value. Though some of them are only in the order of a trace, it is sufficient to determine the magnitude of the velocity. In the preliminary experiment, the author also sometimes has encountered such a difficulty that the determination of the velocity is not exactly decided in consequence of the distorsion of the wave form at great distances from the source. This fact is simply explained by the assumption that the propagation velocity of each harmonics contained in the original wave at the source depends on their wave length respectively. To ascertain experimentally this assumption of the dispersive phenomena, the author has commenced this study.

Method of Measurement.

As the source of the disturbance through the sand there were used, a buzzer, a tuning fork or a vibrator driven by a magnetic coil

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fed by alternating current with the various cycles. As the receiver of the disturbance, a carbon microphone was used such as is used as the transmitter in an ordinary telephone.

Two microphones were half imbedded perpendicularly on the surface of the sand mass at different distances from the source of the vibration and the wave is propagated from the source. The electromagnetic oscillograph is used for recording or observing the vibration thus produced in the sand.

The disturbance arriving on the microphone through the sand causes change in the current of the microphone applied as a constant potential. And these changes in the microphone current were directly observed on the rotating mirror by the naked eye or recorded on the rotating film.

This oscillograph with three string elements can record the arrival of the wave at two microphones. However, sometimes when the amplitude of the receiving wave is small, a high sensitive amplifier is inserted between the microphone and the oscillograph.

From the difference of the arrival of the wave at two points and also the difference of the paths between the source and each microphone, the propagation velocity can be easily determined. Seeing the images on the rotating mirror of the oscillograph, one can easily adjust the difference of the distances between the source and each microphones as to be just n or $n+\frac{1}{2}$ times the wave length of the source. Finding the distance to correspond to n or $n+\frac{1}{2}$ times of the wave length, the propagation velocity is also decided by this method.

It is important in the present experiment to avoid the reflection of the wave from the bottom of the sand mass. For this purpose, a rug of cotton wool is spread at the bottom of the sand mass. By these arrangements, the reflection of the wave at the bottom seems to be practically excluded.

The sand at Iwanai is chosen as the suitable one for studying the propagation of the periodic disturbance which, in case of need, could be compared with the one obtained there by the shock disturbance.

The Detection of the Surface Wave.

Whether the wave now under consideration is really a surface wave or not, will be easily ascertained by the following method.

(a) At the instant when the pressure is applied on the surface of the sand mass as shown in the position a in Fig. 1, the amplitude of the oscillating current through the microphone is considerably diminished and sometimes decreased to a value near zero.

However, the current is no sooner brought back to its original value than the pressure is constantly applied or perfectly removed.

However, when the portion where the pressure is applied or the groove is cut, in the position shown by b of Fig. 1, the microphone current decreases a little, while in the position shown by c of the same figure, the microphone current will not be at all decreased.

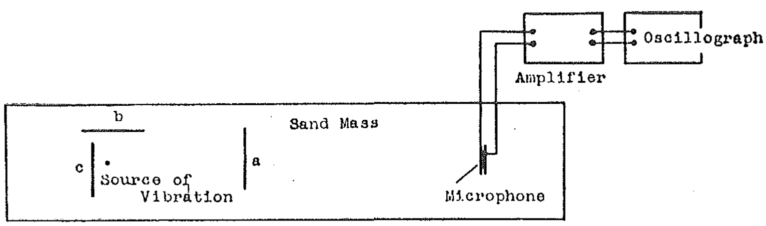


Fig. 1.

(b) At the instant when the thin and long groove was cut in the portion a in Fig. 1, perpendicular to the line connecting the source and receiving point on the surface, the amplitude is also considerably decreased.

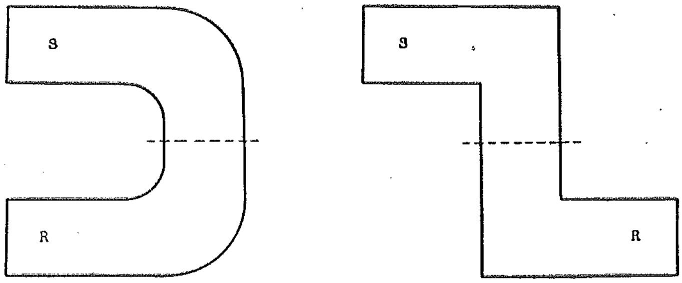
(c) At first one can easily detect the oscillation by the microphone at a small distance from the source of the vibration. However, increasing the distance between the source and the receiving point step by step, one can observe that at first the oscillation at the receiving position does not appear at once but gradually appears with the lapse of time.

Setting the vibrator once more in oscillation, after the oscillation is once cut off, the microphone current will soon appear again.

(d) Even when the shape of the boundary of the sand mass is Z, L or U from with uniform depth as shown in Fig. 2, one can easily detect at point R the periodic disturbance propagated from point S. This fact will be considered as the diffraction phenomena of this wave.

Also at the instant when a thin groove is cut or pressure is applied at the portion a as shown in Fig. 2, on the sand surface the microphone current will be diminished to a rather small value.

(e) When the depth of the sand mass is not approximately uniform, the experiments show that the disturbance does not propagate easily.



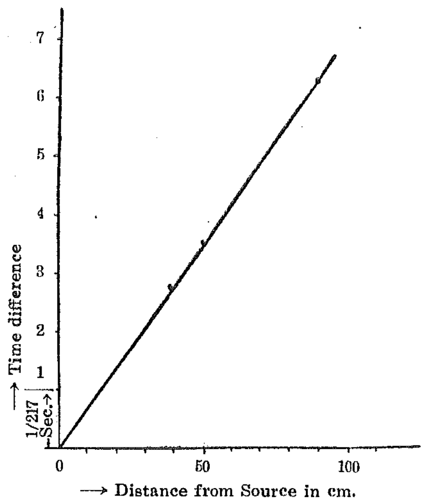
S: Source
R: Receiving point

Fig. 2.

(f) When the thickness of the sand mass is greater than 10 cm., a solitary wave always appears in about one or two seconds after the vibration is cut off.

The Velocity Determination.

1. The dispersion of the wave.



Distance-Time Curve.

Fig. 3.

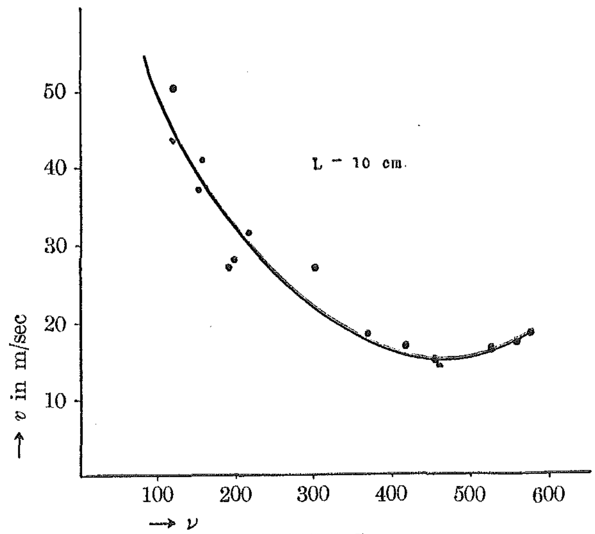
By changing the frequency of the source, the propagation velocity is consequently altered.

When the frequency of the buzzer is 217 cycles/sec. the arrival differences of the waves are indicated as 2.8, 3.5 and 6.2 times $\frac{1}{217}$ sec. for the distances of 38, 50 and 90 cm. respectively from the source. These data give the distance-time curve as shown in Fig. 3 which is the straight line through the origin. From this curve the velocity for 217 cycles/sec. will be determined as 31.5 m./sec.. These results are summarized in Table 1.

Table 1.

Frequency of Source ν per Sec.	Measured Wave length λ in cm.	Velocity V in m/sec.
120	36.0	43.2
120	42.0	50.4
150	24.7	37.0
157	26.2	41.1
192	14.0	26.9
196	14.4	28.2
217	14.5	31.5
300	9.0	27.0
365	5.05	18.4
414	4.05	16.8
450	3.28	15.0
523	3.24	16.9
554	3.20	17.7
575	3.17	18.3

When the velocity is plotted against the frequency, as can be seen from the curve in Fig. 4, the velocity takes its minimum value when the frequency is estimated as about 500 cycles. Below this frequency,



Dispersion Curve.

Fig. 4.

the velocity decreases, with increasing frequency, and above this minimum value, the velocity increases with the frequency.

2. The variation of the velocity with the thickness of the sand mass.

When the thickness of the sand mass is variable but the frequency of the source is kept constant, the velocity is increased with the thickness of the sand mass. The results are tabulated in Table 2 and shown in the curve of Fig. 5.

Table 2. Effect of thickness of sand mass.

$\nu = 120$

Thickness of the Sand Mass L in cm.	Measured Wave length λ in cm.	Velocity V in m/sec.
3.5	21.5	25.8
7.0	34.3	41.16
10.0	39.8	47.76

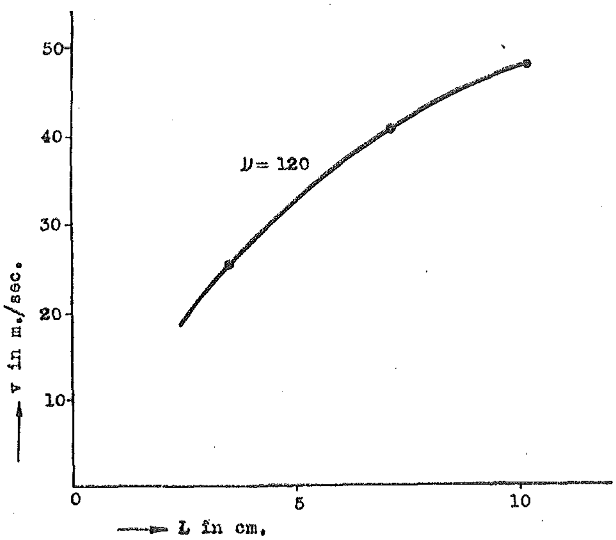


Fig. 5.

It is observed that the relation between H and $\sqrt{L}$ is indicated as one straight line as shown in Fig. 6 within the range of measurement.

3. The effect on the packing.

It is also shown that the propagation velocity in the case of the rough packing is less than that in the case of ordinary one. The results are tabulated in Table 3.

Table 3.
The Effect of the Packing
$\nu = 120$
L = 4 cm.

Packing	Velocity V in m/sec.
dense	18.5
loose	13.2

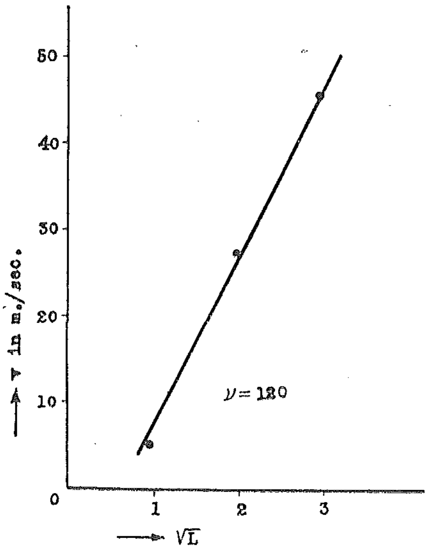


Fig. 6.

Existence of a Stationary Wave.

Using a buzzer of 450 cycles per second as the source of the vibration. Measurement has been made of the intensity of the vibration by the microphone at various distances from the source. The amplitude of the vibration at these points can be indicated by the magnitude of the image on the rotating mirror attached to the oscillograph. The amplitudes thus obtained are plotted against the distance from the source. One of them is shown in Fig. 7.

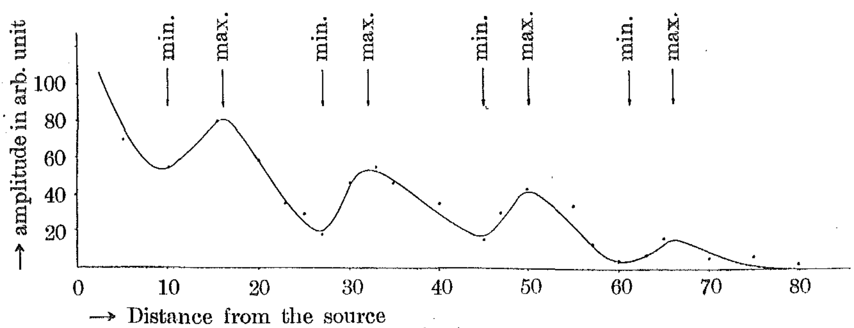


Fig. 7.

The maximum and minimum of the vibration amplitude are distinctly shown on the surface of the sand mass. These maxima and the minima seem to correspond to the loops and nodes of the stationary wave produced on the surface of the sand mass. In this case, it can not be found that each node is at the middle point of the distance between two successive loops.

The cause of this deviation from the centre is simply explained by the assumption of the attenuation of the amplitude which depends on the distance from the source of the vibration.

The amplitudes at each loop and node are plotted against the distance and the respective results are shown in the curves of Fig. 8.

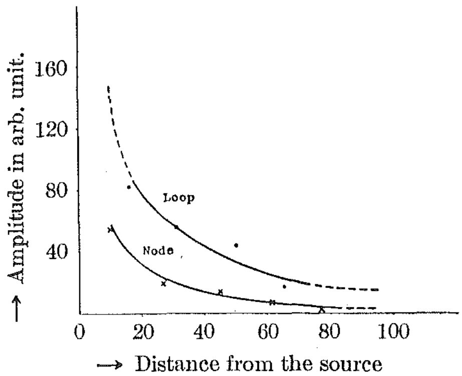


Fig. 8.

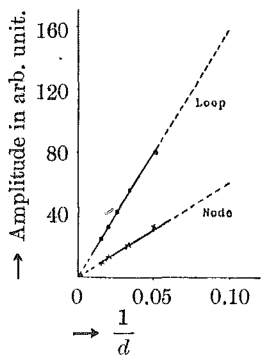


Fig. 9.

If the amplitude at each loop and node be taken against the inverse of their distance from the source, the relations between them are shown as two straight lines through the origin of Fig. 9. From this result, the amplitude of the vibration is attenuated so as to be inversely proportional to the distance from the source of the vibration. In other words, it signifies that the oscillation is propagated as a plane wave.

The proof as the plane wave is exactly that which is expected if the wave produced in the sand mass is propagated as a surface one.

Conclusion.

When the position of the source of the vibration is near the surface of the sand mass, it is shown by various methods that the wave produced in the sand mass is chiefly propagated as a surface one. The determination of the velocity for the various cycles gave the dispersion curve. The experiments also show that the velocity is greatly affected by the thickness and packing of the sand mass. These data may be useful when the statics or dynamics of the granular mass are theoretically treated.

In conclusion, the writer wishes to express his cordial thanks to Professor Y. IKEDA for his kind guidance and also to the late Professor T. TERADA who has offered many valuable suggestions.

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Reference.

(1) K. YONETA, Kwagaku (in Japanese), Vol. 4, No. 8, p. 329 (1934).

(2) Y. IKEDA and M. ARAMATA, On the Propagation of Shock through Sand on the Beaches of Iwanai and Isikari. Mem. Fac. Eng., Hokkaido Imp. Univ., Vol. 4, No. 3, p. 275 (1938).