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# Experimental Investigations on the Weakest Point in the Resistance of Tall Building Frames Against Earthquake.

By

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In this paper the authors describe from both experimental and theoretical standpoints, which story shall be most severely damaged in a tall steel skeleton or reinforced concrete building frame under the action of seismic disturbances, especially by the horizontal movement of the foundation ground.

The authors divided the state of the damage by earthquake into two classes as follows:

- (I) Damage of the first order,
- (II) Damage of the second order.

Damage of the first order is taken to mean such that structures do not suffer from injury to the main frames but the cracks appear merely in the concrete walls. This failure of walls in the damage of the first order tends to occur in the story where column-deflection has the maximum value and this story belongs most probably to the second or third layer in the ordinary building frames.

Damage of the second order is such a case as that when not only are the walls shaken down but columns and girders are also destroyed. In such a severe damage, failures in the main frames seem to occur, in the authors' opinion and from the results of experiments, at the points where the fibre stresses and shearing stresses are at their maximums as obtained by ordinary statical calculation, and even if the free vibration period of a building frame is smaller than that of the earthquake, damage seems to extend to girders in the upper stories as well as to the fixed ends of the lowest columns, depending upon the stiffness ratio of girders and columns.

## Introduction.

In the tall building frames dealt with in this paper there are treated no wooden framed constructions; the study is limited to a rigid frame such as a steel skeleton or a reinforced concrete structure.

Earthquake proof structures are usually designed under the current assumptions that a seismic force acts uniformly at each joint as statical forces for the sake of convenience for calculations.

From the standpoint of strict theory, it will not yield a perfect calculation to solve the seismic problem as a statical one.

However, the results of the present experiments on the vibration damage to a high storied rigid frame like a concrete structure give the idea that the statical treatment of the seismic problem is important for the practical design of earthquake proof structures, because it determines the problems which even the dynamical solution may not solve.

For such frame models of relatively rigid materials as used in the present experiments, the dynamical explanation ordinarily indicates the weakest point against seismic disturbances to be at the lowest story of the structure, while the experimental investigation has shown other characteristics which have rather good coincidence with statical calculation results showing the weakest point on the several stories.

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Also, to Mr. Hideo Matusaka and Suekiti Ono, assistants in the Institute of Structural Engineering of the Hokkaido Imperial University, acknowledgement is due for assistance in the tests of this study.

In general, the seismic damages in tall building frames are very complicated. They are subject to the varied nature of earthquake motion, the kind of structures, load distributions on the structures and the conditions of foundation ground.

On the location of the seismic damage to tall building frames, various discussions have been published by many authorities, such as Dr. F. Omori, Dr. R. Sano, Dr. T. Naito, Dr. Mononobe, A. Mizuhara, Dr. Taniguchi etc.

Among them, Dr. Taniguchi proposed his theory from a point of view quite distinct from the other quoted theories. By the investi-

gations on damages to buildings in the Great Kwanto Earthquake in 1923, he found that even buildings of a mere three stories, whose free vibration periods were so small as 0.2 sec. were destroyed at the second story. When such rigid reinforced concrete buildings were shaken at the period of 1.35 sec. which was the period of principal motion of the Great Kwanto Earthquake the greatest bending moment or shearing force in columns should be expected at the lowest story, according to almost all previous theories. In former days all suggestions in the discussions of vibration of tall buildings assumed that the deformation curves of buildings subjected to lateral forces are similar to the elastic curves of the cantilever or uniform string. But Dr. Taniguchi pointed out that the deformation curves of buildings are peculiar and quite different from those of the cantilever or string. After some theoretical and experimental investigations of the deflection curves of tall building frames he concluded that when a tall building is subjected to earthquake motion, the maximum slope in the deflection curve will most probably occur at the second or at the third story and at these stories the shearing forces in the walls attain their maximum and when the walls are laid of hollow brick, or terracotta they are extensively cracked in "X" form and shaken down, and columns are sometimes damageable at this story.

From the consideration of the results of the present writers' many breaking tests of models and the statical calculations of building frames, the following conclusions may be reached:

In the present opinion of the authors, seismic damages might be classified into two sorts: damage of the first order and that of the second order. By damage of the first order is meant such as the cracking of building walls while the structures do not suffer from injury of the main frame of columns and girders. By damage of the second order is meant such cases as when not only the walls are shaken down but columns and girders are broken.

The above classification of damages was made for convenience in mechanical treatment. It is probable that in actual cases the damages are often in an intermediate mode between the above two classes.

In the future when the study of mechanics has made more progress and the rigorous theoretical solution of building frames with wall taken into consideration has been found, the above classification should be revised.

These two sorts of seismic damages to buildings as defined by the present writers will be discussed in the following sections.

### I. Investigation on the Location of Failure due to Damage of the First Order.

One may be able to determine the correct deformation and correct stresses of model frames only if remarkable progress of rigorous vibration theory is made. However, it will be very difficult and laborious to apply that theory to the actual building frames under the complicated boundary conditions.

In the design of buildings of to-day, one usually treats seismic force as a statical one for the sake of simplicity and assumes that seismic force is concentrated horizontally at every joint, having a magnitude equal to the product of the total mass on each floor and acceleration.

For any given load acting at any joint, the chief elements of deformation of the building frames are the joint-rotation angles and member-revolution angles which are simply called "slope" and "deflection" respectively in current use. These two are derived very easily in the present stage of progress of the statical solution of high building frames. The authors believe that the senior one is one of the men to whom the progress in the statical solution of high building frames is under a heavy debt.

The amounts of slope and deflection vary complicatedly with the rigidity and length of columns and girders, the number of stories and bays, the position and distribution of loads, the distribution of seismic force which depends upon the vibration mode, consequently upon the free vibration period of the structure and forced vibration period of the earthquake, and many other boundary conditions. It is possible also to design imaginary building frames with slopes and deflections of arbitrary values for the given horizontal loads as shown in Figs. 1 to 5.

For ordinary building frames and load distributions the maximum deflection will most probably occur between the second and fourth stories and not at the lowest story under the conditions of the columns fixed at the bases. Also the joint-rotation angle, i.e., the slope, generally takes its maximum value in the same story where the maximum member-revolution angle, i.e., the maximum deflection, occurs.

In the building frames with a great number of bays, the most outside columns have the greatest value of slope and second columns from the outside have the next largest amount of slope. Several

columns near the central part have approximately equal amounts of slope in the same story.

The values of slopes and deflections for twenty eight different kinds of building frames symmetrical to the central vertical axis of frame, having constant stiffness and equal height of columns in each story are given in Takabeya's "Moment Diagram of Building Frames" Vol. II.

These building frames are as follows and the values of slopes and deflections are tabulated in Tables 1 to 5.

- I.
  - a) Building frame of one story and single bay.
  - b) Building frame of two stories and single bay.
  - c) Building frame of three stories and single bay.
  - d) Building frame of four stories and single bay.
  - e) Building frame of five stories and single bay.
  - f) Building frame of six stories and single bay.
  - g) Building frame of seven stories and single bay.
- II.
  - a) Building frame of one story and two bays.
  - b) Building frame of two stories and two bays.
  - c) Building frame of three stories and two bays.
  - d) Building frame of four stories and two bays.
  - e) Building frame of five stories and two bays.
  - f) Building frame of six stories and two bays.
- III.
  - a) Building frame of one story and three bays.
  - b) Building frame of two stories and three bays.
  - c) Building frame of three stories and three bays.
  - d) Building frame of four stories and three bays.
  - e) Building frame of five stories and three bays.
- IV.
  - a) Building frame of one story and four bays.
  - b) Building frame of two stories and four bays.
  - c) Building frame of three stories and four bays.
  - d) Building frame of four stories and four bays.
  - e) Building frame of five stories and four bays.
- V.
  - a) Building frame of one story and five bays.
  - b) Building frame of two stories and five bays.
  - c) Building frame of three stories and five bays.
  - d) Building frame of four stories and five bays.
  - e) Building frame of five stories and five bays.

Table 1.

| (a) Frame of One Story<br>and Single Bay.     |                | (d) Frame of Four Stories<br>and Single Bay. |                 |
|---|----------------|--|-----------------|
| Slope   | Deflection     | Slope  | Deflection      |
| 0.4286  | (2.1429)       | 0.5633                                       | (3.7589)        |
|   |                | 0.0713                                       | (0.8555)        |
|   |                | 0.0088                                       | (0.1056)        |
|   |                | 0.00088                                      | (0.01055)       |
|   |                | Total 0.64428                                | (Total 4.73055) |
| (b) Frame of Two Stories<br>and Single Bay.   |                |  |                 |
| Slope   | Deflection     |  |                 |
| 0.5455  | (3.5456)       | 0.9426                                       | (4.3793)        |
| 0.0545  | (0.6545)       | 0.4990                                       | (3.6293)        |
| Total 0.6000                                  | (Total 4.2001) | 0.0616                                       | (0.8184)        |
|   |                | 0.00615                                      | (0.0818)        |
| 0.8182  | (2.7273)       | Total 1.50935                                | (Total 8.9088)  |
| 0.3818  | (2.0727)       |  |                 |
| Total 1.2000                                  | (Total 4.8000) | 0.9769                                       | (4.2737)        |
| (c) Frame of Three Stories<br>and Single Bay. |                | 0.9205                                       | (4.1784)        |
| Slope   | Deflection     | 0.4840                                       | (3.4417)        |
| 0.5612  | (3.7346)       | 0.0484                                       | (0.6441)        |
| 0.0693  | (0.8315)       | Total 2.4298                                 | (Total 12.5379) |
| 0.0069  | (0.0831)       |  |                 |
| Total 0.6374                                  | (Total 4.6492) | 0.8722                                       | (2.8083)        |
|   |                | 0.8651                                       | (2.7977)        |
| 0.9285  | (4.1919)       | 0.8105                                       | (2.7158)        |
| 0.4850  | (3.4434)       | 0.3810                                       | (2.0715)        |
| 0.0485  | (0.6443)       | Total 2.9288                                 | (Total 10.3933) |
| Total 1.4620                                  | (Total 8.2796) |  |                 |
|   |                | 0.8661                                       | (2.7992)        |
| 0.8106  | (2.7159)       |  |                 |
| 0.381   | (2.0715)       |  |                 |
| Total 2.0577                                  | (Total 7.5866) |  |                 |

(e) Frame of Five Stories  
and Single Bay.

| Slope          | Deflection      |
|----------------|-----------------|
| 0.5635         | (3.7620)        |
| 0.0715         | (0.8583)        |
| 0.00904        | (0.1085)        |
| 0.00111        | (0.0134)        |
| 0.00011        | (0.00134)       |
| Total 0.64526  | (Total 4.74354) |
| 0.9444         | (4.4031)        |
| 0.5007         | (3.6528)        |
| 0.0633         | (0.8419)        |
| 0.00783        | (0.1040)        |
| 0.000783       | (0.0104)        |
| Total 1.517013 | (Total 9.0122)  |
| 0.9910         | (4.4610)        |
| 0.9345         | (4.3655)        |
| 0.4980         | (3.6276)        |
| 0.0615         | (0.8183)        |
| 0.00615        | (0.0818)        |
| Total 2.49115  | (Total 13.3542) |
| 0.9830         | (4.2837)        |
| 0.9758         | (4.2717)        |
| 0.9204         | (4.1783)        |
| 0.4840         | (3.4417)        |
| 0.0484         | (0.6441)        |
| Total 3.4116   | (Total 16.8195) |
| 0.8728         | (2.8092)        |
| 0.8720         | (2.8080)        |
| 0.8651         | (2.7977)        |
| 0.8105         | (2.7158)        |
| 0.3810         | (2.0715)        |
| Total 3.8014   | (Total 13.2022) |

(f) Frame of Six Stories  
and Single Bay.

| Slope           | Deflection      |
|-----------------|-----------------|
| 0.5635          | (3.7620)        |
| 0.0715          | (0.8586)        |
| 0.00908         | (0.1060)        |
| 0.00115         | (0.0138)        |
| 0.000142        | (0.00171)       |
| 0.0000142       | (0.00017)       |
| Total 0.6453862 | (Total 4.74528) |
| 0.9445          | (4.4060)        |
| 0.5009          | (3.6558)        |
| 0.0636          | (0.8451)        |
| 0.00805         | (0.1070)        |
| 0.0010          | (0.01323)       |
| 0.000099        | (0.00132)       |
| Total 1.518149  | (Total 9.02845) |
| 0.9927          | (4.4846)        |
| 0.9363          | (4.3893)        |
| 0.4998          | (3.6513)        |
| 0.0633          | (0.8419)        |
| 0.00782         | (0.1040)        |
| 0.00078         | (0.0104)        |
| Total 2.5007    | (Total 13.4815) |
| 0.9970          | (4.4712)        |
| 0.9899          | (4.4592)        |
| 0.9344          | (4.3653)        |
| 0.4980          | (3.6276)        |
| 0.0615          | (0.8183)        |
| 0.00615         | (0.0818)        |
| Total 3.48695   | (Total 17.8234) |
| 0.9838          | (4.2852)        |
| 0.9829          | (4.2836)        |
| 0.9758          | (4.2717)        |
| 0.9204          | (3.1783)        |
| 0.4840          | (3.4417)        |
| 0.0484          | (0.6442)        |
| Total 4.3953    | (Total 21.1046) |
| 0.8730          | (2.8095)        |
| 0.8728          | (2.8092)        |
| 0.8720          | (2.808)         |
| 0.8651          | (2.7977)        |
| 0.8105          | (2.7158)        |
| 0.3810          | (2.0715)        |
| Total 4.6744    | (Total 16.0117) |



## (g) Frame of Seven Stories and Single Bay.

| Slope           | Deflection       | Slope         | Deflection      |
|-----------------|------------------|---------------|-----------------|
| 0.5635          | (3.7620)         | 0.9981        | (4.4732)        |
| 0.0716          | (0.8591)         | 0.9970        | (4.4712)        |
| 0.0091          | (0.1091)         | 0.9899        | (4.4592)        |
| 0.00114         | (0.0138)         | 0.9344        | (4.3653)        |
| 0.000146        | (0.00175)        | 0.4980        | (3.6276)        |
| 0.0000182       | (0.00022)        | 0.0615        | (0.8183)        |
| 0.0000018       | (0.000022)       | 0.00615       | (0.0818)        |
| Total 0.645506  | (Total 4.745992) | Total 4.48505 | (Total 22.2966) |
| 0.9445          | (4.4063)         | 0.9840        | (4.2857)        |
| 0.5011          | (3.6567)         | 0.9838        | (4.2852)        |
| 0.0636          | (0.8454)         | 0.9829        | (4.2836)        |
| 0.00808         | (0.1074)         | 0.9758        | (4.2717)        |
| 0.00102         | (0.01359)        | 0.9204        | (4.1783)        |
| 0.000127        | (0.00169)        | 0.4840        | (3.4417)        |
| 0.0000127       | (0.000168)       | 0.0484        | (0.6441)        |
| Total 1.5184397 | (Total 9.031248) | Total 5.3793  | (Total 25.3903) |
| 0.9930          | (4.4883)         | 0.8731        | (2.8097)        |
| 0.9367          | (4.3928)         | 0.8730        | (2.8095)        |
| 0.5000          | (3.6543)         | 0.8728        | (2.8092)        |
| 0.0635          | (0.8450)         | 0.8720        | (2.8080)        |
| 0.00804         | (0.1070)         | 0.8651        | (2.7977)        |
| 0.00100         | (0.01323)        | 0.8105        | (2.7158)        |
| 0.0000992       | (0.00132)        | 0.3810        | (2.0715)        |
| Total 2.5023392 | (Total 13.50195) | Total 5.5475  | (Total 18.8214) |
| 0.9991          | (4.4967)         |               |                 |
| 0.9918          | (4.4832)         |               |                 |
| 0.9362          | (4.3892)         |               |                 |
| 0.4998          | (3.6513)         |               |                 |
| 0.0633          | (0.8419)         |               |                 |
| 0.00782         | (0.1040)         |               |                 |
| 0.000781        | (0.01042)        |               |                 |
| Total 3.498801  | (Total 17.97672) |               |                 |

Table 2.

## (a) Frame of One Story and Two Bays.

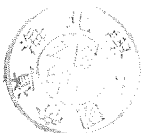
| Slope  | Slope  | Deflection |
|--------|--------|------------|
| 0.3125 | 0.1250 | (1.3750)   |

## (b) Frame of Two Stories and Two Bays.

| Slope        | Slope        | Deflection     |
|--------------|--------------|----------------|
| 0.3569       | 0.1826       | (2.1330)       |
| 0.0190       | 0.0296       | (0.3593)       |
| Total 0.3759 | Total 0.2122 | (Total 2.4923) |
| 0.5229       | 0.3237       | (1.6848)       |
| 0.2537       | 0.1435       | (1.3255)       |
| Total 0.7766 | Total 0.4672 | (Total 3.0103) |

## (c) Frame of Three Stories and Two Bays.

| Slope        | Slope        | Deflection     |
|--------------|--------------|----------------|
| 0.3659       | 0.1837       | (2.2127)       |
| 0.0278       | 0.0303       | (0.4346)       |
| 0.0045       | 0.0001       | (0.0359)       |
| Total 0.3982 | Total 0.2141 | (Total 2.6832) |
| 0.5655       | 0.3788       | (2.4721)       |
| 0.2930       | 0.1972       | (2.0731)       |
| 0.0180       | 0.0266       | (0.3565)       |
| Total 0.8765 | Total 0.6026 | (Total 4.9017) |
| 0.5474       | 0.3395       | (1.7172)       |
| 0.5247       | 0.3135       | (1.6815)       |
| 0.2534       | 0.1435       | (1.3252)       |
| Total 1.3255 | Total 0.7965 | (Total 4.7239) |



## (d) Frame of Four Stories and Two Bays.

| Slope |          | Slope |         | Deflection |           |
|-------|----------|-------|---------|------------|-----------|
|       | 0.3661   |       | 0.1847  |            | (2.2214)  |
|       | 0.0281   |       | 0.0313  |            | (0.4435)  |
|       | 0.0046   |       | 0.0011  |            | (0.0442)  |
|       | 0.000016 |       | 0.00054 |            | (0.00393) |
| Total | 0.398816 | Total | 0.21764 | (Total     | 2.71303)  |
|       | 0.5724   |       | 0.3810  |            | (2.5505)  |
|       | 0.3000   |       | 0.1995  |            | (2.1514)  |
|       | 0.0248   |       | 0.0284  |            | (0.4303)  |
|       | 0.00333  |       | 0.00062 |            | (0.03477) |
| Total | 0.90053  | Total | 0.60952 | (Total     | 5.16697)  |
|       | 0.5904   |       | 0.3944  |            | (2.5079)  |
|       | 0.5676   |       | 0.3680  |            | (2.4680)  |
|       | 0.2928   |       | 0.1970  |            | (2.0728)  |
|       | 0.01794  |       | 0.02636 |            | (0.35618) |
| Total | 1.46874  | Total | 0.98576 | (Total     | 7.40488)  |
|       | 0.5493   |       | 0.3419  |            | (1.7203)  |
|       | 0.5461   |       | 0.3405  |            | (1.7164)  |
|       | 0.5247   |       | 0.3135  |            | (1.6815)  |
|       | 0.2533   |       | 0.1435  |            | (1.32505) |
| Total | 1.8734   | Total | 1.1394  | (Total     | 6.44325)  |

## (e) Frame of Five Stories and Two Bays.

| Slope |         | Slope |          | Deflection |           |
|-------|---------|-------|----------|------------|-----------|
|       | 0.3662  |       | 0.1846   |            | (2.2219)  |
|       | 0.0282  |       | 0.0313   |            | (0.4442)  |
|       | 0.0047  |       | 0.0011   |            | (0.0450)  |
|       | 0.0002  |       | 0.00049  |            | (0.0048)  |
|       | 0.00008 |       | 0.000043 |            | (0.0004)  |
| Total | 0.39938 | Total | 0.217533 | (Total     | 2.7163)   |
|       | 0.5725  |       | 0.3818   |            | (2.5590)  |
|       | 0.3002  |       | 0.2003   |            | (2.1600)  |
|       | 0.0251  |       | 0.0292   |            | (0.4390)  |
|       | 0.0036  |       | 0.00155  |            | (0.04334) |
|       | 0.00008 |       | 0.0005   |            | (0.0041)  |
| Total | 0.90148 | Total | 0.61335  | (Total     | 5.20544)  |
|       | 0.5973  |       | 0.3965   |            | (2.5863)  |
|       | 0.5745  |       | 0.3702   |            | (2.5463)  |
|       | 0.2997  |       | 0.1992   |            | (2.1508)  |
|       | 0.0249  |       | 0.0282   |            | (0.4302)  |
|       | 0.0034  |       | 0.0007   |            | (0.0351)  |
| Total | 1.4998  | Total | 0.9948   | (Total     | 7.7487)   |
|       | 0.5923  |       | 0.3968   |            | (2.5113)  |
|       | 0.5890  |       | 0.3954   |            | (2.5069)  |
|       | 0.5675  |       | 0.3680   |            | (2.4679)  |
|       | 0.2928  |       | 0.1970   |            | (2.0727)  |
|       | 0.0181  |       | 0.0264   |            | (0.3564)  |
| Total | 2.0597  | Total | 1.3836   | (Total     | 9.9152)   |
|       | 0.5496  |       | 0.3420   |            | (1.7206)  |
|       | 0.5494  |       | 0.3416   |            | (1.7202)  |
|       | 0.5461  |       | 0.3405   |            | (1.7164)  |
|       | 0.5247  |       | 0.3135   |            | (1.6814)  |
|       | 0.2533  |       | 0.1436   |            | (1.3251)  |
| Total | 2.4231  | Total | 1.4812   | (Total     | 8.1637)   |

## (f) Frame of Six Stories and Two Bays.

| Slope |           | Slope |          | Deflection |            |
|-------|-----------|-------|----------|------------|------------|
|       | 0.3662    |       | 0.1847   |            | (2.2222)   |
|       | 0.0282    |       | 0.0313   |            | (0.4443)   |
|       | 0.00473   |       | 0.0011   |            | (0.0451)   |
|       | 0.0002    |       | 0.0005   |            | (0.0048)   |
|       | 0.00008   |       | 0.000015 |            | (0.00051)  |
|       | 0.0000055 |       | 0.000014 |            | (0.000052) |
| Total | 0.3994155 | Total | 0.217629 | (Total     | 2.716962)  |
|       | 0.5727    |       | 0.3818   |            | (2.5598)   |
|       | 0.3003    |       | 0.2003   |            | (2.1608)   |
|       | 0.0252    |       | 0.0292   |            | (0.4398)   |
|       | 0.0037    |       | 0.0015   |            | (0.0440)   |
|       | 0.00021   |       | 0.00044  |            | (0.00477)  |
|       | 0.00006   |       | 0.00002  |            | (0.00037)  |
| Total | 0.90217   | Total | 0.61326  | (Total     | 5.20954)   |
|       | 0.5975    |       | 0.3974   |            | (2.5950)   |
|       | 0.5748    |       | 0.3711   |            | (2.5552)   |
|       | 0.3000    |       | 0.2000   |            | (2.1595)   |
|       | 0.0251    |       | 0.0290   |            | (0.4388)   |
|       | 0.00359   |       | 0.00150  |            | (0.0431)   |
|       | 0.00009   |       | 0.00045  |            | (0.00406)  |
| Total | 1.50108   | Total | 0.99945  | (Total     | 7.79566)   |
|       | 0.5993    |       | 0.3990   |            | (2.5899)   |
|       | 0.5960    |       | 0.3977   |            | (2.5857)   |
|       | 0.5744    |       | 0.3702   |            | (2.5462)   |
|       | 0.2996    |       | 0.1992   |            | (2.1507)   |
|       | 0.0247    |       | 0.0282   |            | (0.4301)   |
|       | 0.0034    |       | 0.00069  |            | (0.03505)  |
| Total | 2.0974    | Total | 1.39499  | (Total     | 10.33765)  |
|       | 0.5926    |       | 0.3969   |            | (2.5117)   |
|       | 0.5925    |       | 0.3966   |            | (2.5115)   |
|       | 0.5890    |       | 0.3954   |            | (2.5069)   |
|       | 0.5675    |       | 0.3680   |            | (2.4678)   |
|       | 0.2928    |       | 0.1969   |            | (2.0727)   |
|       | 0.0181    |       | 0.0264   |            | (0.3564)   |
| Total | 2.6525    | Total | 1.7802   | (Total     | 12.4270)   |
|       | 0.5496    |       | 0.3421   |            | (1.7207)   |
|       | 0.5496    |       | 0.3421   |            | (1.7207)   |
|       | 0.5494    |       | 0.3416   |            | (1.7202)   |
|       | 0.5460    |       | 0.3405   |            | (1.7163)   |
|       | 0.5247    |       | 0.3135   |            | (1.6815)   |
|       | 0.2533    |       | 0.1436   |            | (1.3251)   |
| Total | 2.9726    | Total | 1.8234   | (Total     | 9.8845)    |

Table 3.

## (a) Frame of One Story and Three Bays.

| Slope  | Slope  | Deflection |
|--------|--------|------------|
| 0.2222 | 0.1111 | (1.000)    |

## (b) Frame of Two Stories and Three Bays.

| Slope        | Slope        | Deflection     |
|--------------|--------------|----------------|
| 0.2476       | 0.1448       | (1.5060)       |
| 0.0108       | 0.0168       | (0.2412)       |
| Total 0.2584 | Total 0.1616 | (Total 1.7472) |
| 0.3709       | 0.2447       | (1.2117)       |
| 0.1814       | 0.1126       | (0.9705)       |
| Total 0.5523 | Total 0.3573 | (Total 2.1822) |

## (c) Frame of Three Stories and Three Bays.

| Slope        | Slope        | Deflection     |
|--------------|--------------|----------------|
| 0.2530       | 0.1463       | (1.5539)       |
| 0.0161       | 0.0181       | (0.2867)       |
| 0.0027       | 0.0005       | (0.0217)       |
| Total 0.2718 | Total 0.1649 | (Total 1.8623) |
| 0.3957       | 0.2769       | (1.7360)       |
| 0.2042       | 0.1439       | (1.4710)       |
| 0.0104       | 0.0153       | (0.2396)       |
| Total 0.6103 | Total 0.4361 | (Total 3.4466) |
| 0.3860       | 0.2561       | (1.2316)       |
| 0.3727       | 0.2405       | (1.2099)       |
| 0.1812       | 0.1126       | (0.9704)       |
| Total 0.9399 | Total 0.6092 | (Total 3.4119) |

## (d) Frame of Four Stories and Three Bays.

| Slope |           | Slope |         | Deflection |          |
|-------|-----------|-------|---------|------------|----------|
|       | 0.2530    |       | 0.1467  |            | (1.5587) |
|       | 0.0162    |       | 0.0186  |            | (0.2916) |
|       | 0.0027    |       | 0.0010  |            | (0.0263) |
|       | 0.0000025 |       | 0.00021 |            | (0.0023) |
| Total | 0.2719025 | Total | 0.16651 | (Total     | 1.8789)  |
|       | 0.3999    |       | 0.2786  |            | (1.7832) |
|       | 0.2084    |       | 0.1456  |            | (1.5182) |
|       | 0.0146    |       | 0.0168  |            | (0.2845) |
|       | 0.0021    |       | 0.0007  |            | (0.0214) |
| Total | 0.6250    | Total | 0.4417  | (Total     | 3.6073)  |
|       | 0.4109    |       | 0.2882  |            | (1.7576) |
|       | 0.3977    |       | 0.2725  |            | (1.7339) |
|       | 0.2041    |       | 0.1438  |            | (1.4708) |
|       | 0.0104    |       | 0.0153  |            | (0.2396) |
| Total | 1.0231    | Total | 0.7198  | (Total     | 5.2019)  |
|       | 0.3871    |       | 0.2573  |            | (1.2333) |
|       | 0.3852    |       | 0.2564  |            | (1.2312) |
|       | 0.3727    |       | 0.2405  |            | (1.2099) |
|       | 0.1811    |       | 0.1126  |            | (0.9703) |
| Total | 1.3261    | Total | 0.8668  | (Total     | 4.6447)  |

## (e) Frame of Five Stories and Three Bays.

| Slope |          | Slope |          | Deflection |          |
|-------|----------|-------|----------|------------|----------|
|       | 0.2531   |       | 0.1467   |            | (1.5591) |
|       | 0.0163   |       | 0.0185   |            | (0.2920) |
|       | 0.0028   |       | 0.0010   |            | (0.0268) |
|       | 0.00007  |       | 0.0002   |            | (0.0027) |
|       | 0.000045 |       | 0.000006 |            | (0.0002) |
| Total | 0.272315 | Total | 0.166406 | (Total     | 1.8808)  |
|       | 0.4000   |       | 0.2790   |            | (1.7881) |
|       | 0.2085   |       | 0.1460   |            | (1.5229) |
|       | 0.0147   |       | 0.0172   |            | (0.2893) |
|       | 0.0022   |       | 0.0011   |            | (0.0260) |
|       | 0.00003  |       | 0.0002   |            | (0.0023) |
| Total | 0.62543  | Total | 0.4435   | (Total     | 3.6286)  |
|       | 0.4152   |       | 0.2900   |            | (1.8052) |
|       | 0.4018   |       | 0.2742   |            | (1.7811) |
|       | 0.2083   |       | 0.1455   |            | (1.5179) |
|       | 0.0146   |       | 0.0168   |            | (0.2844) |
|       | 0.0021   |       | 0.0007   |            | (0.0214) |
| Total | 1.0420   | Total | 0.7272   | (Total     | 5.4100)  |
|       | 0.4122   |       | 0.2895   |            | (1.7598) |
|       | 0.4102   |       | 0.2886   |            | (1.7575) |
|       | 0.3976   |       | 0.2725   |            | (1.7338) |
|       | 0.2040   |       | 0.1438   |            | (1.4708) |
|       | 0.0104   |       | 0.0153   |            | (0.2396) |
| Total | 1.4344   | Total | 1.0097   | (Total     | 6.9615)  |
|       | 0.3873   |       | 0.2574   |            | (1.2335) |
|       | 0.3873   |       | 0.2572   |            | (1.2334) |
|       | 0.3852   |       | 0.2564   |            | (1.2312) |
|       | 0.3727   |       | 0.2405   |            | (1.2099) |
|       | 0.1812   |       | 0.1126   |            | (0.9704) |
| Total | 1.7137   | Total | 1.1241   | (Total     | 5.8784)  |



Table 4.

## (a) Frame of One Story and Four Bays.

| Slope  | Slope  | Slope  | Deflection |
|--------|--------|--------|------------|
| 0.1756 | 0.0848 | 0.1029 | (0.7871)   |

## (b) Frame of Two Stories and Four Bays.

| Slope        | Slope        | Slope        | Deflection     |
|--------------|--------------|--------------|----------------|
| 0.1916       | 0.1101       | 0.1240       | (1.1652)       |
| 0.0065       | 0.0129       | 0.0100       | (0.1809)       |
| Total 0.1981 | Total 0.1230 | Total 0.1340 | (Total 1.3461) |
| 0.2886       | 0.1890       | 0.2013       | (0.9470)       |
| 0.1423       | 0.0871       | 0.0954       | (0.7663)       |
| Total 0.4309 | Total 0.2761 | Total 0.2967 | (Total 1.7133) |

## (c) Frame of Three Stories and Four Bays.

| Slope        | Slope         | Slope         | Deflection     |
|--------------|---------------|---------------|----------------|
| 0.1959       | 0.1107        | 0.1260        | (1.1990)       |
| 0.0107       | 0.0134        | 0.0119        | (0.2130)       |
| 0.0022       | 0.00013       | 0.00094       | (0.0154)       |
| Total 0.2088 | Total 0.12423 | Total 0.13884 | (Total 1.4274) |
| 0.3048       | 0.2129        | 0.2219        | (1.3383)       |
| 0.1571       | 0.1103        | 0.1152        | (1.1408)       |
| 0.0066       | 0.0115        | 0.0095        | (0.1799)       |
| Total 0.4685 | Total 0.3347  | Total 0.3466  | (Total 2.6590) |
| 0.2996       | 0.1971        | 0.2104        | (0.9611)       |
| 0.2907       | 0.1857        | 0.2000        | (0.9458)       |
| 0.1421       | 0.0872        | 0.0953        | (0.7662)       |
| Total 0.7324 | Total 0.4700  | Total 0.5057  | (Total 2.6731) |

## (d) Frame of Four Stories and Four Bays.

| Slope |           | Slope |         | Slope |         | Deflection     |
|-------|-----------|-------|---------|-------|---------|----------------|
|       | 0.1958    |       | 0.1111  |       | 0.1261  | (1.2022)       |
|       | 0.0106    |       | 0.0137  |       | 0.0120  | (0.2162)       |
|       | 0.0021    |       | 0.0005  |       | 0.0011  | (0.0185)       |
|       | -0.000075 |       | 0.0002  |       | 0.00005 | (0.0015)       |
| Total | 0.20843   | Total | 0.1255  | Total | 0.13925 | (Total 1.4384) |
|       | 0.3081    |       | 0.2138  |       | 0.2236  | (1.3716)       |
|       | 0.1602    |       | 0.1113  |       | 0.1169  | (1.1740)       |
|       | 0.0098    |       | 0.0123  |       | 0.0112  | (0.2115)       |
|       | 0.0016    |       | 0.00033 |       | 0.0008  | (0.0151)       |
| Total | 0.4797    | Total | 0.33773 | Total | 0.3525  | (Total 2.7722) |
|       | 0.3160    |       | 0.2208  |       | 0.2311  | (1.3537)       |
|       | 0.3071    |       | 0.2093  |       | 0.2206  | (1.3369)       |
|       | 0.1569    |       | 0.1103  |       | 0.1151  | (1.1407)       |
|       | 0.0067    |       | 0.0114  |       | 0.0095  | (0.1798)       |
| Total | 0.7867    | Total | 0.5518  | Total | 0.5763  | (Total 4.0111) |
|       | 0.3003    |       | 0.1980  |       | 0.2111  | (0.9623)       |
|       | 0.2989    |       | 0.1974  |       | 0.2103  | (0.9607)       |
|       | 0.2907    |       | 0.1857  |       | 0.2000  | (0.9458)       |
|       | 0.1420    |       | 0.0872  |       | 0.0953  | (0.7661)       |
| Total | 1.0319    | Total | 0.6683  | Total | 0.7167  | (Total 3.6349) |

## (e) Frame of Five Stories and Four Bays.

| Slope         | Slope         | Slope          | Deflection      |
|---------------|---------------|----------------|-----------------|
| 0.1958        | 0.1111        | 0.1261         | (1.2024)        |
| 0.0106        | 0.0137        | 0.0120         | (0.2164)        |
| 0.0021        | 0.00047       | 0.0012         | (0.0187)        |
| -0.0000075    | 0.00017       | 0.000078       | (0.00174)       |
| 0.000041      | -0.000017     | 0.0000146      | (0.000111)      |
| Total 0.20853 | Total 0.12542 | Total 0.13939  | (Total 1.43935) |
| 0.3080        | 0.2141        | 0.2238         | (1.3748)        |
| 0.1602        | 0.1116        | 0.1170         | (1.1773)        |
| 0.0098        | 0.0126        | 0.0113         | (0.2147)        |
| 0.0016        | 0.00063       | 0.00093        | (0.0182)        |
| -0.000035     | 0.00016       | 0.000058       | (0.00148)       |
| Total 0.47957 | Total 0.33909 | Total 0.353088 | (Total 2.78648) |
| 0.3192        | 0.2218        | 0.2328         | (1.3871)        |
| 0.3103        | 0.2103        | 0.2224         | (1.3703)        |
| 0.1601        | 0.1112        | 0.1168         | (1.1738)        |
| 0.0098        | 0.0122        | 0.0112         | (0.2114)        |
| 0.0016        | 0.00032       | 0.00079        | (0.0150)        |
| Total 0.8010  | Total 0.55582 | Total 0.58399  | (Total 4.1576)  |
| 0.3168        | 0.2217        | 0.2319         | (1.3551)        |
| 0.3154        | 0.2212        | 0.2310         | (1.3537)        |
| 0.3071        | 0.2093        | 0.2206         | (1.3369)        |
| 0.1569        | 0.1103        | 0.1151         | (1.1407)        |
| 0.0066        | 0.0114        | 0.0095         | (0.1797)        |
| Total 1.1028  | Total 0.7739  | Total 0.8081   | (Total 5.3661)  |
| 0.3005        | 0.1980        | 0.2112         | (0.9625)        |
| 0.3005        | 0.1979        | 0.2112         | (0.9624)        |
| 0.2989        | 0.1974        | 0.2103         | (0.9609)        |
| 0.2907        | 0.1857        | 0.2000         | (0.9458)        |
| 0.1420        | 0.0871        | 0.0953         | (0.7661)        |
| Total 1.3326  | Total 0.8661  | Total 0.9280   | (Total 4.5977)  |

Table 5.

## (a) Frame of One Story and Five Bays.

| Slope   | Slope   | Slope   | Deflection |
|---------|---------|---------|------------|
| 0.14463 | 0.07025 | 0.08264 | (0.64876)  |

## (b) Frame of Two Stories and Five Bays.

| Slope         | Slope         | Slope         | Deflection      |
|---------------|---------------|---------------|-----------------|
| 0.15596       | 0.08996       | 0.09947       | (0.94989)       |
| 0.00435       | 0.0100        | 0.0081        | (0.1442)        |
| Total 0.16031 | Total 0.09996 | Total 0.10757 | (Total 1.09409) |
| 0.23607       | 0.15467       | 0.16366       | (0.7772)        |
| 0.1168        | 0.0716        | 0.0775        | (0.6330)        |
| Total 0.35287 | Total 0.22627 | Total 0.24116 | (Total 1.4102)  |

## (c) Frame of Three Stories and Five Bays.

| Slope         | Slope         | Slope          | Deflection      |
|---------------|---------------|----------------|-----------------|
| 0.15937       | 0.09042       | 0.10077        | (0.97579)       |
| 0.00777       | 0.0104        | 0.00934        | (0.1690)        |
| 0.00176       | 0.00009       | 0.000588       | (0.0118)        |
| Total 0.16890 | Total 0.10091 | Total 0.110698 | (Total 1.15659) |
| 0.24789       | 0.17314       | 0.17999        | (1.08868)       |
| 0.1275        | 0.0897        | 0.0932         | (0.9316)        |
| 0.00466       | 0.00891       | 0.00759        | (0.1435)        |
| Total 0.38005 | Total 0.27175 | Total 0.28078  | (Total 2.16378) |
| 0.24465       | 0.16105       | 0.17064        | (0.78817)       |
| 0.2381        | 0.1522        | 0.1625         | (0.7764)        |
| 0.1167        | 0.0717        | 0.0775         | (0.6330)        |
| Total 0.59945 | Total 0.38495 | Total 0.41064  | (Total 2.19757) |

## (d) Frame of Four Stories and Five Bays.

| Slope |          | Slope |          | Slope |           | Deflection      |
|-------|----------|-------|----------|-------|-----------|-----------------|
|       | 0.15926  |       | 0.09070  |       | 0.10090   | (0.97819)       |
|       | 0.0078   |       | 0.0108   |       | 0.0095    | (0.1717)        |
|       | 0.00162  |       | 0.000375 |       | 0.000712  | (0.0141)        |
|       | -0.00009 |       | 0.00015  |       | 0.0000587 | (0.00110)       |
| Total | 0.16859  | Total | 0.10203  | Total | 0.11117   | (Total 1.16509) |
|       | 0.25047  |       | 0.17384  |       | 0.18120   | (1.11432)       |
|       | 0.1304   |       | 0.0904   |       | 0.0946    | (0.9575)        |
|       | 0.00725  |       | 0.00951  |       | 0.00874   | (0.1680)        |
|       | 0.00131  |       | 0.000233 |       | 0.000539  | (0.01162)       |
| Total | 0.38943  | Total | 0.27398  | Total | 0.28508   | (Total 2.25144) |
|       | 0.25664  |       | 0.17947  |       | 0.18701   | (1.10065)       |
|       | 0.2502   |       | 0.1708   |       | 0.1788    | (1.088)         |
|       | 0.1274   |       | 0.0897   |       | 0.0933    | (0.9316)        |
|       | 0.00471  |       | 0.00886  |       | 0.00758   | (0.14348)       |
| Total | 0.63895  | Total | 0.44883  | Total | 0.46669   | (Total 3.26373) |
|       | 0.24523  |       | 0.16173  |       | 0.17122   | (0.78909)       |
|       | 0.2442   |       | 0.16125  |       | 0.1707    | (0.788)         |
|       | 0.2381   |       | 0.1522   |       | 0.1625    | (0.7764)        |
|       | 0.11667  |       | 0.07167  |       | 0.07746   | (0.63290)       |
| Total | 0.8442   | Total | 0.54685  | Total | 0.58188   | (Total 2.98639) |

## (e) Frame of Five Stories and Five Bays.

| Slope         |  | Slope         |  | Slope         |  | Deflection      |
|---------------|--|---------------|--|---------------|--|-----------------|
| 0.15933       |  | 0.09068       |  | 0.10092       |  | (0.97838)       |
| 0.00771       |  | 0.010621      |  | 0.009471      |  | (0.17147)       |
| 0.00168       |  | 0.00036       |  | 0.00072       |  | (0.01425)       |
| -0.000024     |  | 0.000126      |  | 0.000068      |  | (0.00126)       |
| 0.0000352     |  | -0.000013     |  | 0.0000048     |  | (0.0000804)     |
| Total 0.16873 |  | Total 0.10177 |  | Total 0.11118 |  | (Total 1.16544) |
| 0.25042       |  | 0.17407       |  | 0.18133       |  | (1.1167)        |
| 0.13002       |  | 0.09056       |  | 0.09455       |  | (0.95948)       |
| 0.00718       |  | 0.00972       |  | 0.00884       |  | (0.17017)       |
| 0.00124       |  | 0.00046       |  | 0.000657      |  | (0.01389)       |
| -0.000044     |  | 0.000119      |  | 0.0000586     |  | (0.00111)       |
| Total 0.38882 |  | Total 0.27493 |  | Total 0.28544 |  | (Total 2.26135) |
| 0.25920       |  | 0.18017       |  | 0.18821       |  | (1.12627)       |
| 0.25258       |  | 0.17124       |  | 0.1800        |  | (1.11328)       |
| 0.12993       |  | 0.09032       |  | 0.09435       |  | (0.95695)       |
| 0.00727       |  | 0.00945       |  | 0.00871       |  | (0.16781)       |
| 0.00130       |  | 0.000239      |  | 0.000583      |  | (0.01161)       |
| Total 0.65028 |  | Total 0.45142 |  | Total 0.47181 |  | (Total 3.36989) |
| 0.25721       |  | 0.18015       |  | 0.18759       |  | (1.10165)       |
| 0.25602       |  | 0.17973       |  | 0.18698       |  | (1.10046)       |
| 0.2500        |  | 0.17052       |  | 0.17878       |  | (1.08764)       |
| 0.12739       |  | 0.08963       |  | 0.09316       |  | (0.93147)       |
| 0.00471       |  | 0.00886       |  | 0.00758       |  | (0.14347)       |
| Total 0.89533 |  | Total 0.62889 |  | Total 0.65409 |  | (Total 4.36469) |
| 0.24531       |  | 0.16176       |  | 0.17127       |  | (0.78917)       |
| 0.24531       |  | 0.16166       |  | 0.17121       |  | (0.78909)       |
| 0.24406       |  | 0.16130       |  | 0.17062       |  | (0.78799)       |
| 0.23804       |  | 0.15221       |  | 0.16250       |  | (0.77638)       |
| 0.11667       |  | 0.07167       |  | 0.07745       |  | (0.63289)       |
| Total 1.08939 |  | Total 0.7086  |  | Total 0.75305 |  | (Total 3.77552) |

In Tables 1 to 5, the coefficients of slopes and deflections numerically indicated respectively  $\frac{W \cdot h}{12EK}$  and  $\frac{W \cdot h}{36EK}$ , where  $W$  indicates intensity of the horizontal joint load,  $h$  height of column,  $E$  modulus of elasticity and  $K$  stiffness. Numerical values of the first row in each frame show the slope and deflection due to a horizontal load  $W$  concentrated at the first joint from the top on the left side of the structure. Those of the second row in each frame show the slope and deflection due to a horizontal load  $W$  concentrated at the second joint from the top on the left side of the structure and so on. Numerical values of the lowest row in each frame show the slope and deflection due to horizontal loads concentrated at every joint on the left side of the structure having the intensity of  $W$  and they are indicated by "Total".

These tables are applicable to determinations of slopes and deflections due to any desired load distribution.

In regard to the slopes and deflections for the frames of variable stiffness, these are shown in Figs. 19 to 50 in a later section.

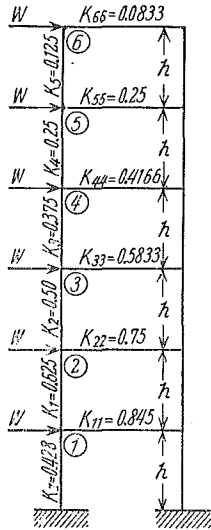


Fig. 1.

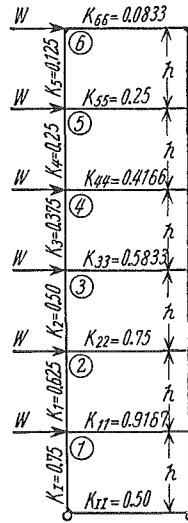


Fig. 2.

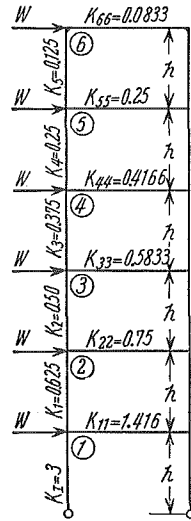


Fig. 3.

Figs. 1 to 5 show the imaginary frames whose slopes in every joint are equal and deflections in every story are approximately the same for the given loads. That is, frames of Figs. 1 to 3 are respectively fixed, partially fixed and hinged at their bases; all the slopes and

deflections are respectively  $\frac{W \cdot h}{2E}$  and  $\frac{5 W \cdot h}{6E}$  for the horizontal loads which act at all joints of the outside of the frame, having equal intensity  $W$ . For the frame of Fig. 4, all slopes and deflections are

$\frac{W \cdot h}{2E}$  and  $\frac{10 W \cdot h}{6E}$  respectively for the same loadings. In the frame of Fig. 5, all slopes and deflections are respectively  $\frac{W \cdot h}{2E}$  and  $\frac{5 W \cdot h}{6E}$  for a single load which acts at the top.

In general, one is able to design frames which have any desired slopes and deflections. Thus for example, for the frame of a single bay, fixed at its base, the values of  $K$  may be determined by the following formulae:

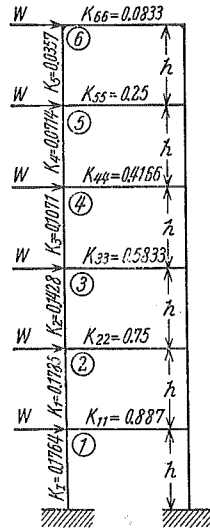


Fig. 4.

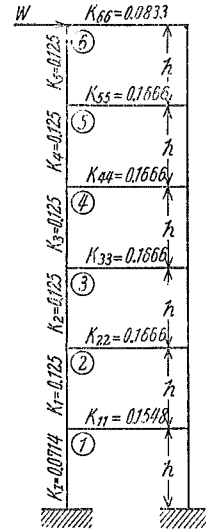


Fig. 5.

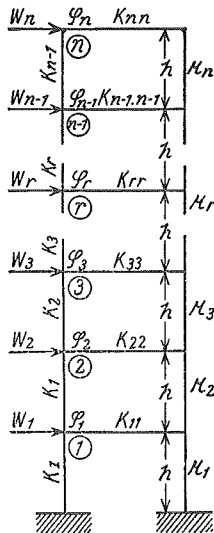


Fig. 6.

$$\left. \begin{aligned}
 K_I &= \frac{\frac{R_1}{2}}{\frac{2}{3}\mu_1 + \varphi_1} \\
 K_I &= \frac{\frac{R_2}{2}}{\frac{2}{3}\mu_2 + (\varphi_1 + \varphi_2)} \\
 K_2 &= \frac{\frac{R_3}{2}}{\frac{2}{3}\mu_3 + (\varphi_2 + \varphi_3)} \\
 &\dots\dots\dots \\
 K_{n-1} &= \frac{\frac{R_n}{2}}{\frac{2}{3}\mu_n + (\varphi_{n-1} + \varphi_n)}
 \end{aligned} \right\} \dots\dots (1)$$



$$\left. \begin{aligned}
 K_{11} &= -\frac{2\varphi_1(K_1+K_1)+\varphi_2 K_1+\mu_1 K_1+\mu_2 K_1}{3\varphi_1} \\
 K_{22} &= -\frac{2\varphi_2(K_1+K_2)+\varphi_1 K_1+\varphi_3 K_2+\mu_2 K_1+\mu_3 K_2}{3\varphi_2} \\
 K_{33} &= -\frac{2\varphi_3(K_2+K_3)+\varphi_2 K_2+\varphi_4 K_3+\mu_3 K_2+\mu_4 K_3}{3\varphi_3} \\
 &\dots\dots\dots \\
 K_{nn} &= -\frac{2\varphi_n K_{n-1}+\varphi_{n-1} K_{n-1}+\mu_n K_{n-1}}{3\varphi_n}
 \end{aligned} \right\} \dots\dots (2)$$

In the above equations

$K$  = stiffness of member ;

$\varphi$  = slope multiplied by  $2E$  ;

$\mu$  = deflection multiplied by  $-6E$  ;

$$R_r = -\frac{1}{3}Q_r h_r$$

where,

$Q_r$  = total shearing force in the  $r$ -th story ;

$h_r$  = height of the column in the  $r$ -th story ;

In equations (1) and (2), there must be the following relations between  $\mu$  and  $\varphi$  :

$$\left. \begin{aligned}
 \frac{2}{3}|\mu_1| &> \varphi_1 \\
 \frac{2}{3}|\mu_2| &> \varphi_1+\varphi_2 \\
 \frac{2}{3}|\mu_3| &> \varphi_2+\varphi_3 \\
 &\dots\dots\dots \\
 \frac{2}{3}|\mu_n| &> \varphi_{n-1}+\varphi_n
 \end{aligned} \right\} \dots\dots\dots (3)$$

In the special case of

$$\varphi_1 = \varphi_2 = \varphi_3 = \dots\dots\varphi_n = \varphi$$

and

$$\mu_1 = \mu_2 = \mu_3 = \dots\dots\mu_n = \mu$$

equations (1), (2) and (3) become

$$\left. \begin{aligned} K_1 &= \frac{\frac{R_1}{2}}{\frac{2}{3}\mu + \varphi}, \\ K_1 &= \frac{\frac{R_2}{2}}{\frac{2}{3}\mu + 2\varphi}, \\ K_2 &= \frac{\frac{R_3}{2}}{\frac{2}{3}\mu + 2\varphi}, \\ &\dots\dots\dots \\ K_{n-1} &= \frac{\frac{R_n}{2}}{\frac{2}{3}\mu + 2\varphi}. \end{aligned} \right\} \dots\dots\dots (4)$$

$$\left. \begin{aligned} K_{11} &= -\frac{(3\varphi + \mu)(K_1 + K_1)}{3\varphi} + \frac{K_1}{3} \\ K_{22} &= -\frac{(3\varphi + \mu)(K_1 + K_2)}{3\varphi} \text{ or } -\frac{R_2 + R_3}{4\varphi} \\ K_{33} &= -\frac{(3\varphi + \mu)(K_2 + K_3)}{3\varphi} \text{ or } -\frac{R_3 + R_4}{4\varphi} \\ &\dots\dots\dots \\ K_{nn} &= -\frac{(3\varphi + \mu)K_{n-1}}{3\varphi} \text{ or } -\frac{R_n}{4\varphi} \end{aligned} \right\} \dots\dots\dots (5)$$

and

$$|\mu| > 3\varphi \dots\dots\dots (6)$$

Again, for the frame of a single bay with hinged base,

$$K_1 = \frac{R_1}{\frac{\mu_1}{3} + \varphi_1} \dots\dots\dots (7)$$

$$K_I = \frac{\frac{R_1}{2}}{\frac{2}{3}\mu + 2\varphi},$$

$$K_{II} = -\frac{R_1 + R_2}{4\varphi},$$

and 
$$K_{II} = -\frac{R_1}{4\varphi}.$$

The expressions of stiffnesses of the rest are the same as in the frame of fixed base.

When the model of frame constructed of rectangular elements is papered with thin Japanese paper like a Japanese paper sliding door and subjected to a statical force system, the deformation of the model is slightly disturbed by this paper covering. But wrinkles due to slope and deflection of frame can be observed on the paper covering with the naked eye. And also when this model with paper covering is placed on a shaking platform which can be shaken to any desired amplitude and period and is subjected to oscillation, it is possible to observe the amount of slope and deflection in each story of the frame by the size of the wrinkles on the paper as shown in Fig. 8. This figure was a snapshot while the frame was shaking.

In the writers' models the distributions and densities of wrinkles on paper due to both statical and seismic force are much the same. Wrinkles in the story of large slope and deflection are larger than those in the story of small slope and deflection.

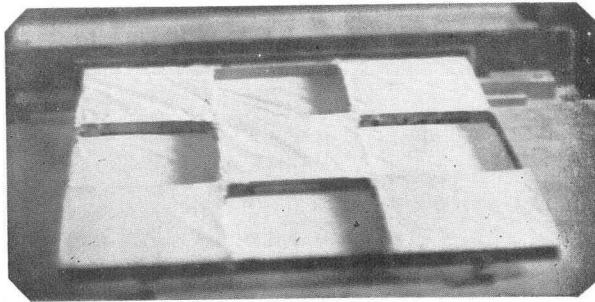


Fig. 8.

Now when this consideration is extended to buildings subjected to seismic action and the above theory of the paper screen is considered to be applicable to concrete building walls, the position where cracks appear most early and severely is in a story where elastic deformation, that is, slope and deflection have the maximum values in all stories. At the same time, for ordinary building frames the maximum deflec-

tion may be expected probably to take place between the second and fourth stories as related before. Therefore, it may be recognized that the damage to walls of buildings due to earthquake will probably appear most early and severely in the second to fourth stories and not in the lowest story, notwithstanding that the free vibration period is considerably smaller than the period of forced vibration of earthquake.

This theory agrees with Dr. Taniguchi's in substance. The authors pay their homage to his eminent opinion which was proposed not long after the Great Kwanto Earthquake of 1923.

But for the damage of the second order it is not always most severe in the second to fourth stories. In regard to the damage of the second order, it is treated in the following section.

It seems that Dr. Taniguchi called public attention to the maximum deflection alone, while the present authors call public attention to the maximum joint rotation-angle, i.e. the maximum slope too, as the bending moment at the end of a member of a frame is derived from slope and deflection and this end moment is important to the second order damage. The slope has generally the maximum value in the story of the maximum deflection as recognized from Tables 1 to 5 and Figs. 19 to 50.

## II. Investigation on the Location of Failure due to Damage of the Second Order.

(1) *Introduction.* Damage of the second order is taken to be such as when columns and girders are broken as well as when walls are largely cracked and shaken down. With the occurrence of damage of the first order, columns and girders can continue to exist in sound bodies, imperfect as they are, but in the case of damage of the second order they can not exist longer in sound bodies.

The breakage of the columns and girders has close relation to the stresses which are induced in their bodies. But actual stress distribution and the state of damages to building frames are complicated problems concerned with the nature of earthquake motion, kind of structures, distribution of loads, state of foundation etc.

So, neglecting these complicated boundary conditions, the authors, using many models, have investigated experimentally the location and state of the failure of building frames due to a simple harmonic motion. Of course that experiments with real building frames are desirable. But there are many difficulties in the construction of many

homogeneous models, testing facilities and economy for experiments with real building frames. The authors therefore obliged to satisfy themselves with models as next described.

(2) *Materials for Model of Building Frames.* As the authors' experiment is concerned with a steel skeleton or reinforced concrete building frame, it seemed to be desirable to use concrete models. But, it is difficult to make up a small and homogeneous model with concrete. The homogeneity of the model is a very important factor in this experiment and it is not always necessary to use concrete for this purpose. Any material will do, so long as with it one can easily construct homogeneous models of any desired form provided that the material is so brittle as easily to break like concrete. This brittleness is also a very important point in this experiment, as the experiment is concerned with the breaking of the model.

The present authors used a good quality of gypsum, namely "dental plaster". A model of any desired form can be very easily made having a high degree of homogeneity with this gypsum.

The mechanical properties of gypsum vary, depending upon the percentage of water used in mixing the gypsum milk, the completeness of drying out and the kind of gypsum, that is, the ingredients in the gypsum and the process of calcination used. From 64 to 120 percent, water was mixed for model making and the age of the plaster cast ranged from 2 to about 20 days.

Test pieces of 30 cm. length, 1.5 cm. width and 1 cm. thickness were made for some models of building frames for the purpose of measurement of the mechanical properties. The results are set down in the tables of the experimental results in an after coming section.

For working loads the modulus of elasticity  $E$  of gypsum ranges from about 20000 to 30000 kg. per sq. cm., the weight from 0.65 to 1.25 gr. per cub. cm. and the bending strength from about 20 to 30 kg. per sq. cm.

Here the bending strength means to value of  $\frac{M}{I}e$ , where  $M$  is the maximum bending moment by which a test piece is broken,  $I$  the moment of inertia of the section and  $e$  the distance of the extreme fibre from the neutral axis of the section.

The tensile strength was not measured directly but it may be estimated from the bending strength of materials.

(3) *Model Making.* The model of the building frame was made as follows. Forms were made of wooden pieces on a thick wooden plank and were wiped with an oil before using. Then a thick milk of gypsum was poured into these forms. About 100 percent of water was required to produce good results in model making. After the gypsum had quite set, the upper face of the model was shaved to a plane and then the forms were removed. The time for the complete setting of gypsum was 5 to 15 minutes. The time for the natural drying out of models was from 2 days to 3 weeks as already mentioned, but for most of the models this time was from 4 days to one week.

(4) *Form and Dimensions of Models.* As the standard model, heights of columns were determined all equally at 10 cm., lengths of girders at 15 cm. and their sections at 1 cm. thickness by 1.5 cm. width.

The number of stories was 6 and number of bays was from 1 to 3.

Fig. 9 shows the standard form and dimensions of a model of 6 stories and 2 bays. The form and dimensions of all models used in the present experiments are shown in Tables 6 to 18 in a later section.

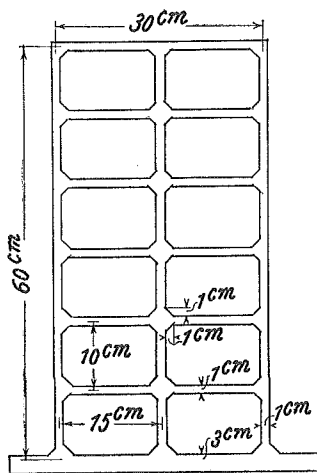


Fig. 9.

A fine wire was embedded in the centre of the section of all the members in such a manner as to prevent the complete falling to pieces during the breaking test.

(5) *Equipment and Method of Experiment.* The model was set up horizontally and supported on rollers which were fitted to a shaking platform and the base of the model was fixed to the platform so as to be subjected to harmonic oscillation as shown in Fig. 10.

In the experiment the amplitude of oscillation was kept to a constant magnitude of 1.5 cm. and the period was changed gradually until the model was destroyed. The period of oscillation was measured with a tachometer.

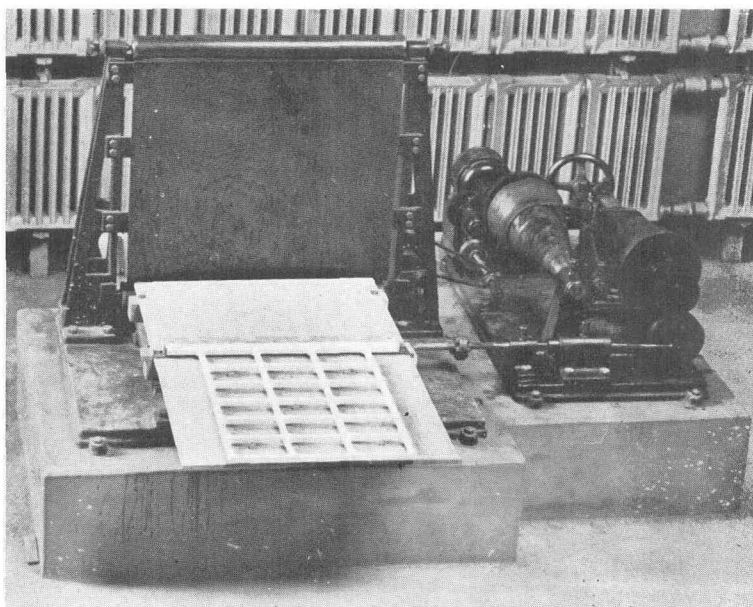


Fig. 10.

(6) *Experimental Results.* When every story was equally loaded with load of  $m$  gr., the periods of free vibration for the models of standard form were, for the most part, such as are expressed by the following formulae;

$$T = 0.0046\sqrt{m} \quad \text{for the model of 6 stories and single bay,}$$

$$T = 0.0034\sqrt{m} \quad \text{for the model of 6 stories and 2 bays,}$$

$$T = 0.0029\sqrt{m} \quad \text{for the model of 6 stories and 3 bays.}$$

In the above formulae  $T$  is the period and it is indicated in seconds.

For the story-loads of 450 gr., 310 gr., and 50 gr. the period of free vibration of the model of 6 stories and single bay becomes 0.0975 sec., 0.081 sec. and 0.033 sec. respectively. The weight of the model itself was 50 gr. for each story and the period of the free vibration due to the weight of the model itself became therefrom 0.033 sec.

Figs. 11 to 13 show the mode of the free vibration of the model of the standard form with 6 stories and single bay. These vibration curves were recorded with optical apparatus.



In the experiments, the period of external vibration which was applied to a model to destroy it was, for the most part, from 0.12 to 0.22 sec. which is fairly long compared with that of the free vibration of the model. Consequently the mode of the vibration of a model under the forced vibration was the same as that of the shaking platform, excepting the increase of the amplitude as shown in Figs. 16 and 17.

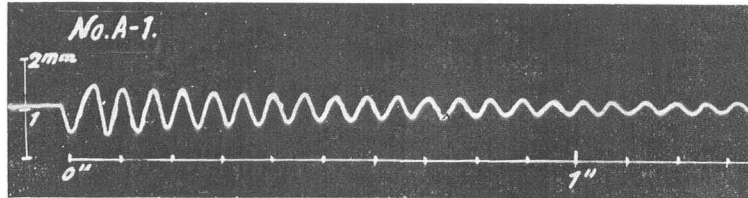


Fig. 11. Frame No. A: 6 stories and single bay (Age: 3 days).  
Live load: 130 gr. on each story. Dead load: 50 gr. on each story.  
Free vibration period: 0.0615 sec.

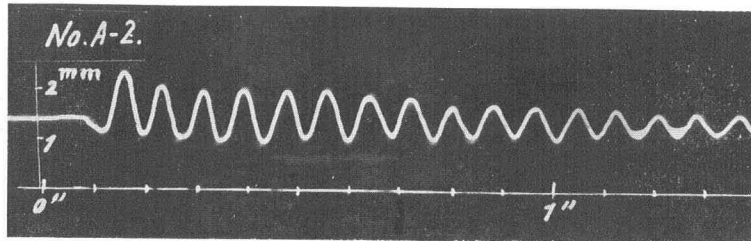


Fig. 12. Frame No. A: 6 stories and single bay (Age: 3 days).  
Live load: 260 gr. on each story. Dead load: 50 gr. on each story.  
Free vibration period: 0.0815 sec.

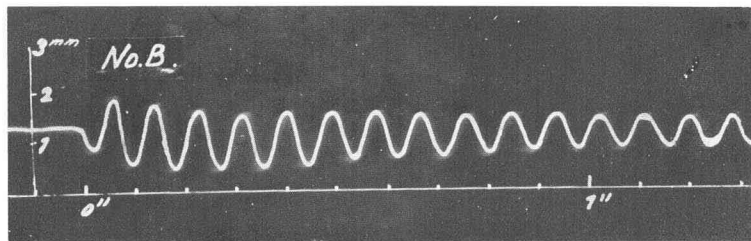


Fig. 13. Frame No. B: 6 stories and single bays (Age: 13 days).  
Live load: 400 gr. on each story. Dead load: 50 gr. on each story.  
Free vibration period: 0.0900 sec.

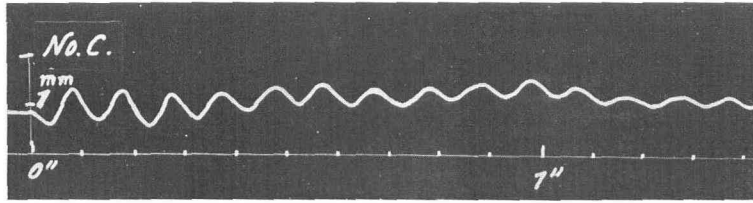


Fig. 14. Frame No. C: 6 stories and 2 bays (Age: 6 days).

Live load: 780 gr. on each story. Dead load: 100 gr. on each story.

Free vibration period: 0.100 sec.

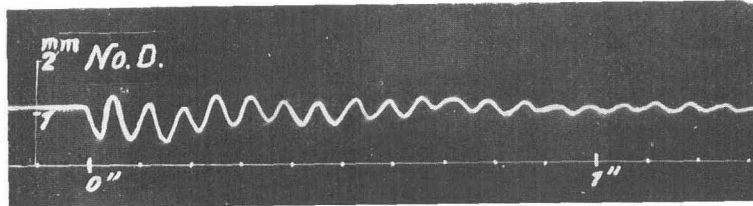


Fig. 15. Frame No. D: 6 stories and 3 bays (Age: 6 days).

Live load: 390 gr. on each story. Dead load: 120 gr. on each story.

Free vibration period: 0.066 sec.

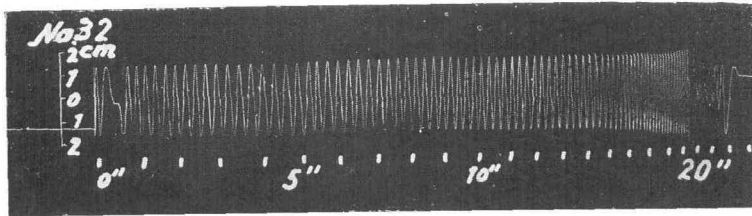


Fig. 16. Frame No. 32: 6 stories and single bay (Age: 7 days).

Live load: 100 gr. on the 5th story. Dead load: 50 gr. on each story.

Amplitude of shaking platform: 1.5 cm.

Destruction period: 0.171 sec.

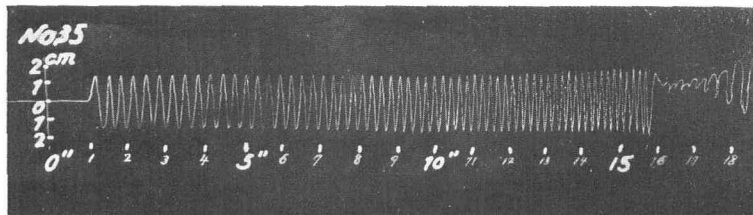


Fig. 17. Frame No. 35: 6 stories and 3 bays (Age: 4 days).

Live load: 0 gr. Dead load: 120 gr. on each story.

Amplitude of shaking platform: 1.5 cm.

Destruction period: 0.158 sec.

Experimental results of the breaking test of the frame are as tabulated in Tables 6 to 18. In the Appendix, there are shown 101 photographs of the features of the damage to the frame.

The deformations of every story and the bending moments at the ends of every member due to a horizontal oscillation may be estimated from the assumption that seismic forces are concentrated horizontally at every story in the same direction with each other having the magnitude of the product of the total mass on each story and the acceleration in each story.

This assumption may be permissible for the frames whose free vibration period is considerably smaller than the period of an earthquake motion such as the frames used in this experiment.

To repeat, for the authors' experiments, the accelerations of the horizontal oscillations of all stories might be assumed to be approximately equal to each other, as the horizontal deformation of the frames is very small compared with the amplitude of the shaking platform.

Fig. 19 to 50 show the results of the statical calculation under the above stated assumption. In these figures, slopes are written in the parentheses at the corresponding joints and deflections at the right side of the corresponding stories. The values at both ends of each member in Fig. 19 to 26 show the bending moments and those in Figs. 27 to 50 the bending stresses. In regard to signs of bending moments, the moment is considered positive when the couple acts in a clockwise direction upon the portion of the member considered and also the sign of bending stress indicates the direction of moment, by which bending stress is caused.

Now, comparing with the results of the statical calculation, the present authors propose to describe the features of the damages to frames, dividing their models into several kinds.

#### *A. Standard Frames.*

(a) *Frames of Six Stories and Single Bay.* Fig. 19 shows that the second story has the maximum deflection and in the girders at the top and bottom of this story very large moments are induced compared with the others. The values of these bending moments are respectively  $2.01 W \cdot h$  and  $2.07 W \cdot h$ . Also it seems that the bending moment at the fixed ends of the lowest columns is the largest among the columns, having the value of  $1.94 W \cdot h$ .  $W$  shows the intensity of a seismic force at every joint and  $h$  the height of column.

As the magnitude of the direct stresses in all the members is relatively small, the extreme fibre stresses of the members may be

estimated from the bending moments only at the ends of members neglecting the effect of the direct stresses.

In the experiments, three characteristic sorts of damage were usual to frames as shown in Ref. Nos. 1 to 6 in Table 6. Fig. 18 shows these 3 types of damage.

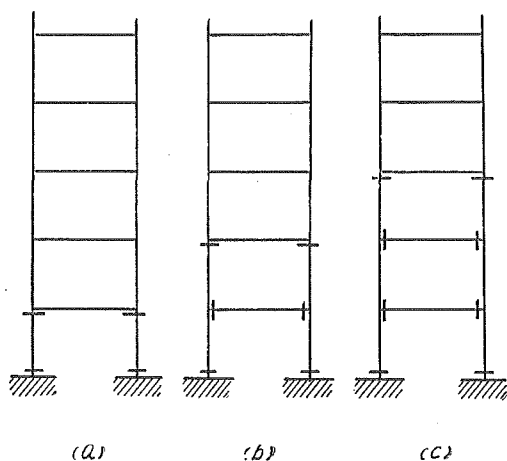


Fig. 18.

These features of damage are all considered to take place respectively as the magnitudes of the moments at the first and second girders and the lowest columns are nearly equal to each other.

The features of the damage to the frame of Ref. No. 4 show the possibility of the extension of the damage to the third girder itself as the result of the faultiness in model making.

(b) *Frames of Six Stories and Single Bay Carrying a Load on Upper Story.* Figs. 20 and 21 show the results of the statical calculation for two frames carrying a load on the top and fifth girders respectively. As the effect of a load on the upper girder, the position of the maximum deflection changes from the second to the third story and at the ends of the girders of both top and bottom of this story the maximum bending moment is induced.

In this case, the moment at the fixed end of the lowest columns is comparatively smaller than that of the second and third girders and accordingly there may be only slight damage of type (a). In the experiments, the features of damages belonged, for the most part, to type (c) as shown in Ref. Nos. 7 to 13 in Tables 6 and 7.

There were occasionally such damages as shown in Ref. Nos. 7 and 8, that is, in these frames none of the members in the first story suffered from injury but they suffered destruction at both of the ends of the second and third girders, at the upper ends of the columns in the fourth story and at the lower ends of the columns in the second story.

(c) *Frames of Six Stories and Two Bays.* The results of the statical calculation become as shown in Figs. 22 and 23 and it is observed that the weakest points are at the fixed ends of the lowest columns and at the ends of the first and second girders each; accordingly all the features of damages of types (a), (b) and (c) may appear.

In the experiments, type (b) damage has most frequently taken place.

(d) *Frames of Six Stories and Three Bays.* For the frame of 6 stories and 3 bays, the results of the statical calculation become as shown in Figs. 24 and 25. Similarly to the above case, it is observed that the weakest points are at the fixed ends of the lowest columns and at both ends of the first and second girders; accordingly it is possible that all the features of damages of types (a), (b) and (c) may appear.

In the experiments, type (b) damage has most frequently been observed and types (a) and (c) occasionally likewise.

When a frame of 3 bays is subjected to only one force on upper story, the maximum deflection takes place upper rather than when it is subjected to forces at every joint. But, the magnitudes of the moments at the fixed ends of the lowest columns and ends of the first girders are not so different from those of the girders which belong to the story of the maximum deflection as in a frame of single bay.

Therefore, even when the frame of 3 bays is loaded on the upper story with a comparatively large load, the features of damage are like those for the frame subjected to loads at every joint. In the experiments, the features of damages took the type of (a) as shown in Ref. Nos. 24 and 25.

#### *B. Frames of Irregular Form, but of Standard Section.*

Ref. Nos. 26 to 28 show the frame of irregular form but whose members have the standard section each and Fig. 26 shows the results of the statical calculation of the frame of Ref. No. 26.

Even in such frames, they were also destroyed at the expected points, considering after the results of the statical calculation, too.

#### *C. Effects of the Stiffness Ratio of Girders and Columns on Features of Damage.*

(a) *Frames of Six Stories and Single Bay.* Ref. Nos. 29 to 41 in Tables 9 to 11 show the effect of the stiffness ratio of girders and

columns on the features of damages to the frames of 6 stories and single bay.

Ref. Nos. 29 to 31 in Table 9 show effect upon frames with girder of 1.5 cm. thickness and in them the stiffness ratio of girder and column is 2.25.

The features of damage to these frames have coincided with type (a) in the experiments, that is, the frames suffered destruction at both ends of the lowest columns, while all other members did not suffer from injury.

The results of the statical calculation of the frames of this sort become as shown in Fig. 27 and they show that the magnitude of the bending stress at the fixed ends of the lowest columns is rather large compared with the other ones, notwithstanding the fact that the deflection takes the maximum value at the second story. Therefore, it may be fully expectable that the features of damage were of type (a).

In the frames of Ref. Nos. 32 to 33, the girders are all 1 cm. thick and stiffness ratio of girders and columns 0.666. In the experiments, every damage type (a), (b) and (c) appeared in the features of the damage to these frames as mentioned already.

Ref. Nos. 34 to 36 show the frames with girders and columns of respectively 2 cm. and 1.5 cm. thickness and their stiffness ratio is 0.282. In the experiments, the failure of girders has extended to the upper ones. For example, in the frames of Ref. Nos. 34 and 35 this failure has extended up to the fourth girder.

In the frames of Ref. Nos. 38 to 41, the stiffness ratios are smaller and in the experiments the failure of girders has extended to all of them.

It may therefore be concluded that the smaller the stiffness ratio of the girders and columns becomes, the higher in the building the failure of girders extends. And it is quite within the bounds of possibility that the failure extends so far as the topmost girder dependent upon the magnitude of the stiffness ratio of girder and column.

Such features of the damages may be reasonable considering from the results of the statical calculation in Figs. 28 to 31. For example, in the frame with girders and columns of respectively 1 cm. and 3 cm. thickness, the bending stress in the topmost girder is greater than that in the fixed ends of the lowest columns. Therefore, before the stress in the fixed ends of the lowest columns reaches the breaking point, all the girders may be destroyed.

(b) *Frames of Six Stories and Two Bays.* Ref. Nos. 42 to 44 show the effect of the stiffness ratio of girders and columns upon the features of damage to the frames of 6 stories and 2 bays.

Ref. No. 42 shows a frame with girders 1.5 cm. thick and the stiffness ratio of 2.25, Ref. No. 43 a frame with girders 1 cm. thick and the stiffness ratio of 0.666 and Ref. No. 44 a frame with columns 1.5 cm. thick and the stiffness ratio of 0.198. In the experiment with the frame of Ref. No. 42, none of the girders suffered from injury but the frame suffered destruction at the bottom ends of the lowest columns.

In the frame of Ref. No. 43 the failure extended to the first girder and in the frame of Ref. No. 44 it extended to the second girder. The results of the statical calculation of these frames are shown in Figs. 32 to 34 and these results show the reasonableness of such features of the damage.

(c) *Frames of Six Stories and Three Bays.* Ref. Nos. 45 to 56 show the effect of the stiffness ratio of girders and columns on the features of damages to the frames of 6 stories and 3 bays. Their stiffness ratios vary from 2.25 to 0.0833. Ref. Nos. 45 to 47 show frames with the stiffness ratio of 2.25, Ref. Nos. 48 to 50 frames with the stiffness ratio of 0.666, Ref. Nos. 51 to 53 frames with the stiffness ratio of 0.198 and Ref. Nos. 54 to 56 the frames with stiffness ratio of 0.0833.

In the frames with the stiffness ratio of 2.25, no girder suffered from injury but the frames suffered destruction at both ends of the lowest columns. In the frames with the stiffness ratio of 0.666, some were destroyed at the lowest columns and every girder escaped from injury, while in other frames the failure extended to the second or even to the third girders. In the frames with the stiffness ratio of 0.198, the failure extended to the second or even to the third girders. In the frames with the stiffness ratio of 0.0833, the damage extended to the second and third or even to the fourth girders.

The results of the statical calculation for these frames are shown in Figs. 35 to 38 and they indicate the possibility of the above mentioned features of damages.

In the frame of 3 bays, the effect of the stiffness ratio of girders and columns on the features of damage is smaller than that in the frame of a single bay.

*D. Partially Stiffened Frames.*

(a) *Frames Specially Stiffened by Partial Rooms.* Ref. Nos. 57 to 62 show the features of the damages to the frames specially stiffened by partial room which consist of members of 2 cm. thickness. Among them, frames of Ref. Nos. 57 to 59 have the especial stiffening of one room at the left side of the lowest story.

Thanks to this stiffening room the lowest story did not suffer from injury at all and these frames were destroyed at the second story in almost all cases.

Frames of Ref. Nos. 60 and 61 have two rooms for stiffening at the left side of the lowest and second stories. In the experiments with these frames, also the first and second stories were not injured at all.

When the stiffening rooms were placed in the space from the lowest story to the third, the first and second stories were destroyed as shown in Ref. No. 62.

The above features of the damages all seem to coincide with the results of the statical calculation of Figs. 39 and 40.

(b) *Frames Specially Stiffened by Partial Columns.* Ref. Nos. 63 to 79 in Tables 13 to 15 show the features of damages to the frames specially stiffened by partial columns which consist of 3 cm. thickness. Among them, the frames of Ref. Nos. 63 to 65 have one stiffening column at the left side of the lowest story.

On account of this stiffening column, in some cases the lowest story did not suffer from injury at all, while in others the frames were destroyed at the first and second stories.

Next, in Ref. Nos. 66 to 79 there are shown the features of damages to frames with various sorts of specially stiffening partial columns. They show that the damages to the frames with such columns extend to the upper stories and the damages of the same kind occur to the frames whose columns all consist of thick members. It is an interesting fact that even when the stiffening columns are arranged as shown in Ref. Nos. 77 to 79, the damages extend considerably to the upper stories.

In Figs. 41 to 43 there are shown some of the results of the statical calculation for these frames.

(c) *Frames Specially Stiffened by Partial Girders.* In Ref. Nos. 80 to 90 are shown the features of damages to the frames specially stiffened by partial girders which consist of members 2 cm. thick.



Frames of Ref. Nos. 80 to 82 have an especially stiffening girder at the lowest part. With this girder, the lowest room of these frames of single bay is stiffened as a whole. Therefore, in the experiments, the frames were more frequently destroyed at the upper stories than at the first story.

For the frames of 3 bays, the lowest story could not be sufficiently strengthened with specially stiffening girders alone, however stiff they might be, and accordingly, in the experiments the frames were always destroyed at the lowest columns. In these cases, on the other hand, the specially stiffening girders prevented the extension of the damage to the upper girders as shown in Ref. Nos. 83 to 90.

In Figs. 44 to 47 there are shown the results of the statical calculation of these frames and these results indicate that the above experimental results are all reasonable.

(d) *Frames with Columns and Girders of Different Stiffness in Each Story.* In the frames of Ref. Nos. 91 to 95, all members of the first and second stories have the thickness of 2 cm., those of the third and fourth stories the thickness of 1.5 cm. and those of the fifth and sixth stories the thickness of 1 cm. .

For these frames the results of the statical calculation become as shown in Figs. 48 and 49. These calculation results show that the most dangerous points are the fixed ends of the lowest columns when the frame carry no loads. When the frames carry loads on every story both ends of the columns in the fifth story become also the most dangerous points.

The experimental results with frames of Ref. Nos. 91, 92, 94 and 95 were all in accord with the calculation results. The features of damage to the frame of Ref. No. 93 were different from the above results, while the fact that all fibre stresses in the lowest columns and the first to third girders are similar in their magnitude shows the reasonableness of such features of damage.

(e) *Frames with Members of Uniform Strength.* Frames of Ref. Nos. 96 to 101 are so designed that all the members may have equal strength against horizontal seismic forces. In the experiments on such frames, the features of damage took no fixed form and the frames were destroyed in a haphazard as shown in Ref. Nos. 96 to 101.

## Summary and Conclusions.

In this experiment, the building frames were dealt with which have a free vibration period comparatively smaller than the period of the earthquake motion. The general conclusion drawn from the investigations described in this paper are as follows:

In an earthquake of such a degree that structures do not suffer from injury of the main frame, that is to say in the definition of the present authors, in an earthquake of the first order, the position where cracks appear most early and severely is in a story where the deflection or the slope has the maximum value. Here, the words "deflection" and "slope" mean the member revolution angle and the joint rotation angle respectively. For the ordinary building frames the maximum deflection most probably occurs between the second and fourth stories.

Therefore, it might be recognized as reasonable that the failure of the walls of buildings due to an earthquake appears most early and severely in the second to fourth stories and not in the lowest story, even though the free vibration period is considerably smaller than that of the earthquake motion.

In an earthquake so strong that the main frames of structures are destroyed, that is to say in the definition of the present authors, in an earthquake of the second order, the most dangerous positions among all members of girders and columns have no close relation to the position of the maximum deflection or slope. When the stiffness of the girders is comparatively greater than that of the columns, the frames are always destroyed at the lowest columns. When the stiffness of the girders is comparatively smaller than that of the columns the damages extend to the upper stories and in some cases damages extend up as far as the topmost girder.

These features of the damages to the main frames seem all to coincide with the results of the statical calculation which is usually employed in the current practice.

Therefore, it seems to be reasonable that the damages to the main frames themselves should extend to the upper stories even in some rigid reinforced concrete buildings whose free vibration period is considerably smaller than that of the earthquake motion.

The features of damages to the various kinds of frames in the experiments are shown in Tables 6 to 18 and in the Appendix, Photographs 1 to 14.

Table 6.

— Breaking point.

○ Position of maximum deflection.

• Position of maximum deflection due to a single load.

× Position of maximum bending stress.

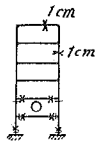
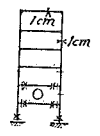
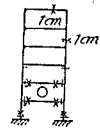
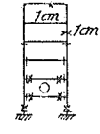
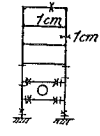
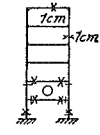
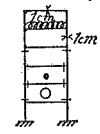
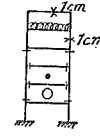
| Ref. No. | Frame No. | Type of frame & features of damage  | Date of construction & test  | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks   |
|----------|-----------|---|------------------------------|--------------------|----------------|--------------------------|---|
| (1)      | No. 11    |    | 1935 :<br>Oct. 12<br>Oct. 29 | —                  | 1.5            | —                        |   |
| (2)      | No. 17    |    | Nov. 2<br>Nov. 4             | 64                 | 1.5            | 0.182                    | Test piece :<br>$E = 25200 \text{ kg/cm}^2$<br>bending strength<br>= $21.75 \text{ kg/cm}^2$<br>weight = $1.25 \text{ kg/cm}^3$ |
| (3)      | No. 20    |    | Nov. 5<br>Nov. 7             | 76                 | 1.5            | 0.193                    |   |
| (4)      | No. 22    |   | Nov. 5<br>Nov. 7             | 76                 | 1.5            | 0.176                    |   |
| (5)      | No. 24    |  | Nov. 8<br>Nov. 11            | 67                 | 1.5            | 0.176                    | Test piece :<br>$E = 25200 \text{ kg/cm}^2$<br>bending strength<br>= $24 \text{ kg/cm}^2$<br>weight : $1.14 \text{ gr/cm}^3$    |
| (6)      | No. 25    |  | Nov. 8<br>Nov. 11            | 67                 | 1.5            | 0.187                    | Test piece :<br>$E = 27000 \text{ kg/cm}^2$<br>bending strength<br>= $24 \text{ kg/cm}^2$<br>weight = $1.14 \text{ kg/cm}^3$    |
| (7)      | No. 9     |  | Oct. 10<br>Oct. 29           | —                  | 3              | —                        | Load on 5th girder<br>= 100 gr.   |
| (8)      | No. 15    |  | Oct. 14<br>Oct. 31           | —                  | 1.5            | —                        | Load on 5th girder<br>= 100 gr.   |

Table 7.

— Breaking point.

○ Position of maximum deflection.

• Position of maximum deflection due to a single load.

× Position of maximum bending stress.

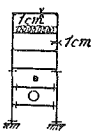
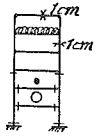
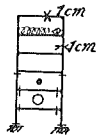
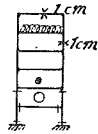
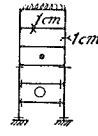
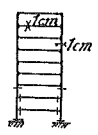
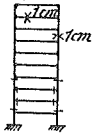
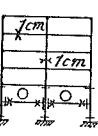
| Ref. No. | Frame No. | Type of frame & features of damage  | Date of construction & test | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks   |
|----------|-----------|---|-----------------------------|--------------------|----------------|--------------------------|---|
| (9)      | No. 27    |    | 1935 :<br>Nov. 9<br>Nov. 15 | 120                | 1.5            | 0.230                    | Load on 5th girder = 200 gr.  |
| (10)     | No. 31    |    | Nov. 12<br>Nov. 19          | 125                | 1.5            | 0.206                    | Load on 5th girder = 240 gr.<br>Test piece :<br>$E = 25000 \text{ kg/cm}^2$<br>bending strength = $18 \text{ kg/cm}^2$<br>weight = $0.7 \text{ gr/cm}^3$    |
| (11)     | No. 26    |    | Nov. 9<br>Nov. 15           | 120                | 1.5            | 0.171                    | Load on 5th girder = 100 gr.<br>Test piece :<br>$E = 28000 \text{ kg/cm}^2$<br>bending strength = $18 \text{ kg/cm}^2$<br>weight = $0.7 \text{ gr/cm}^3$    |
| (12)     | No. 32    |   | Nov. 12<br>Nov. 19          | 125                | 1.5            | 0.206                    | Load on 5th girder = 240 gr.<br>Test piece :<br>$E = 22000 \text{ kg/cm}^2$<br>bending strength = $18 \text{ kg/cm}^2$<br>weight = $0.65 \text{ gr/cm}^3$   |
| (13)     | No. 19    |  | Nov. 2<br>Nov. 4            | 64                 | 1.5            | 0.190                    | Load on 6th girder = 75 gr.<br>Test piece :<br>$E = 25200 \text{ kg/cm}^2$<br>bending strength = $21.75 \text{ kg/cm}^2$<br>weight = $1.25 \text{ gr/cm}^3$ |
| (14)     | No. 30    |  | Nov. 9<br>Nov. 14           | 120                | 1.5            | 0.222                    | Test piece:<br>$E = 27000 \text{ kg/cm}^2$<br>bending strength = $22.5 \text{ kg/cm}^2$<br>weight = $0.76 \text{ gr/cm}^3$                                  |
| (15)     | No. 29    |  | Nov. 9<br>Nov. 14           | 120                | 1.5            | 0.230                    | Test piece :<br>$E = 27000 \text{ kg/cm}^2$<br>bending strength = $22.5 \text{ kg/cm}^2$<br>weight = $0.75 \text{ gr/cm}^3$                                 |
| (16)     | No. 40    |  | Nov. 21<br>Nov. 26          | 85                 | 1.5            | 0.200                    |   |

Table 8.

— Breaking point.

○ Position of maximum deflection.

× Position of maximum bending stress.

| Ref. No. | Frame No. | Type of frame & features of damage | Date of construction & test | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks   |
|----------|-----------|------------------------------------|-----------------------------|--------------------|----------------|--------------------------|---|
| (17)     | No. 51    |                                    | 1935:<br>Dec. 6<br>Dec. 11  | 100                | 1.5            | 0.188                    |   |
| (18)     | No. 56    |                                    | Dec. 16<br>Dec. 26          | 120                | 1.5            | 0.194                    | Test piece:<br>bending strength<br>= 21 kg/cm <sup>2</sup><br>weight = 0.84 gr/cm <sup>3</sup>  |
| (19)     | No. 50    |                                    | Dec. 7<br>Dec. 18           | 120                | 1.5            | 0.163                    | Load on 5th girder<br>= 550 gr.   |
| (20)     | No. 34    |                                    | Nov. 16<br>Nov. 20          | 95                 | 1.5            | 0.158                    |   |
| (21)     | No. 35    |                                    | Nov. 16<br>Nov. 20          | 95                 | 1.5            | 0.158                    |   |
| (22)     | No. 45    |                                    | Nov. 28<br>Dec. 3           | 120                | 1.5            | 0.162                    |   |
| (23)     | No. 52    |                                    | Dec. 7<br>Dec. 18           | 120                | 1.5            | 0.166                    | Test piece:<br>bending strength<br>= 25.5 kg/cm <sup>2</sup><br>weight = 0.9 gr/cm <sup>3</sup> |
| (24)     | No. 42    |                                    | Nov. 22<br>Nov. 27          | 120                | 1.5            | 0.220                    | Load 400 gr.  |

Table 9.

— Breaking point.

○ Position of maximum deflection.

× Position of maximum bending stress.

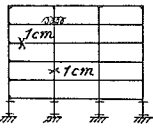
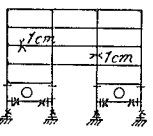
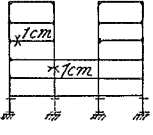
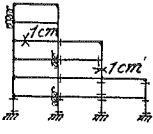
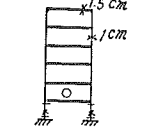
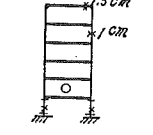
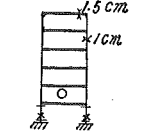
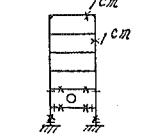
| Ref. No. | Frame No. | Type of frame & features of damage  | Date of construction & test | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks  |
|----------|-----------|---|-----------------------------|--------------------|----------------|--------------------------|--|
| (25)     | No. 43    |    | 1935:<br>Nov. 22<br>Nov. 27 | 120                | 1.5            | 0.206                    | Load on 5th girder<br>= 400 gr.  |
| (26)     | No. 54    |    | Dec. 14<br>Dec. 23          | 120                | 1.5            | 0.170                    | Test piece:<br>bending strength<br>= 18.6 kg/cm <sup>2</sup><br>weight = 0.73 gr/cm <sup>3</sup> |
| (27)     | No. 58    |    | Dec. 21<br>Dec. 26          | 120                | 1.5            | 0.170                    | Test piece:<br>bending strength<br>= 25.8 kg/cm <sup>2</sup><br>weight = 0.83 gr/cm <sup>3</sup> |
| (28)     | No. 57    |   | Dec. 16<br>Dec. 23          | 120                | 1.5            | 0.166                    | Load: 100 gr. each   |
| (29)     | No. 78    |  | 1936:<br>May 5<br>May 12    | 110                | 1.5            | 0.200                    | Stiffness ratio<br>$K_b/K_c = 2.25$  |
| (30)     | No. 112   |  | July 7<br>July 15           | 100                | 1.5            | 0.182                    | Stiffness ratio<br>$K_b/K_c = 2.25$  |
| (31)     | No. 113   |  | July 7<br>July 15           | 100                | 1.5            | 0.200                    | Stiffness ratio<br>$K_b/K_c = 2.25$  |
| (32)     | No. 11    |  | 1935:<br>Oct. 12<br>Oct. 29 | —                  | 1.5            | —                        | Stiffness ratio<br>$K_b/K_c = 0.666$   |

Table 10.

— Breaking point.

○ Position of maximum deflection.

× Position of maximum bending stress.

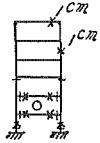
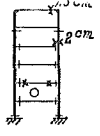
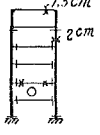
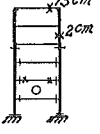
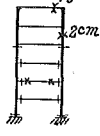
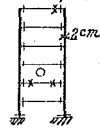
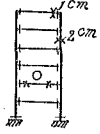
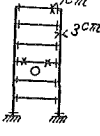
| Ref. No. | Frame No. | Type of frame & features of damage  | Date of construction & test | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks                              |
|----------|-----------|---|-----------------------------|--------------------|----------------|--------------------------|--------------------------------------|
| (33)     | No. 17    |    | 1935 :<br>Nov. 2<br>Nov. 4  | 64                 | 1.5            | 0.182                    | Stiffness ratio<br>$K_b/K_c = 0.666$ |
| (34)     | No. 72    |    | 1936 :<br>Apr. 24<br>May 4  | 100                | 1.5            | 0.176                    | Stiffness ratio<br>$K_b/K_c = 0.282$ |
| (35)     | No. 120   |    | July 10<br>July 15          | 100                | 1.5            | 0.230                    | Stiffness ratio<br>$K_b/K_c = 0.282$ |
| (36)     | No. 121   |   | July 10<br>July 15          | 100                | 1.5            | 0.214                    | Stiffness ratio<br>$K_b/K_c = 0.282$ |
| (37)     | No. 122   |  | Sept. 1<br>Sept. 4.         | 100                | 1.5            | 0.214                    | Stiffness ratio<br>$K_b/K_c = 0.282$ |
| (38)     | No. 71    |  | Apr. 24<br>May 4            | 110                | 1.5            | 0.158                    | Stiffness ratio<br>$K_b/K_c = 0.083$ |
| (39)     | No. 114   |  | July 7<br>July 15           | 100                | 1.5            | 0.166                    | Stiffness ratio<br>$K_b/K_c = 0.083$ |
| (40)     | No. 77    |  | May 5<br>May 12             | 110                | 1.5            | 0.171                    | Stiffness ratio<br>$K_b/K_c = 0.025$ |

Table 11.

— Breaking point.

○ Position of maximum deflection.

× Position of maximum bending stress.

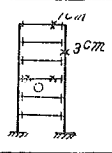
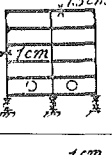
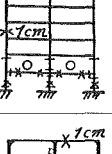
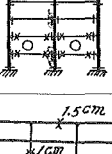
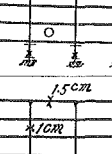
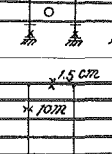
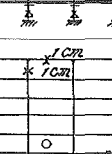

| Ref. No. | Frame No. | Type of frame & features of damage  | Date of construction & test  | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks                              |
|----------|-----------|---|------------------------------|--------------------|----------------|--------------------------|--------------------------------------|
| (41)     | No. 115   |    | 1936 :<br>July 7<br>July 15  | 110                | 1.5            | 0.162                    | Stiffness ratio<br>$K_b/K_c = 0.025$ |
| (42)     | No. 55    |    | 1935 :<br>Dec. 16<br>Dec. 26 | 100                | 1.5            | 0.166                    | Stiffness ratio<br>$K_b/K_c = 2.25$  |
| (43)     | No. 56    |    | Dec. 16<br>Dec. 26           | 120                | 1.5            | 0.194                    | Stiffness ratio<br>$K_b/K_c = 0.666$ |
| (44)     | No. 53    |   | Dec. 14<br>Dec. 23           | —                  | 1.5            | 0.170                    | Stiffness ratio<br>$K_b/K_c = 0.198$ |
| (45)     | No. 76    |  | 1936 :<br>May 2<br>May 6     | 110                | 1.5            | 0.230                    | Stiffness ratio<br>$K_b/K_c = 2.25$  |
| (46)     | No. 117   |  | July 8<br>July 14            | 100                | 1.5            | 0.200                    | Stiffness ratio<br>$K_b/K_c = 2.25$  |
| (47)     | No. 119   |  | July 10<br>July 14           | 100                | 1.5            | —                        | Stiffness ratio<br>$K_b/K_c = 2.25$  |
| (48)     | No. 50    |  | 1935 :<br>Dec. 7<br>Dec. 18  | 120                | 1.5            | 0.166                    | Stiffness ratio<br>$K_b/K_c = 0.666$ |



Table 12.

— Breaking point.

○ Position of maximum deflection.

× Position of maximum bending stress.

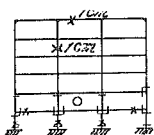
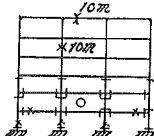
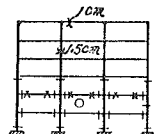
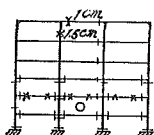
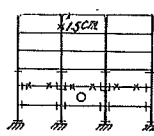
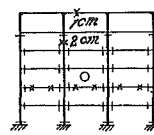

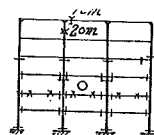
| Ref. No. | Frame No. | Type of frame features of damage  | Date of construction & test | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks                              |
|----------|-----------|---|-----------------------------|--------------------|----------------|--------------------------|--------------------------------------|
| (49)     | No. 45    |    | 1935 :<br>Nov. 28<br>Dec. 3 | 120                | 1.5            | 0.162                    | Stiffness ratio<br>$K_b/K_c = 0.666$ |
| (50)     | No. 34    |    | Nov. 16<br>Nov. 20          | 95                 | 1.5            | 0.158                    | Stiffness ratio<br>$K_b/K_c = 0.666$ |
| (51)     | No. 75    |    | 1936 :<br>May 2<br>May 6    | 110                | 1.5            | 0.206                    | Stiffness ratio<br>$K_b/K_c = 0.198$ |
| (52)     | No. 87    |   | June 3<br>June 16           | 110                | 1.5            | 0.158                    | Stiffness ratio<br>$K_b/K_c = 0.198$ |
| (53)     | No. 105   |  | June 30<br>July 14          | 100                | 1.5            | 0.158                    | Stiffness ratio<br>$K_b/K_c = 0.198$ |
| (54)     | No. 74    |  | May 2<br>May 6              | 110                | 1.5            | 0.193                    | Stiffness ratio<br>$K_b/K_c = 0.083$ |
| (55)     | No. 111   |  | July 6<br>July 14           | 100                | 1.5            | 0.200                    | Stiffness ratio<br>$K_b/K_c = 0.083$ |
| (56)     | No. 109   |  | July 4<br>July 14           | 100                | 1.5            | 0.200                    | Stiffness ratio<br>$K_b/K_c = 0.083$ |



Table 14.

— Breaking point.

○ Position of maximum deflection.

× Position of maximum bending stress.

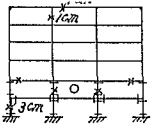
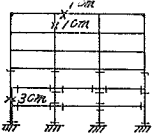
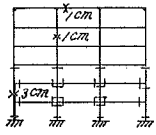
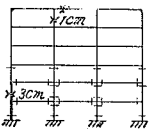
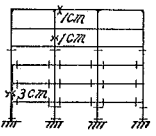
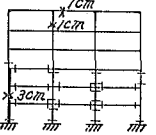
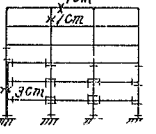
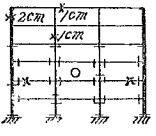
| Ref. No. | Frame No. | Type of frame & features of damage  | Date of construction & test  | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks           |
|----------|-----------|---|------------------------------|--------------------|----------------|--------------------------|-------------------|
| (65)     | No. 86    |    | 1936 :<br>June 11<br>June 17 | 110                | 1.5            | 0.180                    | Stiffening column |
| (66)     | No. 80    |    | May 8<br>May 16              | 110                | 1.5            | 0.200                    | Stiffening column |
| (67)     | No. 90    |    | May 30<br>June 17            | 110                | 1.5            | 0.133                    | Stiffening column |
| (68)     | No. 98    |   | June 20<br>June 29           | 110                | 1.5            | 0.176                    | Stiffening column |
| (69)     | No. 81    |  | May 8<br>May 10              | 110                | 1.5            | 0.176                    | Stiffening column |
| (70)     | No. 88    |  | May 30<br>June 17            | 100                | 1.5            | 0.161                    | Stiffening column |
| (71)     | No. 92    |  | June 11<br>June 18           | 110                | 1.5            | 0.166                    | Stiffening column |
| (72)     | No. 59    |  | Feb. 24<br>Mar. 4            | —                  | 1.5            | 0.171                    | Stiffening column |

Table 15.

— Breaking point.

○ Position of maximum deflection.

× Position of maximum bending stress.

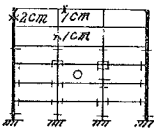
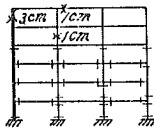

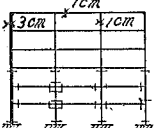
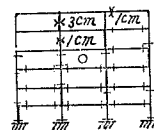
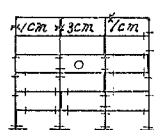

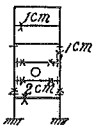
| Ref. No. | Frame No. | Type of frame & features of damage  | Date of construction & test | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks           |
|----------|-----------|---|-----------------------------|--------------------|----------------|--------------------------|-------------------|
| (73)     | No. 60    |    | 1936 :<br>Feb. 27<br>Mar. 4 | —                  | 1.5            | 0.200                    | Stiffening column |
| (74)     | No. 82    |    | May 11<br>May 16            | 110                | 1.5            | 0.200                    | Stiffening column |
| (75)     | No. 93    |    | June 10<br>June 18          | 110                | 1.5            | 0.206                    | Stiffening column |
| (76)     | No. 94    |   | June 11<br>June 18          | 110                | 1.5            | 0.206                    | Stiffening column |
| (77)     | No. 83    |  | May 11<br>May 16            | 110                | 1.5            | 0.171                    | Stiffening column |
| (78)     | No. 89    |  | June 3<br>June 17           | 110                | 1.5            | 0.166                    | Stiffening column |
| (79)     | No. 91    |  | June 11<br>June 18          | 110                | 1.5            | 0.181                    | Stiffening column |
| (80)     | No. 125   |  | Nov. 6<br>Nov. 12           | 100                | 1.5            | 0.162                    | Stiffening column |

Table 16.

— Breaking point.

○ Position of maximum deflection.

× Position of maximum bending stress.

| Ref. No. | Frame No. | Type of frame & features of damage | Date of construction & test | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks           |
|----------|-----------|------------------------------------|-----------------------------|--------------------|----------------|--------------------------|-------------------|
| (81)     | No. 129   |                                    | 1936 :<br>Nov. 6<br>Nov. 12 | 100                | 1.5            | 0.176                    | Stiffening girder |
| (82)     | No. 126   |                                    | Nov. 9<br>Nov. 12           | 100                | 1.5            | 0.172                    | Stiffening girder |
| (83)     | No. 123   |                                    | Nov. 5<br>Nov. 13           | 100                | 1.5            | 0.139                    | Stiffening girder |
| (84)     | No. 127   |                                    | Nov. 9<br>Nov. 13           | 100                | 1.5            | 0.182                    | Stiffening girder |
| (85)     | No. 134   |                                    | Nov. 18<br>Nov. 30          | 100                | 1.5            | 0.150                    | Stiffening girder |
| (86)     | No. 124   |                                    | Nov. 5<br>Nov. 13           | 100                | 1.5            | 0.171                    | Stiffening girder |
| (87)     | No. 128   |                                    | Nov. 9<br>Nov. 20           | 100                | 1.5            | 0.120                    | Stiffening girder |
| (88)     | No. 130   |                                    | Nov. 11<br>Nov. 20          | 100                | 1.5            | 0.143                    | Stiffening girder |

Table 17.

× Position of maximum bending stress.

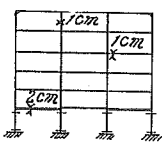
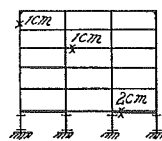
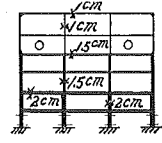
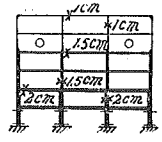
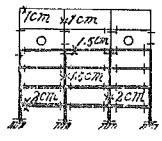
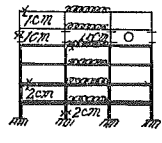
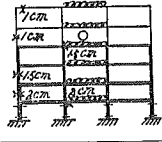
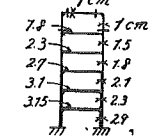
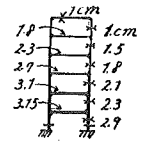
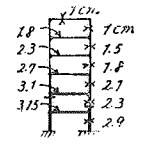
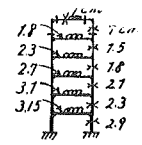
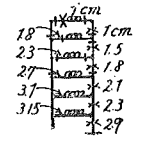
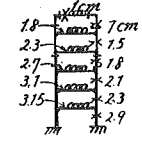
| Ref. No. | Frame No. | Type of frame & features of damage  | Date of construction & test  | Per-cent. of water | Ampli-tude cm. | Destruction period sec. | Remarks                      |
|----------|-----------|---|------------------------------|--------------------|----------------|-------------------------|------------------------------|
| (89)     | No. 132   |    | 1636 :<br>Nov. 15<br>Nov. 20 | 100                | 1.5            | 0.171                   | Stiffening girder            |
| (90)     | No. 131   |    | Nov. 11<br>Nov. 20           | 100                | 1.5            | 0.182                   | Stiffening girder            |
| (91)     | No. 64    |    | Mar. 7<br>Mar. 10            | —                  | 1.5            | 0.162                   | Stiffening column and girder |
| (92)     | No. 65    |   | Mar. 7<br>Mar. 10            | —                  | 1.5            | 0.200                   | Stiffening column and girder |
| (93)     | No. 67    |  | Apr. 20<br>Apr. 30           | 125                | 1.5            | 0.181                   | Stiffening column and girder |
| (94)     | No. 63    |  | Feb. 28<br>Mar. 4            | —                  | 1.5            | 0.201                   | Stiffening column and girder |
| (95)     | No. 66    |  | Apr. 20<br>Apr. 30           | 125                | 1.5            | 0.166                   | Stiffening column and girder |
| (96)     | No. 95    |  | June 6<br>June 23            | 110                | 1.5            | 0.103                   | Uniform strength             |

Table 18.

— Breaking point.

○ Position of maximum deflection.

× Position of maximum bending stress.

| Ref. No. | Frame No. | Type of frame & features of damage  | Date of construction & test | Per-cent. of water | Ampli-tude cm. | Dest-ruction period sec. | Remarks          |
|----------|-----------|---|-----------------------------|--------------------|----------------|--------------------------|------------------|
| (97)     | No. 96    |    | 1936 :<br>June 9<br>June 23 | 110                | 1.5            | 0.138                    | Uniform strength |
| (98)     | No. 97    |    | June 4<br>June 23           | 110                | 1.5            | 0.094                    | Uniform strength |
| (99)     | No. 110   |    | July 4<br>Aug. 12           | 100                | —              | —                        | Uniform strength |
| (100)    | No. 116   |  | July 7<br>Aug. 18           | 100                | —              | 0.200                    | Uniform strength |
| (101)    | No. 118   |  | July 8<br>Aug. 12           | 100                | —              | 0.188                    | Uniform strength |

Slope, Deflection and Moment.

Coefficients:  
 Slope:  $W \cdot h / 2EK_c$   
 Deflection:  $W \cdot h / 6EK_c$   
 Moment:  $W \cdot h$ .

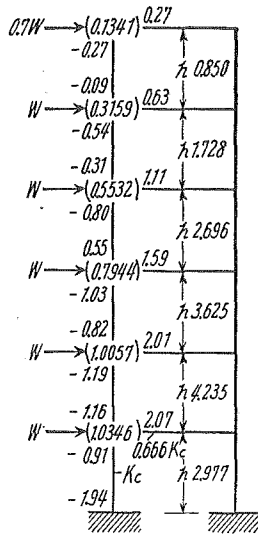


Fig. 19.

Slope, Deflection and Moment.

Coefficients:  
 Slope:  $W \cdot h / 2EK_c$   
 Deflection:  $W \cdot h / 6EK_c$   
 Moment:  $W \cdot h$ .

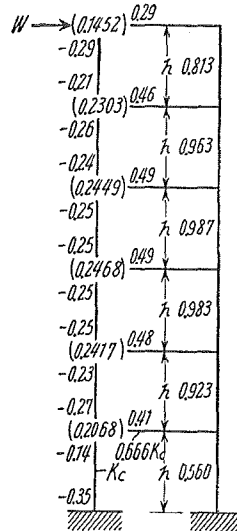


Fig. 20.

Slope, Deflection and Moment.

Coefficients:  
 Slope:  $W \cdot h / 2EK_c$   
 Deflection:  $W \cdot h / 6EK_c$   
 Moment:  $W \cdot h$ .

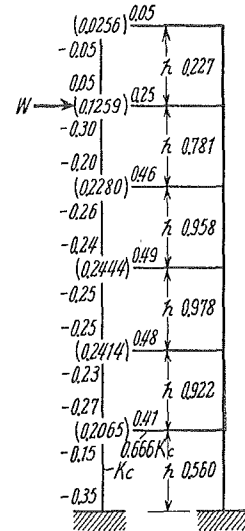


Fig. 21.

Slope, Deflection and Moment.

Coefficients:  
 Slope:  $W \cdot h / 2EK_c$   
 Deflection:  $W \cdot h / 6EK_c$   
 Moment:  $W \cdot h$ .

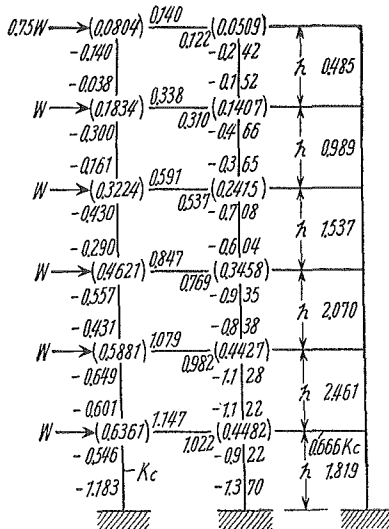


Fig. 22.

Slope, Deflection and Moment.

Coefficients:  
 Slope:  $W \cdot h / 2EK_c$   
 Deflection:  $W \cdot h / 6EK_c$   
 Moment:  $W \cdot h$ .

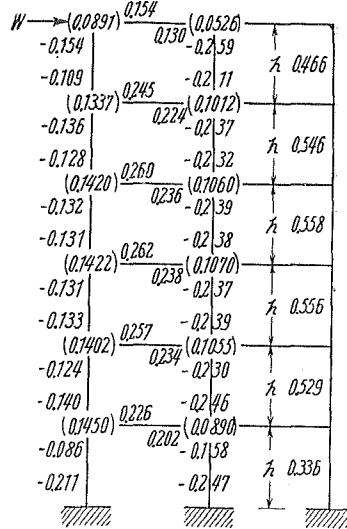


Fig. 23.



## Slope, Deflection and Moment.

Coefficients:

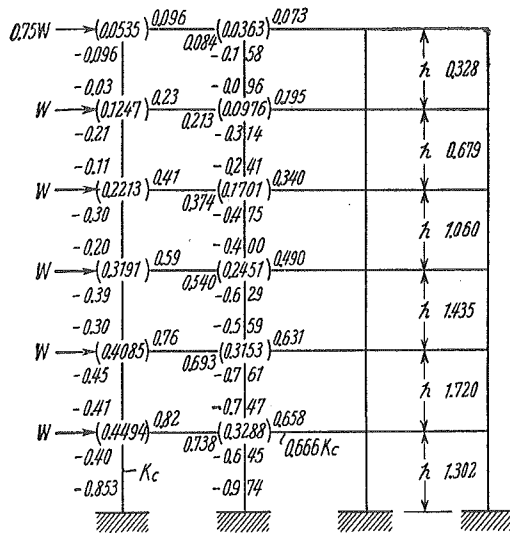
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Moment:  $W \cdot h$ .

Fig. 24.

## Slope, Deflection and Moment.

Coefficients:

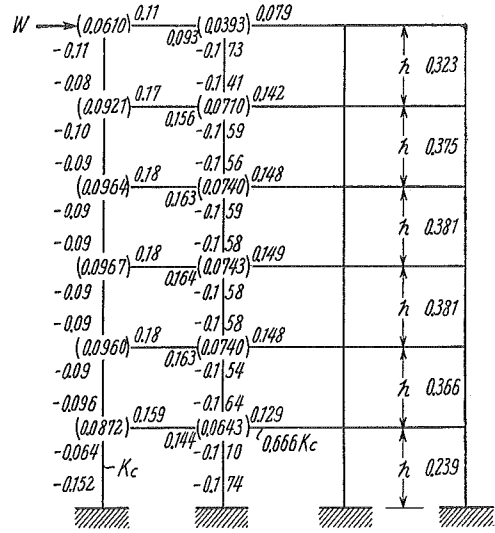
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Moment:  $W \cdot h$ .

Fig. 25.

## Slope, Deflection and Moment.

Coefficients:

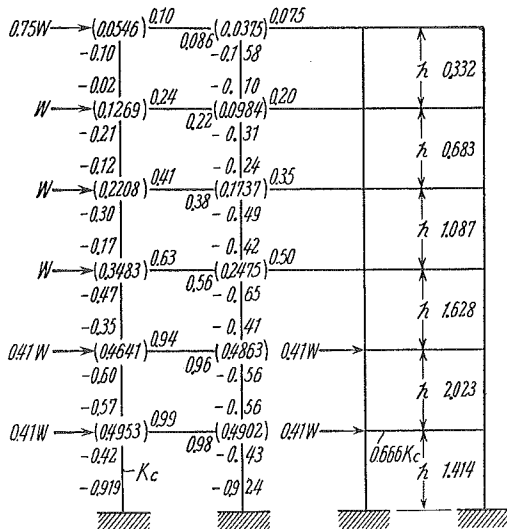
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Moment:  $W \cdot h$ .

Fig. 26.

## Slope, Deflection and Bending Stress.

Coefficients:

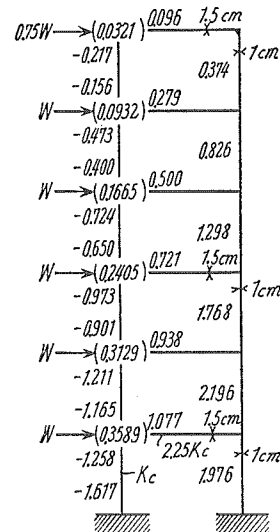
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$ . (kg./cm<sup>2</sup>)

Fig. 27.

Slope, Deflection and Bending Stress.

Coefficients:

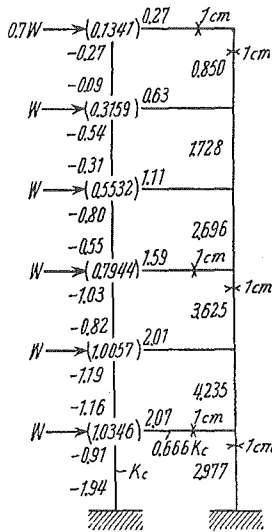
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 28.

Slope, Deflection and Bending Stress.

Coefficients:

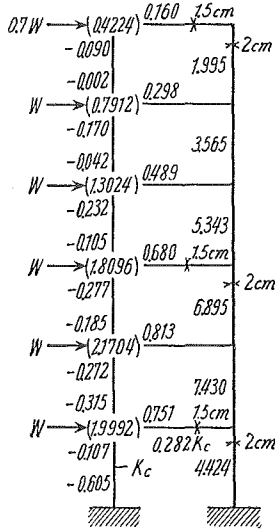
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 29.

Slope, Deflection and Bending Stress.

Coefficients:

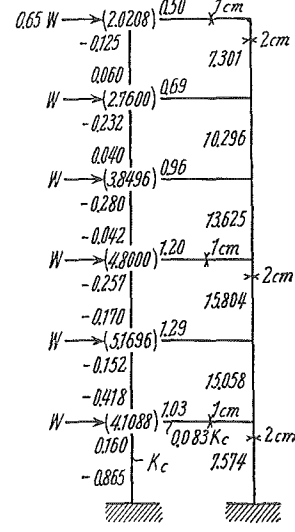
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 30.

Slope, Deflection and Bending Stress.

Coefficients:

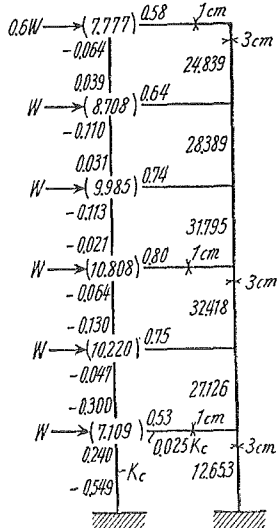
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 31.

Slope, Deflection and Bending Stress.

Coefficients:

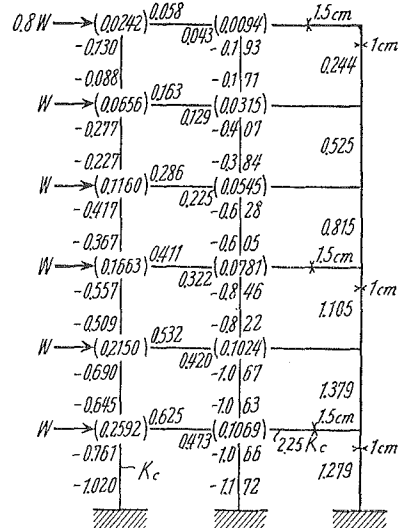
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 32.

### Slope, Deflection and Bending Stress.

Coefficients:

Slope :  $W \cdot h / 2EK_c$

Deflection :  $W \cdot h / 6EK_c$ 

Stress :  $6W \cdot h$  (kg/cm<sup>2</sup>)

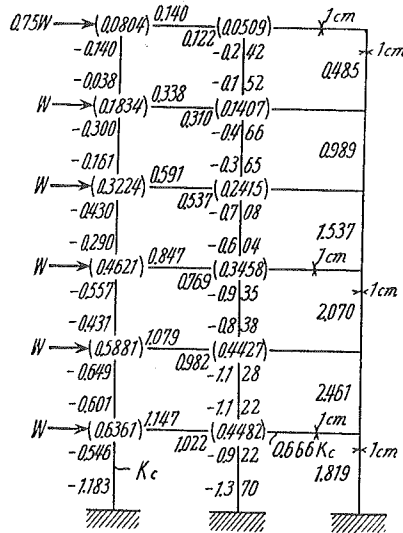


Fig. 33.

### Slope, Deflection and Bending Stress.

Coefficients :

Slope:  $W \cdot h / 2EK_c$

Deflection :  $W \cdot h / 6EK_c$

Stress:  $6Wh$  (kg/cm<sup>2</sup>)

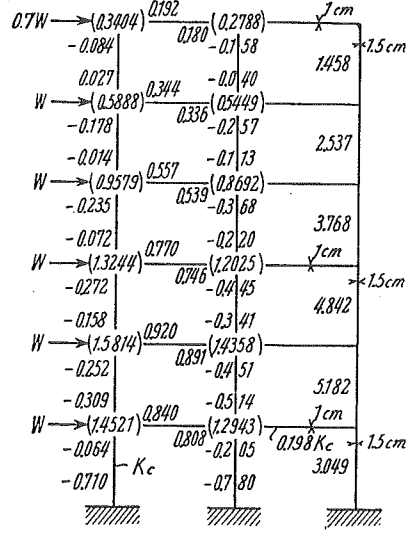


Fig. 34.

### Slope, Deflection and Bending Stress.

Coefficients :

Slope :  $W \cdot h / 2EK_c$

Deflection:  $W \cdot h / 6EK_c$ 

Stress :  $6 W \cdot h$  (kg/cm<sup>2</sup>)

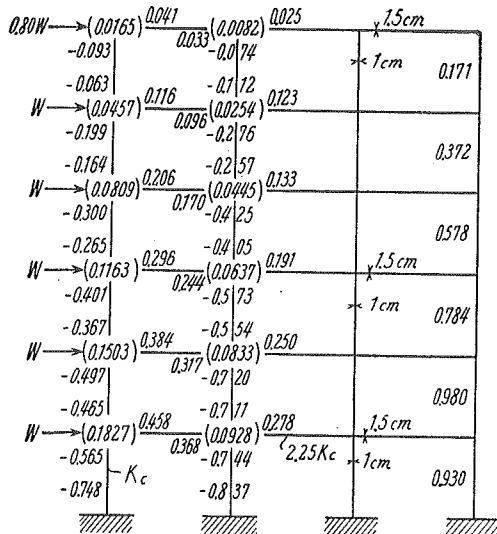


Fig. 35.

### Slope, Deflection and Bending Stress.

Coefficients :

Slope:  $W \cdot h / 2 E K_c$

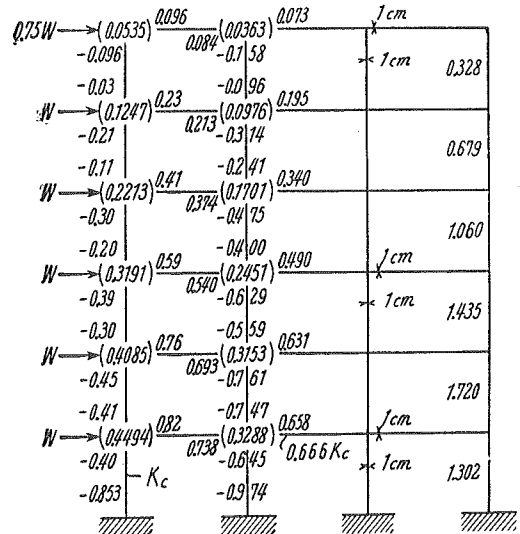
Deflection :  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 36.

## Slope, Deflection and Bending Stress.

Coefficients:

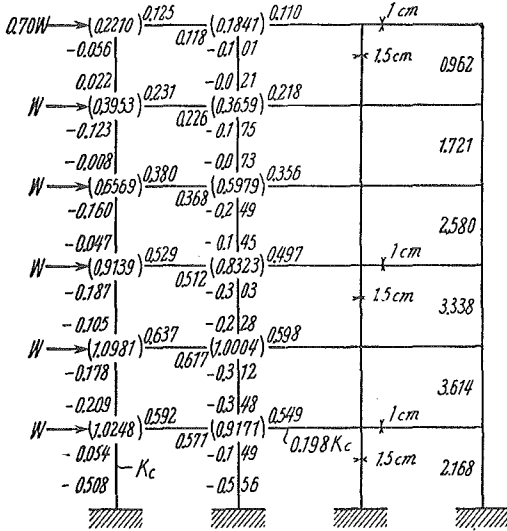
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (cm<sup>2</sup>)

Fig. 37.

## Slope, Deflection and Bending Stress.

Coefficients:

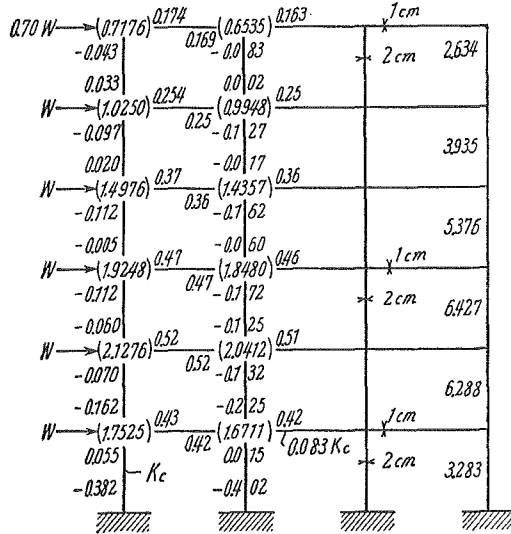
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 38.

## Slope, Deflection and Bending Stress.

Coefficients:

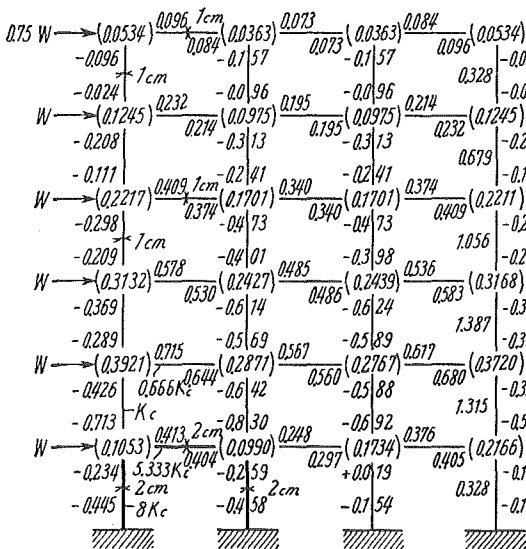
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 39.

## Slope, Deflection and Bending Stress.

Coefficients:

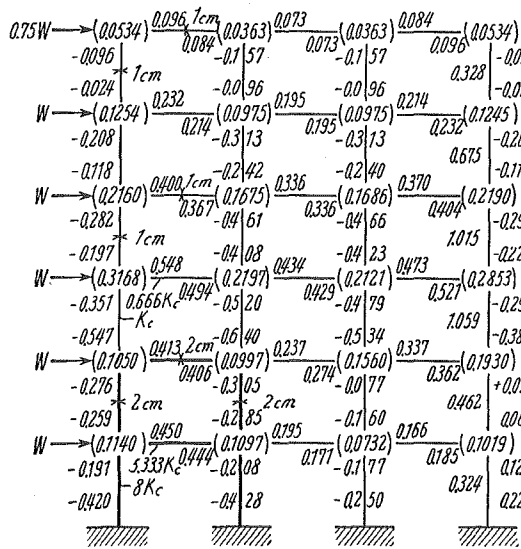
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 40.



## Slope, Deflection and Bending Stress.

Coefficients:

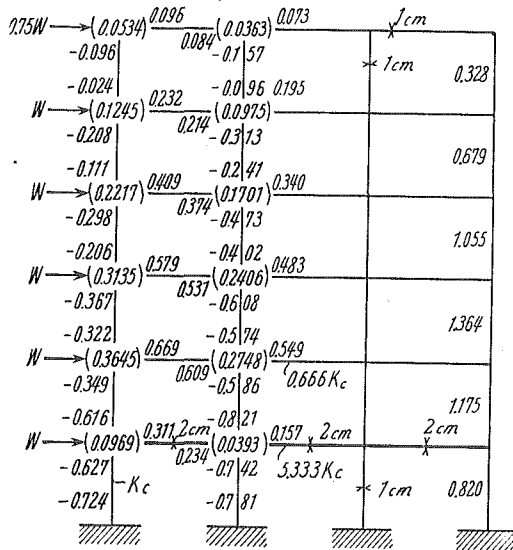
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 45.

## Slope, Deflection and Bending Stress.

Coefficients:

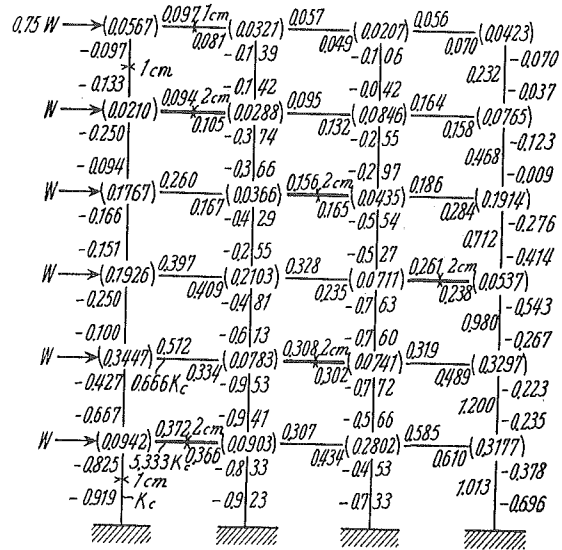
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 46.

## Slope, Deflection and Bending Stress.

Coefficients:

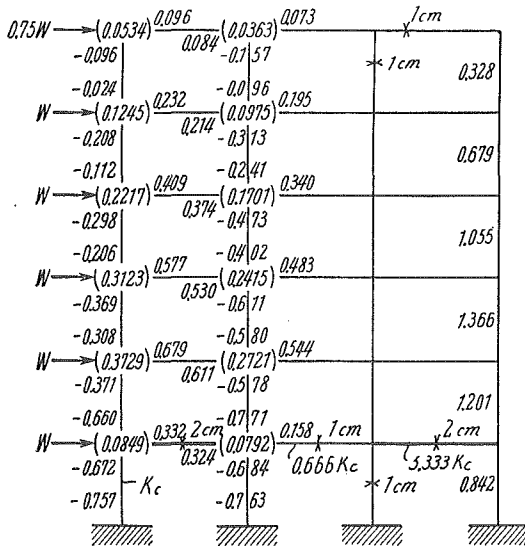
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 47.

## Slope, Deflection and Bending Stress.

Coefficients:

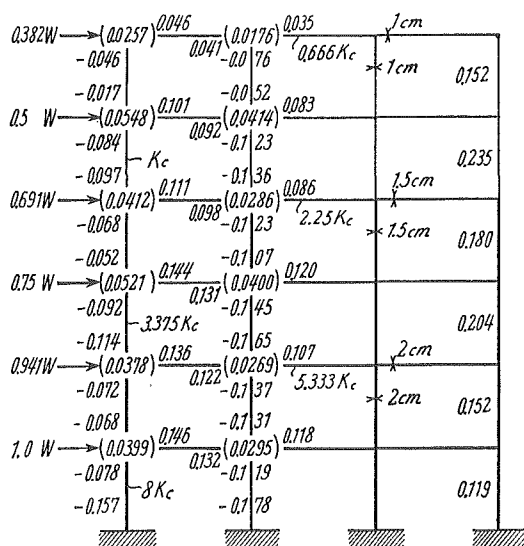
Slope:  $W \cdot h / 2EK_c$ Deflection:  $W \cdot h / 6EK_c$ Stress:  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 48.

Slope, Deflection and Bending Stress.

Coefficients :

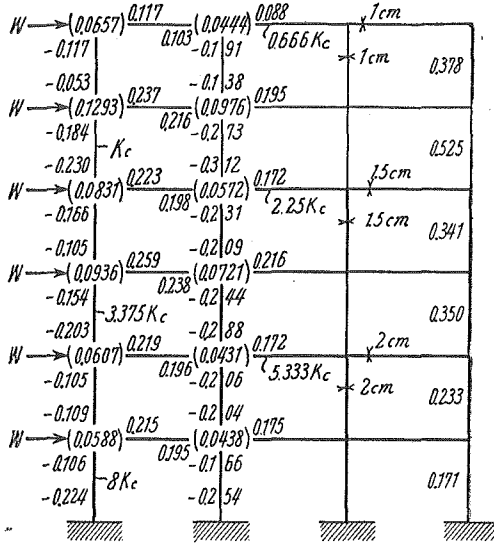
Slope :  $W \cdot h / 2EK_c$ Deflection :  $W \cdot h / 6EK_c$ Stress :  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 49.

Slope, Deflection and Bending Stress.

Coefficients :

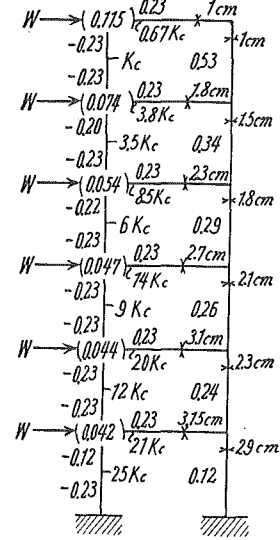
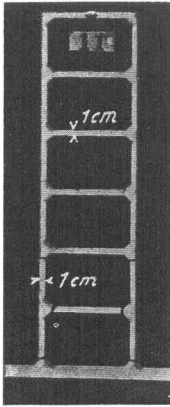
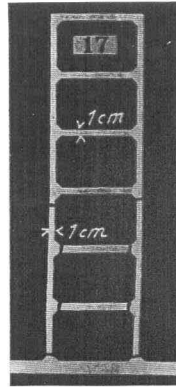
Slope :  $W \cdot h / 2EK_c$ Deflection :  $W \cdot h / 6EK_c$ Stress :  $6W \cdot h$  (kg/cm<sup>2</sup>)

Fig. 50.

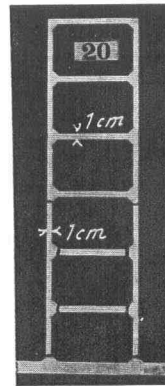
Appendix, Photograph 1.  
Features of Damage to Frames.



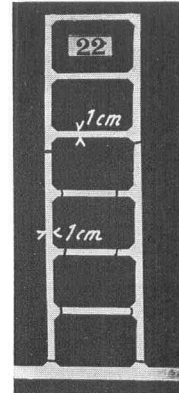
Ref. No. 1.



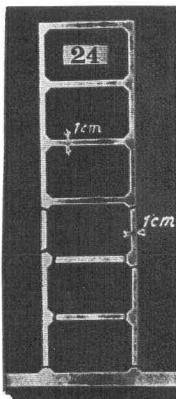
Ref. No. 2.



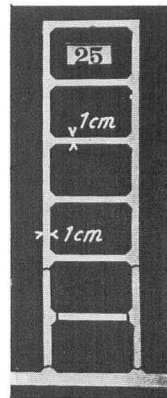
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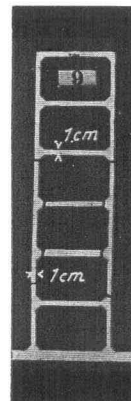
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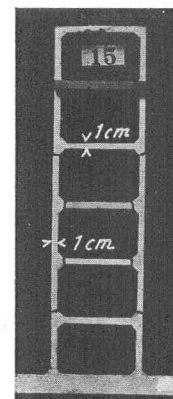
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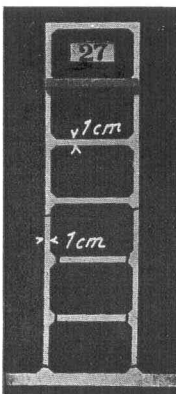
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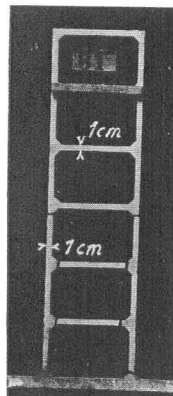
Ref. No. 7.



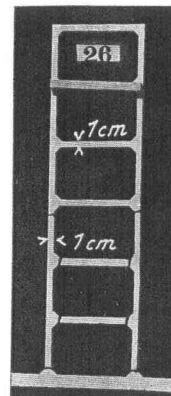
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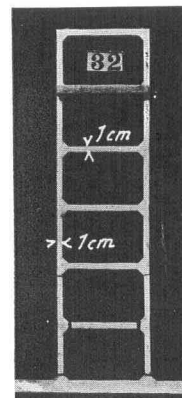
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Ref. No. 10.



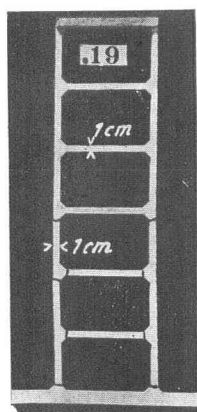
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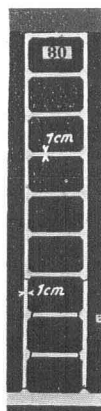
Ref. No. 12.



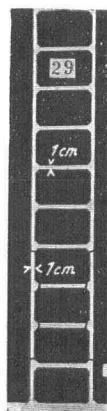
Appendix, Photograph 2.  
Features of Damage to Frames.



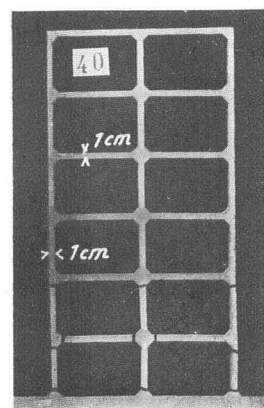
Ref. No. 13.



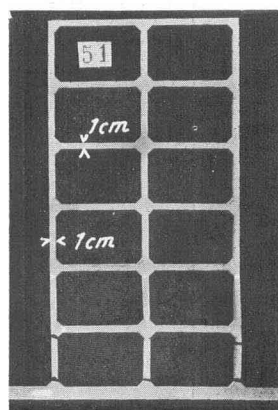
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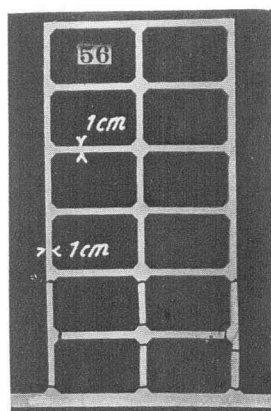
Ref. No. 15.



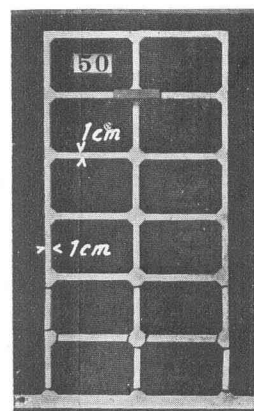
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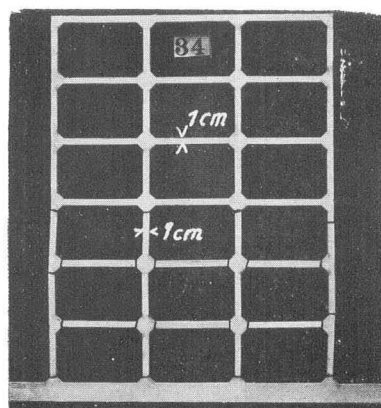
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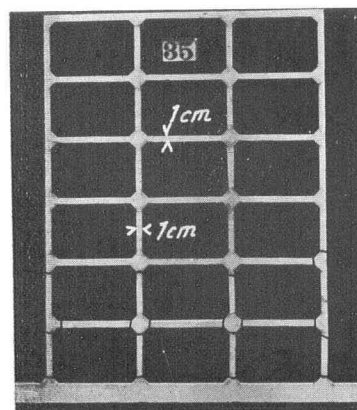
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Ref. No. 19.

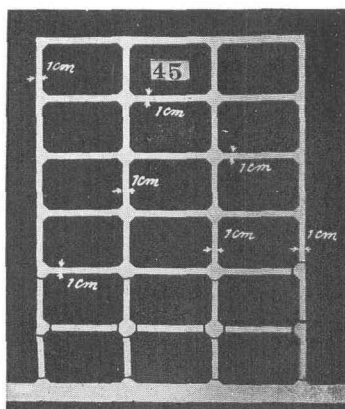


Ref. No. 20.

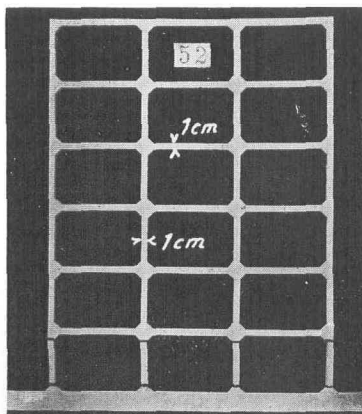


Ref. No. 21.

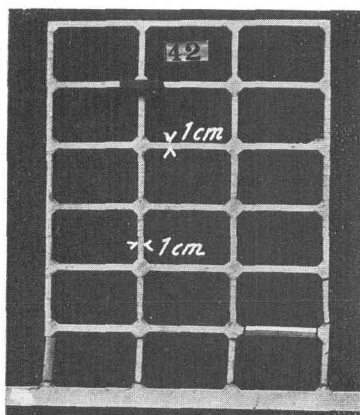
Appendix, Photograph 3.  
Features of Damage to Frames.



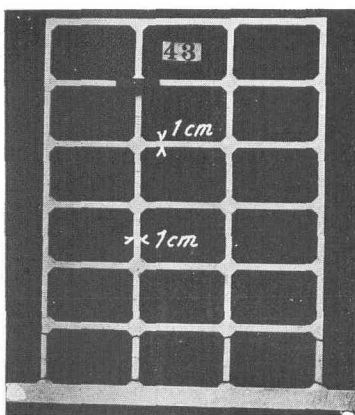
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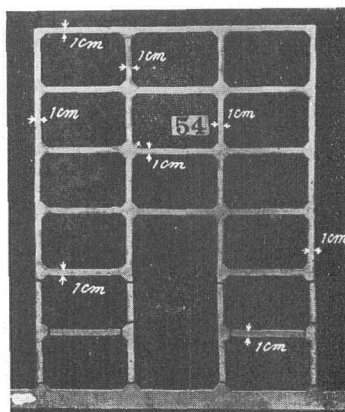
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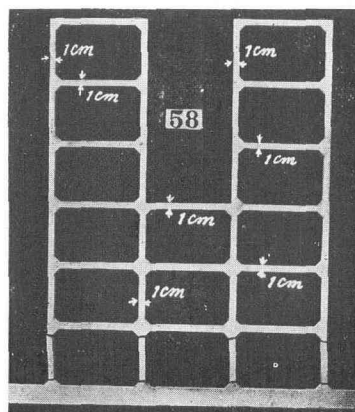
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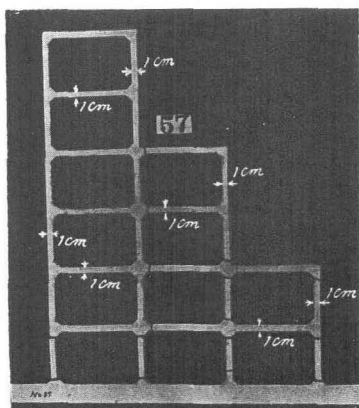


Ref. No. 26.

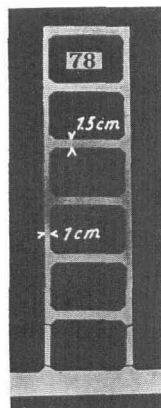


Ref. No. 27.

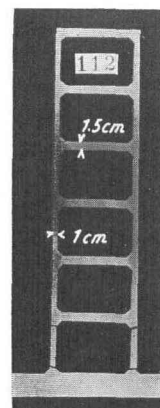
Appendix, Photograph 4.  
Features of Damage to Frames.



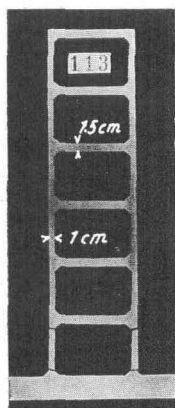
Ref. No. 28.



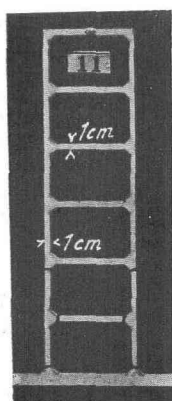
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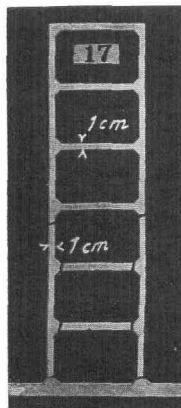
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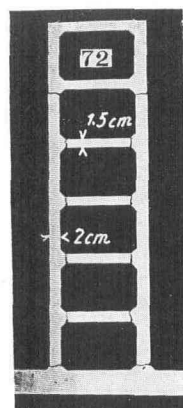
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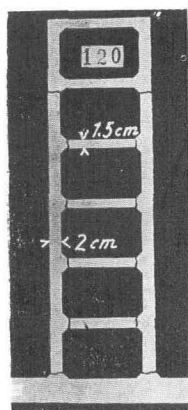
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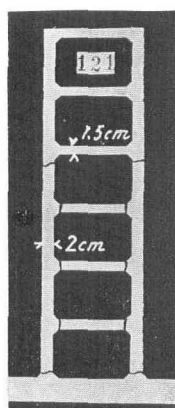
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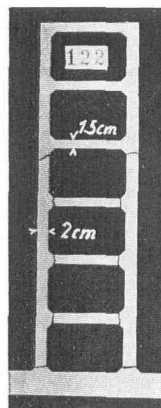
Ref. No. 34.



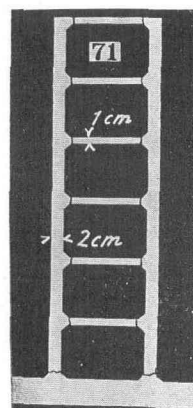
Ref. No. 35.



Ref. No. 36.

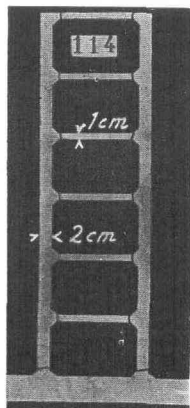


Ref. No. 37.

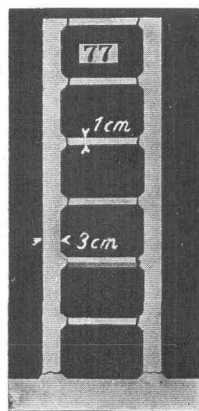


Ref. No. 38.

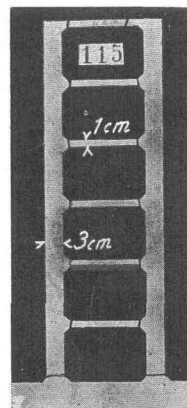
Appendix, Photograph 5.  
Features of Damage to Frames.



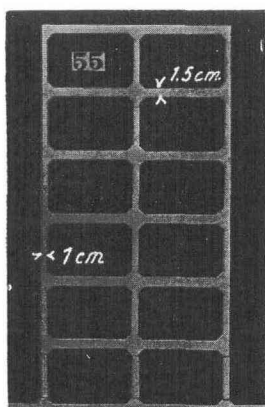
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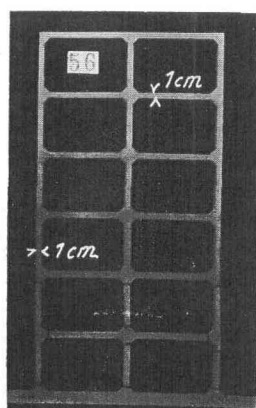
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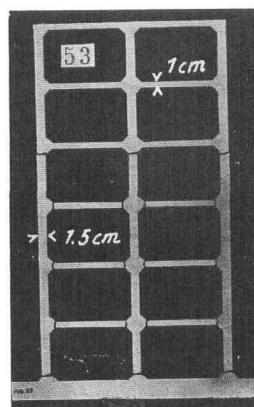
Ref. No. 41.



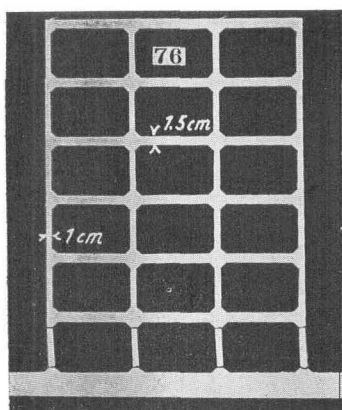
Ref. No. 42.



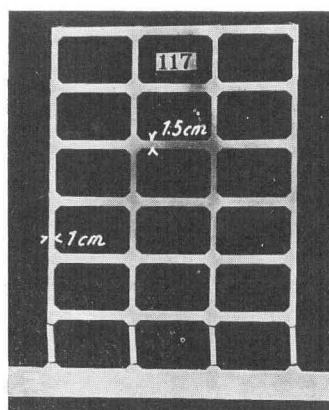
Ref. No. 43.



Ref. No. 44.

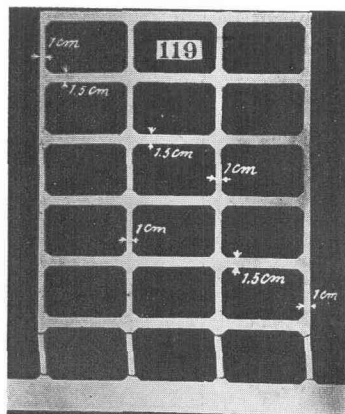


Ref. No. 45.

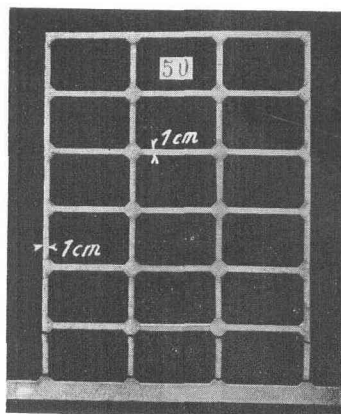


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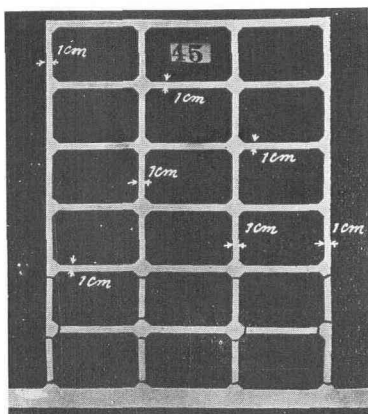
Appendix, Photograph 6.  
Features of Damage to Frames.



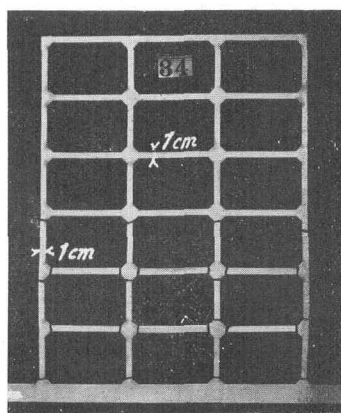
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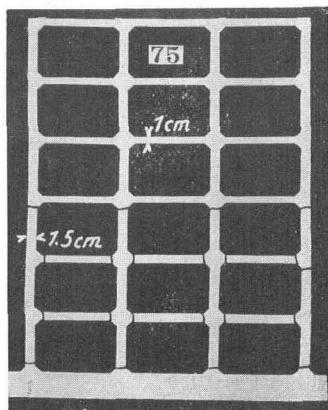
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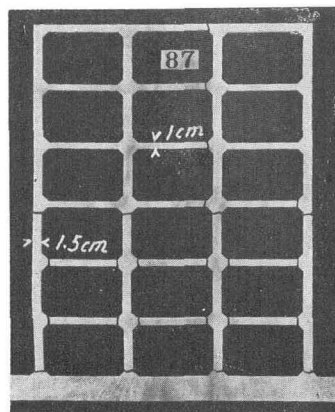
Ref. No. 49.



Ref. No. 50.

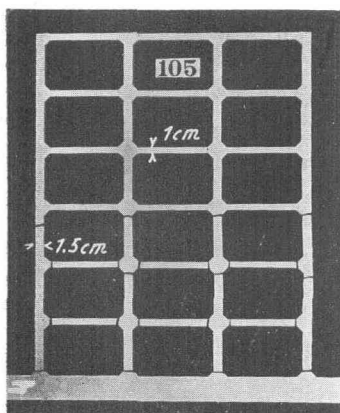


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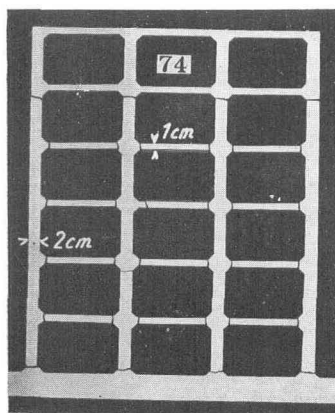


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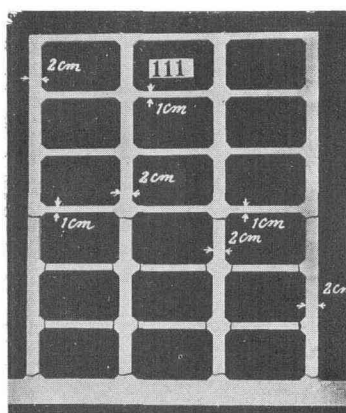
Appendix, Photograph 7.  
Features of Damage to Frames.



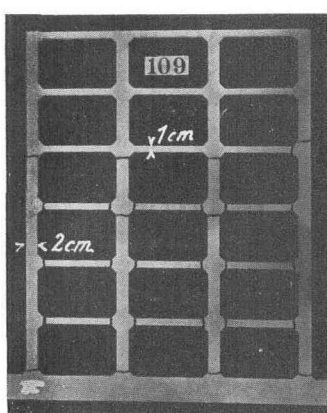
Ref. No. 53.



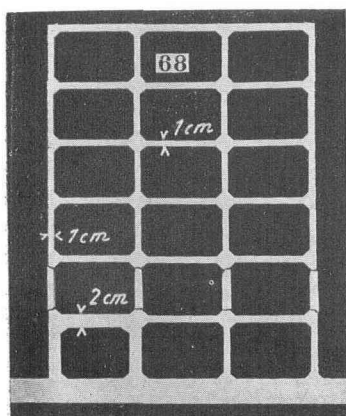
Ref. No. 54.



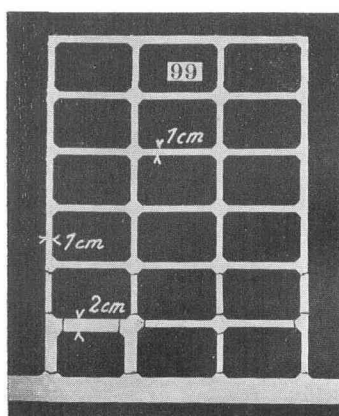
Ref. No. 55.



Ref. No. 56.

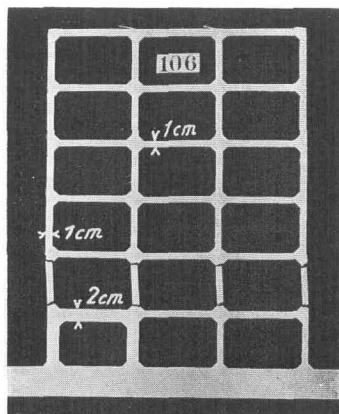


Ref. No. 57.

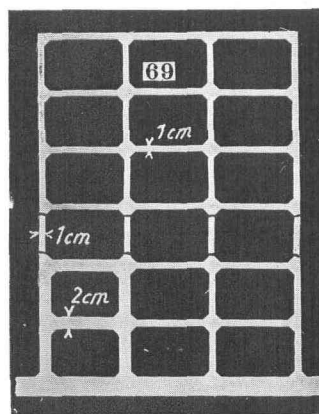


Ref. No. 58.

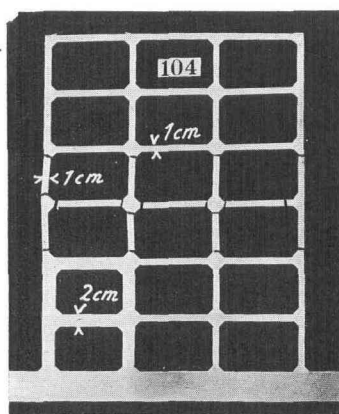
Appendix, Photograph 8.  
Features of Damage to Frames.



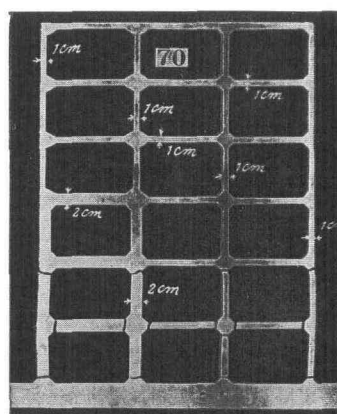
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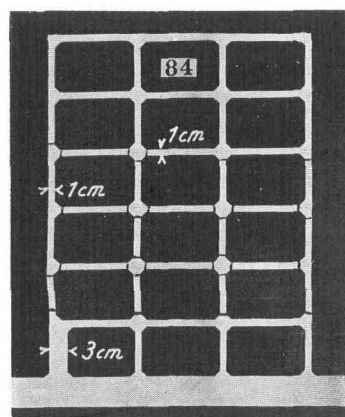
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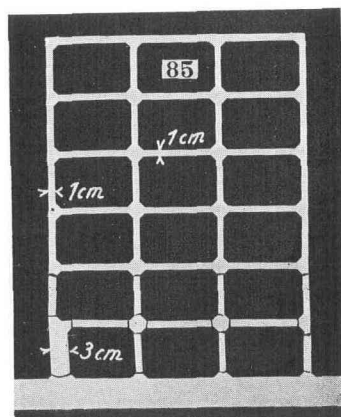
Ref. No. 61.



Ref. No. 62.



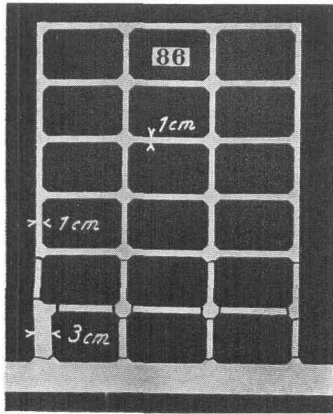
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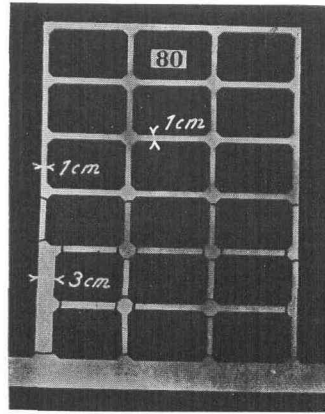
Ref. No. 64.



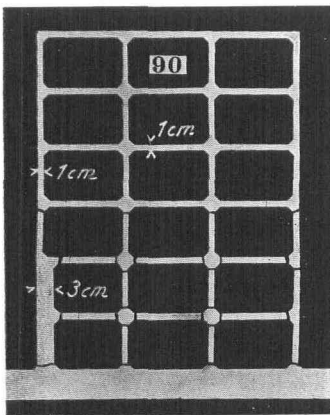
Appendix, Photograph 9.  
Features of Damage to Frames.



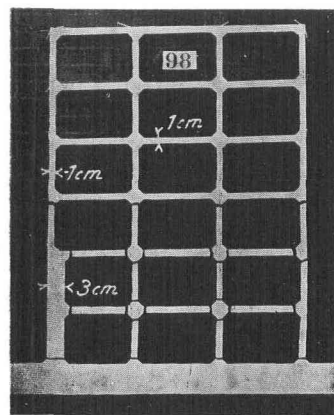
Ref. No. 65.



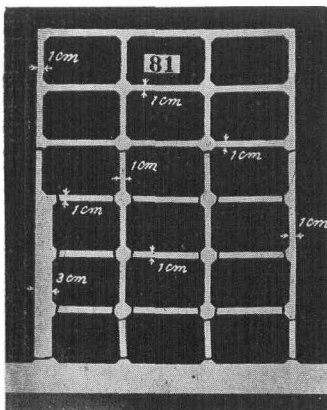
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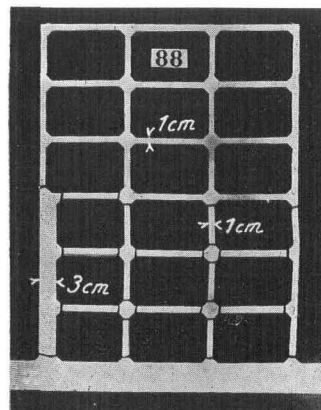
Ref. No. 67.



Ref. No. 68.



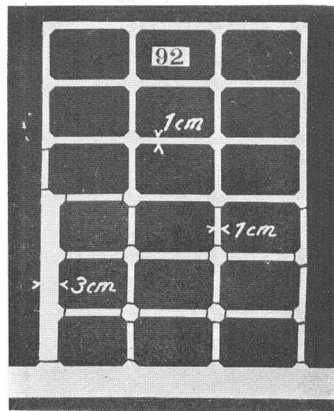
Ref. No. 69.



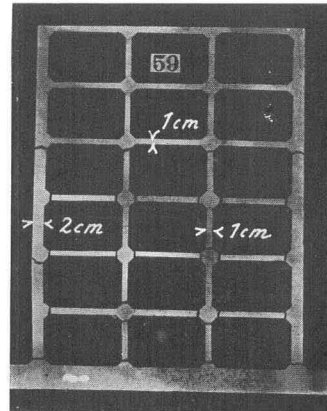
Ref. No. 70.



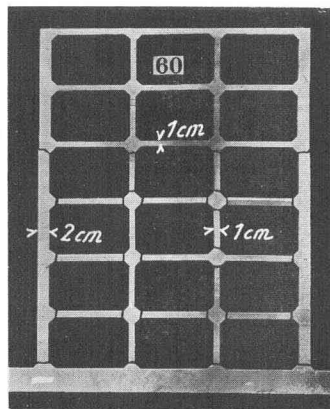
Appendix, Photograph 10.  
Features of Damage to Frames.



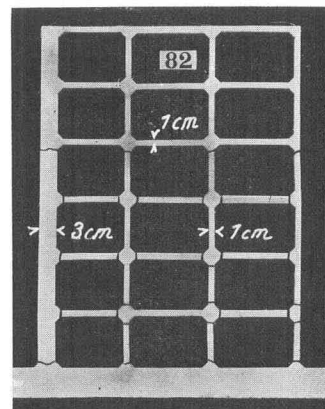
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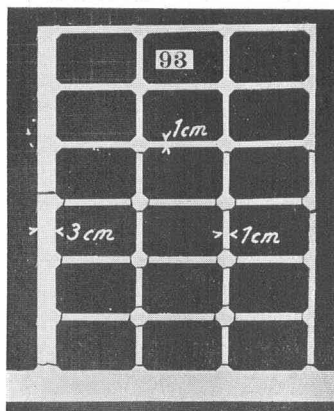
Ref. No. 72.



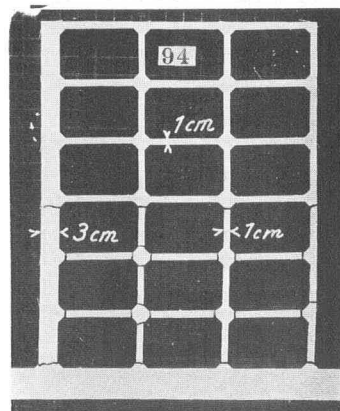
Ref. No. 73.



Ref. No. 74



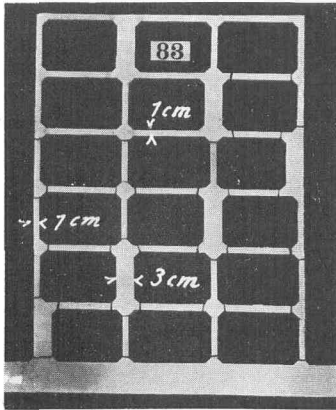
Ref. No. 75.



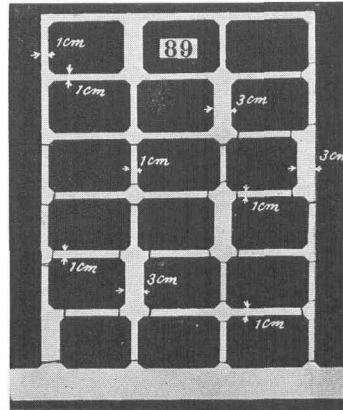
Ref. No. 76.

## Appendix, Photograph 11.

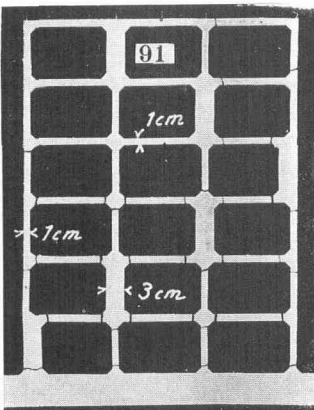
## Features of Damage to Frames.



Ref. No. 77.



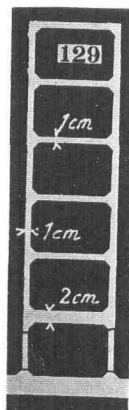
Ref. No. 78.



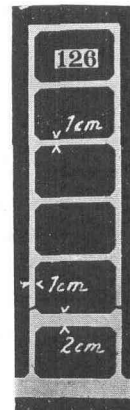
Ref. No. 79.



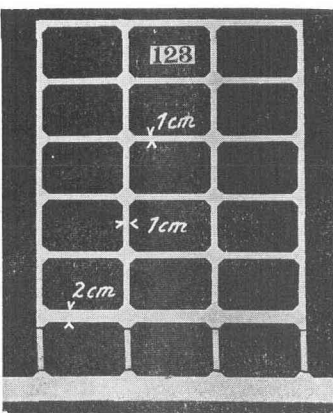
Ref. No. 80.



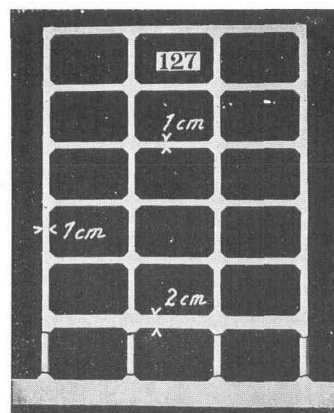
Ref. No. 81.



Ref. No. 82.



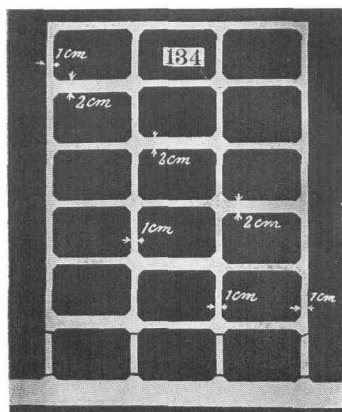
Ref. No. 83.



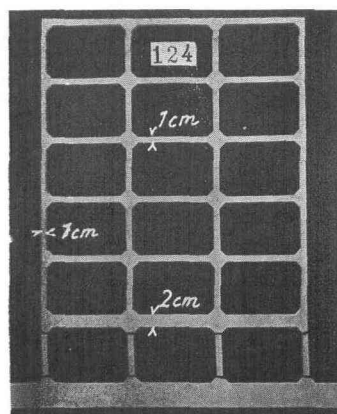
Ref. No. 84.

## Appendix, Photograph 12.

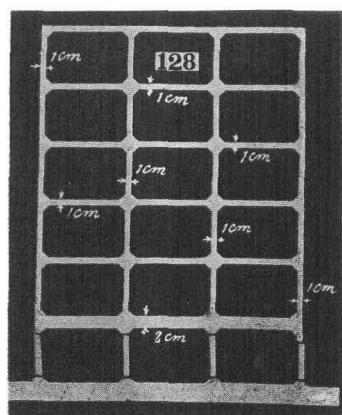
## Features of Damage to Frames.



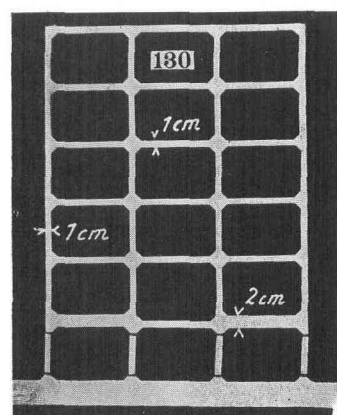
Ref. No. 85.



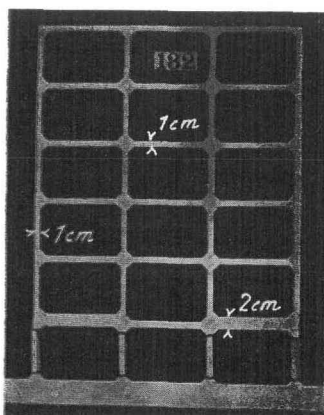
Ref. No. 86.



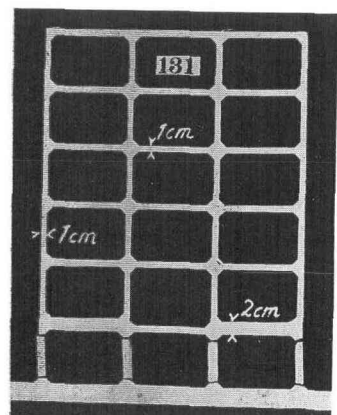
Ref. No. 87.



Ref. No. 88.

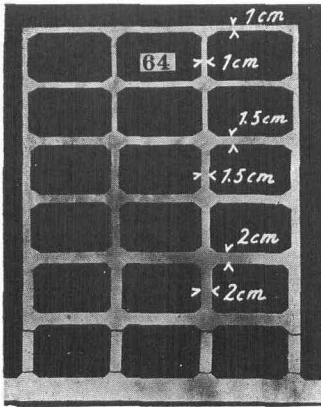


Ref. No. 89.

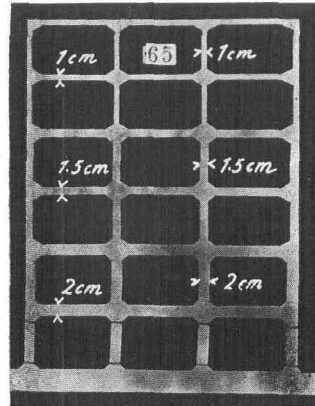


Ref. No. 90.

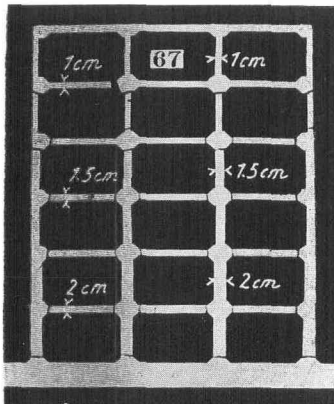
Appendix, Photograph 13.  
Features of Damage to Frames.



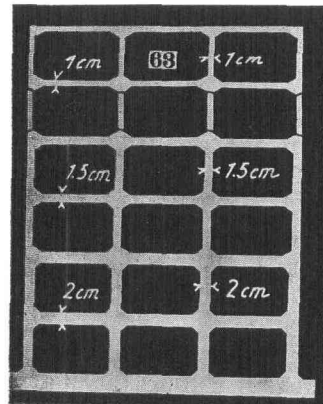
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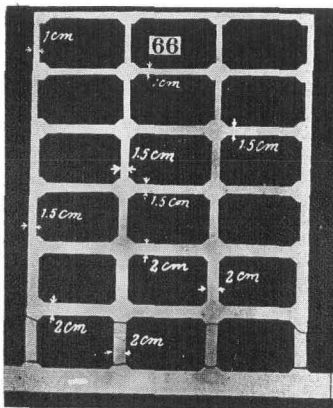
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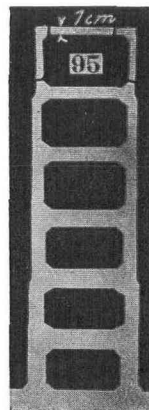
Ref. No. 93.



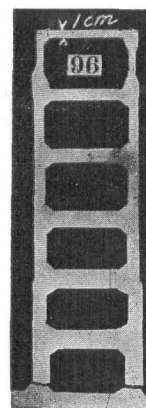
Ref. No. 94.



Ref. No. 95.

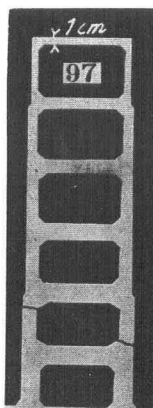


Ref. No. 96.

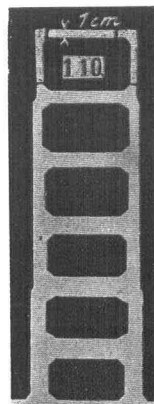


Ref. No. 97.

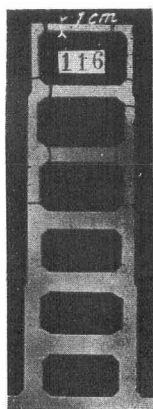
Appendix, Photograph 14.  
Features of Damage to Frames.



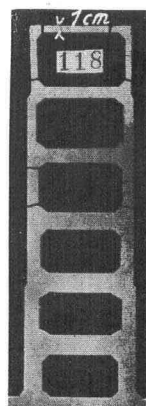
Ref. No. 98.



Ref. No. 99.



Ref. No. 100.



Ref. No. 101.