Theory of the Slot Oscillator

by

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Contents

Introduction

This is a theoretical study on the radiation from magnetic current oscillator or slot oscillator.

The experimental studies on electro-magnetic screening had been investigated by Dr. Y. Asami, who found the fact that intense electro-magnetic wave emerges through narrow slot of suitable length on metal plate or metal enclosure.

Upon this fact, some experimental studies to utilize such slot as a kind of aerials were performed with various interesting results. On the other hand, a theoretical discussion on these phenomena which the author has treated assures their theoretical foundation. As will be mentioned later, slot oscillator must be a magnetic current oscillator, hence properties of the magnetic current oscillator must be clarified. In the following paragraphs these properties will be explained in detail.

1. Fundamental Equations of Magnetic Current Oscillator.

The fundamental equations of Magnetic current oscillator can be deduced similarly to those of electric current oscillator.

Now, let ρ^* be the volume density of free magnetic charges and $\boldsymbol{P}^*$ be the vector quautity defined by,

$$\rho^* = -\nabla \boldsymbol{P}^* \qquad \cdots\cdots (1,\ 1)$$

Then the magnetic current density $\boldsymbol{i}^*$ is expressed as

$$i^* = \frac{\partial}{\partial t} \boldsymbol{P}^* \quad \cdots\cdots (1,\ 2)$$

and field equations becomes as follows,

$$\left.\begin{array}{ll} \nabla \times \boldsymbol{H} = \varepsilon \dfrac{\partial \boldsymbol{E}}{\partial t} & \nabla \times \boldsymbol{E} = -\boldsymbol{i}^* - \mu \dfrac{\partial \boldsymbol{H}}{\partial t} \\ \nabla \cdot \boldsymbol{E} = 0 & \nabla \cdot \boldsymbol{H} = \dfrac{1}{\mu} \rho^* \end{array}\right\} \quad \cdots\cdots (1,\ 3)$$

If the vector $\boldsymbol{\Pi}^*$ satisfies the equation

$$-\nabla^2 \boldsymbol{\Pi}^* + \varepsilon\mu \partial^2 \boldsymbol{\Pi}^*/\partial t^2 = \boldsymbol{P}^*/\mu \quad \cdots\cdots (1,\ 4)$$

then, the field can be expressed as follows,

$$\boldsymbol{H} = \nabla \times \nabla \times \boldsymbol{\Pi}^* - \boldsymbol{P}^*/\mu \qquad \boldsymbol{E} = -\mu \partial/\partial t \nabla \times \boldsymbol{\Pi}^* \quad \cdots\cdots (1,\ 5)$$

If the rectagular coordinates of the point of observation and the current elements are (x_1, x_2, x_3) and (ξ_1, ξ_2, ξ_3) respectively, $\boldsymbol{\Pi}^*$ will become

$$\boldsymbol{\Pi}^* (x, t) = \frac{e^{jwt}}{jw\mu} \int_V \boldsymbol{P}_0^* (\xi) \frac{e^{-jkr}}{r} dV = \frac{1}{jw\mu} \frac{e^{jwt}}{4\pi} \int_{V} \boldsymbol{i}_0^* (\xi) \frac{e^{-jkr}}{r} dV \quad \cdots\cdots (1,\ 6)$$

where,

$$\boldsymbol{P}^* = \boldsymbol{P}_0^* (\xi) e^{jwt}, \quad \boldsymbol{i}^* = \boldsymbol{i}_0^* (\xi) e^{jwt}, \quad r^2 = (x_1 - \xi_1)^2 + (x_2 - \xi_2)^2 + (x_3 - \xi_3)^2, \quad k = \frac{w}{v}$$

Next we shall try to compare the equations of magnetic current oscillator with those of electric current oscillator. Let $\boldsymbol{E}, \boldsymbol{H}$ and $\boldsymbol{E}', \boldsymbol{H}'$ be the radiation fields of electric and magnetic current oscillators respectively, then

$$\boldsymbol{E} = \nabla \times \nabla \times \boldsymbol{\Pi}, \quad \boldsymbol{H} = jw\varepsilon \cdot \nabla \times \boldsymbol{\Pi} \quad \cdots\cdots (1,\ 7)$$

$$\boldsymbol{H}' = \nabla \times \nabla \times \boldsymbol{\Pi}^*, \quad \boldsymbol{E}' = -jw\mu \cdot \nabla \times \boldsymbol{\Pi}^* \quad \cdots\cdots (1,\ 8)$$

Now let the axis of the linear electric or magnetic current oscillator coincide with the z-axis and let the current distribution along them be as follows

$$\left.\begin{array}{l} I_z = I_0 \sin (k\xi - \alpha) e^{jwt} \\ I_z^* = I_0^* \sin (k\xi - \alpha) e^{jwt} \end{array}\right\} \quad \cdots\cdots (1,\ 9)$$

then Π and Π^* become as follows

$$\Pi_z = \frac{I_0}{jw\varepsilon} \frac{e^{jwt}}{4\pi} \int_{\xi_1}^{\xi_2} \frac{e^{-jkr}}{r} \sin (k\xi - \alpha)\, d\xi \quad \cdots\cdots (1,\ 10)$$

$$\Pi_z^* = \frac{I_0^*}{jw\mu} \frac{e^{jwt}}{4\pi} \int_{\xi_1}^{\xi_2} \frac{e^{-jkr}}{r} \sin (k\xi - \alpha)\, d\xi \quad \cdots\cdots (1,\ 11)$$

Thus we can find the similarities between both electric and magnetic fields of the current oscillators whose current distributions are quite the same. For example, the fields at the point far enough from the sources of the electric current element $I_0 e^{jwt} dz$ and the magnetic current element $I_0^* e^{jwt} dz$ become as follows, respectively,

$$H_\varphi = j \frac{I_0 dz}{2\lambda} \frac{e^{j(wt - kr)}}{r} \sin\theta, \quad E_\theta = \eta H_\varphi, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} \quad \cdots\cdots (1,\ 12)$$

$$E'_\varphi = -j \frac{I_0^* dz}{2\lambda} \frac{e^{j(wt - kr)}}{r} \sin\theta, \quad H'_\theta = -E'_\varphi/\eta \quad \cdots\cdots (1,\ 13)$$

In the case of the magnetic current oscillator, the direction of the electric field is at right angle to the oscillator. If the current distributions in electric and magnetic current oscillators are same, the radiation patterns of both systems become quite the same.

2. Fundamental Cosiderations for slot Oscillator.

There are several ways to proof that the slot oscillator must be a magnetic current oscillator. One of these will be shown as follows.

Let us consider the hole on an infinitely large conductor plane, upon which x–y axes are taken and let the area of the hole be $S = \Delta x \times \Delta y$ (Fig. 1) Assume the electro-magnetic field over this area to be uniform which is expressed by (E_x, H_y). Adopting the electric current density and magnetic current density i and i^* in stead of H_y and E_x on the surface of the hole as below,

$$i_x = -H_y\ , \quad i_y^* = -E_x \qquad \cdots\cdots\cdots\cdots (2,\ 1)$$

and putting

$$H_y = H = H_0\, e^{j\omega t}\ , \quad E_x = E = E_0\, e^{j\omega t} \quad (2,\ 2)$$

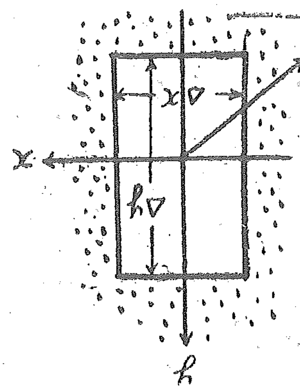

Fig. 1

then, this hole may be deemed as an electric current element $i_x \Delta y \Delta x = -HS$ parallel to the x-axis and a magnetic current element $i_y^* \Delta x \Delta y = -ES$ parallel to the y-axis. Hence the radiation field at a distant point from these sources will be

$$\left.\begin{aligned} E_\theta &= j\,(H_0\eta \cos\theta + E_0)\,\frac{S}{2\lambda}\cos\varphi\,\frac{e^{j(\omega t - kr)}}{r} \\ E_\varphi &= -j\,(H_0\eta + E_0\cos\theta)\,\frac{S}{2\lambda}\sin\varphi\,\frac{e^{j(\omega t - kr)}}{r} \\ H_\theta &= -E_\varphi/\eta\ , \quad H_\varphi = E_\theta/\eta \end{aligned}\right\} \quad\cdots\cdots\cdots\cdots (2,\ 3)$$

But this field does not satisfy the boundary conditions on the conductor surface. Hence, to avoid this inconsistency this field is considered to be reflected on the conductor surface, and the actual field E_θ, E_φ will be obtained by superposition of the original field $E'_\theta(\theta)$, $E'_\varphi(\theta)$ and the reflected field $E'_\theta(\pi-\theta)$, $-E'_\varphi(\pi-\theta)$ that is

$$\left.\begin{aligned} E_\theta &= E'_\theta(\theta) + E'_\theta(\pi-\theta) \\ E_\varphi &= E'_\varphi(\theta) - E'_\varphi(\pi-\theta) \end{aligned}\right\} \quad\cdots\cdots\cdots\cdots (2,\ 4)$$

thus, we get

$$\left.\begin{aligned} E_\theta &= j\,\frac{E_0 S}{\lambda}\cos\varphi\,\frac{e^{j(\omega t - kr)}}{r} \\ E_\varphi &= -j\,\frac{E_0 S}{\lambda}\cos\theta\cdot\sin\varphi\,\frac{e^{j(\omega t - kr)}}{r} \\ H_\theta &= -E_\varphi/\eta\ , \quad H_\varphi = E_\theta/\eta \end{aligned}\right\} \quad\cdots\cdots\cdots\cdots (2,\ 5)$$

It can now be shown that this field satisfies the boundary conditions. This idea had been shown first by Stratton.[1)]

We see that the above field is equal to the field radiated by the magnetic current element $-2ES = 2V\cdot\Delta y$ along the y-axis, where $V = -E\Delta x$ and, also, the source corresponding to the reflected wave can be considered as the image of the original source and its current densities will be $i_x = H$, $i^* = -E$. Thus, the current density of the superposed source of both the original and the image fields will be nothing more than $i_y^* = -2E$.

Hence, in general, denoting by $\boldsymbol{E}(\xi, \eta)$ the electric field given over the surface of the hole and $\boldsymbol{n}$ the outward normal on this surface, then the current density of this surface is given by the magnetic current density $\boldsymbol{i}^*(\xi, \eta)$ only and this is equal to

$$\boldsymbol{i}^*(\xi, \eta) = \boldsymbol{n} \times 2\boldsymbol{E}(\xi, \eta) \qquad \cdots\cdots (2, 6)$$

3. Calculation of Radiation Impedance of the Magnetic Current Oscillator.

For the first place, let us consider the case of the electric current oscillator. In this case the radiation impedance can be calculated by the wellknown e.m. f. method which is shown as follows.

Suppose that there are n linear conductors carrying electric current and are parallel to one another and to the z-axis. The current in the jth conductor is I_j. The z-component of the electric field at the jth conductor due to ith current is E_{ji}. Then the total mean power of this syatem is

$$\bar{P} = -\frac{1}{2} R_e \sum_{j=1}^{n} \sum_{i=1}^{n} \int_{\xi_{1j}}^{\xi_{2j}} E_{ji} \tilde{I}_j \, d\xi_j \qquad \cdots\cdots (3, 1)$$

where, the integration to be extended the over length of each conductor. In the e.m.f. method, $\bar{P}$ is also considered as the total radiation power of this system.

Writing

$$\left. \begin{aligned} I_j &= I_{0j} \sin(k\xi_j - \alpha_j) e^{j\omega t} \\ E_{ji} &= I_{0i} U_{ji} e^{j\omega t} \end{aligned} \right\} \qquad \cdots\cdots (3, 2)$$

then the expression of $\bar{P}$ becomes

$$\bar{P} = \frac{1}{2} R_e \sum_{j=1}^{n} \sum_{i=1}^{n} I_{0i} \tilde{I}_{0i} Z_{ji} \qquad \cdots\cdots (3, 3)$$

where,

$$Z_{ji} = -\int_{\xi_{1j}}^{\xi_{2j}} U_{ji} \sin(k\xi_j - \alpha_j) \, d\xi_j \qquad \cdots\cdots (3, 4)$$

The quantities Z_{ji} are called the mutual radiation impedances between the ith conductor and the jth conductor.

Next let us consider the case of magnetic current oscillator. In this case we can introduce the m.m.f. method instead of the e.m.f. method. Supposing that there are n liner magnetic current oscillators and are placed in the same manner as in the previous case. Let H'_{ji} the z-component of the magnetic field at the jth oscillator due to ith oscillator, then the work necessary to maintain the magnetic current I_j^* against the m.m.f. due to the field H'_{ji} can be calculated, and total mean power $\bar{P}^*$ of this system must be equal to total sum of these works. That is,

$$\bar{P}^* = -\frac{1}{2} R_e \sum_j \sum_i \int_{\xi_{1j}}^{\xi_{2j}} \tilde{H}'_{ji} I_j^* \, d\xi_j \qquad \cdots\cdots (3, 5)$$

Putting

$$\left.\begin{aligned} I_j^* &= I_j^* \sin(k\xi_j - \alpha_j)\, e^{j\omega t} \\ H'_{ji} &= I_i^* U_{ji}^* e^{j\omega t} \end{aligned}\right\} \quad \cdots\cdots (3,\ 6)$$

the expression of $\bar{P}^*$ becomes

$$\bar{P}^* = \frac{1}{2} R_e \sum \sum \tilde{I}_i^* I_j^* \; Y_{ji}^* \quad \cdots\cdots (3,\ 7)$$

where,

$$Y_{ji}^* = -\int_{\xi_{1j}}^{\xi_{2j}} \tilde{U}_{ji}^* \sin(k\xi_j - \alpha_j)\, d\xi_j \quad \cdots\cdots (3,\ 8)$$

The quantities Y_{ji}^* are the mutual radiation admittances. Comparison of these easily lead to the co-relation between Z_{ji} and Y_{ji}^* that is,

$$U_{ji} = \eta^2 U_{ji}^*$$

hence

$$Y_{ji}^* = \tilde{I}_{ji}/\eta^2, \quad \eta = 120\pi \quad \cdots\cdots (3,\ 9)$$

Putting $V(\xi)$ the voltage between two longer sides of the slot in case of the slot aerial may be considered as a magnetic current aerial carrying a current of $2V(\xi)$, hence, the mutual radiation admittance $Y_{ji}^{(s)}$ of the slot is

$$Y_{ji}^{(s)} = 4Y_{ji}^* \quad \cdots\cdots (3,\ 10)$$

Let V_0 be the voltage amplitude at the feeding point of the slot, then the electric current amplitude I_0 at the same point will be shown as follows,

$$I_{0i} = V_{0j} Y_{ji}^{(s)}, \quad I_{0j} = V_{0i} Y_{ji}^{(s)} \quad \cdots\cdots (3,\ 11)$$

Hence, the radiation power $\bar{P}^{(s)}$ of the slot system can be shown in the following form.

$$\bar{P}^{(s)} = \frac{1}{2} R_e \sum \sum I_{0i} \tilde{I}_{0j} Z_{ji}^{(s)} \quad \cdots\cdots (3,\ 12)$$

where

$$Z_{ji}^{(s)} = 1/\tilde{Y}_{ji}^{(s)} = \eta^2/4Z_{ji} \quad \cdots\cdots (3,\ 13)$$

The quantities $Z_{ji}^{(s)}$ are the mutual radiation impedances of slot system, and can be calculated from the electric current oscillator system which has the same structure as the slot system.

For example, radiation impedance of a single linear conductor of length $\lambda/2$ is

$$Z_{11} = 73.1 + j\,42.5 \quad \Omega$$

Hence, the radiation impedance of a single linear slot of length $\lambda/2$ will be calculated as follows,

$$Z_{11}^{(s)} = (120\pi)^2/4Z_{11} = 363.5 - j\,211 \quad \Omega \quad \cdots\cdots (3,\ 14)$$

4. Radiation Impedance when Length and Width of the Slot are varied.

In the previous case, we have considered a linear oscilltor which cross section is very small.

Let us next consider the case that the oscillator has a finite cross section.

In the case of the electric current oscillator of finite cross section, current flows along the surface by the skin effect. The radiation power from such conductor will be considered.

If we divide the total current I_0 into many linear current elements parallel to the axis, then the radiation power can be calculated as follows

$$\bar{P} = \frac{1}{2} \oint \oint Z_{12} \left(I_0 \frac{ds_1}{s} \right) \left(\bar{I}_0 \frac{ds_2}{s} \right) \qquad \cdots\cdots (4,\ 1)$$

where s is the periphery of the cross section, and Z_{12} is the mutual radiation impedance between two linear current elements $I_0 \frac{ds_1}{s}$ and $I_0 \frac{ds_2}{s}$.

Expressing the radiation power $\bar{P}$ as

$$\bar{P} = \frac{1}{2} Z_{11} I_0^2 \qquad \cdots\cdots (4,\ 2)$$

where,

$$Z_{11} = \frac{1}{s^2} \oint \oint Z_{12}\, ds_1\, ds_2 \qquad \cdots\cdots (4,\ 3)$$

then, Z_{11} is the radiation impedance of the conductor of finite cross section with s as its peripheral length.

Thus, if we can find the radiation impedance $Z_{11}^{(s)}$ of the slot which has the same dimension as the metal plate oscillator will be calculated by the eq. (3.13)

Assuming that the phase constant k along the oscillator to be the same as that of free space, though the whole length of the oscillator would be $\frac{\lambda}{2} \pm \Delta$, the current distribution along the oscillator vanishes at both ends, but at the center the derivative of the amplitude curve will be discontinuous[2]. Under this assumption it is possible to calculate the radiation impedance of the slot oscil'ator whose length and breadth are $L = \frac{\lambda}{2} \pm \Delta$ and a, respectivily.

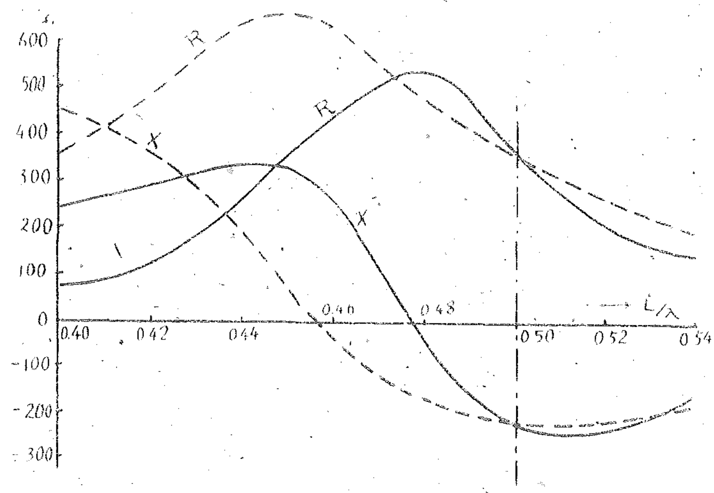

Fig. 2 $Z_{11}^{(s)} = R + jX$

Fig. 2 shows such radiation impedance $Z_{11}^{(s)} = R + jX$ of slot. when total length L is just equal to $\lambda/2$ radiation impedance is always 363.5−j211 Ω, in spite of different values of a and in the case when L is varied around $\lambda/2$, R and X both vary but their rate of change are diffeient corresponding to the values of a.

For example, if the width of the slot is $a = \lambda e/1000$, then $X = 0$ at $L = 0{,}4778\lambda$ and $Z_{11}^{(s)}$ takes a pure resistive value, while at longer or shorter L, $Z_{11}^{(s)}$ becomes capacitive or inductive respectively.

In the case of electric current oscillator the real part of the radiation impedance varies slowly when its length is varied from $\lambda/2$ but in the case of the slot oscillator we see that both real and imaginary parts vary extensively.

5. Resonance Character of Radiation Power.

The resonance character of radiation power of the slot oscillator when its length is varied will be considered next.

Assuming that the slot is made on an infinitely large metal plate and is excited at its center by a lecher wire the reflecting coefficient at the feeding point will be

$$\mathring{m} = (Z_{11}^{(s)} - Z_0)/(Z_{11}^{(s)} + Z_0)$$

where, Z_0 is the surge impedance of the feeder.

Hence, the power radiated by the slot becomes

$$P = 1 - |\mathring{m}|^2$$

Since $Z_{11}^{(s)}$ is the function of length and width of the slot, the relation between the radiation power and the dimensions of the slot may be obtained as shown in Fig. 3. It is seen from the figure that the radiation power becomes maximum near the length where the radiation impedance becomes pure resistive with the tendency that the resonance character becomes flat into a wide band characteristics as the width of the slot is increased.

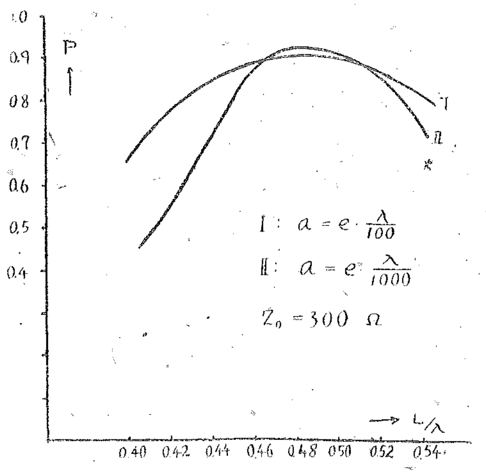

Fig. 3

6. Electric Current Distribution on the Conductor Surface around the Slot.

Let us choose x-y plane on the conductor surface upon which a linear slot of length $\lambda/2$ is made with its center located at the origin and the slot itself coincided with the y-axis. (Fig. 4) If the magnetic current distribution along this slot is

$$I^* = I_0^* \cos ky \cdot e^{j\omega t} \quad \cdots\cdots (6,\ 1)$$

the tangential components of the exact field on the conductor surface on right hand side of y-axis will be as follows,

$$H_y = j\frac{I_0^*}{\eta}\frac{e^{j\omega t}}{4\pi}\left[-\frac{e^{-jkr_2}}{r_2} - \frac{e^{-jkr_1}}{r_1}\right]$$
$$H_x = -j\frac{I_1^*}{\eta}\frac{e^{j\omega t}}{4\pi}\left[-\frac{\left(y-\frac{l}{2}\right)}{x}\frac{e^{-jkr_2}}{r_2} - \frac{\left(y+\frac{l}{2}\right)}{x}\frac{e^{-jkr_1}}{r_1}\right] \quad \cdots\cdots\cdots (6,\ 2)$$

Thus, the current density i on the conductor surface leads to

$$i_x = -H_y\ ,\quad i_y = H_x \quad \cdots\cdots\cdots (6,\ 3)$$

we must use the real parts of H_x and H_y in eq. (6. 2), hence i_x and i_y take the following forms,

$$i_x = i_{x0}\sin(\omega t - \theta_1)$$
$$i_y = i_{y0}\sin(\omega t - \theta_2)$$

Here, i_{x0} and i_{y0} are the amplitude distributions of i_x and i_y, respectively, and $\theta_1 =$ const. and $\theta_2 =$ const. give the equiphase lines of i_x and i_y respectively.

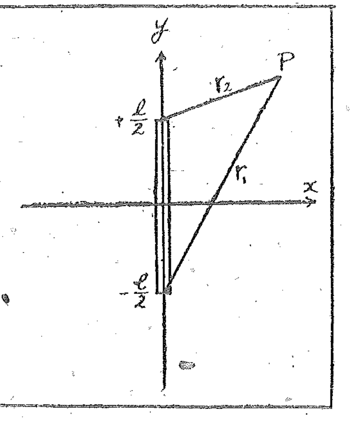

Fig. 4

The current distsibutions on the front and back of the plane are exactly same. i_{x0} and i_{y0} are shown in Fig. 5 and Fig. 6 respectively.

In Fig. 5 i_x flows down from above and this current crosses the slot at right angle. The magnitude of this current density is a maximum in the x-axis and it becomes larger the nearer the slot is approached. and the current density is seen to be very large at both ends of the slot.

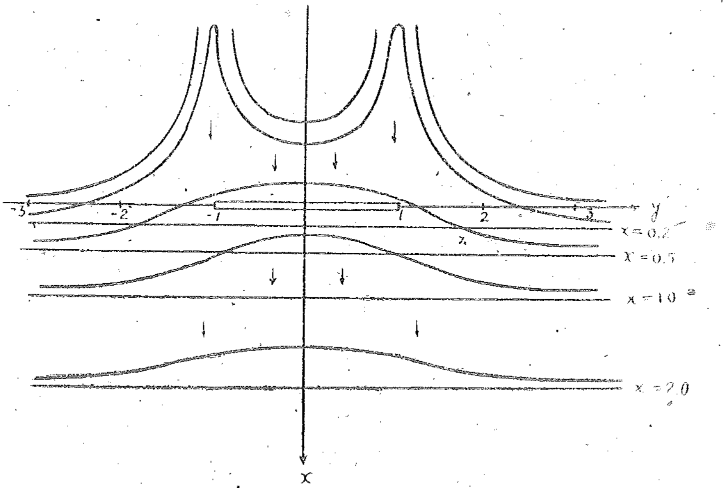

Fig. 5

The curves show the magnitude of i_{x0} along the lines $x = 0,\ 0.2, \cdots\cdots$ when slot length is taken equal to 2. → shows the direction of i_{x0}

The equiphase lines of i_x at a distant position are groups fo confocal ellipses which foci are at both ends of the slot. Near the slot they form a dumbell shape swelling around both ends of the slot.

On the other hand the distrubution of i_y is shown in Fig. 6. The fact that i_{y0} becomes large near both ends of the slot but the directions of the currents along both longer sides of the slot are opposite to each other, means that i_y does not contribute so much to radiation. Thus we can explain the experimental fact that the dimension of the conductor plate in y direction has little effect on the radiation pattern.

Conclusion.

In this paper the author has clarified the properties of the magnetic current oscillator or the slot oscillator, namely, the radiation field, and its pattern, radiation impedance, resonance charactor of the radiation power to oscillator length and the effect of dimensions of conductor plate.

Acknowledgement.

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1) Stratton & Chu ; Phy. Rev. 56, 99 (1939)
2) Bechmann ; I. R. E. Vol. 19 p. 1471 (1931)