Power Circle Diagram Of Interconnected Electric Power Transmission System

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Outline : Expressing power distribution in interconnected electric transmission system with many power stations and substations of different voltages by using matrix, it is shown that circle diagram of this system is given the main diagonal terms of matrix as centers and the other terms as radii. A numerical example is taken from the data of the main high tension transmission line of Hokkaido district.

1. General Equation Or Matrix Of Power Distribution In Interconnected Transmission System

Generally speaking, any kind of electric power transmission network can be transfomed into an equivalent circuit as shown by Figure 1. which has many branches and terminals mutually connected, acting as real generating stations or transfoming stations. In order to solve such a network, the value of impedance of every branch line, which connects stations with each other, and the value of every transformer shoued be converted into the value of standard voltage, and the value of equivalent π circuits between every two terminals should be obtained separately.

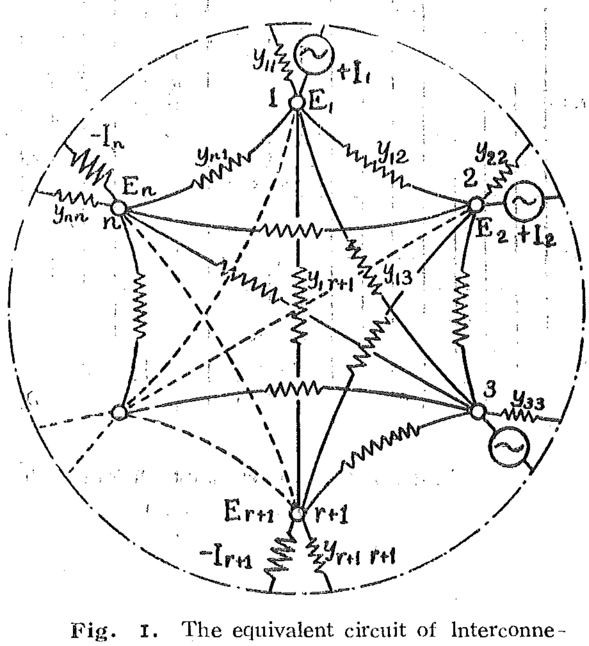


Fig. 1. The equivalent circuit of Interconnected transmissoin network.
Terminal 1 — r mean sending ends, terminal (r + 1) — n recieving ends.

The equivalent π circuit between terminals can be described as follows when general electric transmission constants are used:

Admittance on both legs of the circuit,

$$y_1=\frac{D}{B}-\frac{1}{B} \qquad y_2=\frac{A}{B}-\frac{1}{B}$$

Series impedance of the circuit, $Z=B$.

The forming of these equivalent π circuits is for convenience sake of inducing admittance between terminals and short-eircuited abmittance at each terminal will be related later. However, it is not the only way.

Out from these equivalent π circuits, we can get y_{ij}, the driving abmittance between i anb j terminals, as it is equivalent to the reciprocal of value of series impedance of the π circuit, and y_{ii} also, the admittance at terminal i, will be got by adding up all supplement values belonged to the terminal only: That is-.

$$y_{ij}=\frac{1}{B} \qquad Y_{ii}=\Sigma\left(\frac{D}{B}-\frac{1}{B}\right),\left(\frac{A}{B}-\frac{1}{B}\right).$$

Besides, there are loads or generating powers to be taken into consideration. Therefore it will become as shown by Figure 1.

Getting $E_1, E_2, \cdots\cdots E_n$ stand for voltage of each terminal, terminal suffix $1\cdots\cdots$ r for sending elements and their currents for $+I_1,\ I_2,\ \cdots+Ir$; terminal suffix(r+1) $\cdots$n for receiving elements and their currents $-Ir_{+1}\ -I_{r+2},\ \cdots\cdots-I_n$ respectively, we get the following equation according to the Kirchhoff's law.

$$\begin{aligned}
+I_1&=y_{11}E_1+y_{12}(E_1-E_2)+\cdots\cdots\cdots\cdots+y_{1n}(E_1-E_n)\\
+I_2&=y_{21}(E_2-E_1)+y_{22}E_2+\cdots\cdots\cdots\cdots+y_{2n}(E_2-E_n)\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
+I_r&=y_{r1}(E_r-E_1)+\cdots\cdots\cdots\cdots\cdots\cdots+y_{rn}(E_r-E_n)\\
-I_{r+1}&=y_{(r+1)1}(E_{r+1}-E_1)+y_{(r+1)(r+1)}E_{r+1}\cdots\cdots+y_{(r+1)n}(E_{r+1}-E_n)\\
-I_{r+2}&=y_{(r+2)1}(E_{r+2}-E_1)+\cdots\cdots\cdots\cdots+y_{(r+2)n}(E_{r+2}-E_n)\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
-I_n&=y_{n1}(E_n-E_1)+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots+y_{nn}E_n \qquad (1)
\end{aligned}$$

The nature of transmission network will be explained more clearly when we describe the above equation by using matrix as follows:

$$\begin{pmatrix}+I_1\\+I_2\\\vdots\\+I_r\\\hline -I_{r+1}\\-I_{r+2}\\\vdots\\-I_n\end{pmatrix}=\left(\begin{array}{c|c}
Y_{11}-y_{12}-y_{13}\cdots\cdots & -y_{1(r+1)}\cdots\cdots\cdots-y_{1n}\\
-y_{21}\ Y_{22}\ -y_{23}\cdots\cdots & \cdots\cdots\cdots\cdots-y_{2n}\\
\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots\\
-y_{r1}\cdots\cdots\cdots\cdots Y_{rr} & \cdots\cdots\cdots\cdots-y_{rn}\\
\hline
-y_{(r+1)1}\cdots\cdots\cdots\cdots & Y_{(r+1)(r+)}\cdots-y_{(r+1)n}\\
-y_{(r+2)1}\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots\\
\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots\\
-y_{n1}\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots Y_{nn}
\end{array}\right)\times\begin{pmatrix}E_1\\E_2\\\vdots\\E_r\\\hline E_{r+1}\\E_{r+2}\\\vdots\\E_n\end{pmatrix}\cdots(2)$$

But $Y_{ii}=y_{ii}+y_{i1}+y_{i2}\cdots\cdots\cdots\cdots+y_{in}$

The elements in the above equation will be described also in the form of so-called power circle diagram constants by using generl circuit constants between terminals as follows:

$$y_{12}=\frac{1}{B_{12}},\quad y_{13}=\frac{1}{B_{13}},\ \cdots\cdots y_{21}=\frac{1}{B_{21}}=\frac{1}{B_{12}},\quad y_{31}=\frac{1}{B_{13}},\ \cdots\cdots$$

$$Y_{11}=\frac{D_{12}}{B_{12}}+\frac{D_{13}}{B_{13}}+\frac{D_{14}}{B_{14}}+\cdots\cdots\cdots\cdots+\frac{D_{1n}}{B_{1n}},$$

$$Y_{22}=\frac{A_{12}}{B_{12}}+\frac{D_{23}}{{}_{23}B}+\frac{D_{24}}{B_{24}}+\cdots\cdots\cdots\cdots+\frac{D_{2n}}{B_{2n}},$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$Y_{(r+1)\,(r+1)}=\frac{A_{1\,(r+1)}}{B_{1\,(r+1)}}+\frac{A_{2\,(r+1)}}{B_{2\,(r+1)}}+\cdots\cdots\frac{A_{r\,(r+1)}}{B_{r\,(r+1)}}+\frac{D_{(r+1)\,(r+2)}}{B_{(r+1)\,(r+2)}}+\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots+\frac{D_{(r+1)\,n}}{B_{(r+1)\,n}}.$$

In the matrix (2), Y_{ii}, the terms upon a main diagonal, mean the short-circuited abmittance at the terminal, that is the admittance in case of all other terminals being short-circuited, excepting only the term of i order, and of which the value will never become zero. Other admittances excepting that upon the diagonal mean the driving admittance between terminals, and of which the value between only noncombinated terminals become zero. However, terms located at symetrical positions of the diagonal are equal with each other, and terms in the upper side of the diagonal mean the sending driving abmittance and that of the lower side mean the receiving driving admittance.

Equation (2) is shown with the current unit, but if we want it with the power unit we must multiply each terminal current, shown by a horizontal rank of the matrix, by E_{ik}, the conjugated value of each terminal voltage; that is ——

$$\begin{pmatrix} +P_{11}+j\,Q_{11}\cdots\cdots\cdots \\ +P_{22}+j\,Q_{22}\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ +P_{rr}+j\,Q_{rr}\cdots\cdots\cdots \\ \hline -(P_{(r+1)(r+1)}j\,Q_{(r+1)(r+1)}) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -(P_{nn}+j\,Q_{nn})\cdots\cdots \end{pmatrix}$$

$$=\left|\begin{array}{l|l} Y_{11}|E_1|^2,-y_{12}E_2\,E_{1k},\cdots\cdots\cdots & \cdots y_{(r+1)}E_{(r+1)}E_{1k},\cdots y_{1n}E_nE_1 \\ -y_{21}E_1\,E_{1k},Y_{22}|E_2|^2,\cdots\cdots\cdots & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ -y_{r1}E_1E_{rk},\cdots\cdots+Y_{rr}|E_r|^2 & \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \hline \end{array}\right|\cdots(3)$$

$$\begin{vmatrix} -y_{(r+1)1}E_1E_{(r+1)k}, \cdots\cdots & Y_{(r+1)(r+1)}|E_{(r+1)}|^2, \cdots\cdots \\ \cdots\cdots & \cdots\cdots \\ -y_{n1}E_1E_{nk}, \cdots\cdots & \cdots\cdots Y_{nn}|E_n|^2 \cdots\cdots \end{vmatrix}$$

If the network is based on toe constant voltage transmission system, namely, the absolute value of every terminal voltage is constant, then every term upon the main diagonal of the above power matrix gets constant value and other terms are to have different phase angles individually. Accordingly, the vector locus forms circle with changes of load value, and we will have nany circle diagrams corresponding to each term. The number of the circles will be the twice of that of ointing transmission lines between terminals.

The sending power circle diagrams are made with the upper terms of the main diagonal, and the receiving power circle diagrams are with the lower terms of the diagonal.

If the terminal of i orber is taken for an example, the total generating power of this terminal is

$$P_{11} + j\ Q_{11} = Y_{11}|E_1|^2 - y_{12}E_2E_{1k} - y_{13}E_3E_{1k} \cdots\cdots - y_{1n}E_nE_{1k}.$$

which is shown on the first line of the equation (3). However, from this terminal, power is branched out to the terminals connected, so the above equation can be shown also as follows, havlng divided Y_{ii} to each component.

$$\begin{aligned}(P_{12} + j\ Q_{12}) + (P_{13} + j\ Q_{13}) + \cdots\cdots + (P_{1n} + j\ Q_{1n}) \\ = \{(y_{112} + y_{12})|E_1|^2 - y_{12}E_3E_{1k}\} \\ + \{(y_{113} + y_{13})|E_1|^2 - E_{13}E_3E_{1k}\} \\ \cdots\cdots \\ + \{(y_{11n} + y_{1n})|E_1|^2 - y_{1n}E_nE_{1n}\}\end{aligned}$$

If this equation is shown with circle diagram constants which are used in most cases,

$$\begin{aligned}(P_{12} + j\ Q_{12}) + (P_{13} + j\ Q_{13}) + \cdots\cdots + (P_{1n} + j\ Q_{1n}) \\ = \left(\frac{D_{12}}{B_{12}}|E_1|^2 - \frac{1}{B_{12}}E_2E_{1k}\right) + \left(\frac{D_{13}}{B_{13}}|E_1|^2 - \frac{1}{B_{13}}B_3E_{1k}\right) + \cdots\cdots \\ \cdots\cdots + \left(\frac{D_{1n}}{B_{1n}}|E_1|^2 - \frac{1}{B_{1n}}E_nE_{1k}\right),\end{aligned}$$

The right side and the left side of this equation contain many terms contrasted with each other, and admittances appeared on the right side are obtained by using equivalent π circuit constants as was related before. As for the first term, it is explained as follows.

$$(P_{12} + j\ Q_{12}) = (y_{112} + y_{12})|E_1|^2 - y_{12}E_2E_{1k}.$$

This shows power which is sent to terminal 2 from terminal 1. As is based on the constant voltage transmission system, the factor changeable due to the load change is only phase angle or "power angle" of voltage. And $(y_{112}+y_{12})|E_1|^2$, $|y_{12}E_2E_1|$, show center and radius of circle respectively, which means the well-known sending end circle. The same discourse can be made for many other terms in like manner, and many circle diagrams are made.

However, we are discussing these theories always basing upon the transmission network, and it will make us notice thefollowing fact. Namely, the circle diagrams of the terms located on symetrical positions of the main diagonal of matrix (3) have equal radius value and contrasted power angles. For an example, $-y_{12}E_2E_{1k}$ and $-y_{21}E_1E_{2k}$ have equal radius value but contrasted with each other as to their power angles. The adove fact shows us that a half of the total number of circles will be enough to determine all diagrams we want. This can be utilized in case we map out the diagrams altogether. While, power and voltage in the transmission network are recorded usually on the power distribution diagrams in generating stations or transformimg stations. These recorded voltages and line constants which are obtained separately will supply enough values needed for equation (3) to draw every power diagram. And points of action on the diagrams can be determined by effective sending power or otherwise by effective receiving power obtained from power distribution diagrams. Instead of receving voltage, power factor or load admittance may be applied.

The above explanation related up to now is the same with that of the ordinary power diagram theories, however, the use of matrix will explain mutual relations as an interconnected transmission system. In next chapter, will be related method of getting the total sending power and total receiving power.

2. Total Sending and Receiving Power and Power Circle Diagram of Interconnected Transmisson System

Total sending power is the sum of power from 1 order to r order of sending end terminal shown by equation (3); and

$$\begin{aligned}P_1+jQ_1 &= (P_{11}+jQ_{11})+(P_{22}+jQ_{22})+\cdots\cdots+(P_{rr}+jQ_{rr})\\ &= (y_{11}|E_1|^2+Y_{22}|E_2|^2+\cdots\cdots+Y_{rr}|E_r|^2)\\ &+(-Y_{12}E_2E_{1k}-Y_{13}E_3E_{1k}\cdots\cdots-y_{1r}E_rE_{1k})\\ &+(-y_{21}E_1E_{2k}-y_{31}E_1E_{3k}\cdots\cdots-y_{r1}E_1E_{rk})\\ &+(-y_{1(r+1)}E_{r+1}E_{1k}\cdots\cdots-y_{1n}E_nE_{1k}\\ &\quad -y_{2(r+1)}E_{r+1}E_{2k}\cdots\cdots=y_{1n}E_nE_{rk})\cdots\cdots(4)\end{aligned}$$

The first term of the right side of the equation means short circuited power at each sending end. The second and the third term means driving power which is relayed and sent or received between each sending end, and every corresponding terms have contrasted power angle and the terms will become zero by being cancelled if sending loss being negligible. The fourth term means real driving power actually sent from sending ends towards receiving ends. As was already related,we may be able to attain the object by drawing each circle individually between terminals, but it is pracially impossible to draw a lot of circles on the s ame paper. So, author introduce here a different method that claims to draw deve loped circlesas shown by Figure 3. based on the equation (4). The total powera lso can be obtained easily if we have a gance at it. Basically speaking, howeve it is just an application of the orbinary circle diagram char.

Now, constant value of vector sum $(y_{11}|E_1|^2 + y_{22}|E_2|^2 \cdots + Y_{rr}|E_r|^2)$ is drawn on a co-ordinate of total sending power $(P_1 + j\ Q_1)$, and from the end of the vector all other vector terms $(Y_{12} E_2 E_{1k}, \ldots\ldots$etc.) are subtracted in the vectorical manner, and the end finally got will show the total sending power as shown by the equation (4). However, as power angles of vector $y_{12} E_2 E_{1k} \ldots\ldots$ etc. are unknown, they should be obtained by utilizing effective powers given by the power distribution diagrams. For instance, in case of $y_{12} E_2 E_{1k}$, the equation of the circle between 1 and 2 is $(p_{12} + j\ Q_{12}) = (y_{112} + Y_{12})|E_1|^2 - y_{12} E_2 E_{1k}$, and the circle center andthe radius are, respectively, $(y_{112} + y_{12})|E_1|^2$, $(y_{12} E_2 E_{1k})$. If we apply value P_{12} to the above equation, we can get j Q_{12} or power angle immediately by figure. Utilizing this reason, we subtract the value $-y_{12} E_2 E_{1k} = (-y_{112} + y_{12}) \times |E_1|^2 + P_{12} + j\ Q_{12}$. from the end of the sum of short-circuited power $(y_{11}|E_1|^2 + \cdots\cdots + y_{rr}|E_r|^2)$. Namely, we subtract a constant value $(Y_{112} + Y_{12})|E_1|^2$ first, vectorically, and secondly, from the end of the vector toward a horizontal axis we measure P_{12}, the effective power given by distribution diagram ; thirdly, passing this point draw a vertical line then catch a point intersected by the line and the circle centered on an end of $(y_{11}|E_1|^2 + \cdots\cdots + y_{rr}|E_r|^2)$ with radius $y_{12} E_2 E_{1k}$. In the same way we subtract $y_{13} E_3 E_{1k}$ starting from the intersecting point found by the above method now. Drawing of circles with radius $|y_{12} E_2 E_{1k}|$, $|y_{13} E_3 E_{1k}|, \ldots\ldots$etc. will show ordinary power circle diagrams between terminals. If we utilize calculation example, Figure 3a, in order to explain it in detail, the sum of short-circuited power of sending end is shown No. 1 2 3 4 5 6 7 8 9 17 and point 17 is the end. From this point, we subtract $y_{9,\,17} E_{17} E_{9}$, namely, subtract vector $\overline{17,\ 9}$, $(y_{9,\ 17} E_{17} + Y_{9,\ 17})|E_9|^2$, and get a point 9 first, and make a vertical line at a place where the effective power is $(P_{9,\ 17})$ which is obtained from distribution diagrams, with the point 9 as its starting point. The vertical line then intersects with circle (9, 17) of radius $(y_{9,\ 17} E_9 E_{17})$ at the point 17, which shows the point of action. In the same way we draw a circle (9, 7) by starting from the point 17. If effective power P on distribution diagrams takes a negative sign, it means relayed power reception from other ends, even though it is actually a sending end. The same method is repeated with all terms of the equation until the final point showing the total sending power is found. The final circle diagrams contain ordinary circle diagrams, and this shows not only the total sum but also the nature of their mutual relations. So, distributing or mutual relations of power, voltage, power factor, etc. in interconnected transmission system can be glanced at instantly by this. The second and the third term of equation (4) have only a difference of the power angle being contrasted, and so one was determined the other will be found naturally.

Likewise, the total receiving power is the sum of power from (r +1) order to n order of terminals in euqatiou (3); and its value is—

$$
\begin{aligned}
P_2 + j\,Q_2 &= (P_{(\gamma+1)\,(\gamma+1)} + j\,Q_{(\gamma+1)\,(\gamma+1)})\cdots\cdots + (P_{nn} + j\,Q_{nn}) \\
&= (\boldsymbol{Y}_{(\gamma+1)\,(\gamma+1)}\,|\boldsymbol{E}_{\gamma+1}|^2 \cdots\cdots\cdots\cdots\cdots\cdots + \boldsymbol{Y}_{nn}\,|\boldsymbol{E}_n|^2) \\
&+ (-\boldsymbol{y}_{(\gamma+1)\,(\gamma+2)}\,\boldsymbol{E}_{\gamma+2}\,\boldsymbol{E}_{(\gamma+1)\,k} \cdots\cdots\cdots\cdots - \boldsymbol{y}_{(\gamma+1)}\,\boldsymbol{E}_{n(\gamma+1)\,k} \\
&+ (-\boldsymbol{y}_{(\gamma+2)\,(\gamma+1)}\,\boldsymbol{E}_{\gamma+1}\,\boldsymbol{E}_{(\gamma+2)\,k} \cdots\cdots\cdots\cdots - \boldsymbol{y}_{n\,(\gamma+1)}\,\boldsymbol{E}_{\gamma+1}\,\boldsymbol{E}_{nk}) \\
&+ (-\boldsymbol{y}_{(\gamma+1)\,1}\,\boldsymbol{E}_1\,\boldsymbol{E}_{(\gamma+1)\,k} \cdots\cdots\cdots\cdots - \boldsymbol{y}_{n1}\,\boldsymbol{E}_1\,\boldsymbol{E}_{nk} \\
&\quad - \boldsymbol{y}_{(\gamma+1)\,2}\,\boldsymbol{E}_2\,\boldsymbol{E}_{(\gamma+1)\,k} \cdots\cdots\cdots\cdots - \boldsymbol{Y}_{n\gamma}\,\boldsymbol{E}_\gamma\,\boldsymbol{E}_{nk}) \cdots\cdots (5)
\end{aligned}
$$

The first term means the sum of receiving ends short-circuited power which is constant value, the second and the third terms mean driving power which is sent or received between receiving ends by relay and has contrasted power angle having nature to cancel each other, and the fourth term means driving power actually sent from sending ends to receiving ends. The fourth term is determined of itself if the fourth term of equation (4) is given.

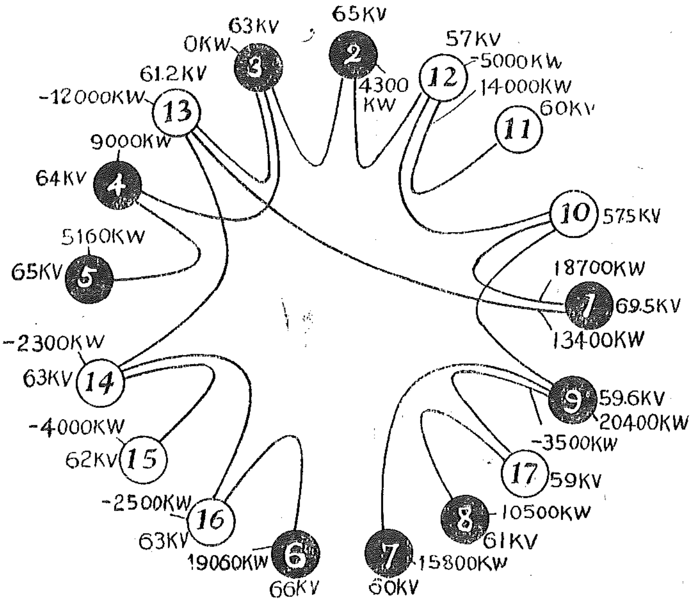
According to an usual practice, a receiving end power circle diagram is drawn with its negative power value locating at positive side of coordinate. Accordingly, total sending power and receiving power can be drawn on a same coordinate as shown by Figure 3a, b, and so it seems that an interconnected transmission system with many terminals works in a similar way with the well-known diagram of simple transmission line. It immediately shows the total sending efficiency and the total loss.

3. Example of Calculation

The main part of power transmission system ot high tension in Hokkaido District is simplified as shown by Fig. 2. Terminal 1 — 9 mean generating stations and 10—17 mean transforming stations. The power matrix of this system is shown by Table I. when calculated using the equation (3).

Short-circuited admittance, driving admittance and line constants of equivalent π circuit between terminals of this system are shown by Table II together with voltage and power of distribution diagram of Fig. 2.

Fig. 2. Power Distribution Diagram.
It shows terminal vcetage of each station and effective power at each line, when the standard voltage is 66 kv.



The number means generating station and transforming stations locoted at the places shown below

Generating Station.	Transforming Station.
1. Uryu.	10. Sunagawa.
2. Ebetsu.	11. Coal mines.
3. Kariki.	12. Bibai.
4. Moiwa.	13. Sapporo.
5. Jozankei.	14. Otaru.
6. Kombu.	15. Nagahama.
7. Kamikawa.	16. Yoichi.
8. Antaruma.	17. Asahigawa.
9. Eoroshi.	

Using this table, each term of the power matrix of Table I can be calculated, and so sending power circle diagram by equation (4) and receiving power circle diagram by equation (5) can be drawn as Figure 3 a, b, by the method related before. The drawing of circle diagrams needs value of admittance of every π circuit between terminals, absolute valne of voltages, and also effective powers numbering as much as branch lines which connect terminals. The effective power is unnecessary in case of drawing a simple transmission line system, but becomes necessary when interconncted system is drawn by the above manner. In this calculation example, there are 17 branch lines, so 17 effective power will be necessary as described on the power distribution diagram. The points of action of other effective power and reactive power will be got spontaneously on their diagrams by this drawing as related before. Without receiving voltage, voltage can be determined also by using loads or power factor. However, this method, after all, is euqal with the ordinary circle diagram method, so it may be neglected here. On the other hand, when the value of load varies, its vector moves on to the circumference of the circle as the system is based on a constant voltage system, and the situation of linking of circles will vary introducing changes of mutual relations of electric power distribution. The method shown here give us a great convenience to clarify the distribution of power and voltage, but, it necessitates rather complicated troubles because it is the same in principle with the method of ordinary circle diagram. Hower, this method seems to be an only method if we want to show the variation condition of load together with other characters.

Figure 3 a, b, are constructed only by ordinary circle diagrams that are developed by

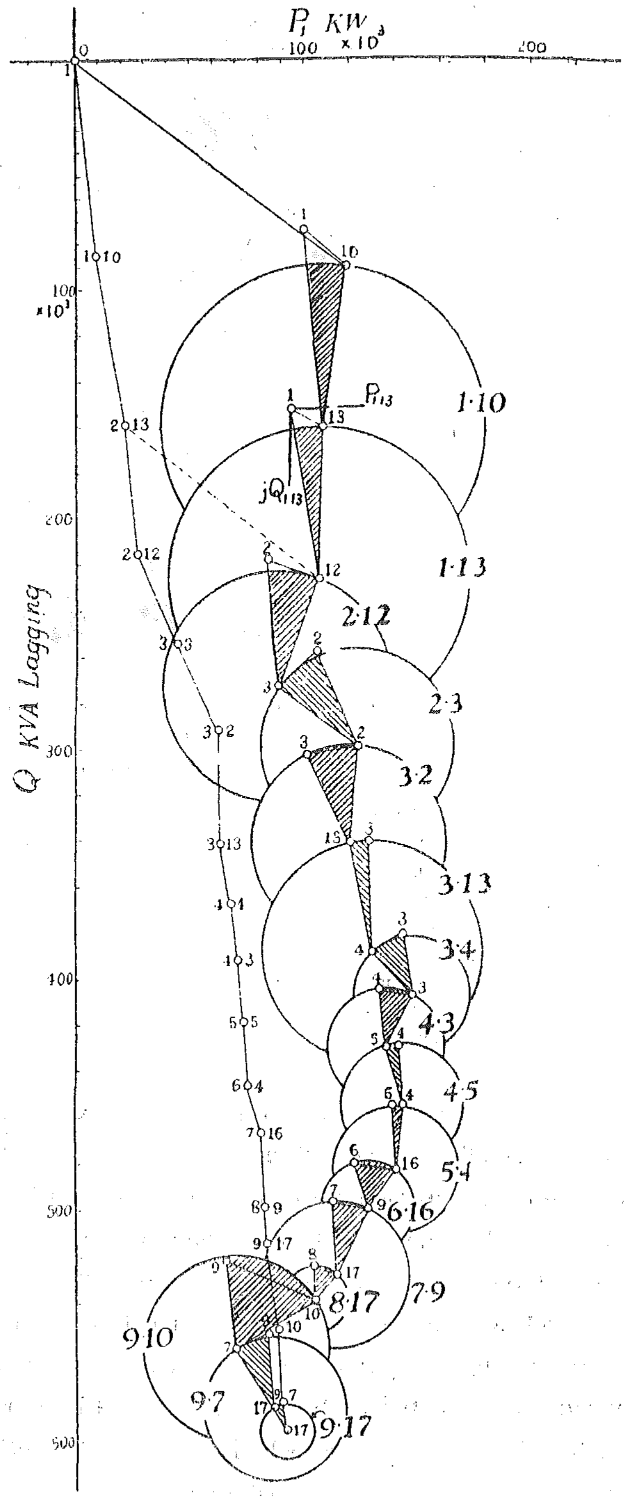


Fig. 3, a. Power Circle Diagram at Sending end.

Fig 3 a, b, indicate power matrix on Tab. 1. Main diagonal terms of matrix are indicated by indented line on the left side, other terms by radii of circles and power angles with oblique lines.

Tab. 1. Power Matrix

It shows power distribution of network shown by Fig. 2. The main diagonal terms mean short-circuited power, other terms driving power of sending or recieving.

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	$\left(\frac{D_{110}}{B_{110}}+\frac{D_{113}}{B_{113}}\right)\lvert E_1\rvert^2$									$-\frac{1}{B_{110}}E_{10}E_{1K}$			$-\frac{1}{B_{113}}E_{13}E_{1K}$				
2		$\left(\frac{D_{23}}{B_{23}}+\frac{D_{212}}{B_{212}}\right)\lvert E_2\rvert^2$	$-\frac{1}{B_{23}}E_3E_{2K}$									$-\frac{1}{B_{212}}E_{12}E_{3K}$					
3		$-\frac{1}{B_{32}}E_2E_{3K}$	$\left(\frac{A_{32}}{B_{32}}+\frac{D_{34}}{B_{34}}+\frac{D_{313}}{B_{313}}\right)\lvert E_3\rvert^2$	$-\frac{1}{B_{34}}E_4E_{3K}$									$-\frac{1}{B_{313}}E_{13}E_{3K}$				
4			$-\frac{1}{B_{43}}E_3E_{4K}$	$\left(\frac{A_4}{B_{43}}+\frac{D_{45}}{B_{45}}\right)\lvert_4E\rvert^2$	$-\frac{1}{B_{45}}E_5E_{4K}$												
5				$-\frac{1}{B_{54}}E_4B_{5K}$	$\frac{A_{54}}{B_{54}}\lvert E_5\rvert^2$												
6						$\frac{D_{616}}{B_{616}}\lvert E_6\rvert^2$										$-\frac{1}{B_{616}}E_{16}E_{6K}$	
7							$\frac{D_{79}}{B_{79}}\lvert E_7\rvert^2$		$-\frac{1}{B_{79}}E_9E_{7K}$								
8								$\frac{D_{817}}{B_{817}}\lvert E_8\rvert^2$									$-\frac{}{B_{817}}E_{17}E_{8K}$
9							$-\frac{1}{B_{97}}E_7E_{9K}$		$\left(\frac{A_{97}}{B_{97}}+\frac{D_{910}}{B_{910}}+\frac{D_{917}}{B^{917}}\right)\lvert E_9\rvert^2$	$-\frac{1}{B_{910}}E_{10}E_{9K}$							$-\frac{1}{B_{917}}E_{17}E_{9K}$
10	$-\frac{1}{B_{101}}E_1E_{10K}$								$-\frac{1}{B_{109}}E_9E_{10K}$	$\left(\frac{A_{101}}{B_{101}}+\frac{A_{109}}{B_{109}}+\frac{D_{1012}}{B_{1012}}\right)\lvert E_{10}\rvert^2$		$-\frac{1}{B_{1012}}E_{12}E_{10K}$					
11											$\frac{D_{1112}}{B_{1112}}\lvert E_{11}\rvert^2$	$-\frac{1}{B_{1112}}E_{12}E_{11K}$					
12		$-\frac{1}{B_{122}}E_2E_{12K}$								$-\frac{1}{B_{1210}}E_{10}E_{12K}$	$-\frac{1}{B_{1211}}E_{11}E_{12K}$	$\left(\frac{A_{122}}{B_{122}}+\frac{B_{1210}}{B_{1210}}+\frac{A_{1211}}{B_{1211}}\right)\lvert E_{12}\rvert^2$					
13	$-\frac{1}{B_{131}}E_1E_{13K}$		$-\frac{1}{B_{133}}E_3E_{13K}$										$\left(\frac{A_{131}}{B_{131}}+\frac{A_{133}}{B_{133}}+\frac{D_{1214}}{B_{1314}}\right)\lvert E_{13}\rvert^2$	$-\frac{1}{B_{1314}}E_{14}E_{13K}$			
14													$-\frac{1}{B_{1413}}E_{13}E_{14K}$	$\left(\frac{A_{1413}}{B_{1413}}+\frac{D_{1415}}{B_{1415}}+\frac{D_{1416}}{B_{1416}}\right)\lvert E_{14}\rvert^2$	$-\frac{1}{B_{1415}}E_{15}E_{14K}$	$-\frac{1}{B_{1416}}E_{16}E_{14K}$	
15														$-\frac{1}{B_{1514}}E_{14}E_{15K}$	$\frac{A_{1514}}{B_{1514}}\lvert E_{15}\rvert^2$		
16						$-\frac{1}{B_{166}}E_6E_{16K}$								$-\frac{1}{B_{1614}}E_{14}E_{16K}$		$\left(\frac{A_{166}}{B_{166}}+\frac{A_{1614}}{B_{1614}}\right)\lvert E_{16}\rvert^2$	
17								$-\frac{1}{B_{178}}E_8E_{17K}$	$-\frac{1}{B_{179}}E_9E_{17K}$								$\left(\frac{A_{178}}{B_{178}}+\frac{A_{179}}{B_{179}}\right)\lvert E_{17}\rvert^2$

Tab. II. Figures for Drawing Circle Diagrams.

Terminal Order	Admittance mho	Voltage kv	Effective Power kw
1	$\frac{1}{B_{110}} = 0.00203 - j\ 0.0176$ $\frac{1}{B_{113}} = 0.00254 - j\ 0.0155$ $\frac{D_{110}}{B_{110}} = 0.00203 - j\ 0.001604$ $\frac{D_{113}}{B_{113}} = 0.00254 - j\ 0.01542$	$E_1 = 69.5$	$P_{110} = 18700$ $P_{113} = 13400$ $P_{11} = 32100$
2	$\frac{1}{B_{23}} = 0.0042 - j\ 0.0093$ $\frac{1}{B_{212}} = 0.0012 - j\ 0.0135$ $\frac{D_{23}}{B_{23}} = 0.0042 - j\ 0.00931$ $\frac{D_{212}}{B_{212}} = 0.0012 - j\ 0.0\ 365$	$E_2 = 65$	$P_{23} = - \sim$ $P_{212} = + \sim$ $P_{22} = 4300$
3	$\frac{1}{B_{32}} = \frac{1}{B_{23}}$ $\frac{1}{B_{34}} = 0.00095 - j\ 0.00628$ $\frac{1}{B_{313}} = 0.00014 - j\ 0.0124$ $\frac{A_{32}}{B_{32}} = 0.0042 - j\ 0.00931$ $\frac{D_{34}}{B_{34}} = 0.00095 - j\ 0.0063$ $\frac{D_{313}}{B_{313}} = 0.00014 - j\ 0.01241$	$E_3 = 63$	$P_{32} = + \sim$ $P_{34} = - \sim$ $P_{313} = - \sim$ $P_{33} = 0$
4	$\frac{1}{B_{43}} = \frac{1}{B_{34}}$ $\frac{1}{B_{45}} = 0.00045 - j\ 0.0095$ $\frac{A_{43}}{B_{43}} = 0.00095 - j\ 0.00629$ $\frac{D_{45}}{B_{45}} = 0.00045 - j\ 0.00652$	$E_4 = 64$	$P_{43} = + \sim$ $P_{45} = - \sim$ $P_{44} = 9000$
5	$\frac{1}{B_{54}} = \frac{1}{B_{45}}$ $\frac{A_{54}}{B_{54}} = 0.00045 - j\ 0.00652$	$E_5 = 65$	$P_{54} = 5160$ $P_{55} = P_{54}$
6	$\frac{1}{B_{616}} = 0.00135 - j\ 0.0048$ $\frac{D_{616}}{B_{616}} = 0.00135 - j\ 0.00478$	$E_6 = 66$	$P_{616} = 19060$ $P_{66} = P_{616}$
7	$\frac{1}{B_{79}} = 0.00036 - j\ 0.00884$ $\frac{D_{79}}{B_{79}} = 0.00036 - j\ 0.008713$	$E_7 = 60$	$P_{79} = 15800$ $P_{77} = P_{79}$
8	$\frac{1}{B_{817}} = 0.000312 - j\ 0.0040$ $\frac{D_{817}}{B_{817}} = 0.000312 - j\ 0.00397$	$E_8 = 61$	$P_{817} = 10500$ $P_{88} = P_{817}$
9	$\frac{1}{B_{97}} = \frac{1}{B_{79}}$ $\frac{1}{B_{910}} = 0.00127 - j\ 0.01104$ $\frac{1}{B_{917}} = 0.00041 - j\ 0.00267$ $\frac{A_{97}}{B_{97}} = 0.00036 - j\ 0.008586$ $\frac{D_{910}}{B_{910}} = 0.00127 - j\ 0.001079$ $\frac{D_{917}}{B_{917}} = 0.00041 - j\ 0.00237$	$E_9 = 59.6$	$P_{97} = - \sim$ $P_{910} = + \sim$ $P_{917} = - 3500$ $P_{99} = 20400$
10	$\frac{1}{B_{101}} = 0.00203 - j\ 0.0176$ $\frac{1}{B_{109}} = 0.00127 - j\ 0.01104$ $\frac{1}{B_{1012}} = 0.00336 - j\ 0.0275$ $\frac{A_{101}}{B_{101}} = 0.0023 - j\ 0.01752$ $\frac{A_{109}}{B_{109}} = 0.00127 - j\ 0.01079$ $\frac{D_{1012}}{B_{1012}} = 0.00336 - j\ 0.02744$	$E_{10} = 57.5$	$P_{101} = - \sim$ $P_{109} = - \sim$ $P_{1012} = - \sim$ $P_{1010} = - \sim$
11	$\frac{1}{B_{1112}} = 0.00058 - j\ 0.0084$ $\frac{D_{1112}}{B_{1112}} = 0.00058 - j\ 0.008513$	$E_{11} = 60$	$P_{1112} = - \sim$ $P_{1111} = - \sim$
12	$\frac{1}{B_{122}} = \frac{1}{B_{212}}$ $\frac{1}{B_{1210}} = \frac{1}{B_{1012}}$ $\frac{1}{B_{1211}} = \frac{1}{E_{1112}}$ $\frac{A_{122}}{B_{122}} = 0.0012 - j\ 0.01335$ $\frac{A_{1210}}{B_{1210}} = 0.00336 - j\ 0.02744$ $\frac{A_{1211}}{B_{1211}} = 0.00058 - j\ 0.008513$	$E_{12} = 57$	$P_{122} = - \sim$ $P_{1210} = + \sim$ $P_{1211} = 14000$ $P_{1212} = -5000$
13	$\frac{1}{B_{131}} = \frac{1}{B_{113}}$ $\frac{1}{B_{133}} = \frac{1}{B_{313}}$ $\frac{1}{B_{1314}} = 0.00185 - j\ 0.0184$ $\frac{A_{131}}{B_{131}} = 0.00254 - j\ 0.01539$ $\frac{A_{133}}{B_{133}} = 0.00014 - j\ 0.01239$ $\frac{D_{1314}}{B_{1314}} = 0.00185 - j\ 0.01844$	$E_{13} = 61.2$	$P_{131} = - \sim$ $P_{133} = + \sim$ $P_{1314} = - \sim$ $P_{1313} = - 12000$
14	$\frac{1}{B_{1413}} = \frac{1}{E_{1314}}$ $\frac{1}{B_{1415}} = 0.00047 - j\ 0.0085$ $\frac{1}{B_{1416}} = 0.0012 - j\ 0.0052$ $\frac{A_{1413}}{B_{1413}} = 0.00185 - j\ 0.01844$ $\frac{D_{1415}}{B_{1415}} = 0.00047 - j\ 0.00851$ $\frac{D_{1416}}{B_{1416}} = 0.0012 - j\ 0.00521$	$E_4 = 63$	$P_{1413} = + \sim$ $P_{1415} = + \sim$ $P_{1416} = - \sim$ $P_{1414} = - 2300$
15	$\frac{1}{B_{1515}} = \frac{1}{B_{1415}}$ $\frac{A_{1514}}{B_{1514}} = 0.00047 - j\ 0.00851$	$E_{15} = 62$	$P_{1514} = - 400$ $P_{1515} = - P_{1514}$
16	$\frac{1}{B_{166}} = \frac{1}{B_{616}}$ $\frac{1}{B_{1614}} = \frac{1}{B_{1416}}$ $\frac{A_{166}}{B_{166}} = 0.00135 - j\ 0.00482$ $\frac{A_{1614}}{B_{1614}} = 0.0012 - j\ 0.00521$	$E_{16} = 63$	$P_{166} = - \sim$ $P_{1614} = + \sim$ $P_{1616} = - 2500$
17	$\frac{1}{B_{178}} = \frac{1}{B_{817}}$ $\frac{1}{B_{179}} = \frac{1}{B_{917}}$ $\frac{A_{178}}{B_{178}} = 0.000312 - j\ 0.00402$ $\frac{A_{179}}{B_{179}} = 0.00041 - j\ 0.00264$	$E_{17} = 59$	$P_{178} = - \sim$ $P_{179} = + \sim$ $P_{1717} = - \sim$

Foot-Note. P_{110} stands for effective sending power from terminal 1 to terminal 10, P_{11} for total power on terminal 1, and it is the same with the following correspondingly. Sign− means sending, sign+ recieving, sign∼ that the figure is determined by circle diagram.

power matrix, but some explanations will be necessary to be given as author fear they are not well-known yet : -

(1) Circle Group :

Sign (1, 10), (1, 13), etc. in Figure 3 a. means a sending end power circle from sending end 1 to receiving end 10. Power angle with oblique lines means short-circuited power betwen terminals, of which the one situated on the left side of the circle means electric receiving by relay from other terminals though is originally a sending end and that situated on the right side of the circle means electric sending As for the circle (1, 13), for instance, it should be situated on the origin of coordinate (P_1, j Q_1), but the same consequence will be got if the origin is transformed to point 1, of which the coordinate is named ($P_{1, 13}$, j $Q_{1, 13}$).

Horizontal axis takes effective power and vertical axis

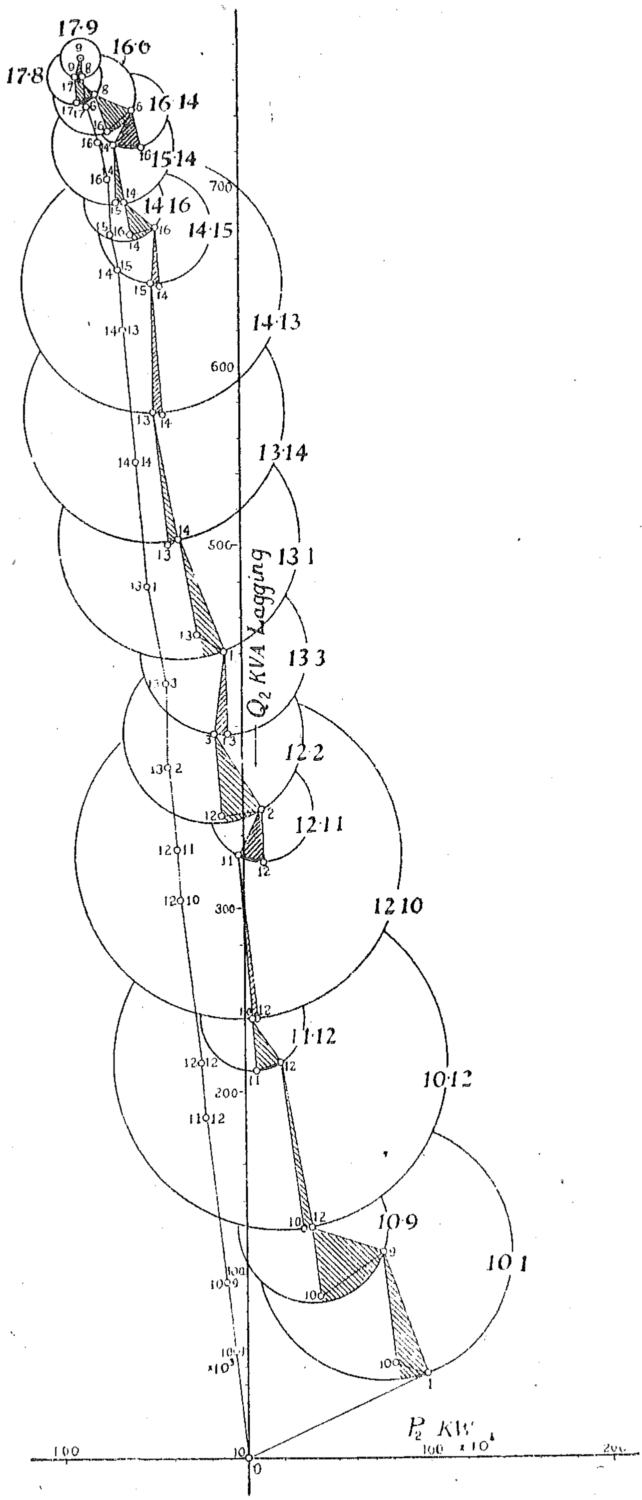


Fig. 3, b. Power Circle Diagram at Recieving end.

Circles (10, 1), (12, 10), etc. means a circle recieved from terminal 1 to terminal 10, etc. Total recieving power is shown by effective component of vector $\overline{0.1}$, and total recieving power by effective component of vector $\overline{1,10}$ on Fig. 3 a.

reactive power. Center point of the circle is obtained by transitting in parallel the vector $\overline{1, 13}$, a term in the main diagonal of power matrix. Radius of the circle is obtained from the Table II.

The point of action 13 is determined by radius of circle and effective power which is obtained from power distribution diagrams or mutual relation of this circle group. Circle (2, 3) and circle (3, 2) show that they have a contrast power angle each other, and that a sending end terminal 3 is the one receiving power from other terminals by relay.

The more transmission system becomes complicated, the more increases the number of circles, therefore proper dividing becomes necessary in this case. In the Figure 3 a, b, there exist circle (i, j) and circle (j, i) which have an equal radius value with different center distances, but, these circles respectively correspond to term (i, j) of the upper side of main diagonal and term (j, i) of the lower side.

(2) Sum of short-circuited power:

It is shown on left side of the Figure 3 as an indented line, of which components are vectors to indicate circle centers, and the component's slant means the vector angle of short-circuited admittance ($y_{iif} + y_{ij}$). With an increase of line resistance, the angle of slant becomes wider, which means an increase of line loss. In case of a system which has an ideal, line loss should be consrant in every part of the system and the indented line becomes straight. In Figure 3 a, most part of line is straight except parts $\overline{2, 3}$, $\overline{3, 2}$, $\overline{6, 16}$.

(3) Total transmission efficiency:

The effective components of vector $\overline{0, 10}$ and $\overline{0, 1}$ in the Figure 3 a, b, are the total generating power and the total receiving power, hereupon: -

$$\text{Total transmission efficiency} := \frac{99{,}000\text{KW}}{118{,}000\text{KW}} 100 = 85\%$$

(4) Phase improving condenser:

The total receiving power is 99,000 KW and its reactive power is 45,000 $\overline{\text{KW}}$ as shown by vector $\overline{0, 1}$. Even if a condenser fitted for this figure was prepared, it is unsufficient because of iregularity of current phases at each terminal,namely,because of lagging or leading which differs in reality. we take an example on circle (10, 1) of Figure 3, b. Cood efficiency can be got for electricity sending between terminals when the power factor is improved, that is approximately 1. Therefore, if we place a condenser onto a receiving end, terminal 10, and the voltage on terminal 10 is left as it is, sending voltage on terminal 1 is to become low and transmission efficiency becomes higher, It can be seen also from the Figure 3a, namely, power factor becomes better owing to sending voltage dropped. However, in case the receiving voltage on terminal 10 got dropped and is desired to be raised, and voltage on terminal 1 is left as it is, receiving voltage on terminal 10 becomes high by the condenser, and so, power factor, consequently transmission efficiency will be impr-oved. This is understood from circle (10, 1) and (1, 10). On the other hand, however, the above treatment spoils terminal 9 because excessive leading current is flowed out from this terminal, which causes reduction of transmission efficiency as it is julged from circle (10, 9). To raise this terminal voltage is not easy as thet-erminal is a relayed generating station. Accordingly, there must exist suitable values of condenser capacity fit for improvement of the power factors. To solve this difficulty, we must take efforts making calculations in various way on figures or equarions.

(5) Transmission Capacity:

This is shown as a radius of each circle if line loss is neglected. If the capacity is proportional to the radius of circle, irregularity of dimention of each circle is quite negligible. However, circles (9, 10), (6, 16), etc. which are relatively small compared with their sending power are impossible to send enough power. As the situation of circle combination on Figure 3 is irregular in order, electric power transmitted from a sending end to an aparted receiving end must be followed or obtained in accordance with the order of that of the Figure 2, and not the Figure 3. That is, sending power is not transmitted in order of the circle order of Figure 3. If sending power is proportional to the radii of circles, all power angles and line losses become equal,and an indented line showing short-circuited power becomes straight. Then Figure 3a, b, will become like an ordinal transmission circle diagram of a single transmission line.

4. Conclusion

Distribution of power and voltage of an interconnected power transmisson system can be converted into an equivalent junction points network, and if it is developed in matrix the main diagonal means short circuited power, terms of the upper side of the diagonal mean sending driving power, and the lower side mean receiving driving power. Also, if we divide the system into sending ends and receiving ends and sum them up with variable factors of each of their voltage's phase angle, "power angle," we can draw a power circle dragiam of the net-work, which is an assembly of ordinary power circle diagram between terminals. The diagram does show not merely the total sending and receiving power, but all other kinds of electric nature of the system. Even though there are some presumptive values in the network taken up as an example and the real situation is different from that calculation, this shows that we can improve the line system furthermore.