Experimental Studies on Reinforced Concrete Rockers—On the Strength of Contact surface

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Synopsis

In this paper, the results of tests on the strength of concrete at contact surface of reinforced concrete rocker, a kind of movable supports of reinforced concrete beam, and the effect of bearing plate are described, and the method of design of contact surface is showed.

CONTENTS

Introduction

Formerly, some tests were made by the writer upon the reinforced concrete rocker, a kind of movable supports of reinforced concrete beams, to find the distribution of horizontal tensile stress in vertical symmetrical section, the effect of arrangement of reinforcements, the effect of steel bearing plate at contact surface, and the method of design and safety factor for working load. And from these results, recommended method of design of reinforced concrete rockers was showed[1]. Further experiments and calculations have been continued on the strength of contact surface, and on the effect of bearing plate.

In this paper, the results of further studies are described, with some reduplications

Note—[1]The writer, "Experimental Studies on Reinforced Concrete Rockers," Journal of the Civil Engineering Society of Japan, Vol. 27. No. 7. July 1941, Vol. p. 630~651.

of the former report.

1. Tests on Allowable Bearing Unit Stress of Concrete at Contact Surface

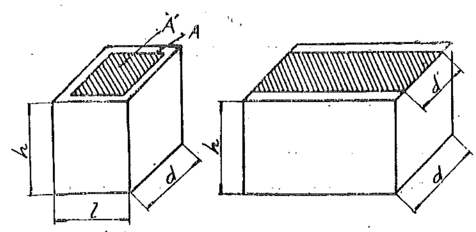
Because the concrete near at contact surface of reinforced concrete rocker is suffered to partial loading, that causes high compressive stress in concrete. In the case of partial loading on concrete surface, higher allowable bearing unit stress can be taken compared to the case of full surface loading. This allowable bearing unit stress depends upon not only the strength of strut itself, but the relation between loading area (A') and total area of the strut (A). In the "Standard Specification for Reinforced Concrete" of the Civil Engineering Society of Japan, allowable bearing unit stresses are specified as following.

When the surface area of strut (A) is wider than the loaded area (A'), the allowable bearing unit stress σ_{ca}' can be based on the following formulas (Fig. 1):

In case (a) $\sigma_{ca}' = \sigma_{ca} \sqrt[3]{\frac{A}{A'}}$

In case (b) $\sigma_{ca}' = \sigma_{ca} \sqrt[3]{\frac{d}{d'}}$

$$\left(\sigma_{ca} = \frac{\sigma_{28}}{3.5} \right)$$



(a) $h \geq d$ ($d > l$) (b) $h \geq d$

Fig. 1.

But σ_{ca}' shall not be greater than 120 kg/cm².

These formulas are based upon tests made by Bauschinger and Bach on sand stone, and by Graf on concrete, and these are adopted in standard specifications of some countries. But in our country, no test had been made about this problem.

Though the limit of σ_{ca}' (120kg/cm²) was specified by the range of above tests, actually, in design of reinforced cocnrete rockers which have circular end surfaces, it is quite difficult to keep the unit stress at contact surface below 120 kg/cm². Further, from the results of compression tests on rockers by the writer, it could be expected that the limit of σ_{ca}' would be far more raised. At the same time, there is room for discussions about the formulas themselves.

For such reasons, tests were intended by the writer.

1. The Former Experiments and Discussions on the Present Specification.

As the former experiments about the bearing strength of concrete when partially loaded, there are those by Bauschinger, Bach and Graf. But two formers tested on sand stone, and only Graf tested on concrete.[2)]

(1) Tests by Bauschinger and Bach (1876)

Test specimens of sand stone of about 10cm × 10cm × 10cm were used. Bearing area was varied by steel plates.

Fig. 2 shows the results by Bauschinger, and Fig. 3 shows the results by Bach.

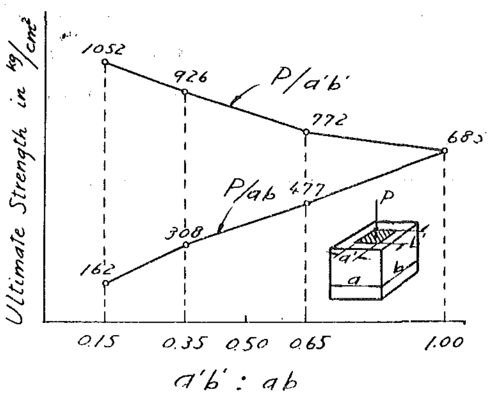


Fig. 2. Test Results by Bauschinger on Sand Stone Specimens.

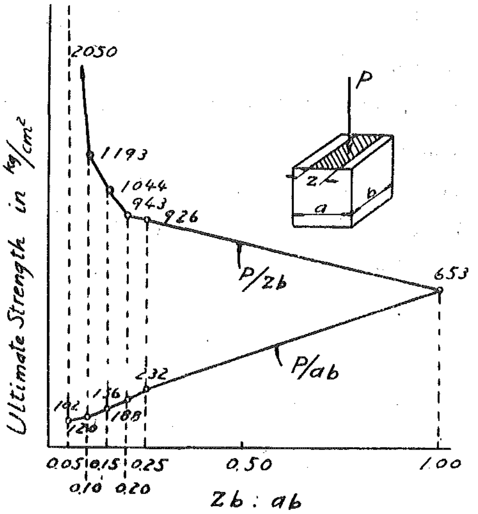


Fig. 3. Test Results by Bach on Sand Stone Specimens.

(2) Tests by Graf (1921)

In his tests, specimens of concrete were used, and the results are shown in Fig. 4.

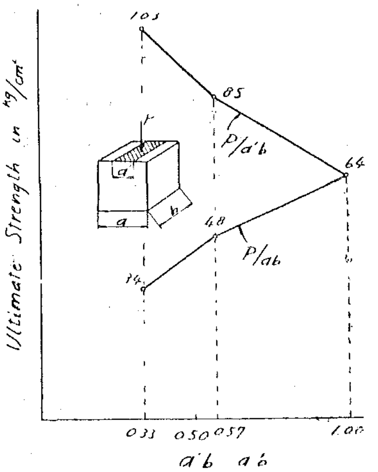


Fig. 4. Test Results by Graf on Concrete Specimens.

In the tests, strength of concrete was very poor (64 kg/cm²), and the range of tests was comparatively limited.

Effects of height and width of strut, effects of partial loading on both ends, etc, were tested too. The results show that the bearing strength decreases as the height of strut increases, and increases as the width increases, and when the strut is partially loaded on both ends the bearing strength is nearly independent of loaded area. Here the bearing strength means the value of load divided by loaded area.

As mentioned at the start, in the "Specification for Reinforced Concrete"

Note – 2) Bach-Baumann, Elastizität und Festigkeit, 9. Aufl. s. 213.
Emperger, Handbuch für Eisenbetonbau, 3. Aufl. I. Bd., s. 350.

of the Civil Engineering Society of Japan, $\sigma_{ca}' = \sigma_{ca} \sqrt[3]{\frac{d}{d'}}$ for line load, and $\sigma_{ca}' = \sigma_{ca} \sqrt[3]{\frac{A}{A'}}$ for point load are specified as the formulas of allowable bearing unit stresses. That is, the general type of the formulas is $K' = K \sqrt[n]{\frac{A}{A'}}$ $(n=3)$, and this type is adopted in the specification for concrete in Germany too. This type of formula is based on above mentioned tests by Bauschinger, Bach and Graf, but, some of these tests were made on sand stone, and the range of tests was limited, and there is still room for discussions about the value of n. And it is doubtful whether the formula for line load is the same as that for point load. Further, in case of consolidation of test results in formulas, the general type of formulas must be continuous at $A = A'$. That is, it must be $K' = \beta K = \alpha \sigma_{28}$.

Therefore the writer adopted the general type of formulas as following,

$$K' = \beta K \sqrt[n]{\frac{A}{A'}} = \alpha \sigma_{28} \sqrt[n]{\frac{A}{A'}}$$

in which K' : bering strength

K : strength of strut itself

σ_{28} : strength of concrete at 28 days

A : sectional area of strut

A' : bearing area

It was intended to determine the value of n, β and α in above formula by tests, in each case of line loading and point loading.

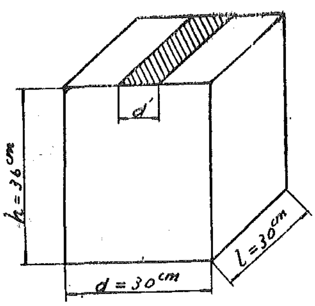


Fig. 5. Dimensions of Test Specimens (in Case of Line Load).

2. Allowable Bearing Unit Stress of Concrete in Case of Line Loading.

The contact surface of usual reinforced concrete rocker, which is movable in one direction, is suffered to line load. Though the contact surface has some width, we will call it line load for convenience sake.

The dimensions of test specimens used in this test are, width $d = 30$ cm, length $l = 30$cm and height $h = 36$ cm (1.2 d), as shown in Fig. 5.

The strength of concrete used in the test is as shown in Table 1.

Tabele 1 Strength of Concrete Used in Line Loading Test

No. of Specimen	Proportions of Mixture by Volume	Water Cement Ratio (%)	Slump (cm)	Flow	Age (days)	Dimensions of Specimens (cm) Diameter	Height	Weight of Unit Volume (kg/m³)	Ultimate Load (t)	Compressive Strength (kg/cm²)
1	1:2:4	61	6.5	185	28	15	30	2435	47.0	266
2	〃	〃	〃	〃	〃	〃	〃	2424	46.3	262

3	1:2:4	61	6.5	185	28	15	30	2411	48.0	272
4	〃	〃	8.5	190	〃	〃	〃	2418	51.5	291
5	〃	〃	〃	〃	〃	〃	〃	2416	52.5	297
6	〃	〃	〃	〃	〃	〃	〃	2411	51.0	289
Mean								2419		280

The specimens for test of bearing strength were manufactured in wooden forms, and unmolded after 3 days, then cured in water of standard temperature (about 20°C) till the day before test. Then, the top and bottom surfaces of specimen were rubbed with grindstone. They were all tested at the age of 28 ~ 30 days. For each loading area 3 specimens were tested, and the loading area was varied by steel plates with several breadths. The range of d/d' is from 1 to 30.

The test results are summerized in Table 2.

Table 2. Test Results of Bearing Strength (in Case of Line Load)

No. of Speci-mens	Breath of Loading d'(cm)	Age (days)	Weight of Unit Vol-ume (kg/m³)	Load P (t)		P/dl (kg/cm²)		$P/d'l$ (kg/cm²)	
				Crack	Failure	Crack	Failure	Crack	Failure
1	30	28	2441	315	338	350	376	350	376
2	〃	29	2451	303	372	337	413	337	413
3	〃	〃	2444	360	373	400	414	400	414
4	15	28	〃	187	224	208	249	415	498
5	〃	29	2441	—	225	—	250	—	500
6	〃	〃	2426	174	220	193	244	387	489
7	25	28	2441	295	330	328	367	393	440
8	〃	29	2429	290	297	322	330	387	396
9	〃	30	2426	242	302	269	336	323	403
10	10	28	〃	115	166	128	184	383	553
11	〃	29	〃	125	178	139	198	417	593
12	〃	〃	〃	103	181	114	201	343	603
13	20	28	2433	146	262	162	291	243	437
14	〃	29	2432	215	253	240	281	358	422
15	〃	30	2435	146	245	162	272	243	408
16	5	28	2426	84	119	93.3	132	560	790
17	〃	〃	2432	83	102	92.2	113	553	677
18	〃	〃	2420	194	126	104	140	627	841
19	2.5	〃	2423	75	75	83.3	83.3	1000	1000
20	〃	〃	2426	82	82	91.6	91.6	1099	1099
21	〃	〃	2398	70	70	77.6	77.6	931	931
22	1	〃	2407	60	60	66.7	66.7	2000	2000
23	〃	〃	2426	59	59	65.6	65.6	1967	1967
24	〃	〃	2423	55	61	61.1	68.0	1833	2040

The mean values of strengths for each beaing breadth are as shown in Table 3, in which $A = dl$ and $A' = d'l$.

Table 3. Mean Values for Each Breadth of Loading.

Breadth of Loading d' (cm)	$A'/A = d'/d$	$A/A' = d/d'$	P/A (kg/cm²)		P/A' (kg/cm²)	
			Crack	Failure	Crack	Failure
30	1.00	1.00	362	401	362	401
25	0.83	1.20	306	344	368	413
20	0.67	1.50	188	281	281	422
15	0.50	2.00	201	248	401	463
10	0.33	3.00	127	194	381	583
5	0.17	6.00	95.6	128	580	769
2.5	0.08	12.00	84.2	84.2	1010	1010
1	0.03	30.00	64.5	66.8	1933	2002

Almost in all cases, failure occured in wedge shape under the loading surfaces.

Table 3 is illustrated in Fig. 6, which shows the relation between $A'/A = d'/d$ and P/A or P/A'.

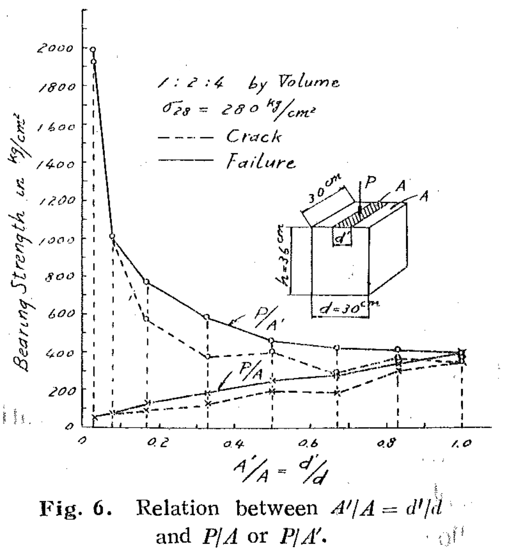


Fig. 6. Relation between $A'/A = d'/d$ and P/A or P/A'.

The relation between $P/A' = K$ and A/A' is nearly in straight line on logarithmic section paper, therefore, the relation can be represented in general as

$$K' = r \sqrt[n]{\frac{A}{A'}}$$

Now put

$$K' = \beta K \sqrt[n]{\frac{A}{A'}} = \beta K \left(\frac{A}{A'}\right)^m$$
$$= \alpha\, \sigma_{28} \sqrt[n]{\frac{A}{A'}} \quad \cdots\cdots(1)$$

and β, α and m can be determined from the test results.

By method of least square, we get

$$m = 0.46$$
$$n = \frac{1}{m} = 2.16$$
$$\beta = 0.90$$

Therefore, equation (1) becomes

$$K' = 0.90\, K \sqrt[2.16]{\frac{A}{A'}} \quad \cdots\cdots(2)$$

Next, put

$$K' = \alpha\, \sigma_{28} \sqrt[2.16]{\frac{A}{A'}} \quad \cdots\cdots (3)$$

then $\qquad \alpha = 1.28$

Therefore, equation (2) becomes

$$K' = 1.28\, \sigma_{28} \sqrt[2.16]{\frac{A}{A'}} \quad \cdots\cdots (4)$$

in which, coefficient 1.28 depends upon the dimensions of strut, especially upon the height.

Now, equation (4) will be compared with the equation specified in "Standard Specification for Reinforced Concrete."

In Fig. 7, line (1) shows the test results, line (2) represents the formula specified in "Standard Specification" when σ_{28} is 210 kg/cm^2 (maximum strength of concrete considered in "Standard Specification"), and line (3) represents the formula obtained from the test if we put $\sigma_{28} = 210$ kg/cm^2.

Line (3) shows larger K' than line (2), and the difference between these increases as A/A' increases. In the range where A/A' is from 1 to 2, the test results show the tendency close to line (2).

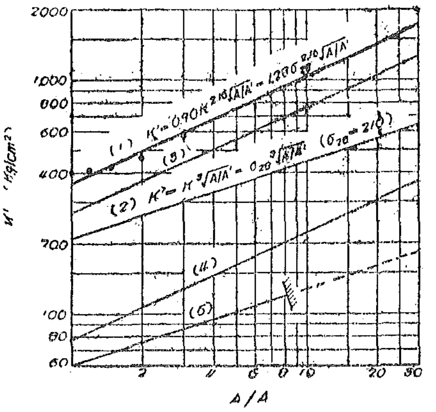


Fig. 7. Comparison between the Fomula Specified in "Standard Specification" and the Test Results (in Case of Line Loading).

The formula specified in "Standard Specification" is applicable only in the range to $A/A' \doteqdot 8$, but the formula here obtained is applicable up to $A/A' = 30$, that is, the limit of allowable bearing unit stress σ_{ca}' can be raised more than the present specification.

Line (5) in Fig. 7 shows the allowable bearing unit stress with safety factor of 3.5 in respect to line (2), but in this case, applicable limit is only up to $A/A' \doteqdot 8$.

On the other hand, line (4) shows the allowable bearing unit stress with safety factor of 3.5 in respect to line (3), which is the line obtained from test results when $\sigma_{28} = 210$ kg/cm^2. The line (4) is applicable in all the range of tests, A/A' is from 1 to 30, and it shows that the maximum value of σ_{ca}' is 367 kg/cm^2.

When we put $\alpha = 1$ in equation (4) on safety side,

$$\sigma_{ca}' = \sigma_{ca} \sqrt[2.16]{\frac{A}{A'}} \quad \cdots\cdots (5)$$

which is shown in Fig. 8. The formula obtained from tests for point load, later mentioned, is shown together in Fig. 8.

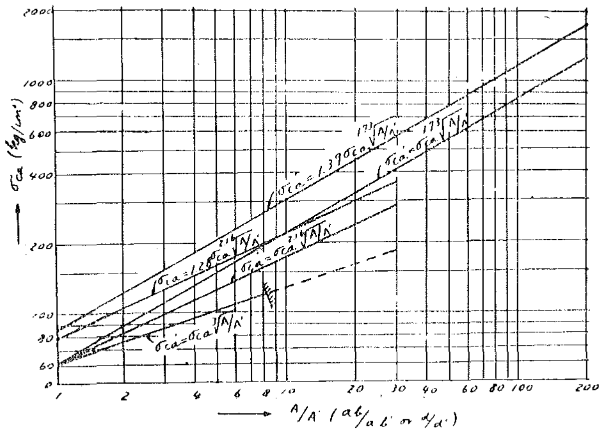


Fig. 8. Allowable Bearing Unit Stress when We Put $a = 1$.

Further, varying the exponent on the safety side to simplify the formula, the writer propose next formula of allowable bearing unit stress in case of line loading.

$$\sigma_{ca}' = \sigma_{ca} \sqrt[2.2]{\frac{A}{A'}} \cdots\cdots (6)$$

Equation (6) is compared with the present specified formula in Fig. 9, in which assumed $\sigma_{28} =$ 210 kg/cm², that is, $\sigma_{ca} = 60$ kg/cm².

The formula for point loading is together shown in Fig. 9.

From Fig. 9, it is proved that the allowable limit of σ_{ca}' can be taken up to 280 kg/cm² in case of line loading.

Thus, by adopting the larger bearing unit stress, it grows far easier to design the contact surfaces of reinforced concrete rockers.

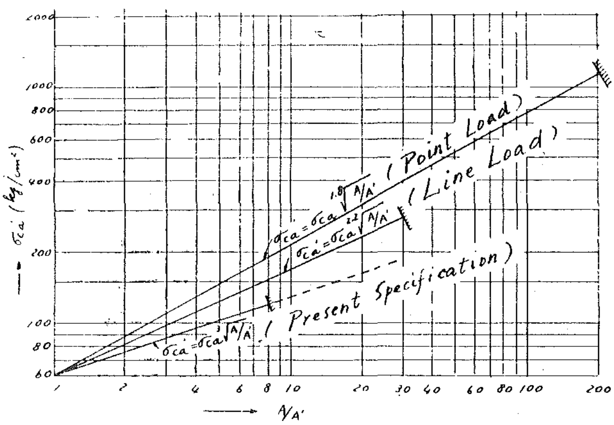


Fig. 9. Comparison between the Present Specified Formula and the Formula Proposed by the Writer.

3. Allowable Bearing Unit Stress of Concrete in Case of Point Loading.

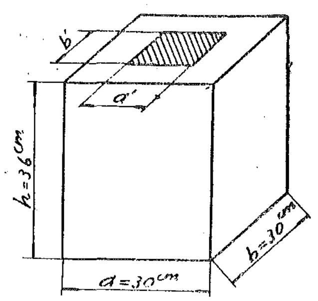


Fig. 10. Dimensions of Test Specimens (in Case of Point Load).

The contact surface of rocker, which is movable in any direction, is suffered to point load. Strictly speaking, it has some area, but for convenience sake we will call it point load.

The dimensions of test specimens used in this test are the same as those used in case of line loading, that is, width $a = 30$cm, length $b = 30$cm and heigt $h = 36$ cm, as shown in Fig. 10.

The strength of concrete used in this test is as shown in Table. 4, and the mean value of strength is 156 kg/cm², which is smaller than that in line loading test.

Table 4. Strength of Concrete Used in Point Loadig Test.

No. of Speci-men	Propor-tions of Mixture by Volume	Water Cement Ratio	Slump (cm)	Flow	Age (days)	Dimensions of Specimens (cm)		Weight of Unit Volume	Ultimate Load (t)	Compres-sive Strength (kg/cm^2)
						Diameter	Height			
1	1:2:4	62	6.5	195	28	15	30	2403	27.5	156
2	〃	〃	〃	〃	〃	〃	〃	2403	26.2	148
3	〃	〃	〃	〃	〃	〃	〃	2399	28.8	163
Mean								2401		156

The dimensions of steel plates used for loading were 30cm × 30cm (total area of strut), 25cm × 25cm, 18cm × 18cm, 9cm × 9cm, 5cm × 5cm, and 2cm × 2cm.

The test results are summerized in Table 5.

Table 5. Test Results of Bearing Strength (in Case of Point Load).

No. of Specimens	Area of Loading (cm^2) $a' \times b' = A'$	Weight of Unit Volume (kg/m^3)	Load P (t)		P/ab (kg/cm^2)		$P/a'b'$ (kg/cm^2)	
			Crack	Failure	Crack	Failure	Crack	Failure
1	30×30=900	2392	170	268	189	298	189	298
2	〃	2426	193	235	214	261	214	261
3	〃	2393	113	197	126	219	126	219
4	9× 9= 81	2448	55.0	58.0	61.1	64.4	679	716
5	〃	2438	70.0	70.0	77.8	77.8	864	864
6	〃	2401	66.0	66.0	73.3	73.3	815	815
7	25×25=625	2448	—	168	—	187	—	269
8	〃	2414	—	185	—	206	—	296
9	30×30=900	2444	151	205	168	228	168	228
10	25×25=625	2414	94.0	159	104	177	150	254
11	5× 5= 25	2441	41.0	41.0	45.6	45.6	1640	1640
12	〃	2401	44.0	44.0	48.9	48.9	1760	1760
13	〃	2457	45.0	50.0	50.0	55.6	1800	2000
14	18×18=324	2407	82.0	97.0	91.1	108	253	299
15	〃	2433	85.0	114	94.4	127	262	352
16	〃	2414	90.0	115	100	128	278	355
17	2× 2= 4	2451	11.0	21.4	12.2	23.8	2750	5350
18	〃	2420	11.0	20.8	12.2	23.1	2750	5200
19	〃	2454	14.9	19.8	16.6	22.0	3725	4950
20	ϕ10.2=81.8	2417	45.0	46.5	50.0	51.7	550	568
21	〃	2420	63.0	63.0	70.0	70.0	770	770
22	〃	2401	58.5	58.5	65.0	65.0	715	715

(All specimens were tested at age of 28 days)

No. 20 ∼ 22 were loaded by circular steel plates having diameters of 10.2 cm, for purposes of reference.

When the loading area was $2 \times 2 = 4\,\mathrm{cm}^2$, concrete showed bearing unit stress in excess of $5000\,\mathrm{kg/cm}^2$, far larger value than in case of line loading.

The mean values of strengths for each loading area are as shown in Table 6, in which $A = ab$ and $A' = a'\,b'$.

Table 6. Mean Values for Each Breadth of Loading.

Area of Loading (cm²) $a' \times b' = A'$	A'/A	A/A'	P/A (kg/cm²)		P/A' (kg/cm²)	
			Crack	Failure	Crack	Failure
30×30=900	1.000	1.000	174	252	174	252
25×25=625	0.694	1.440	104	190	150	273
18×18=324	0.360	2.778	95.2	121	264	335
9× 9= 81	0.090	11.111	70.7	71.8	786	798
5× 5= 25	0.028	36.000	48.2	50.3	1733	1800
2× 2= 4	0.004	225.000	13.7	23.0	3075	5167

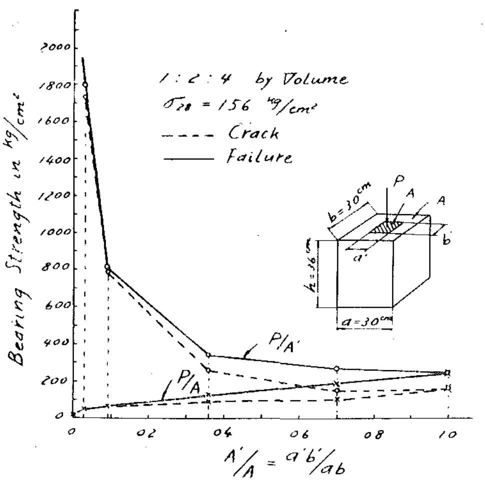


Fig. 11. Relation between $A'/A = a'b'/ab$ and P/A or P/A'

When failure occured, steel plates were pressed in concretes, finally causing longitudinal cracks on sides of specimens almost in all cases.

Table 6 is illustrated in Fig. 11, which shows the relation between $A'/A = a'b'/ab$ and P/A or P/A'.

In this case too, the relation between K' and A/A' can be represented in general by following formula, as same as in case of line loading.

$$K' = \gamma \sqrt[n]{\frac{A}{A'}}$$

Now, β, α and m in equation (1) shall be determined from test results.

By method of least square, we get

$$m = 0.58$$

$$n = \frac{1}{m} = 1.73$$

$$\beta = 0.86$$

Therefore, equation (1) becomes

$$K' = 0.86\,\mathrm{K} \sqrt[1.73]{\frac{A}{A'}} \quad \cdots\cdots (7)$$

Next, putking $\alpha = 1.39$ in regard to σ_{28}, equation (7) becomes

$$K' = 1.39\ \sigma_{28}\ \sqrt[1.73]{\frac{A}{A'}} \quad \cdots\cdots (8)$$

In Fig. 12, equation (7) or (8) is compared with the formula specified in the present specification.

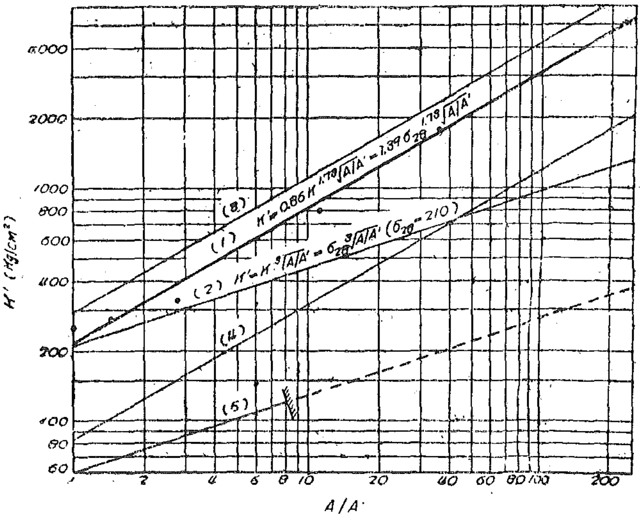


Fig. 12. Comparison between the Formula Specified in "Standard Specification" and the Test Results (in Case of Point Loading).

Line (1) in Fig. 12 shows the test results, line (2) represents the formula specified in "Standard Specification" when σ_{28} is 210 kg/cm^2, and line (3) represents the formula obtained from the test if we put $\sigma_{28} = 210$ kg/cm^2.

Comparing line (2) and (3), the difference of K' increases as A/A' is larger. and the increase of K' is larger than that in case of line loading.

In this case too, in the range where A/A' is from 1 to 3, the test results show the tendency close to line (2).

Further, the formula here obtained is applicable up to $A/A' = 200$.

Line (5) shows the allowable bearing unit stress with safety factor of 3.5 in respect to line (2), in which applicable limit is up to $A/A' = 8$.

On the other hand, line (4) shows the allowable bearing unit stress with safety factor of 3.5 in respect to line (3). The line (4) is applicable in all the range of tests, $A/A' = 1 \sim 200$, and it shows that the maximum value of σ_{ca}' is 1803 kg/cm^2.

As same as in case of line loading, putting $\alpha = 1$ in equation (8) on safety side,

$$\sigma_{ca}' = \sigma_{ca}\ \sqrt[1.73]{\frac{A}{A'}} \quad \cdots\cdots (9)$$

which is shown in Fig. 8.

Further, varying the exponent on the safety side to simplify the formula, in case of point loading, the writer propose next formula of allowable bearing unit stress.

$$\sigma_{ca}' = \sigma_{ca} \sqrt[1.8]{\frac{A}{A'}} \quad \cdots\cdots (10)$$

In Fig. 9, equation (10) is compared with the present specified formula.

Equation (10) is applicable up to $A/A' = 200$, and from Fig. 9, it is proved that in case of point loading the allowable limit of σ_{ci}' can be taken up to 1140 kg/cm^2 (when $\sigma_{28} = 210$ kg/cm^2), far larger value than the present specification.

4. Proposal to Revise the Present Specification and Simplified Formulas.

From the results of tests, which were held in wide range of d/d' or A/A', the writer propose to revise the present spcification as follows.

When the surface area of strut (A) is wider than the loaded area (A'), the allowable bearing unit stress σ_{ca}' shall be based on the following formulas (see Fig.1):

In case (a) $\quad \sigma_{ca}' = \sigma_{ca} \sqrt[1.8]{\frac{A}{A'}}$

In case (b) $\quad \sigma_{ca}' = \sigma_{ca} \sqrt[2.2]{\frac{d}{d'}}$

in which $\sigma_{ca} = \frac{\sigma_{28}}{3.5}$

But σ_{ca}' shall not be greater than 1140 kg/cm^2 in case (a), and 280 kg/cm^2 in case (b).

Next, to simplify the calculation of σ_{ca}', it was tried to derive simplified approximate formulas in the range of A/A' or $d/d' = 1 \sim 30$.

Generally put

$$\sigma_{ca}' = x\,\sigma_{ca} \sqrt{\frac{A}{A'} \text{ or } \frac{d}{d'}}$$

then, next simplified formulas are obtained.

$$\left.\begin{array}{ll} \text{In case (a)} & \sigma_{ca}' = 1.1\,\sigma_{ca} \sqrt{\dfrac{A}{A'}} \\ \text{In case (b)} & \sigma_{ca}' = 0.9\,\sigma_{ca} \sqrt{\dfrac{d}{d'}} \end{array}\right\} \quad \cdots\cdots (11)$$

Errors by equation (11) are as follows:

In case (a), error is $+10 \sim -10\%$ in whole range of A/A', but only $+6 \sim -3\%$ in the range of $A/A' = 2 \sim 10$. In case (b), error is $-10 \sim +5\%$ in whole range of d/d', but only $-7 \sim -0\%$ in the range of $d/d' = 2 \sim 10$.

Practically, errors in these extents are permissible, and above mentioned approximate formulas can be adopted in these ranges of A/A' or d/d'.

II. Effect of Bearing Plate on Stress in Concrete

Usually, on the contact surfaces of reinforced concrete rocker, steel bearing plates are used to prevent the crushing of concrete due to narrow strip load, and to make better contact between the rocker and bottom surface of girder or upper surface of pier or abutment (see Fig. 13).

By the bearing plate, load P is distributed on some width of concrete.

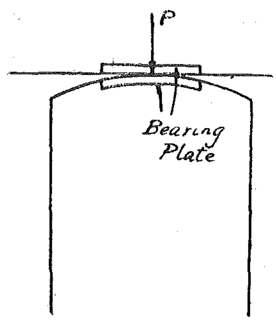


Fig. 13

In this chapter, the width of distributed load on concrete, the distribution of compressive stress, and the relation between these and permissible bearing unit stress of concrete mentioned in the former chapter, are treated.

1. Distribution of Load by Bearing Plate on Concrete.

Taking unit length of rocker, the bearing plate may be treated as a beam placed on elastic foundation. Strictly saying, it is a curved beam, however, because the deformation occurs in narrow part, it can be treated as a straight beam.[3)]

The general differential equation of elastie deformation of a beam is

$$EI\frac{d^2y}{dx^2} = -M$$

Solutions of the equation when a straight beam is laid on elastic foundation loaded with a single concentrated load P on the center, are as follows (Fig. 14):

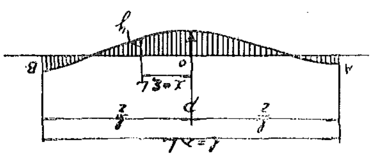


Fig 14

$$\left.\begin{aligned} y &= \frac{P}{2KL}\left[[\xi]_1 + a\cosh\xi\cos\xi + b\sinh\xi\sin\xi\right] \\ p &= \frac{P}{2L}\left[[\xi]_1 + a\cosh\xi\cos\xi + b\sinh\xi\sin\xi\right] \\ M &= \frac{PL}{4}\left[[\xi]_3 - b\cosh\xi\cos\xi + a\sinh\xi\sin\xi\right] \end{aligned}\right\} \cdots\cdots\cdots\cdots(12)$$

in which

$$\left.\begin{aligned} K &= \text{coefficient of elastic foundation} \\ L &= \sqrt[4]{\frac{4EI}{K}} \\ \lambda &= \frac{l}{L} \end{aligned}\right\}$$

Note : [3)]Following calculations are refered to "Theorie des Trägers auf elastischer Unterlage und ihre Anwendung auf den Tiefbau", by Keiichi Hayashi, Berlin, 1921.

$$\left.\begin{aligned}
\xi &= \frac{x}{L} \\
a &= \frac{2+\cos\lambda-\sin\lambda+e^{-\lambda}}{\sinh\lambda+\sin\lambda} \\
b &= \frac{\cos\lambda+\sin\lambda-e^{-\lambda}}{\sinh\lambda+\sin\lambda} \\
[\xi]_1 &= e^{-\xi}[\cos\xi+\sin\xi] \\
[\xi]_3 &= e^{-\xi}[\cos\xi-\sin\xi]
\end{aligned}\right\} \cdots\cdots\cdots\cdots (13)$$

In the case here treated, it should be assumed that the bond between steel plate and concrete can not take any tension in portion where y is negative (Fig. 15).

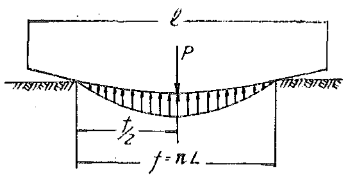


Fig. 15

Then, the width of contact area, f, is given by

$$\begin{aligned} f &= \pi L \\ &= \pi \sqrt[4]{\frac{4EI}{K}} \cdots\cdots\cdots\cdots (14) \end{aligned}$$

and y, p and M are given by following formulas.

$$\left.\begin{aligned}
y &= \frac{P}{2KL}\left[[\xi]_1 + 0.0903(\cosh\xi\cos\xi - \sinh\xi\sin\xi)\right] \\
p &= \frac{P}{2\,L}\left[[\xi]_1 + 0.0903(\cosh\xi\cos\xi - \sinh\xi\sin\xi)\right] \\
M &= \frac{PL}{4}\left[[\xi]_3 + 0.0903(\cosh\xi\cos\xi + \sinh\xi\sin\xi)\right]
\end{aligned}\right\} \cdots\cdots\cdots\cdots (15)$$

Under the point of loading,

$$y_o = 0.545\frac{P}{KL}$$

$$p_o = 0.545\frac{P}{L}$$

$$M_o = 0.273\,PL$$

Now, with reference to the coefficient of elastic bed, K, when a steel beam is laid on concrete bed, it may be considered as follows (Fig. 16).

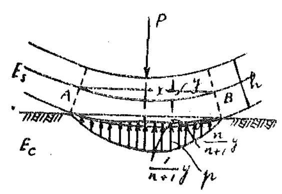


Fig. 16

Let y is vertical displacemnent of axis of a deformed beam, then

$$p = Ky$$

And let y_s = deformation of bottom surface of steel beam

y_c = deformation of concrete

E_s = modulus of elasticity of steel

E_c = modulus of elasticity of concrete

$$n = \frac{E_s}{E_c}$$

then $$\frac{y_s}{y_c} = \frac{E_c}{E_s} = \frac{1}{n}$$

Therefore $$y_s = \frac{1}{n} y_c$$

And $$y = y_s + y_c = y_c \left(\frac{1}{n} + 1\right) = y_c \left(\frac{n+1}{n}\right)$$

Therefore $$y_c = \frac{n}{n+1} y \quad \text{and} \quad y_s = \frac{1}{n+1} y$$

Strain

$$\varepsilon = \frac{y_s}{\frac{h}{2}} = \frac{\frac{1}{n+1} y}{\frac{h}{2}} = \frac{2y}{(n+1)h}$$

Stress $$\sigma = p$$

$$E_s = \frac{\sigma}{\varepsilon} = \frac{p}{\frac{2y}{(n+1)h}} = \frac{(n+1)\,ph}{2y}$$

Therefere $$K = \frac{p}{y} = \frac{2E_s}{(n+1)h} \quad \cdots\cdots (17)$$

Put $n = 10$, then equation (17) becomes

$$K = \frac{2E_s}{11h} \quad \cdots\cdots (18)$$

Moment of inertia of a rectangular section with unit width is

$$I = \frac{h^3}{12}$$

Therefore

$$L = \sqrt[4]{\frac{4E_s I}{K}} = \sqrt[4]{\frac{4E_s h^3 \cdot 11h}{12 \cdot 2E_s}} = \sqrt[4]{\frac{11}{6}}\, h = 1.164\,h \quad \cdots\cdots (19)$$

From equation (14)

$$f = \pi L = 3.1416 \times 1.164\,h \fallingdotseq 3.66\,h \quad \cdots\cdots (20)$$

Equation (20) indicates that concentrated load P is distributed by steel bearing plate on area having a width of $3.66h$.

Substituting equation (19) into equation (16),

$$p_o = \frac{0.545P}{1.164\,h} = 0.468\,\frac{P}{h}$$

If it were uniformly distributed

$$p_o = \frac{P}{\pi L} = \frac{P}{3.66h} = 0.273\,\frac{P}{h}$$

which is of 41.8% smaller than that.

Substituting equation (19) into equation (15), we get

$$p = \frac{P}{2.328\,h}\Big[[\xi]_1 + 0.0903\,(\cosh\xi\cos\xi - \sinh\xi\sin\xi)\Big] \cdots\cdots\cdots\cdots\cdots\cdots (21)$$

Values of p in the contact area, calculated by equation (21) are shown in Table 7.

Table 7. Values of p

ξ	x	p
0	0	0.4683 P/h
$\pi/6$	$f/6$	0.3753 P/h
$\pi/3$	$f/3$	0.1993 P/h
$\pi/2$	$f/2$	0 P/h

Table 8. Values of f and p for Various Thicknesses of Bearing Plates.

h (cm)	f (cm)	p_o	$p_{\frac{f}{6}}$	$p_{\frac{f}{3}}$	$p_{\frac{f}{2}}$
2.0	7.32	0.2342P	0.1877P	0.0997P	0
1·5	5.49	0.3122P	0.2502P	0.1329P	0
1.0	3.66	0.4683P	0.3753P	0.1993P	0
0.5	1.83	0.9366P	0.7506P	0.3986P	0

Values of f and p for various thicknesses of bearing plates are shown in Table 8 (see Fig. 17).

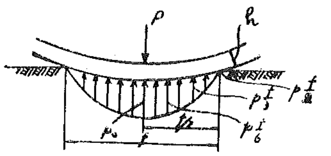


Fig. 17

From Table 8, Fig. 18 is obtained, which shows that the thinner the thickness of bearing plate, the distributing width is smaller and the compressive unit stress under the point of loading is larger.

Assume that P is distributed as $p = p_o \cos\frac{\pi x}{f}$, as Bortsch did, then

$$P = \int_{-\frac{f}{2}}^{\frac{f}{2}} p\,dx = p_o \int_{-\frac{f}{2}}^{\frac{f}{2}} \cos\frac{\pi x}{f}\,dx$$

$$= 2\,p_o\,\frac{f}{\pi}$$

Therefore $\qquad p_o = \dfrac{\pi}{2f}\,P$

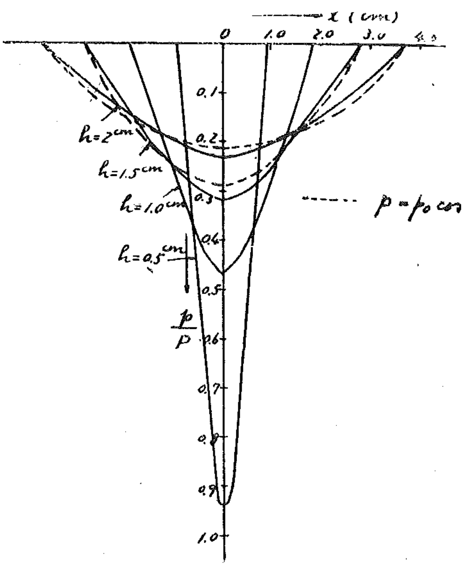


Fig. 18. Distributions of p for Various Thicknesses of Bearing Plates.

Table 9 shows the values of p for verious values of h in the case of cosine distribution.

Table 9. Values of p in Case of Cosine Distribution.

h (cm)	f (cm)	p_o	$p_{\frac{f}{6}}$	$p_{\frac{f}{3}}$	$p_{\frac{f}{2}}$
2.0	7.32	0.2146 P	0.1784 P	0.1030 P	0
1.5	5.49	0.2861 P	0.2478 P	0.1431 P	0
1.0	3.66	0.4292 P	0.3717 P	0.2146 P	0
0.5	1.83	0.8584 P	0.7434 P	0.4292 P	0

In Fig. 18, by broken line, the differences between the two cases are shown for $h = 2.0$cm and 1.5cm.

Ratio of value of p calculated by assuming as consine distribution to the value of p calculated by equation (21) under the point of loading is

$$\frac{p_o = \dfrac{\pi}{2f}P = \dfrac{\pi}{7.32\,h}P}{p_o = 0.4683\,\dfrac{P}{h}} = 0.92$$

Value of p at loaded point calculated by assuming cosine distribution is 8 % smaller than value of p calculated by equation (21). However, approximately, we may consider it as cosine distribution with few error, for convenience of treatment of calculations.

Knowing the distribution of load on concrete by bearing plate, bending moment in bearing plate may be calculated by equation (15), and the unit stress in plate may be found by

$$\sigma = \frac{M}{W}$$

Here too, it is unnecessary to treat it as a curved beam, because the thickness of plate is far thinner compared to the radius of curvature.

2. Uniform Distributed Load Equivalent to Actually Distributed Load.

Though the distribution of load has been found as shown in Fig. 18, this must be translated into equivalant uniform load, to check whether the stress intensity is within the allowable bearing unit stress of concrete loaded by line load mentioned in former chapter. That is, the width b of equivalent uniform load must be determined.

Because the actual distribution of load may be assumed as cosine distribution with neglisible small error, as before mentioned, the writer intended to find the width of uniform distributed load, causing deformation of top boundary surface nearly equal to that caused by cosine distributed load.

It may be assumed that there is few effect of type of load distribution on the side boundary surfaces of a rocker. Therefore, in calculation of the equivalent distribution of load, we may treat this as a plate having infinite length, neglecting the boundary conditions on both sides.

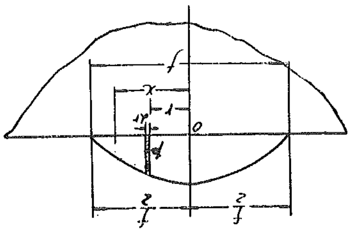


Fig. 19

When a unit load pdr acts on boundary surface of semi-infinite plate, as shown in Fig. 19, the vertical displacement v_x at point x on boundary surface is given by next formula.[4)]

$$v_x = \frac{2\,pdr}{\pi\,E}\,ln\frac{d}{x-r} - \frac{(1+\nu)}{\pi\,E}\,pdr$$

Therefore, the displacement at point x due to any distributed load is obtained by integrating on all the loads.

In the first, in case of uniformly distributed load, let b is the width of distribution, then the displacement becomes as follows.

At the portions where $x \leqq \frac{b}{2}$ (outside of loaded area)

$$v_x = \frac{2}{\pi\,E}\,P\int_{-\frac{b}{2}}^{\frac{b}{2}} ln\frac{d}{x-r}\,dr - \frac{1+\nu}{\pi\,E}\,p\int_{-\frac{b}{2}}^{\frac{b}{2}} dr$$

$$= \frac{P}{\pi\,E}\left[\,ln\,\frac{d^2}{x^2-\left(\frac{b}{2}\right)^2} - \frac{2}{b}\,x\,ln\frac{x+\frac{b}{2}}{x-\frac{b}{2}} + (1-\nu)\,\right] \cdots\cdots\cdots\cdots\cdots\cdots (2)_1$$

At the portions where $0 \leqq x \leqq \frac{b}{2}$ (inside of loaded area)

$$v_x = \frac{2}{\pi\,E}\,p\left[\int_{-\frac{b}{2}}^{x} ln\frac{d}{x-r}\,dr + \int_{x}^{\frac{b}{2}} ln\frac{d}{x-r}\,dr\right] - \frac{1+\nu}{\pi\,E}\,p\int_{-\frac{b}{2}}^{\frac{b}{2}} dr$$

$$= \frac{P}{\pi\,E}\left[\,ln\,\frac{d^2}{\left(\frac{b}{2}\right)^2-x^2} - \frac{2}{b}\,x\,ln\frac{\frac{b}{2}+x}{\frac{b}{2}-x} + (1-\nu)\,\right] \cdots\cdots\cdots\cdots\cdots\cdots (22)_2$$

At $x = \frac{b}{2}$

$$v_x = \frac{P}{\pi\,E}\left[\,ln\,\frac{d^2}{b^2} + (1-\nu)\,\right] \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (22)_3$$

Next, in case of cosine distribution, let f is the width of distribution, then

$$p = p_o \cos\frac{\pi\,r}{f}$$

At the portions outside of loaded area

$$v_x = \frac{2\,p_o}{\pi\,E}\int_{-\frac{b}{2}}^{\frac{b}{2}} \cos\frac{\pi\,r}{f}\,ln\frac{d}{x-r}\,dr - \frac{1+\nu}{\pi\,E}\,p_o\int_{-\frac{b}{2}}^{\frac{b}{2}} \cos\frac{\pi\,r}{f}\,dr$$

To integrate the first term, we expand cosine into series.

Note—[4)]Timoshenko, Theory of Elasticity, 1934.

$$\cos\frac{\pi r}{f}=1-\frac{\left(\frac{\pi r}{f}\right)^2}{2!}+\frac{\left(\frac{\pi r}{f}\right)^4}{4!}-\cdots\cdots+(-1)^n\frac{\left(\frac{\pi r}{f}\right)^{2n}}{(2n)!}$$

$$+\cdots\cdots\cdots\cdots \qquad \left(-\frac{\pi}{2}<\frac{\pi r}{f}<\frac{\pi}{2}\right)$$

Let $\frac{\pi}{f}=a$, then

$$\int\cos ar\,ln\frac{d}{x-r}dr=\int ln\frac{d}{x-r}dr-\frac{a^2}{2!}\int r^2\,ln\frac{d}{x-r}dr$$

$$+\frac{a^4}{4!}\int r^4\,ln\frac{d}{x-r}dr+\cdots\cdots+(-1)^n\frac{a^{2n}}{(2n)!}\int r^{2n}\,ln\frac{d}{x-r}dr+\cdots\cdots$$

$$\int r^{2n}\,ln\frac{d}{x-r}dr=\frac{r^{2n+1}}{2n+1}ln\frac{d}{x-r}-\frac{1}{2n+1}\int\frac{r^{2n+1}}{x-r}dr$$

$$=\frac{r^{2n+1}}{2n+1}ln\frac{d}{x-r}+\frac{1}{2n+1}\left\{-\frac{(x-r)^{2n+1}}{2n+1}+{}_{2n+1}C_1\,x\frac{(x-r)^{2n}}{2n}\right.$$

$$\left.-{}_{2n+1}C_2\,x^2\frac{(x-r)^{2n-1}}{2n-1}+\cdots\cdots+x^{2n+1}ln(x-r)\right\}$$

Therefore

$$\int_{-\frac{b}{2}}^{\frac{b}{2}}\cos\frac{\pi r}{f}\,ln\frac{d}{x-r}dr$$

$$=\left[\frac{f}{2}ln\frac{d^2}{x^2-\left(\frac{f}{2}\right)^2}+f-x\,ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right]$$

$$-\frac{1}{2!}\left(\frac{\pi}{f}\right)^2\left[\frac{\left(\frac{f}{2}\right)^3}{3}ln\frac{d^2}{x^2-\left(\frac{f}{2}\right)^2}+\frac{2}{3}\left(\frac{f}{2}\right)\left\{x^2+\frac{\left(\frac{f}{2}\right)^2}{3}\right\}-\frac{x^3}{3}ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right]$$

$$+\frac{1}{4!}\left(\frac{\pi}{f}\right)^4\left[\frac{\left(\frac{f}{2}\right)^5}{5}ln\frac{d^2}{x^2-\left(\frac{f}{2}\right)^2}+\frac{2}{5}\left(\frac{f}{2}\right)\left\{x^4+\frac{x^2}{3}\left(\frac{f}{2}\right)^2+\frac{1}{5}\left(\frac{f}{2}\right)^4\right\}\right.$$

$$\left.-\frac{x^5}{5}ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right]$$

$$-\frac{1}{6!}\left(\frac{\pi}{f}\right)^6\left[\frac{\left(\frac{f}{2}\right)^7}{7}ln\frac{d^2}{x^2-\left(\frac{f}{2}\right)^2}+\frac{2}{7}\left(\frac{f}{2}\right)\left\{x^6+\frac{1}{3}x^4\left(\frac{f}{2}\right)^2\right.\right.$$

$$\left.\left.+\frac{1}{5}x^2\left(\frac{f}{2}\right)^4+\frac{1}{7}\left(\frac{f}{2}\right)^6\right\}-\frac{x^7}{7}ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right]+\cdots\cdots\cdots\cdots$$

Integrating the second term

$$\int_{-\frac{b}{2}}^{\frac{b}{2}}\cos\frac{\pi r}{f}dr=\frac{2f}{\pi}$$

$$\frac{1+\nu}{\pi E}p_0\int_{-\frac{b}{2}}^{\frac{b}{2}}\cos\frac{\pi r}{f}dr=2\,p_0\frac{f}{\pi}\cdot\frac{1+\nu}{\pi E}$$

Put $p_0=\frac{\pi}{2f}P$ into above formulas, then v_x at the portion outside of loaded area becomes as follows.

$$v_x = \frac{P}{\pi E}\cdot\frac{\pi}{f}\left\{\left[\frac{f}{2}\,ln\frac{d^2}{x^2-\left(\frac{f}{2}\right)^2} + f - x\,ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right]\right.$$

$$-\frac{1}{2!}\left(\frac{\pi}{f}\right)^2\left[\frac{\left(\frac{f}{2}\right)^3}{3}\,ln\frac{d^2}{x^2\left(\frac{f}{2}\right)^2} + \frac{2}{3}\left(\frac{f}{2}\right)\left\{x^2+\frac{\left(\frac{f}{2}\right)^2}{3}\right\} - \frac{x^3}{3}\,ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right]$$

$$+\frac{1}{4!}\left(\frac{\pi}{f}\right)^4\left[\frac{\left(\frac{f}{2}\right)^5}{5}\,ln\frac{d^2}{x^2-\left(\frac{f}{2}\right)^2} + \frac{2}{5}\left(\frac{f}{2}\right)\left\{x^4+\frac{x^2}{3}\left(\frac{f}{2}\right)^2+\frac{1}{5}\left(\frac{f}{2}\right)^4\right\}\right.$$

$$\left.-\frac{x^5}{5}\,ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right]$$

$$-\frac{1}{6!}\left(\frac{\pi}{f}\right)^6\left[\frac{\left(\frac{f}{2}\right)^7}{7}\,ln\frac{d^2}{x^2-\left(\frac{f}{2}\right)^2} + \frac{2}{7}\left(\frac{f}{2}\right)\left\{x^6+\frac{1}{3}x^4\left(\frac{f}{2}\right)^2\right.\right.$$

$$\left.\left.+\frac{1}{5}x^2\left(\frac{f}{2}\right)^4+\frac{1}{7}\left(\frac{f}{2}\right)^6\right\} - \frac{x^7}{7}\,ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right] + \cdots\cdots\Bigg\}$$

$$-P\,\frac{1+\nu}{\pi E}$$

Put $x = \frac{f}{n}$, then in the result, v_x at the portion outside of loaded area becomes as follows.

$$v_x = \frac{P}{\pi E}\left\{\left[ln\frac{d^2}{x^2-\left(\frac{f}{2}\right)^2}\right] + \pi\,1 - \frac{1}{n}\,ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right]$$

$$-\frac{\pi^3}{3!}\left[\left\{\frac{1}{n^2}+\frac{1}{12}\right\} - \frac{1}{n^3}\,ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right] + \frac{\pi^5}{5!}\left[\left\{\frac{1}{n^4}+\frac{1}{12n^2}+\frac{1}{80}\right\}\right.$$

$$\left.-\frac{1}{n^5}\,ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right] - \frac{\pi^7}{7!}\left[\left\{\frac{1}{n^6}+\frac{1}{12n^4}+\frac{1}{80n^2}+\frac{1}{448}\right\} - \frac{1}{n^7}\,ln\frac{x+\frac{f}{2}}{x-\frac{f}{2}}\right]$$

$$+\cdots\cdots\cdots\cdots$$

$$\left.-(1+\nu)\right\}\cdots\cdots\cdots\cdots\cdots(23)_1$$

Similarly, v_x at the portion inside of loaded area is obtained by calculating the formula,

$$v_x = \frac{2p_0}{\pi E}\left[\int_{-\frac{f}{2}}^{x}\cos\frac{\pi r}{f}\,ln\frac{d}{x-r}\,dr + \int_{x}^{\frac{b}{2}}\cos\frac{\pi r}{f}\,lr\frac{d}{r-x}\,dr\right]$$

$$-\frac{1+\nu}{\pi E}\,p_0\int_{-\frac{b}{2}}^{\frac{b}{2}}\cos\frac{\pi r}{f}\,dr$$

In the result,

$$v_x = \frac{P}{\pi E}\left\{\left[ln\frac{d^2}{\left(\frac{f}{2}\right)^2-x^2}\right] + \pi\left[1-\frac{1}{n}\,ln\frac{\frac{f}{2}+x}{\frac{f}{2}-x}\right]\right.$$

$$-\frac{\pi^3}{3!}\left[\left\{\frac{1}{n^2}+\frac{1}{12}\right\} - \frac{1}{n^3}\,ln\frac{\frac{f}{2}+x}{\frac{f}{2}-x}\right] + \frac{\pi^5}{5!}\left[\left\{\frac{1}{n^4}+\frac{1}{12n^2}+\frac{1}{80}\right\}\right.$$

$$- \frac{1}{n^5} ln \frac{\frac{f}{2}+x}{\frac{f}{2}-x} \Bigg] - \frac{\pi^7}{7!} \Bigg[\left\{ \frac{1}{n^6} + \frac{1}{12n^4} + \frac{1}{80n^2} + \frac{1}{448} \right\} - \frac{1}{n^7} ln \frac{\frac{f}{2}+x}{\frac{f}{2}-x} \Bigg] + \cdots$$

$$- (1+\nu) \Bigg\} \cdots\cdots\cdots\cdots (23)_2$$

At $x = \frac{f}{2}$,

$$v_x = \frac{P}{\pi E} \left\{ ln \frac{d^2}{f^2} + \pi - \frac{\pi^3}{3!} \left\{ \frac{1}{n^2} + \frac{1}{12} \right\} \right.$$

$$+ \frac{\pi^5}{5!} \left\{ \frac{1}{n^4} + \frac{1}{12n^2} + \frac{1}{80} \right\} - \frac{\pi^7}{7!} \left\{ \frac{1}{n^6} + \frac{1}{12n^4} + \frac{1}{80n^2} + \frac{1}{448} \right\} + \cdots\cdots\cdots\cdots$$

$$\left. - (1+\nu) \right\} \cdots\cdots\cdots\cdots (23)_3$$

Table 10 shows the displacement of boundary surface computed by eqaution (23) in case of cosine distribution for the thickness of bearing plate h = 2 cm, that is, f = 3.66h = 7.32 cm, assuming that d (depth to the point where the vertical displacement is zero) = 45cm, m (Poisson's number of concrete) = 6, and $\nu = \frac{1}{\mathrm{m}}$ (Poisson's-ratio) = 0.6667. Here, Value of d has effect only upon the relative situation of displacemant curve, and has no effect on the relative quantity of displacement.

Table 10. Values of v_x Due to Load of Cosine Distribution

(h = 2cm, f = 7.32cm, d = 45cm, ν = 0.6667)

x	0	$f/6$	$f/3$	$f/2$	f	$2f$
v_x	6.090 $\frac{P}{\pi E}$	5.857 $\frac{P}{\pi E}$	4.765 $\frac{P}{\pi E}$	3.613 $\frac{P}{\pi E}$	2.048 $\frac{P}{\pi E}$	0.591 $\frac{P}{\pi E}$

Plotting the values of Table 10, the curve shown in Fig. 20 is obtained.

Next, the width b of uniformly distributed load, giving a curve as nearly equal to this displacement curve, should be determined.

Now, the displacements due to uniformly distributed load, computed by equation (22) for $b = f$, 0.8 f, 0.7 f, and 0.6 f, are as shown in Table 11.

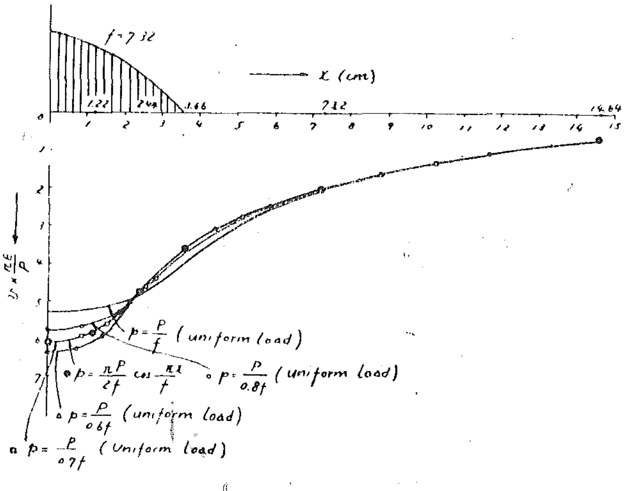


Fig. 20 Comparison of Displacements of Boundary Surface.

Table 11. Values of v_c Due to Uniform Distributed Load ($h = 2\text{cm}$, $f = 7.32\text{cm}$, $d = 45\text{cm}$, $\nu = 0.6667$)

x	0	$b/6$	$b/3$	$b/2$	b	$2b$
$v_{b=f}$	$5.352\,\dfrac{P}{\pi E}$	$5.238\,\dfrac{P}{\pi E}$	$4.866\,\dfrac{P}{\pi E}$	$3.965\,\dfrac{P}{\pi E}$	$2.056\,\dfrac{P}{\pi E}$	$0.600\,\dfrac{P}{\pi E}$
$v_{b=0\cdot 8f}$	5.798 〃	5.685 〃	5.313 〃	4.412 〃	2.502 〃	1.047 〃
$v_{b=0.7f}$	6.065 〃	5.952 〃	5.579 〃	4.656 〃	2.767 〃	1.313 〃
$b_{b=0.6f}$	9.373 〃	6.260 〃	5.888 〃	4.985 〃	3.075 〃	1.620 〃

Plotting the Values of Table 11 on Fig. 20, we can find that the two show nearly equal displacement when b is equal to $0.7\,f$.

For comparison, magnified curve is shown in Fig, 21, in which both are nearly on one curve.

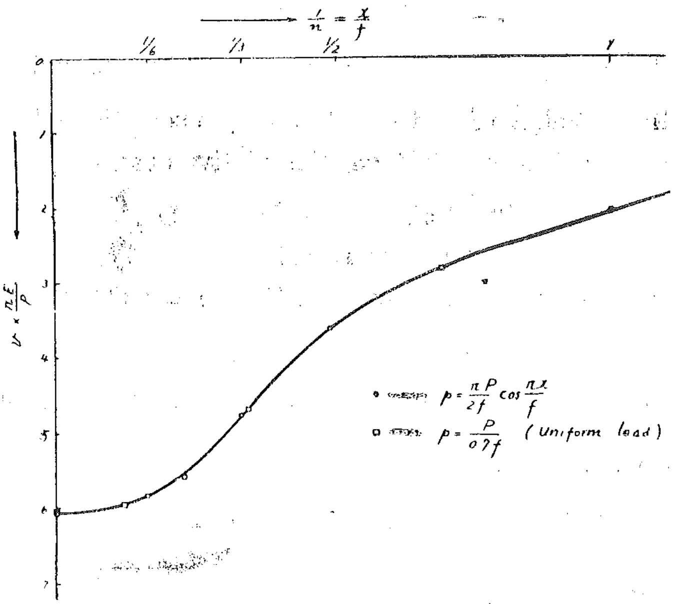


Fig 21. Comparison of Displacements (Magnified Curve).

By above comparative calculations, cosine distributed load can be substituted by uniform distributed load having a width of $0.7\,f$, that is, $p = \dfrac{P}{0.7f}$.

Above calculations are applicable for any values of f, therefore in the end we can put

$$\left.\begin{aligned} b &= 0.7\,f \\ &= 0.7 \times 3.66\,h \\ &= 2.562\,h \end{aligned}\right\} \qquad (24)$$

For change of d or d/f, the curve will vary its relative situation upward or downward.

In design, after calculating

$$\frac{P}{b}=\frac{P}{2.562\,h}$$

the intensity of stress may be checked whether it does not exceed the allowable bearing unit stress, gained in the former chapter. Reversely, strength of concrete and thickness of bearing plate may be determined so that the stress intensity of concrete is within the allowable bearing unit stress.

In above, load P has been treated as a concentrated load, but in actual, P has some width of distribution. Therefore, if necessary, this width of distribution may be taken in consideration.

3. Practical Example of Design.

To design the thickness of bearing plate and the strength of concrete in contact with the bearing plate, in case of the reinforced concrete rocker as shown in

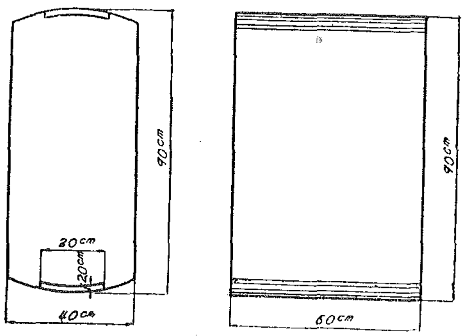


Fig. 22 Shape and Dimensions of Reinforced Concrete Rocker.

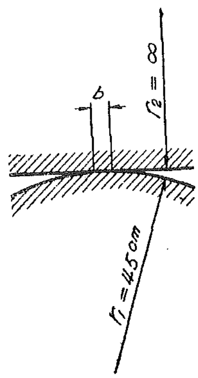


Fig. 23 Contact Condition

Fig. 22.

Both rocking surfaces are circular, having its center at one half of the height of rocker.

a. Design of Steel Bearing Plate.

Because the thickness of steel plate is far thinner compared to the depth of concrete, and the deformation of steel is about one tenth of the deformation of concrete, we may consider the contact condition as same as the case of contact between concrete and concrete (Fig. 23).

The breadth b of contact area is given by Hertz's formula.

$$b=4\sqrt{\frac{2P}{\pi l E}\,\frac{1-\frac{1}{m^2}}{\frac{1}{r_1}+\frac{1}{r_2}}}$$

Put in above formula

$$P = 100{,}000\text{kg},\ l = 60\text{cm},\ E = 210{,}000\text{kg/cm}^2,\ m = 6,$$

$$r_1 = 45\text{cm},\ r_2 = \infty$$

then

$$b = 4\sqrt{\frac{2 \times 100{,}000}{3.1416 \times 60 \times 210{,}000} \cdot \frac{1-\frac{1}{36}}{\frac{1}{45}}} = 1.95\text{cm}$$

Therefore, maximum compressive unit stress in contact area becomes

$$\sigma_{max} = \frac{4P}{\pi bl} = \frac{4 \times 100{,}000}{3.1416 \times 1.95 \times 60} = 1{,}088\ \text{kg/cm}^2$$

Contact unit stress for steel may be permissible up to 6500~7500 kg/cm², therefore, there is no question as to contact stress of plate.

Next, the stress of bearing plate due to bending moment will be determined. From equation (16), bending monent at the loaded point is

$$M_o = 0.273\,PL \text{ (in which } P \text{ represents the load per unit length)}$$

From equation (19)

$$L = 1.164\,h = 1.164 \times 2 = 2.328\text{cm}$$

Therefore

$$\text{M}o = 0.273 \times \frac{100{,}000}{60} \times 2.328 = 1059\text{kg}\cdot\text{cm}$$

Section modulus

$$W = \frac{bh^2}{6} = \frac{4}{6}$$

Therefore,

$$\sigma = \frac{M}{W} = 1059 \times \frac{6}{4} = 1589\text{kg/cm}^2$$

This value is greater than the allowable unit stress of steel, usually of 1200 kg/cm², but this may be permissible if the stress of concrete under the plate is within its allowable bearing unit stress, and if it may be taken into consideration that load P is distributed on some width on its loaded point.

Reversely, when the thickness of bearing plate, h, would be determined, adopting $\sigma = 1200\text{kg/cm}^2$, h becomes 2.3 cm. Of course, it is better to use a bearing plate having thickness of 23mm.

b. Design of Strength of Concrete in Contact with Bearing Plate.

While the width of load on concrete distributed by bearing plate is from equation (20),

$$f = 3.66\,h = 7.32\text{cm}$$

Concentrated load P is already distributed on a width of 1.95cm at the contact surface, therefore, in the end f may be considered approximately as

$$f = 7.32 + 1.95 = 9.27\text{cm}$$

The width of uniformly distributed load equivelent to this is from equation (24)

$$b = 0.7\,f \fallingdotseq 6.5\text{cm}$$

Therefore, the intensity of compressive stress in concrete becomes

$$p = \frac{P}{bl} = \frac{100{,}000}{6.5 \times 60} = 256\text{kg/cm}^2$$

By equation (6), the permissible bearing unit stress of concrete is

$$\sigma'_{ca} = \sigma_{ca} \sqrt[2.2]{\frac{d}{d'}}$$

Put $\quad \dfrac{d}{d'} = \dfrac{40}{6.5} = 6.15$

$$\sqrt[2.2]{\frac{d}{d'}} = 2.283$$

then $\quad \sigma'_{ca} = 2{,}283\,\sigma_{ca}$ (it is necessary to be $p \leqq \sigma'_{ca}$)

$$\sigma_{ca} = \frac{\sigma'_{ca}}{2.283} = \frac{256}{2{,}283} = 112\text{kg/cm}^2$$

Because the allowable bearing uint stress may be raised about 20% by use of reinforcement,

$$\sigma_{ca} = \frac{112}{1.2} = 93.3\text{kg/cm}^2$$

Therefore, adopting safety factor of 3.5,

$$\sigma_{28} = 3.5 \times 93.3 = 326\text{kg/cm}^2$$

It is necessary to use the concrete whose strength is greater than this value.

If the bearing plate does not be used, the intensity of stress in concrete becomes very large as follows.

$$b = 1.95 \times 0.7 = 1.365\text{cm}$$

$$p = \frac{P}{bl} = \frac{100{,}000}{1.365 \times 60} = 1220\text{kg/cm}^2$$

$$\frac{d}{d'} = \frac{40}{1.365} = 29.3 < 30$$

From equation (6),

$$\sigma'_{ca} = \sigma_{ca} \sqrt[2.2]{\frac{d}{d'}} = \sigma_{ca} \sqrt[2.2]{29.3}$$

$$= 433\text{kg/cm}^2 \quad < 1220\text{kg/cm}^2$$

Thus, we find that it is necssary to use the steel bearing plate on the contact surface, or else it will be difficult to keep the stress in concrete within the permissible bearing unit stress.

Conclusion

Contents of this paper are summerized as follows:

1. Tests were made upon the allowable bearing unit stress of concrete when it is suffered to partial loading, and in result of these the experimental formulas were gained, and thus, it became clear that in design far larger bearing unit stress are permissible than the value given by the formula specificedl in pressent "Standard Specification for Renforced Concrete".

2. Treating the steel bearing plate as a beam on elastic foundation, the state of distribution of load in concrete by bearing plate was calculated, and it became possible to design the thickness of steel bearing plate and the strength of concrete in contact with the plate.

3. The width of uniform load equivalent to the calculated state of distribution of load was found by computation, and it was made possible to apply the test results to design of strength of concrete.

4. In the last, a practical example of design was showed to provide better understanding.

Further the rocking resistance of rockers is under studies by the writer now, and the result will be described in next report.

Adding: For this study, a subsidy was granted by the Department of Education.

(The End)