Over-Expansion in the Nozzle

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(Received Sept. 28, 1951)

In this paper some experimental data are presented on the energy losses occurring in over-expansion nozzle, which are important factors in estimating the steam or gas turbine performance under non-design conditions such as over-load, part load, or starting conditions.

The velocity coefficients of a divergent nozzle are measured by the apparatus for testing turbine nozzles by the reaction method. At the same time the pressure distributions along the nozzle axis are taken with the pitot tube and the locations of the shock in the divergent part of nozzle are confirmed. The results of these tests show good agreement qualitatively with the calculated results based upon some assumptions. However, theoretical calculation is difficult at the down stream after the shock when the separation occurs at the shock. These shock waves and the separation from the nozzle wall are photographed by the Schlieren Method

Introduction

Considering the rise of a back pressure in a divergent nozzle, we note that at the slight degree of the rise the pressure along the nozzle axis first lowers as free expansion and then rises to the value of the back pressure. When the back pressure further rises the shock wave appears in the divergent part of the nozzle and the flow shows the different character between before and after the shock wave. That is to say, the pressure drops below the back pressure corresponding to the divergence of the nozzle, over-expansion occurs, and after the shock wave the flow turns to compression up to the exhaust pressure. The location of a shock wave approaches to the throat of the nozzle with the rise of the exhaust pressure. Such over-expansion flow with a shock wave involves considerably higher energy losses and is accompanied by a decrease of the nozzle velocity coefficient.

In this paper, the velocity coefficients of a divergent nozzle are measured and compared with the calculated values. At the same time pressure distributions are measured along the axis of the nozzle and Schlieren photographs are taken to observe the phenomena of the shock.

Theoretical Correlation

The following assumptions are taken concerning the flow in the nozzle.

1. The expansion and compression in the nozzle are in an adiabatic and is-entropic change.
2. It is an one dimensional flow.
3. The pressure at the nozzle exit is equal to the exhaust pressure.

Under these conditions, the calculated values of pressure and velocity in a divergent nozzle at over-expansion state are shown in Fig. 1. The over-expansion occurs when the back pressure P_e exceeds the pressure P_C corresponding to a nozzle divergence ratio. That is to say, the flow pressure in the nozzle drops below the back pressure at once in the divergent part of the nozzle and again rises to the exhaust pressure. In a slight degree of over-expansion the pressure rise occurs at the nozzle exit. Increasing the back pressure the compression flow appears in the divergent part of the nozzle. The sudden jump of the pressure occurs at the point where

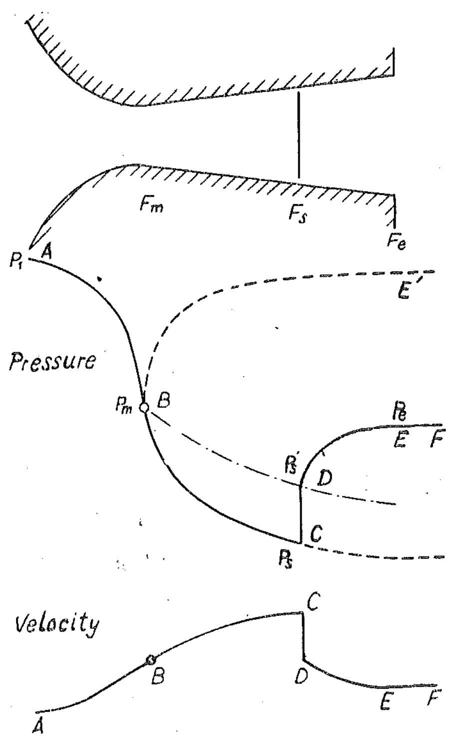

Fig. 1. Pressure and Velocity Curves in a Divergent Nozzle at Over-expansion

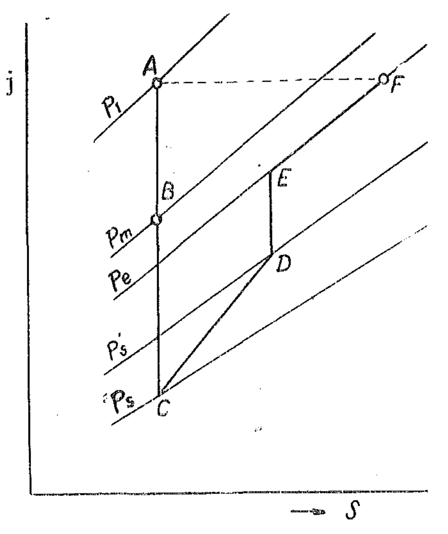

Fig. 2. Over-expansion in i, s-Diagram

the over-expansion flow turns into the compression flow and the velocity of a flow turns from supersonic to subsonic. At this turning point the shock wave appears.

This phenomenon can be represented in i, s-diagram in Fig. 2. Notations in this figure correspond to those in Fig. 1. Point F represents the final state of the flow where the flow velocity becomes zero and all energy turns into the thermal energy, i. e. its enthalpy becomes equal with that in the nozzle inlet. Points C and D respectively present the state before and after the shock wave. These values at points C and D presenting the location of the shock wave in a nozzle can be studied as follows.

Pressure and velocity at the compression flow in the divergent part of nozzle can be calculated using the same idea about the compression flow in a diffusor where the exit state has been given and then line DE has been drawn. Point D represents the critical point corresponding to the throat of a diffusor and it has an acoustic velocity. So the flow state at a nozzle exit does not affect the upperstream of this point. There the shock wave occurs. The higher the exhaust pressure the more the location of the shock wave shifts to the upper stream. The shock wave disappears at the throat of the nozzle when the exhaust pressure rises further. In these cases the flow divides into two kinds of stream. The one is expansive in a convergent part and the other is compressive in a divergent part. The whole flow is subsonic and there is no shock wave.

When the shock wave appears in a divergent part, the flow at the throat should be decided by the initial condition of a nozzle, takes an acoustic velocity, and keeps the

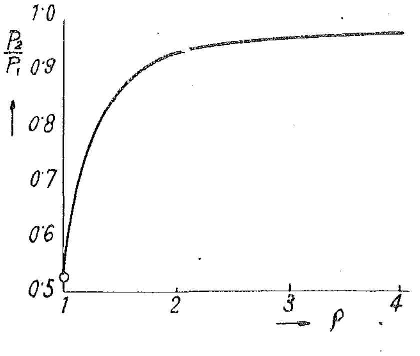

Fig. 3. Limited Divergence Ratios Keeping the Constant Mass Flow

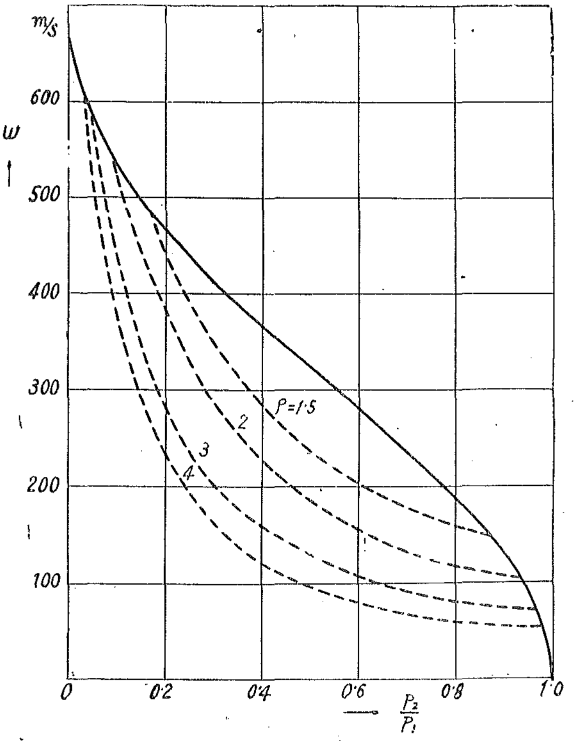

Fig. 4. Exit Velocity for Isentropic Expansion and Theoretical Over-expansion

constant mass flow regardless of the exhaust pressure. It should be noted that these nozzle characteristics suggest some method to utilize the divergent nozzle in a wide range as flow control devices keeping the constant mass flow, even the pressure ratio that is defined as the exit pressure at the nozzle divided by the inlet pressure does not exceed the critical pressure ratio. The limited divergence ratios keeping the constant mass flow in isentropic flow are shown in Fig. 3. where calculations are made for the air whose initial temperature is 15°C.

When the shock wave occurs in the nozzle. The flow shows over-expansion at once in the divergent part of the nozzle and then the mass velocity exceeds the velocity calculated as the isentropic flow. However at the downstream after the shock wave the velocity decreases and the actual exit velocity takes a lower value compared with the isentropic mass velocity.

In Fig. 4, the upper curve shows the velocity for the isentropic expansion plotted against the pressure ratio and the broken curves give the velocities issuing from the frictionless nozzles of various divergence ratios in which the processes are as described above. It is calculated for air of 15°C initial temperature. In this case

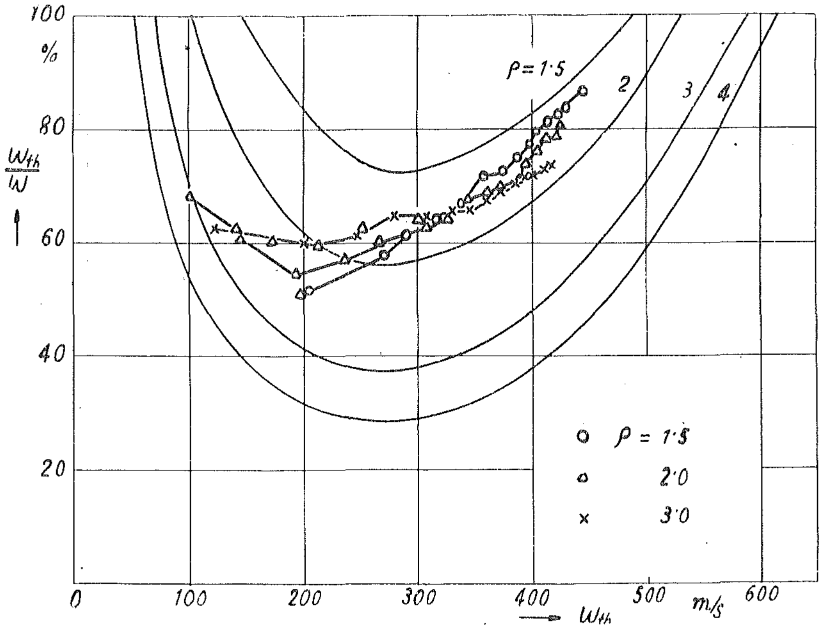

Fig. 5. Measured and Calculated Velocity Coefficients of Over-Expansion Nozzles

the velocities for the isentropic expansion are calculated by the help of the air i, s-chart of Ohga and Saito.

So, the calculated velocity coefficents corresponding to Fig. 4 are shown in Fig. 5. It may be seen that, with decrease of pressure difference between the nozzle inlet pressure and exit pressure in the same nozzle and increase of degree of the over-expansion, the curve of the velocity coefficient shows a decreasing character and passes through a minimum point, and next shows a tendency to increase. A decrease of velocity coefficient appears remarkably in the nozzle of the higher divergence ratio.

Measurements of Nozzle Velocity Coefficients

Test Methods and Apparatus:

The mass velocities from the nozzle were measured by the "Reaction methods" with the apparatus shown in Fig. 6.

The nozzles to be used had a rectangular cross-secition, about 1 cm^2 nozzle throat area, and comparatively long divergent parts whose angles of divergence were 6° to 12° and divergence ratios 1.5 to 3.0. The nozzles have been constructed with

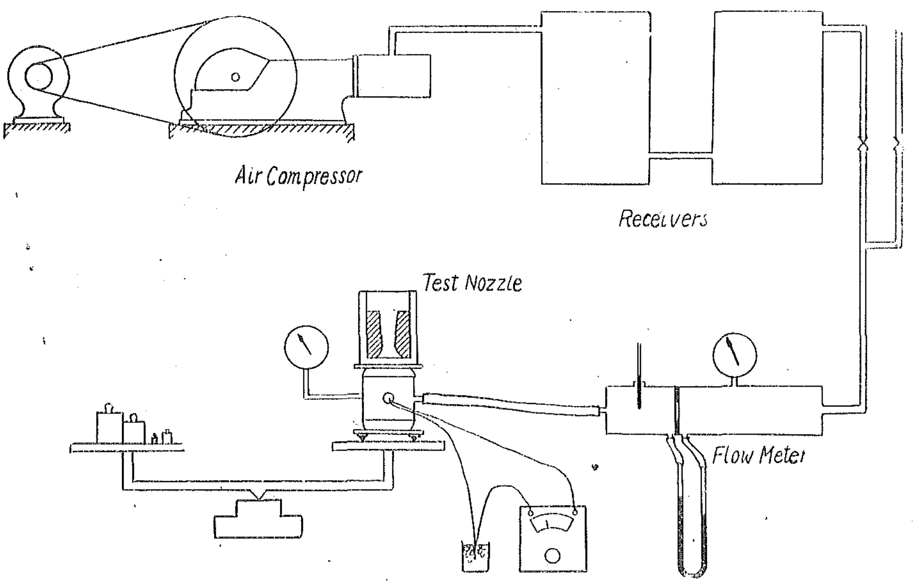

Fig. 6. Test Apparatus for Measuring Nozzle Velocity Coefficients

glass plates and wood block. The former are two flat plates holding the positions opposite to each other and the latter are fine textured wood with polished surface that are made in divergent forms corresponding to the divergence ratios.

Atmospheric air is compressed to a pressure by the compressor having 3.5 m^3/

min capacity. The air stream from this compressor passes first through recievers having 8 m^3 capacity, enters into orifice type flow meter, and passes into the nozzle through the flexible rubber hose as shown in Fig. 6. Air pressure at the inlet is controlled by the by-pass valve blowing off the excessive air to the open air.

Nozzle reactions are measured by the balance in such a manner that the nozzle holder is controlled to show the maximum reading of the reaction force with the angle adjusting device and at the same time the deflection angles of the nozzle axis to perpendicular are measured.

The probable errors at the measurement of reaction force owing to the rigidity of the connecting rubber hose are maintained within 3 grs, that means below 1 % error at the high speed flow and below 3 % even at about 50 m/s flow.

Test Results:

Jet velocity w (m/s) leaving the nozzle can be calculated with the reaction force R (kg) and the quantity G flowing per second (kg/s) as follows

$$w = \frac{R\,g}{G}.$$

Thus the measured values of the velocity coefficients are added in Fig. 5 to compare with the theoretical values. It may be seen that the measured values are something greater than the theoretical values and they show some resemblance in their tendency.

The Pressure Distribution along a Nozzle Axis

Next, the pressure distributions along the nozzle axis were measured and compared with the calculated values.

The pitot tube consisted of brass tube having 3 mm external diameter and 30 cm long, and it had 0.5 mm diameter hole on the tube side to measure the static presssure distribution along the axis of the nozzle for the various nozzle inlet pressure. The first pitot tube was made of 0.8 mm external diameter to avoid as much as possible the effect of disturbance upon the flow state. However, the violent vibration of the measuring tube itself occurred and it became impossible to measure the pressure, so that it became necessary to use a comparatively larger size pitot tube.

Fig. 7 shows the pressure distributions along the axis of the nozzle with a divergence ratio of 1.5. In this case air temperature at the nozzle inlet is 15°C.

A comparison of these values with the calculated values described above is shown in Fig. 8 using the ratios of the pressure to the nozzle inlet pressure as abscissa.

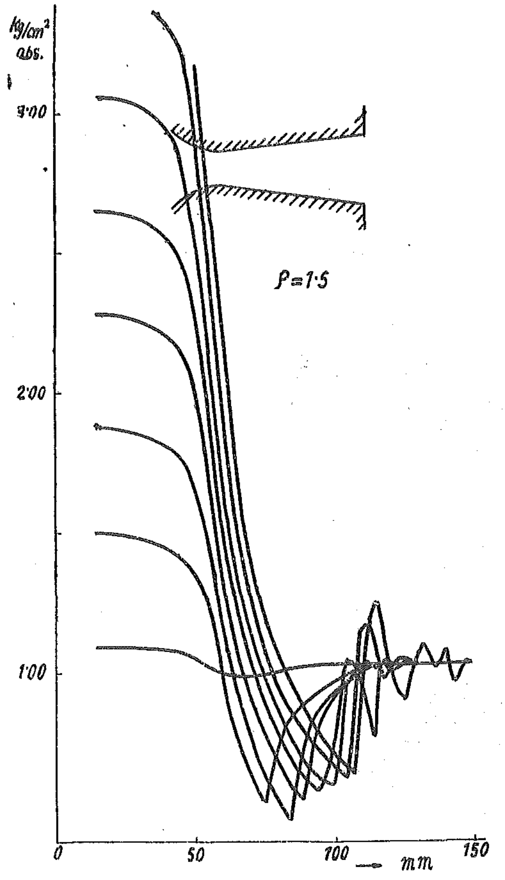

Fig. 7. Static Pressure Along the Nozzle Axis

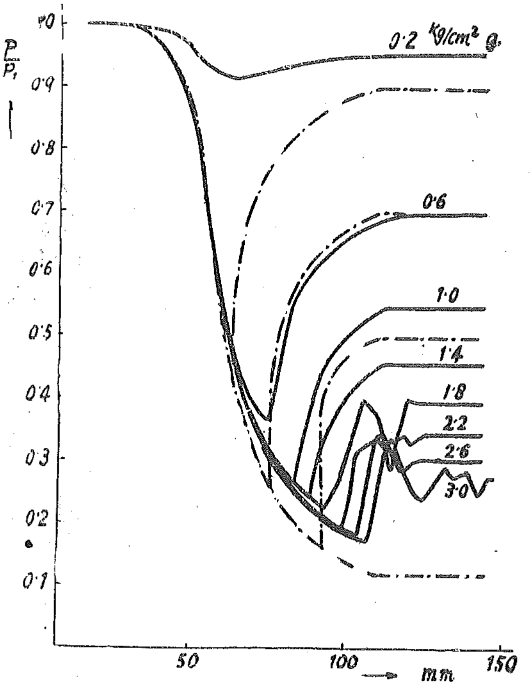

Fig. 8. Static Pressure Along the Nozzle Axis

At the part of upstream to the throat i. e. the convergent part, the expansion should be considered as adiabatic. Therefore the good agreement between the measured values and the calculated ones can be seen. It is found, however, that deviations of both values appear at the divergent part and the energy losses increase in this supersonic region.

The pressure ascending curves after the shock agree qualitatively with the calculated curves and represent the wave forms as being vanishingly small in the down stream, while they appear at high ratios between inlet and outlet pressure.

Schlieren Photograph

Fig. 9 illustrates one of the Schlieren photographs which shows the over-expansion in the divergent nozzle with divergence ratio of 2.0.

It can be seen that a normal shock wave does not exist and the separation from the nozzle wall occurs at the point where the flow turns from expansive to compressive, that is to say, an oblique shock wave appears at this point and the curve of pressure distribution along the nozzle axis represents the wave form shown in Figs. 7 and 8 owing to the respective interferences between compression wave and expansion wave.

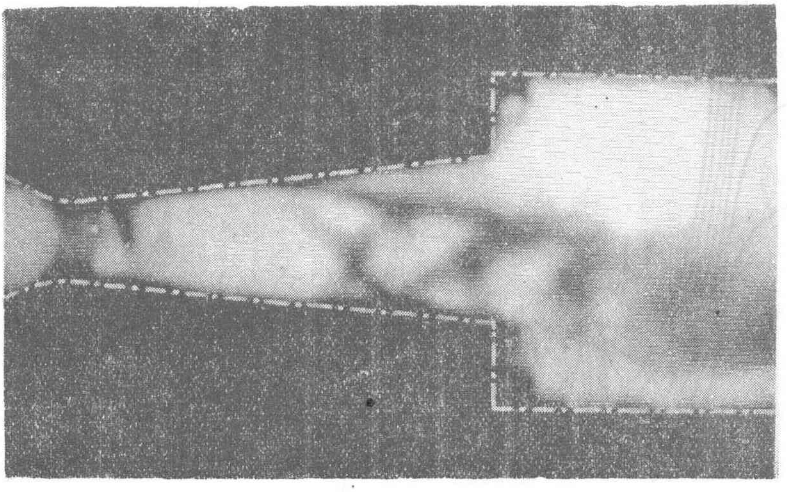

Fig. 9. Schlieren Photograph ($\rho=2.0$, $p=1.85\ kg/cm^2$gauge)

The flow separated from the nozzle wall does not fill the passage of the nozzle and streams through a part of the passage. So the actual exit velocity exceeds the velocity calculated as the flow without a separation. Therefore the measured nozzle coefficients on the over-expansion flow are smaller than the calculated values. The separated flow is deflected being adiacent to one side of the nozzle wall as shown in Fig.9. On the symmetrical divergent nozzle the flow often changes its direction along from one side wall to the other, so that the state of flow is very unstable.

Besides Mach waves are observed at the part of a throat in this photograph. It has been shown that there is sudden change of the density, that is to say, energy loss occurs remerkably in this region corresponding with the degree of deviations between the measured values and the calculated values of the pressure distribution along the axis at the throat.

Conclusions

1. The velocity coefficient for a divergent nozzle at over-expansion decreases with an increase of the degree of over-expansion and with an increase in divergence ratio, and shows the minimum value at some low velocity. These values are higher than the calculated values based on the assumptions that the flow in the nozzle passage is completely isentropic and one dimensional.

2. The measured values of static pressure along the axis of a nozzle show good agreement with the calculated values treated as isentropic expansion from the nozzle inlet to the throat. The deviation of both occurs in the divergent part.

3. The separation of the flow in over-expansion nozzle is observed with Schlieren photographs in this experiment. In this case the direction of the flow become remerkably unstable.

Acknowledgements

The authors wish to express their appreciation to Prof. Ohga for his kind guidance and encouragement throughout the course of this study. Also this work was assisted financially by a grant from the Ministry of Education in Japan, which we wish gratefully to acknowledge.

Appendix 1

Conditions about the Constant Mass Flow through a Nozzle

Conditions about the constant mass flow through a nozzle would be expressed as the function of the limited ratio of inlet and outlet pressure to a divergence ratio. This limited condition is shown as the curve ABE′ in Fig. 1, in which the velocity in the nozzle throat becomes sonic and then turns to compressive flow in the divergent part.

If the flow is assumed to be one-dimensional, steady, and adiabatic, relations between the variables of this flow will be shown as follows.

Where F represents the sectional area (m^2), v the specific volume (m^3/kg), P the pressure (kg/m^2), g the acceleration due to gravity (= 9.80m/s^2), and k the specific heat ratio. And suffix 1, m, and 2 denote the conditions at inlet, throat, and outlet of the nozzle respectively.

The equation of continuity in a steady flow may be written

$$\frac{F_m w_m}{v_m} = \frac{F_2 w_2}{v_2} \qquad (1)$$

The condition of adiabatic change is

$$P_m v_m^K = P_2 v_2^K . \qquad (2)$$

and, on the other hand, the equation of adiabatic compression is

$$\frac{w_m^2 - w_2^2}{2g} = \frac{k}{k-1} p_m v_m \left[\left(\frac{P_2}{P_m} \right)^{\frac{k-1}{k}} - 1 \right] . \qquad (3)$$

as the flow becomes sonic at the throat,

$$w_m^2 = gk P_m v_m \qquad (4)$$

$$P_m = P_1 \left(\frac{2}{k+1} \right)^{\frac{k}{k-1}} \qquad (5)$$

from equations (1) and (2) we obtain

$$\left(\frac{F_2}{F_m} \right) \left(\frac{w_2}{w_m} \right) = \left(\frac{P_m}{P_2} \right)^{\frac{1}{k}} .$$

and from equations (3) and (4), we obtain

$$\frac{w_2}{w_m} = \left[1 - \frac{2}{k-1} \left[\left(\frac{P_2}{P_m} \right)^{\frac{k-1}{k}} - 1 \right] \right]^{\frac{1}{2}}$$

Then the conditions that we want can be expressed as follows from above two equations and equation (5),

$$\left(\frac{F_m}{F_2} \right)^2 = \frac{k+1}{k-1} \times \left(\frac{2}{k+1} \right)^{\frac{-2}{k-1}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{1}{k}} - \left(\frac{P_2}{P_1} \right)^{\frac{k+1}{k}} \right] \qquad (6)$$

Appendix 2

The Exit Velocity from the Nozzle for Over-Expansion

Energy equation between the nozzle exit E and the infinite F can be written in the form

$$i_f - i_e = \frac{A}{2g} (w_e^2 - w_f^2) . \qquad (7)$$

where i represents the enthalpy (kcal/kg), T the temperature (°K), A the heat equivalent of mechanical work (=1/427 kcal/kgm), c_p the specific heat at constant pressure (kcal/kg°C), and R the gas constant.

In this case

$$w_f = 0 .$$

Assuming the flow as perfect gas

$$i_f - i_e = c_p (T_f - T_e) \qquad (8)$$

and

$$c_p = \frac{k}{k-1} AR . \qquad (9)$$

i_f, i_e and c_p can be eliminated from above three equations, so

$$w_e^2 = \frac{2gkR}{k-1}(T_f - T_e). \tag{10}$$

And if G (kg/s) denotes a mass flow in the nozzle,

$$G = \frac{F_e w_e}{v_e}. \tag{11}$$

$$v_e = \frac{R T_e}{P_e}. \tag{12}$$

At the throat the flow becomes sonic,

$$G = \frac{F_m P_1}{\sqrt{R T_1}} \sqrt{gk\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}. \tag{13}$$

From equations (11), (12), and (13) G and v_e can be eliminated.

$$T_e = \frac{F_e}{F_m} \frac{P_e \sqrt{RT_1}}{P_1 R \sqrt{gk\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} w_e. \tag{14}$$

Upon substituting T_e from equation (14) to equation (10)

$$w_e^2 = \frac{2gkR}{k-1}\left[T_f - \frac{F_e}{F_m}\frac{P_e}{P_1}\frac{\sqrt{R T_1}}{R\sqrt{gk\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} w_e\right].$$

So we get

$$w_e = \sqrt{\frac{2gkRT_1}{k-1}}\left(-\sqrt{X} + \sqrt{X+1}\right).$$

where

$$X = \left(\frac{F_e}{F_m}\right)^2\left(\frac{P_e}{P_1}\right)^2 \frac{1}{2(k-1)} \frac{1}{\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}.$$

Appendix 3

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