Suitable Voltages for Minimum Transmission Loss in Interconnected Power Network System.

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Abstract.

In an interconnected transmission network, it is of practical imporance to decide suitable voltages of power generating stations for the purpose of minimizing the transmission loss consumed in the network. In an interconnected power network, the total transmission loss must be accounted for and the individual branch line loss is useless there. By a graphical method with circle diagrams, these suitable voltages can approximately be decided in trials. Though it is an approximating method, reasonably correct values will be attained as a result of repetition of the trials. To clarify the details of this method, a numeric example is shown in the last chapter with explanations illustrated by the procedures of the example. As most do actually a usual generating station may have two transformer stations, though this is of course not necessary; through each two lines, the sum of two line loss is to be taken and minimized, of which the calculation is, therefore, not very complicated.

1. Introduction.

In a power transmission network with many power generating and transformer stations, it is required that each receiving voltage supplying demand loads be kept as constant as possible, therefore water and steam power stations are operated with prescribed outputs. Satisfying these conditions, it is desirable to keep transmission loss at a minimum, improving the system power factor. For this power factor improvement, the present authors underlook to study the suitable voltages at sending ends or receiving ends of which the voltages are adjustable with generator excitation or synchronous or static condensers. As numerical examples, data taken from the high tension transmission network in Hokkaido are simplified and modified for convenience of the caluculation.

2. General Equations of Transmission Networks.

Voltages, currents and powers at sending or receiving ends of a network with many generating stations and transformer stations are expressed using matrices[(1)(2)] by similar formulas as that of a four terminals network with only one receiving and sending end as follows:

$$\left.\begin{aligned} [E_1] &= [A][E_2]+[B][I_2] \\ [I_1] &= [C][E_2]+[D][I_2] \end{aligned}\right\} \cdots\cdots (1)$$

$$\left.\begin{aligned} [P_1+jQ_1] &= [B]^{*-1}[D]^*[E_1][E_1]_k-[B]^{*-1}[E_2][E_1]_k \\ -[P_2+jQ_2] &= [B]^{-1}[A][E_2][E_2]_k-[B]^{-1}[E_1][E_2]_k \end{aligned}\right\} \cdots\cdots (2)$$

but

$$\left.\begin{aligned} [A][D]^*-[B][C]^* &= [1] \\ [D]^*[A]-[B]^*[C] &= [1] \end{aligned}\right\} \cdots\cdots (3)$$

where $[E_1]$, $[I_1]$, $[P_1+jQ_1]$ are voltages, currents and powers respectively at generating ends, and $[E_2]$, $[I_2]$, $[P_2+jQ_2]$ are those of receiving ends respectively. $[A]$, $[B]$, $[C]$, $[D]$ are four terminal matrices between any two terminals in the network. Sign $*$ and k show the transposed matrix and the conjugated matrix. Eq. (2) can be rearranged as the following

(1) K. Ogushi, Power Circle Diagram of Interconnected Electric Power Transmission System. The Memoirs of the Faculty of Engineering, Hokkaido University, Vol. III, No. 3. 1930.

(2) K. Ogushi & G. Miura, Electrical Characteristics of Interconnected Power Transmission Systems. The Memoirs of the Faculty of Engineering, Hokkaido University, (in print).

power matrix as a convenient form for the calculation.

$$\begin{pmatrix} +(P_{ii}+jQ_{ii}) \\ -(P_{jj}+jQ_{jj}) \\ \cdots\cdots\cdots\cdots \end{pmatrix} = \begin{pmatrix} Y_{ii}|E_i|^2, & -y_{ij}E_jE_{ik}, & -y\cdots\cdots \\ -(y_{ji}E_iE_{jk}, & Y_{jj}|E_j|^2, & -y\cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots \end{pmatrix} \quad \cdots (4)$$

where

$$Y_{ii} = \sum_j \frac{A_{ji}}{B_{ji}} + \sum_j \frac{D_{ij}}{B_{ij}}$$

$$y_{ij} = \frac{1}{B_{ij}}$$

Total powers at generating stations and transformer stations have all the components of such sending and receiving powers which flow through the lives between interconnected terminals, thus $\sum(P_{ij}+jQ_{ij}) -\sum(P_{ji}+jQ_{ji})+\cdots$ taking a receiving power with negative signs. In the right side of the matrix (4), the diagonal terms show respectively the sum of short circuit powers between interconnected transmission lines as the sending end $\frac{D}{B}|E_i|^2$ and as the receiving end $\frac{A}{B}|E_i|^2$, and other terms $\frac{1}{B_{ij}}E_iE_{jk}$ show driving powers between terminals. These values correspond exactly with the circle diagrams constants, that is, with the centers and radii. The power matrix (4) expresses all the electrical characteristics of a transmission network.

For the practical calculation, it is convenient to use per unit values, taking impedances B as per unit values for the standard M. V. A. base, voltages E as unit voltages. Then all terms of the matrix are given as multiples of base M. V. A.

3. Total Transmission Loss in a Network and Graphical Solution of Suitable Voltages for Minimum Transmission Loss.

If we sum up each term of matrix (4), we obtain

$$\sum(P_{ij}+jQ_{ij})-\sum(P_{ji}+jQ_{ji}) = v+ju \quad \cdots\cdots\cdots\cdots (5)$$

It is obvious that v is the total transmission loss in the network and u is the total wattless power consumed in the network. This transmission loss can be expressed by real parts of the right side of matrix (4) as follows:

$$v = \sum \left| \frac{A}{B} \right| |E_i|^2 \cos \alpha_{ij} + \sum \left| \frac{D}{B} \right| |E_i|^2 \cos \alpha_{ji} - 2 \sum \left| \frac{1}{B_{ij}} \right| |E_i| \, |E_j| \cos \beta_{ij} \cos \theta_{ij} \quad \cdots\cdots\cdots (6)$$

where α_{ij}, α_{ji}, β_{ij} are the angles of the vector quantities of $\frac{A}{B}$, $\frac{D}{B}$ and $\frac{1}{B}$. θ_{ij} is the phase difference between voltages E_i and E_j or the so-called power angle. θ_{ij} and E_i, E_j are variable due to generating or load conditions and voltage adjustment. But these variables depend on the sending or receiving powers through each interconnected transmission line as shown in matrix (4).

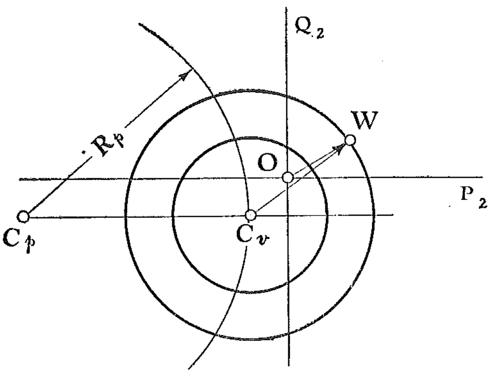

Fig. 1. Line loos circle diagrams.

The most suitable voltages for the minimum transmission loss can be analytically gotten by calculating the minimum value of the loss, Eq. (6), which has many variables of voltages under the condition of given load outputs and some constant terminal voltages. This analytical solution, however, is not easy but almost impossible because of many variables, because it takes fine and skill to solve partial differential equations of considerably high degree.

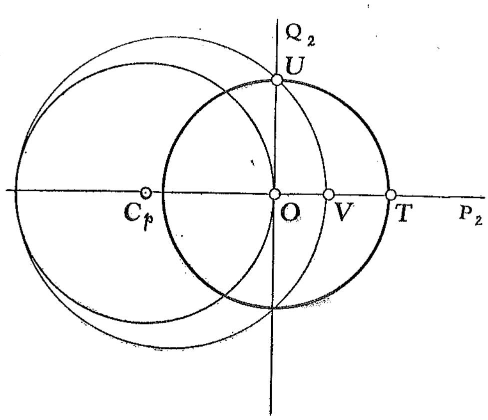

Fig. 2. Line loss circle diagrams for a simplified line.

Now, we will begin to discuss the circle diagram of a transmission line connecting two terminals of which the voltages are E_1, and E_2. In general the transmission loss can be graphically expressed on the power circle diagram[3] as Fig. 1.

(3) K. Ogushi, Denki Kogaku Ensenzu, Shukyosha, 1939.

$$v = P_1 - P_2 = \frac{r_v^2 + 2\alpha_v R_p}{2R_p} \qquad \cdots\cdots (7)$$

where v is the wattage loss consumed in a network, that is, the difference of the power input P_1 and the power output P_2, r_v is a radius of a loss circle diagram drawn as a concentrical centered at a point C_v which is called the minimum loss point. $\alpha_v = \alpha_p - R_p$ where α_p is a magnitude of P component of a point $\boldsymbol{C}_p$ on the power co-ordinate, and R_p is a radius of the zero effective power circle diagram centered at point C_p. The statements above are explained detail in the reference book (3), so are not touched upon forther here.

As is well known, C_p, R_p are defined by the following formulas.

$$\left.\begin{aligned} \boldsymbol{C}_p &= -\frac{\boldsymbol{C}_{ij}\boldsymbol{B}_{ijk} + \boldsymbol{A}_{ij}\boldsymbol{D}_{ijk}}{\boldsymbol{D}_{ij}\boldsymbol{B}_{ijk} + \boldsymbol{D}_{ijk}\boldsymbol{B}_{ij}} \\ \boldsymbol{R}_p &= \left| \frac{1}{\boldsymbol{D}_{ij}\boldsymbol{B}_{ijk} + \boldsymbol{D}_{ijk}\boldsymbol{B}_{ij}} \cdot \frac{\boldsymbol{E}_i}{\boldsymbol{E}_j} \right| \end{aligned}\right\} \qquad \cdots\cdots (8)$$

where i and j are terminals in the concerned transmission line. Eq. (8) is the formula expressed on the receiving end co-ordinate as this is generally convenient for drawing circle diagrams counting the loss. If expressed on the sending end co-ordinate, $\boldsymbol{D}$ and $\boldsymbol{A}$ are mutually interchanged and the negative sign of the first equation is dropped. Also as it is convenient to take admittance for the unit of a co-ordinate in such case, multiples of $|\boldsymbol{E}|^2$ are dropped in Eq. (8), which is usually seen in ordinary power circle diagrams expressed in the power unit. However, those calculations are very complicated excepting in the case of a series impedance of a short line.

In the case of a short line, admittances, such as capacitances, or leakages, can be neglected resulting $\boldsymbol{A} = \boldsymbol{D} = 1$, and $\boldsymbol{C} = 0$. In this case, $\boldsymbol{C}_p$ is an intersecting point of a horizontal axis and a bisecting normal line of a line $\overline{OC_e}$ where O is the origin and $\boldsymbol{C}_e$ is the center of a power circle diagram. Therefore, the minimum loss point is at the origin, and

$$v = \frac{r_v^2}{2R_p} \qquad \cdots\cdots (9)$$

and

$$v = OV \qquad \text{(see Fig. 2)}$$

In general R_p has a very large value, then we take

$$nv = \frac{r_v^2}{\frac{2R_p}{n}} .$$

using $\frac{2R_p}{n}$ circle, in any mediate value as we like, as it is easier in graphical representation.

For the long line, it is practical enough to take capacities of the total line. However, also in this case the above simplified method can be applied if we suppose the line to be a π line. $A=D=1+YZ$, $B=Z$, $C=2Y+ZY^2$† for a symmetrical π circuit with a series total line-impedance and half a total line-capacitance at both terminals. Then, the various quantities expressed on the receiving end admittance co-ordinate are

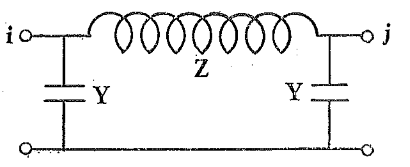

Fig. 3. Equivalent circuit of a long line.

$$C_e = -\frac{A}{B} = -\left(\frac{1}{Z} + Y\right)$$

$$C_i = -\frac{C}{D} = -\left(Y + \frac{1}{Z+\frac{1}{Y}}\right)$$

$$R_p = \left|\frac{1}{DB_k + D_k B}\right| = \left|\frac{1}{Z+Z_k}\right| \quad \text{(Same value as that of the former)}$$

$$C_p = -(CB_k + AD_k)\,R_p = -R_p - Y \qquad \cdots\cdots\cdots\cdots\cdots\cdots (10)$$

Therefore, C_e moves Y, C_p moves Y and C_i moves $Y+\frac{1}{Z+\frac{1}{Y}}$ from original values and R_p is kept at the same value.

$C_{v,min}$ is at point Y. We can easily apply the simplified method in the case of a series impedance line.

As the above method defines the transmission loss by drawing loss circle diagrams, it is rather not straightforward regardless of the simplification. In order to count the transmission loss, it is more beneficial and desirable to use the "loss line" and to measure the vertical distance to the line from spontaneous working points. The loss line is a straight line which satisfies the following conditions:

(1) It is to be at a right angle to a straight line connecting the

† Remark on $Y^2 \neq |Y|^2$.

minimum loss point C_v and the center C_e of a power circle diagram.

(2) It is to pass through a point intersected by the output line[3] and the vertical axis of the co-ordinate.

Then the loss can be found by measuring the length of this vertical distance from a working point to this loss line, that is to say N. (see Fig. 4)

$$v = \frac{N\overline{C_v C_e}}{R_p} \qquad (11)$$

When we find a sending voltage E_i under the condition that N is minimum, the voltage E_i is the suitable voltage of the terminal i. However, this fact is satisfied only on a transmission line with one receiving and one sending end, and is not necessarily satisfied on a general transmission network. If the sending end i is connected with terminals k, l, m, n, …… besides the terminal j…such as as is most usual in a power network,… it is necessary to find the suitable voltages E_{ij}, E_{ik}, E_{il}, E_{im}, …… between each two terminals separately by the method above related. Then, it must be determined what is the most suitable value of E_i from the resultant of various E_i values by mutually checking the values with each other by trials. The details of these points will be clarified in Chapter 5 with a numerical example.

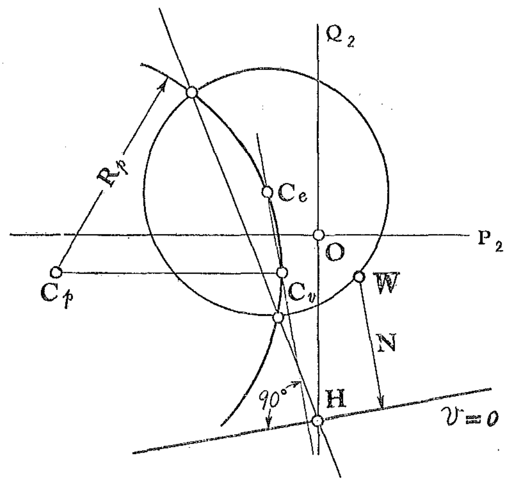

Fig. 4. Loss line.

4. Mathematical Consideration for the Suitable Voltage with Minimum Transmission Loss.

In this case, the simplified method is adopted in which the assumptions $A=D=1$, and $C=0$ are satisfied, and clarify the formulas of minimum transmission loss. In Fig. 5, the center of $C_p = -\frac{A}{B}$ is located on a horizontal axis. The center C_e of a power circle diagram, which is located on a periphery of a circle C_p is put to be

$$C_e = a+jb\,.$$

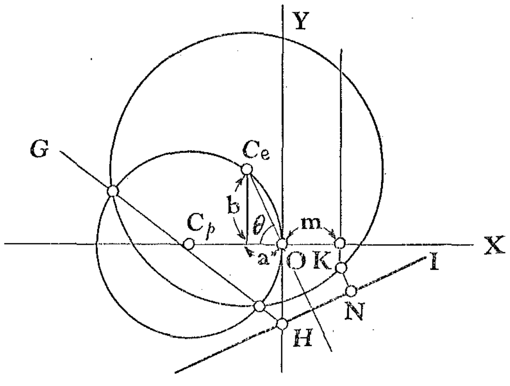

Fig. 5. Notations for a loos line for mathematical analysis.

The radius of the circle C_p is

$$R_a = \left|\frac{1}{B}\,\frac{E_i}{E_j}\right| = R = KE_i\,.$$

In this cace E_i unknwon and E_j, $\frac{1}{B}$ are kuown, while K is a constant.

Then an equation of a point on a periphery of the circle C_e is

$$(x-a)^2+(y-b)^2=K^2E_i^2$$

$$\cos\theta = \frac{\overline{OM}}{\overline{OC_p}}\,.$$

$$\overline{OC_p} = \overline{OM}\times\frac{1}{\cos\theta} = \frac{\sqrt{a^2+b^2}}{2}\cdot\frac{\sqrt{a^2+b^2}}{a} = \frac{a^2+b^2}{2a}\,.$$

While, an equation of a circle C_p is

$$\left(x-\frac{a^2+b^2}{2a}\right)^2+y^2=\left(\frac{a^2+b^2}{2a}\right)^2$$

Finding a straight line $\overline{GH}$ which connects two intersecting points of circles C_e and C_p, the above two equations are subtracted on each side.

$$(x-a)^2+(y-b)^2-K^2E_i^2-\left(x-\frac{a^2+b^2}{2a}\right)^2-y^2+\left(\frac{a^2+b^2}{2a}\right)^2=0\,.$$

Rearranging

$$x\left(\frac{a^2+b^2}{a}-2a\right)-2yb+a^2+b^2-K^2E_i^2=0\,.$$

Therefore, a point intersected by this straight line and a vertical axis of the co-ordinate can be obtained by putting $x=0$ in above the equation. Accordingly, $H\left(x=0,\quad y=\frac{a^2+b^2-K^2E_i^2}{2b}\right)$.

Next, a straight line $\overline{OC_e}$ where the co-ordinate of C_e is (a, b) is represented by

$$\frac{x-a}{0-a}=\frac{y-b}{0-b}$$

therefor

$$bx-ay=0\,.$$

An equation of a straight line which passes through the point H is

$$Ax+B\left(y-\frac{a^2+b^2-K^2E_i^2}{2b}\right)=0\,.$$

In order to be at a right angle to the line $\overline{OC_e}$, it must be satisfied by the following relation as is well known from the knowledge of analytical geometry

$$Ab-Ba=0\,.$$

So, the equation of the above line which passes through the point H and is a right angle to the line $\overline{OC_e}$, that is, of a line $\overline{HI}$ is

$$ax+b\left(y-\frac{a^2+b^2-K^2E_i^2}{2b}\right)=0\,.$$

If the receiving power is to be P, then we put $m=\frac{P}{E_j^2}$ of which the value is known under the given conditions as E_j is the receiving voltage. Then the point K is obtained from an intersection of a line $x=m$ and a circle $(x-a)^2+(y-b)^2=K^2E_i^2$,

$$(m-a)^2+(y-b)^2=K^2E_i^2$$

$$y=b\pm\sqrt{K_2E_i^2-(m-a)^2}=b\pm S$$

where

$$S=\sqrt{K^2E_i^2-(m-a)^2}\,.$$

Fromexperience, it is usual to take a negative sign to the above formula. A straight line $\overline{KN}$ which passes through the point K found now is

$$A'(x-m)+B('y-b+S)=0$$

and from that it is parallel to the line $\overline{OC_e}$, $\frac{A'}{b}=\frac{B'}{-a}$.

Therefore, the equation of a line $\overline{KN}$ is

$$b\,(x-m)-a\,(y-b+S)=0$$

Next, an intersection N is obtained by solving the equations on lines $\overline{HI}$ and $\overline{KN}$ simultaneously as follows:

$$X = \frac{a^3 - aK^2E_i^2 + 2b^2m - ab^2 + 2abS}{2(a^2+b^2)}.$$

$$Y = \frac{b^3 - bK^2E_i^2 - 2abm + 3a^2b - 2a^2S}{2(a^2+b^2)}.$$

Then, the length $\overline{KN} = N$ is obtained from

$$N^2 = (m-X)^2 + (b-S-Y)^2.$$

Substituting the above values of X, Y, and rearranging

$$N = \frac{b^2 - a^2 + 2am + K^2E_i^2 - 2bS}{2\sqrt{a^2+b^2}}.$$

The condition of minimum loss can be reduced from

$$\frac{dN}{dE_i} = \frac{1}{\sqrt{a^2+b^2}}\left(K^2E_i^2 - \frac{bK^2E_i^2}{S} = 0\right)$$

therefore

$$b^2 = S^2$$

or

$$E^i = \frac{\sqrt{b^2+(m-a)^2}}{K} \quad \cdots\cdots (12)$$

The suitable voltage E_i in a case of an ordinary transmission line with two sending and two receiving terminals two terminals is thus found with rather a simple result.

While, the minimum loss in which condition (12) is satisfied is calculated as follows:

$$N = \frac{1}{2\sqrt{a^2+b^2}}\left\{b^2 - a^2 + 2am + K^2E_i^2 - 2b\sqrt{b^2}\right\}$$

$$= \frac{1}{2\sqrt{a^2+b^2}}\left\{K^2E_i^2 - a^2 - b^2 + 2am\right\}.$$

However, from the condition

$$b^2 = S^2$$

or

$$m^2 = K^2E_i^2 - a^2 - b^2 + 2am$$

So

$$N = \frac{m^2}{2\sqrt{a^2+b^2}} \quad \cdots\cdots (13)$$

If we express by S^2 instead of b^2, then

$$N = \frac{b^2 - a^2 + 2am + K^2E_i^2 - 2b\sqrt{S^2}}{2\sqrt{a^2+b^2}} = \frac{m^2 + (S-b)^2}{2\sqrt{a^2+b^2}}$$

From this formula, we can draw a curve expressing the variation of loss versus voltage the same as in the previous graphical solution.

5. Numerical Examples of Graphical Solution of Suitable Voltages for Minimum Transmission Loss.

Fig. 6 is a connection diagram and an admittance map of the numerical exampple in 10 M.V.A. base. These admittances are inverse

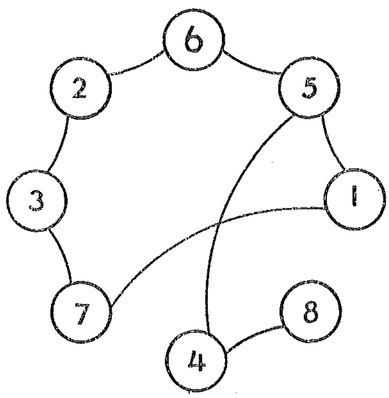

Fig. 6. Connection diagram of the network in the example.

In Fig. 6 values of each line admittance are corrected to per unit by the formula, $Y_{pu} = Y_{mho} \times \frac{(60)^2}{10}$.

$$\frac{1}{B_{15}} = 0.0075 - j\,0.0284 \text{ mho} = 2.70 - j\,10.2 \text{ per unit}$$

$$\frac{1}{B_{17}} = 0.0027 - j\,0.0141 \text{ mho} = 0.97 - j\;5.07 \text{ per unit}$$

$$\frac{1}{B_{23}} = 0.0137 - j\,0.0460 \text{ mho} = 4.94 - j\,16.6 \text{ per unit}$$

$$\frac{1}{B_{26}} = 0.0090 - j\,0.0625 \text{ mho} = 3.24 - j\,22.5 \text{ per unit}$$

$$\frac{1}{B_{37}} = 0.0100 - j\,0.0650 \text{ mho} = 3.60 - j\,23.4 \text{ per unit}$$

$$\frac{1}{B_{45}} = 0.0140 - j\,0.0416 \text{ mho} = 5.04 - j\,15.0 \text{ per unit}$$

$$\frac{1}{B_{48}} = 0.0060 - j\,0.0420 \text{ mho} = 2.16 - j\,15.1 \text{ per unit}$$

$$\frac{1}{B_{56}} = 0.0060 - j\,0.0420 \text{ mho} = 2.16 - j\,15.1 \text{ per unit} \qquad \frac{1}{B_{32}} = \frac{1}{B_{23}} \text{ etc.}$$

TABLE 1. Power Matrix.

Terminal number / Terminal power	1	2	3	4	5	6	7	8
$P_{11}+jQ_{11}$	$\left(\frac{D_{15}}{B_{15}}+\frac{D_{17}}{B_{17}}\right)E_1^2$				$-\frac{1}{B_{15}}E_5E_{1k}$		$-\frac{1}{B_{17}}E_7E_{1k}$	
$P_{22}+jQ_{22}$		$\left(\frac{D_{23}}{B_{23}}+\frac{D_{26}}{B_{26}}\right)E_2^2$	$-\frac{1}{B_{23}}E_3E_{2k}$			$-\frac{1}{B_{26}}E_6E_{2k}$		
$P_{33}+jQ_{33}$		$-\frac{1}{B_{23}}E_2E_{3k}$	$\left(\frac{A_{23}}{B_{23}}+\frac{D_{37}}{B_{37}}\right)E_3^2$				$-\frac{1}{B_{37}}E_7E_{3k}$	
$P_{44}+jQ_{44}$				$\left(\frac{D_{45}}{B_{45}}+\frac{D_{48}}{B_{48}}\right)E_4^2$	$-\frac{1}{B_{45}}E_5E_{4k}$			$-\frac{1}{B_{48}}E_8E_{4k}$
$-(P_{55}+jQ_{55})$	$-\frac{1}{B_{15}}E_1E_{5k}$			$-\frac{1}{B_{45}}E_4E_{5k}$	$\left(\frac{A_{15}}{B_{15}}+\frac{A_{45}}{B_{45}}+\frac{D_{56}}{B_{56}}\right)E_5^2$	$-\frac{1}{B_{56}}E_6E_{5k}$		
$-(P_{66}+jQ_{66})$		$-\frac{1}{B_{26}}E_2E_{6k}$			$-\frac{1}{B_{56}}E_5E_{6k}$	$\left(\frac{A_{26}}{B_{26}}+\frac{A_{56}}{B_{56}}\right)E_6^2$		
$-(P_{77}+jQ_{77})$	$-\frac{1}{B_{17}}E_1E_{7k}$		$-\frac{1}{B_{37}}E_3E_{7k}$				$\left(\frac{A_{17}}{B_{17}}+\frac{A_{37}}{B_{37}}\right)E_7^2$	
$-(P_{88}+jQ_{88})$				$-\frac{1}{B_{48}}E_4E_{8k}$				$\frac{A_{48}}{B_{48}}E_8^2$

values of impedances per unit or equal to $\frac{1}{B}$ per unit in four terminal constants $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$. In this case lines are always very short, $\boldsymbol{A}=\boldsymbol{D}=1$, $\boldsymbol{C}=0$, therefore these admittances show directly the circle diagrams centers and radii. But in the special case of long lines, capacitances are shown, which can easily be obtained by the general circuit constants. Table I is a power matrix of the high voltage network. Power stations and substations are shown by the numbers.

Sending and receiving circle diagram constants can be read in unit of 10 M.V.A. base, taking voltages E_i, E_j, etc per unit of 60 KV base.

TABLE 2. Voltages and effective powers at each station and branch line. E_1, E_2, E_3, E_4 are voltages of sending power stations to be determined by graphical methods as the suitable values.

Terminal number	Voltage		Effective Power	
	KV	p. u.	MW	p. u.
1	$E_1 = ?$		$P_{15} = +17$ $P_{17} = +13$ $P_{11} = +30$	+ 1.7 + 1.3 + 3.0
2	$E_2 = ?$		$P_{23} = +20$ $P_{26} = +28$ $P_{22} = +8$	− 2.0 + 2.8 + 0.8
3	$E_3 = ?$		$P_{32} = +20$ $P_{37} = -3$ $P_{33} = +17$	+ 2.0 − 0.3 + 1.7
4	$E_4 = ?$		$P_{45} = +15$ $P_{48} = +5$ $P_{44} = +20$	+ 1.5 + 0.5 + 2.0
5	$E_5 = 60$	1.	$P_{51} = -17$ $P_{54} = -15$ $P_{56} = -8$ $P_{55} = -40$	− 1.7 − 1.5 − 0.8 − 4.0
6	$E_6 = 60$	1.	$P_{62} = -28$ $P_{65} = +8$ $P_{66} = -20$	− 2.8 + 0.8 − 2.0
7	$E_7 = 60$	1.	$P_{71} = -13$ $P_{73} = +3$ $P_{77} = -10$	− 1.3 + 0.3 − 1.0
8	$E_8 = 60$	1.	$P_{84} = -5$ $P_{88} = -5$	− 0.5 − 0.5

Effective powers listed in Table 2 are taken from tidal power distribution diagrams on many power stations or substations, though those figures are all presumptive here. Concerning notations, P_{15} shows a sending effective power from the therminal 1 to terminal 5, and so on. Sign + means sending electic powers and sign — means receiving powers.

Now, the method will be explained of graphical solutions for suitable sending voltages under certain receiving voltages given, which are to be kept constant.

4 (An. generating station) ~8 (As. transformer station)

$$-(P_{48}+jQ_{48}) = \frac{1}{\boldsymbol{B}_{48}}\boldsymbol{E}_8^2 - \frac{1}{\boldsymbol{B}_{48}}\boldsymbol{E}_4\boldsymbol{E}_{8k}$$

$$\frac{P_{48}+jQ_{48}}{|\boldsymbol{E}_8|^2} = -\frac{1}{\boldsymbol{B}_{48}} + \frac{1}{\boldsymbol{B}_{48}}\frac{\boldsymbol{E}_4}{\boldsymbol{E}_8}.$$

$\boldsymbol{B}_{48}$, $\boldsymbol{E}_8$ and P_{48} are given in the previous tables. Taking into consideration that the coordinate is of an admittance unit, we draw receiving end power circle diagrams for various values of the sending voltage E_i. And the length of the line, N is measured under each voltage E_i and listed as in the following table. Also see Fig. 7.

E_4	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	2.09	0.83	0.43	0.14	0.07	0.07	0.43	1.44	2.45

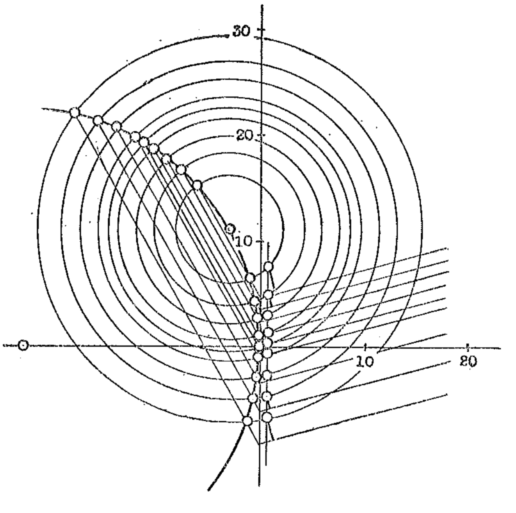

Fig. 7. Circle diagrams between terminals 4 and 8.

4 (An. generating station) ~5 (Su. transformer station)

$$-\frac{P_{45}+jQ_{45}}{E_5^2} = -\frac{1}{B_{45}} + \frac{1}{B_{45}}\frac{E_4}{E_5}.$$

Using a value B_{45} from the former table, Fig. 8 and the following table are similarly obtained.

E_4	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	3.02	1.80	1.15	0.43	0.28	0.43	0.90	2.10	4.07

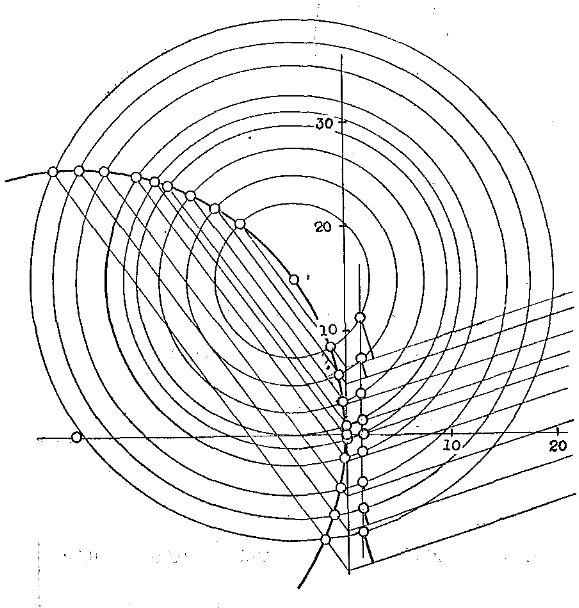

Fig. 8. Circle diagrams between terminals 4 and 5.

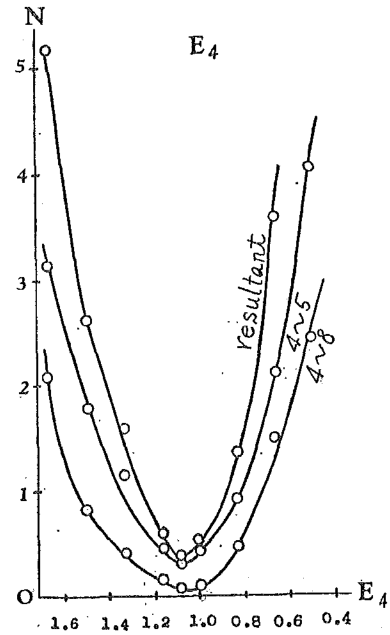

Fig. 9. Loss versus voltage curve for determining E_4.

Then, as the power station 4 is connected with transformer stations 8 and 5, the minimum transmission loss can be obtained by taking the sum of these corresponding values. Then the voltage E_4 for a point is to be read. (Fig. 9) We judge this suitable voltage of station 4 as 1.07 per unit or 64 KV in actual practice.

1 (Ur. generating station) ~5 (Su. transformer station)

$$\frac{P_{15}+jQ_{15}}{E_5^2} = -\frac{1}{B_{15}} + \frac{1}{B_{15}}\frac{E_1}{E_5}.$$

See Fig. 10.

E_1	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	2.53	1.62	0.79	0.47	0.22	0.25	0.36	1.08	2.53

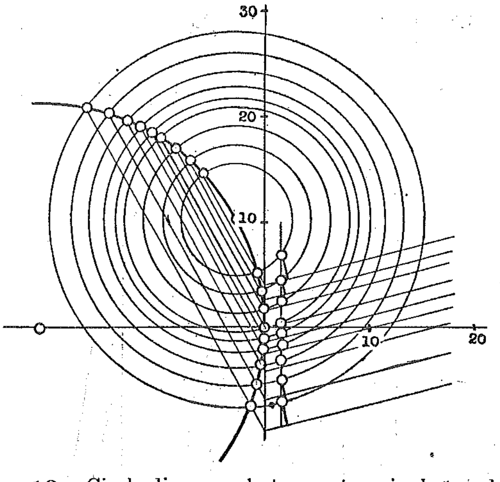

Fig. 10. Circle diagrams between terminals 1 and 5.

1 (Ur. generating station) ~7 (Sp. transformer station)

$$\frac{P_{17}+jQ_{17}}{E_7^2} = -\frac{1}{B_{17}} + \frac{1}{B_{17}}\frac{E_1}{E_7}.$$

See Fig. 11.

E_1	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	2.67	1.55	1.08	0.65	0.54	0.61	0.94	1.23	3.25

just as in the procedures for station 4, we judge the suitable voltage of station 1 to be, E_1 as 1.07 per unit or 64 KV in actual practice. (Fig. 12) That is, if E_1 is adjusted at such a value, the generating reactive power of the Ur. station is just sufficient for the demand of minimum transmission loss.

2 (Eb. generating station) ~6 (Mine. transformer station)

$$\frac{P_{26}+jQ_{26}}{E_6^2} = -\frac{1}{B_{26}} + \frac{1}{B_{26}}\frac{E_2}{E_6}.$$

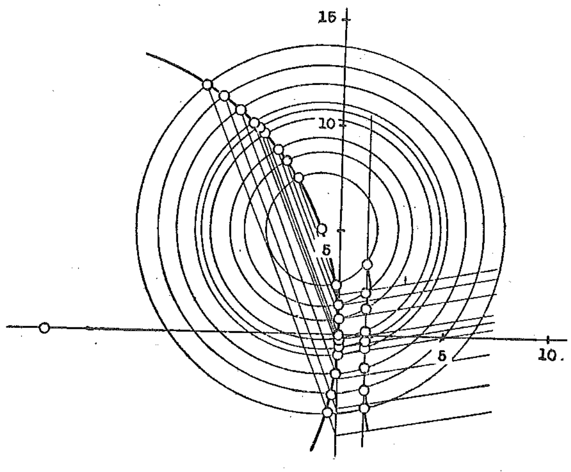

Fig. 11. Circle diagrams between terminals 1 and 7.

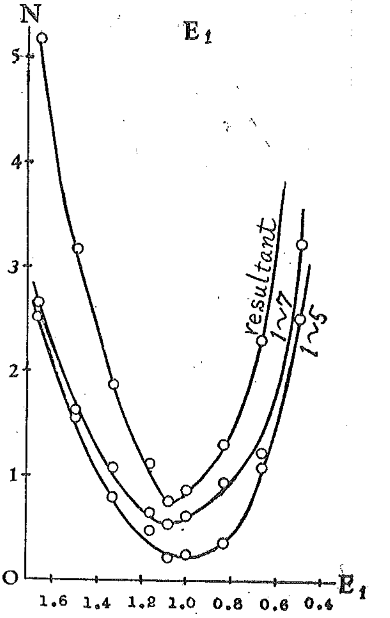

Fig. 12. Loss versus voltage curve for determining E_1.

See Fig. 13.

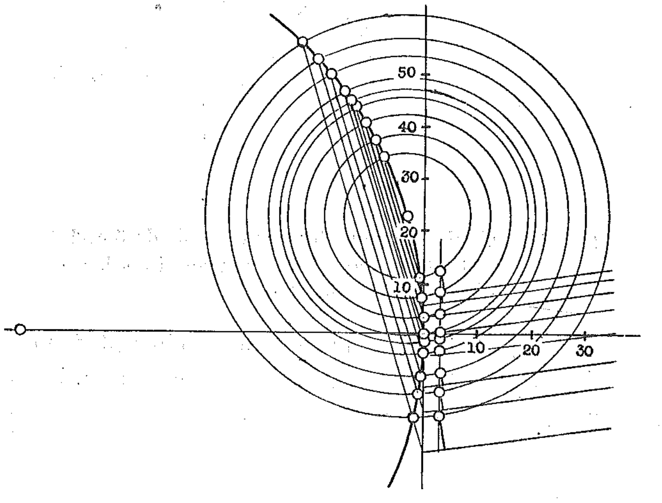

Fig. 13. Circle diagrams between terminals 2 and 6.

E_2	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	5.40	3.25	1.66	0.72	0.51	0.36	0.72	1.44	3.60

3 (Jo. generating station) ~7 (Sp. transformation station)

$$\frac{P_{37}+jQ_{37}}{E_7^2} = -\frac{1}{\boldsymbol{B}_{37}} + \frac{1}{\boldsymbol{B}_{37}}\frac{\boldsymbol{E}_3}{\boldsymbol{E}_7}.$$

See Fig. 14.

E_3	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	5.60	4.17	2.16	0.97	0.36	0.22	0.72	1.44	2.38

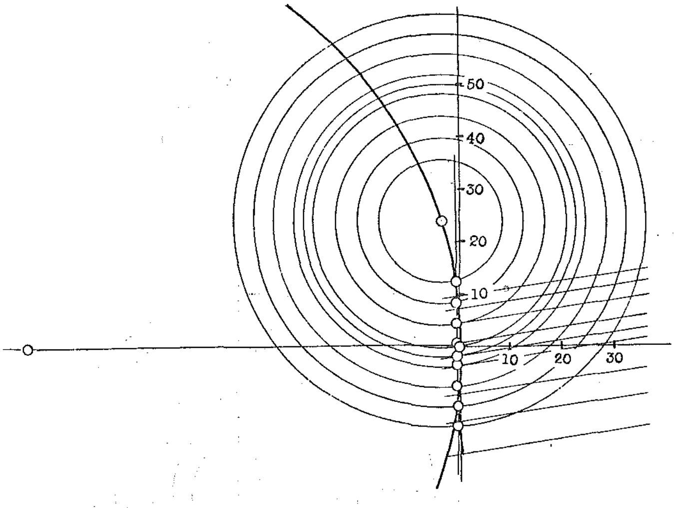

Fig. 14. Circle diagrams between terminals 3 and 7. Output lines are not shown here in order not to complicate the figure.

2 (Eb. generating station) ~3 (Jo. generating station) Nextly, the case where generating stations 2, and 3 are mutually connected is considered. As voltages of both stations are unknown in this case, different treatments must be applied now comcompared with the cases before for getting E_1 and E_4.

However, even in this case, these voltages can be determined by the graphical method as correctly as we can by the actual trials.

First, first, find the suitable voltage E_{3a} of station 3, which is determined from circle diagrams 3∼7. Second, find the suitable voltage E_2' of station 2 from circle diagrams 2∼3*a* by the use of the voltage E_{3a} obtained now. Third, find the suitable voltage of station 2 now from circle diagrams 2∼6, and comparing this with E_2', that is, totalizing it and E_2', find the voltage E_2'' which satisfies the minimum loss at station 2, namely the suitable voltage of 2. Fourth, using E_2'', find the suitable voltage E_{3b} of station 3 again by circle diagrams 2∼3*b*, and next using two groups of circle diagrams 3∼7 and 2∼3*b*, find the most suitable voltage E_{3c} of station 3 which makes the sum of two transmission loss minimum. Fifth, adopting E_{3c} as the lovtage of station 3, find the suitable voltage E_2''' of station 2 by 2∼3*c*, and comparing E_2''' with the voltage gotten by 2∼6, that is, summing up values from diagrams 2∼6 and 2∼3*c*, find the most suitable voltage E_2''''. Sixth, using E_2'''', find that of station 3, E_{3d} by 2∼3*d*, and next comparing E_{3d} with the voltage by 3∼7, find the most suitable voltage E_{3e} of station 3. Thus repeating such trials, it will be possible to get the most suitale voltage of stations 2 and 3 approximately.

Now we will follow the related steps on the example. Circle diagrams 3∼7 and2∼6 are already obtained.

2 ∼ 3*a*

From circle diagrams 3∼7, we take E_{3a} as $E_{3a}=1$.

$$\frac{P_{23}+Qj_{23}}{E_{3a}^2}=-\frac{1}{B_{23}}+\frac{1}{B_{23}}\frac{E_2}{E_{3a}}.$$

See Figs. 15 to 18 in the following.

E_2	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	3.60	2.70	1.30	0.61	0.36	0.18	0.40	1.15	2.35

From the above results, E_2' is 1. Then summing up these two results, that is circle diagrams 2∼6 and 2∼3*a*, it is determined that the suitable voltage E_2'' is 1.02.

2 ∼ 3*b*

using the value E_2'', again draw circle diagrams between 2 and 3.

$$\frac{P_{32}+jQ_{32}}{E_2''^2}=\frac{1}{B_{32}}-\frac{1}{B_{32}}\frac{E_3}{E_2''}.$$

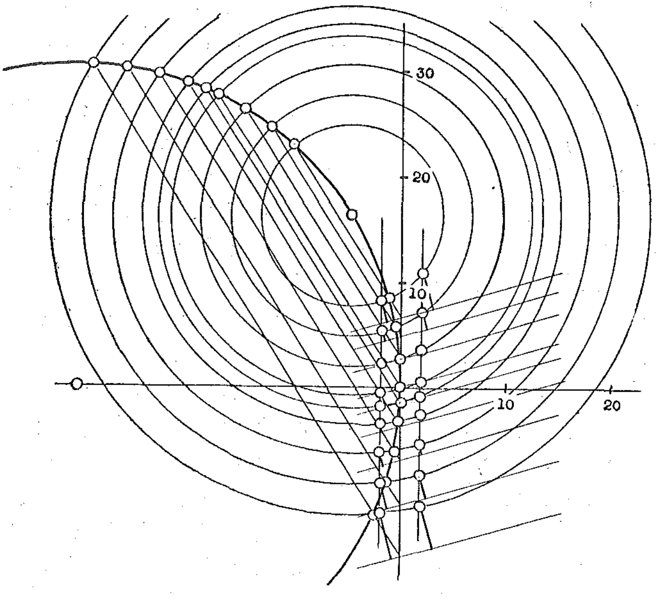

Fig. 15. Circle diagrams between terminals 2 and $3a$, 2 and $3d$.

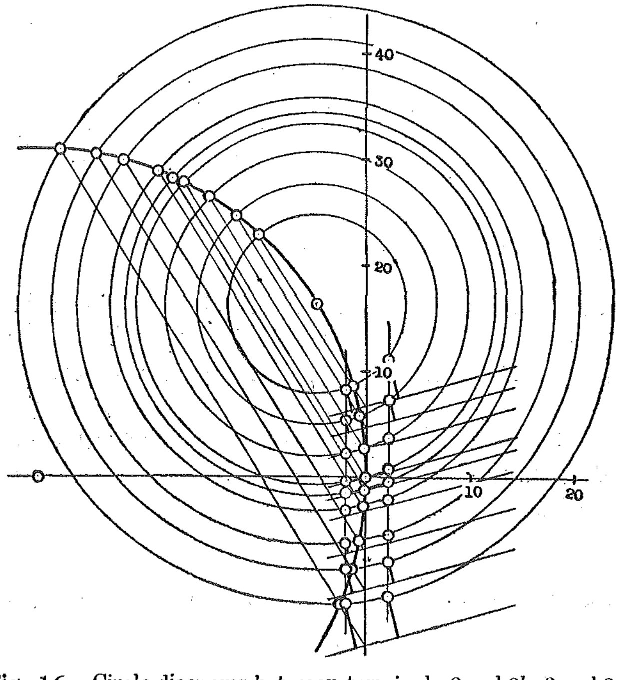

Fig. 16. Circle diagrams between terminals 2 and $3b$, 2 and $3c$.

E_3	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	3.95	2.16	1.19	0.58	0.22	0.29	0.87	1.98	4.15

From the above consequences, E_{3b} is gotten as 1.03. Then summing up loss versus voltage curves of 2~3*b* and 3~7, E_{3c} is determined as 1.02.

2 ~ 3*c*

Using this value, again draw circle diagrams 2~3*c*.

$$\frac{P_{23}+jQ_{23}}{E_{3c}^2} = -\frac{1}{B_{23}} + \frac{1}{B_{23}}\frac{E_2}{E_{3c}}.$$

E_2	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	4.35	2.52	1.44	0.72	0.18	0.14	0.65	1.36	2.52

Comparing with that of 2~6, the suitable voltage E_2'''' is 1.

2 ~ 3*d*

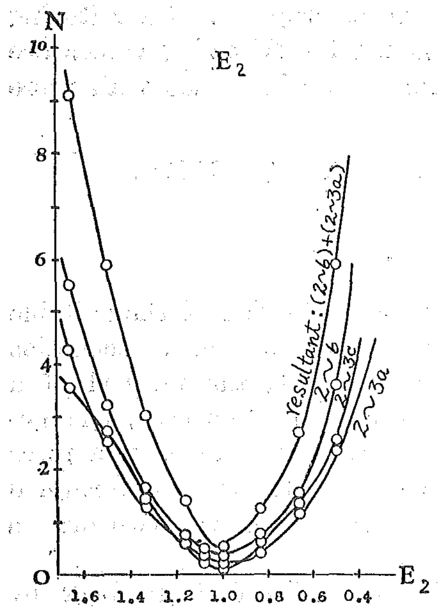

Fig. 17. Loss versus voltage curve for determining E_2.

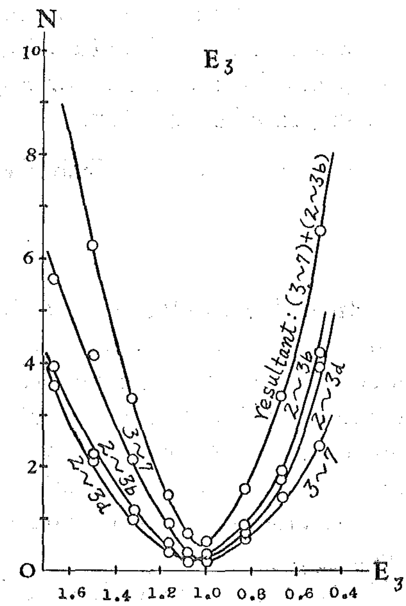

Fig. 18. Loss versus voltage curve for determining E_3.

Adopting E_2'''', the most suitable voltage of the station 3 is determined.

E_3	1.67	1.50	1.33	1.16	1.08	1.00	0.83	0.66	0.50
N	3.60	2.16	1.08	0.43	0.29	0.25	0.72	1.80	3.95

With that of 3∼7, E_{3e} is obtained as 1.04. Repeating such procedures, it is possible to approximate the most preferable value of sending voltages of stations 2 and 3.

From the results above obtained, if it is assumed that values so far gotten are to be the finest values, than the suitable voltages of all the sending power stations are determined as follows:

$$E_1 = 1.07, \quad E_2 = 1, \quad E_3 = 1.04, \quad E_4 = 1.07,$$

or expressed by actual KV,

$$E_1 = 64\ \text{KV}, \quad E_2 = 60\ \text{KV}, \quad E_3 = 62.5\ \text{KV}, \quad E_4 = 64\ \text{KV}.$$

As final checks of these values, the suitable voltages are to be calculated by the use of Eq. (12). Of course, also in this case, concerning stations 2 and 3, several trials must be made similarly to the previous way and final consequences must be sequred. Substituting the wholly identical data with the foregoing, the following values are finally obtained which show a quite remarkable coincidence with those of the graphical solutions.

$$E_1 = 64\ \text{KV}, \quad E_2 = 59.5\ \text{KV}, \quad E_3 = 62.84\ \text{KV}, \quad E_4 = 64\ \text{KV}.$$

6. Conclusion.

In order to clarify and simplify the determination of the suitable voltages of power sending stations in an interconnected transmission network, the authors reported here, as one way, the method of a graphical solution by use of circle diagrams. In this case, one generating station has most usually two transformer stations in a pwer network system, so the suitable voltage of the station is determined by the sum of transmission loss consumed in each of the two branch lines.

The formal analysis of this kind of problem is ordinarily made by the correct solutions of partial differential equations as is just related in the discussion but this is practically impossible. Then it is needful to find some substitute for mathematical analysis. From this point of view, the author think that graphical solutions are the only way to attain this purpose.

From the results of the example, it is found that the suitable voltages of generating stations are the values not greetly differing from the given voltages of the receiving stations, thus approximately closing by the resemblance values.

Reference

K. Ogushi: Circle Diagram for Electrical Engineer, Shukyosha, 1949.