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Tripping Characteristics of Protective Relays on  
Transmission Network,  
Expressed by Circle Diagram Method.

(Part I)

By

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Outline.

Tripping conditions of protective relays can be shown on the co-ordinate plane, expressing the watt and wattless components of electrical values at a relay setting point, a fault locating point or another any point on transmission network, while various electrical quantities can be shown on the same co-ordinate plane by the ordinary circle diagram method. Thus, it is easily possible to get electrical values at any desired point in network, when relay operated.

The object of this paper is to determine the range of relay operation in network and its proper selective action, making the cut off line by tripping as small as possible when fault occurs.

The transmission network may be classified under two sorts (1) network with one sending and one receiving end, and (2) network with many power stations and substations. It is preferable to take watt and wattless power as co-ordinate axis of circle diagram. But, in the case of unknown terminal voltage, admittance plane may be Taken.

The relays for common use operate in the circles as follows. Over-current relay operates in current circle, Power or ground relay operates in watt power circle. Impedance or mho relay operates in admittance circles. The present authors have already fully described and discussed these circles in a previous paper (1).

§ 1. Apparent current circle diagram and over-current relay.

An apparent current at which over-current relay is adjusted to trip, can be expressed on the co-ordinate plane of watt and wattless component of any point in network as an apparent current diagram as well known. Therefore, by using this diagram, it may be possible to get the tripping characteristics at a distant point from the relay setting point. At first, the following simple case will be studied.

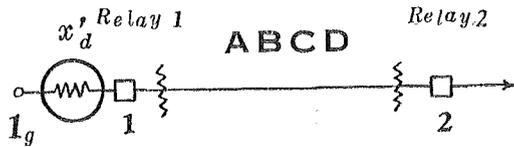


Fig. 1

I General transmission network with one sending and one receiving end, at each of which an over-current relay is set.

Tripping characteristics of relays arranged as fig. 1 will be expressed on the co-ordinate planes,  $W_{1_g} = P_{1_g} + jQ_{1_g}$  and  $W_2 = P_2 + jQ_2$  at air-gap voltages point and receiving point, taking their voltages  $E_{1_g}$  and  $E_2$  as reference respectively.  $E_{1_g}$  and  $W_{1_g}$  were used instead of  $E_1$  and  $W_1$ , the values of the actual sending end. It is convenient to consider the air-gap voltage, because at the short circuit instant, the relays operate by air-gap voltage and the relay 1 current is equal to the generator current.  $I_1$  and  $I_2$  are the relay 1 and 2 currents or the sending and the receiving current respectively.

$A B C D$  are general circuit constants, including the transient generator impedance  $x'_d$

(1) Tripping area of relay 1 shown on  $W_{1_g}$  plane.

It is obvious that relay 1 trips outside of the circle, having its center at origin and its radius  $I_1$  at which it is adjusted to trip. This equation is

$$P_{1_g} + jQ_{1_g} = |I_1| E_{1_g} \epsilon^{j\theta} \dots \dots \dots (1)$$

The power circle diagram is

$$P_{1_g} + jQ_{1_g} = \frac{D}{A} E_{1_g}^2 + \frac{1}{B} E_{1_g} E_2 \epsilon^{j\theta} \dots \dots \dots (2)$$

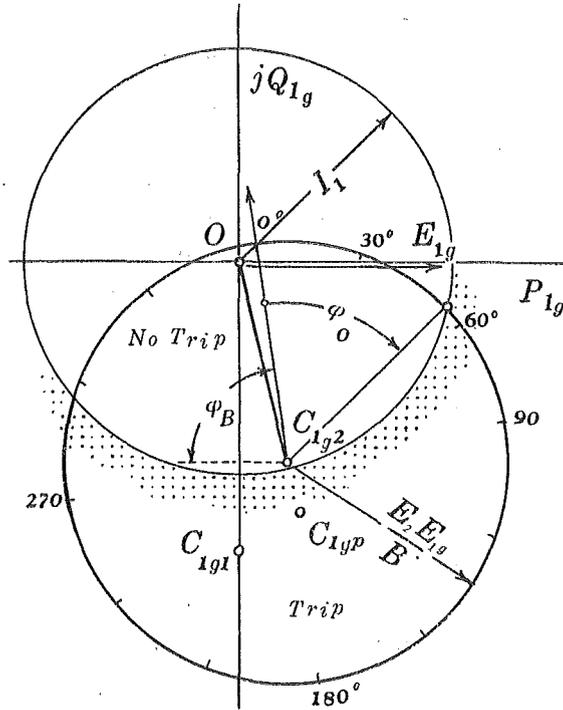


Fig. 2 Trip area of relay 1 on  $W_{10}$ -plane.

These circles are shown in fig. 2. A short circuit fault at receiving end 2 must be cleared out by relay 2, not by relay 1. Then, the center of the power circle diagram which is the short circuit point at the receiving end 2 must be inside the circle of equation (1).

$$|I_1| \geq \left| \frac{D}{A} E_{1g} \right| \dots\dots\dots (3)$$

Relay 1 is caused to trip by generator terminal short circuit, corresponding to the point,  $C_{1g1} = jX'd E_{1g}^2$  on  $W_{1g}$  plane in fig. 2 and by any short circuit fault at an intermediate point between the sending and receiving ends of which the co-ordinate may be outside of the circle of radius  $I = \left| \frac{D}{B} E_{1g} \right|$  as shown by  $C_{1gp}$  in fig. 2. Therefore, if the relay has protection against the above short circuit fault as its object, the trip area is suitable of definition by the circle equation (3).

The relay may operate in case of over load, if the power angle

of  $E_1 E_2 \epsilon^{j\phi_0}$  becomes larger than  $\phi_0$  in fig. 2. In the same way, a large power angle may be caused to occur by the cycle change of a generator, if it were parallel running at receiving end.

(2) Tripping area of relay 2, shown in  $W_{1_0}$  plane.

The apparent current circle to show  $I_2$  on  $W_{1_0}$  plane is as follows

$$W_{1_c} = \frac{C}{A} E_{1_0}^2 + \frac{1}{A} I_2 E_{1_c} \epsilon^{j\theta_2} \dots \dots \dots (4)$$

This center has the co-ordinate of the value, when receiving end 2 has no load.

Relay 2 is adjusted to trip out side of this circle. The center of the power circle diagram  $C_{1_2}$  must be in the tripping area, because relay 2 has a duty to clear out any faults at the receiving end. For any short circuit fault between sending and receiving ends 1 and 2, relay 2 can not operate, although  $C_{1_0}$  or  $C_{1_p}$  are in the tripping area, because relay 2 circuit is already cut out by these faults. The general

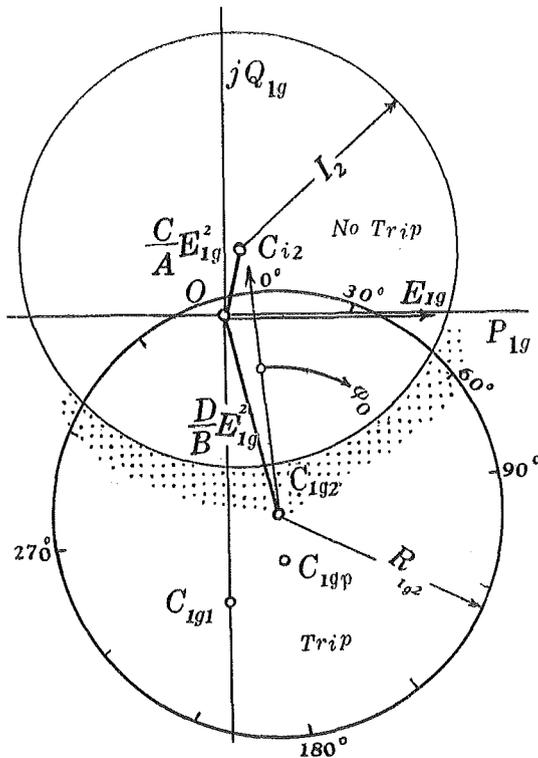


Fig. 3 Trip area of relay 2 on  $W_{1_0}$ -plane.

circuit constant  $C = \infty$  in this case. However, in the case of tripping against the large power angle  $\phi_0$  or due to the loss of synchronism, relays 1 and 2 may operate quite in the same way. Comparing the trip area of relay 1 in fig. 2 and that of relay 2 in fig. 3, it is understood that complete selective action of faults is impossible by this method, because the tripping areas of the two are very similar and  $\frac{C}{A} E_{i_0}^2$  is nearly zero in most cases.

(3) Tripping area of relay 1 and relay 2 on the  $W_2$  plane.

Taking the receiving vortage  $E_2$  as reference phase, receiving power and lagging reactive power as positive values, let the receiving power circle diagram be drawn as in fig. 4. These equations are as follows.

$$P_2 + jQ_2 = I_2 E_2 \varepsilon^{j\theta_2} \dots\dots\dots (5)$$

$$P_2 + jQ_2 = -\frac{C}{D} E_2^2 + \frac{1}{D} I_1 \varepsilon^{j\theta_1} E_2^2 \dots\dots\dots (6)$$

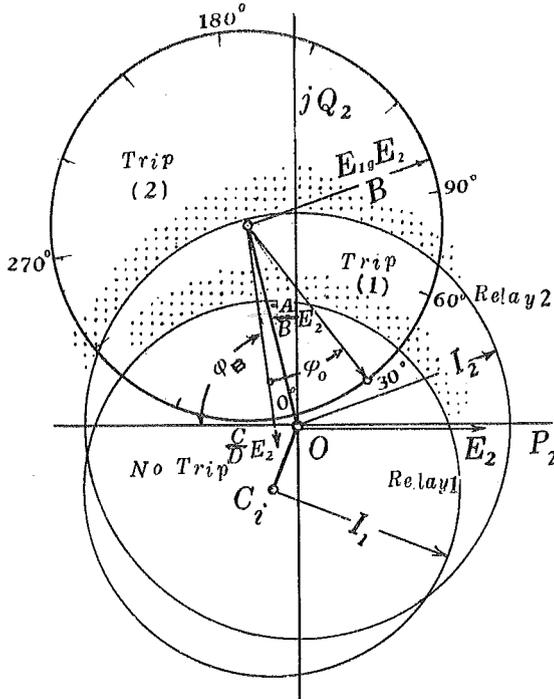


Fig. 4 Trip areas of relay 1 and relay 2 on  $W_2$ -plane.

$$P_2 + jQ_2 = - \frac{A}{B} E_2^2 + \frac{1}{B} E_1 E_2 \varepsilon^{j\theta} \dots\dots\dots (7)$$

The requirements for the relays are the same as explained in the preceding sections.

**II Tripping areas of the relays of transmission network with many power stations and substations.**

The equations of power circle diagrams for this network are known as

$$[P_{1_g} + jQ_{1_g}] = [E_{1_gk}] \frac{[D]^*}{[B]^*} [E_{1_c}] + [E_{2k}] \frac{1}{[B]^*} [E_{1_g}] \dots\dots\dots (8)$$

$$[P_2 + jQ_2] = [E_{2k}] \frac{-[A]}{[B]} [E_2] + [E_{1_gk}] \frac{1}{[B]} [E_2] \dots\dots\dots (9)$$

The apparent current circles are

$$[P_{1_c} + jQ_{1_g}] = [E_{1_gk}] \frac{[C]^*}{[A]^*} [E_{1_g}] + [E_{1_gk}] \frac{1}{[A]^*} [I_2] \dots\dots\dots (10)$$

$$[P_{1_g} + jQ_{1_c}] = [E_{1_gk}] [I_1] \dots\dots\dots (11)$$

$$[P_2 + jQ_2] = [E_{2k}] \frac{-[C]}{[D]} [E_2] + [E_{2k}] \frac{1}{[D]} [I_1] \dots\dots\dots (12)$$

$$[P_2 + jQ_2] = [E_{2k}] [I_2] \dots\dots\dots (13)$$

where (A), (B), (C), (D) are the matrix of general transmission network, [\*] are transposed matrix and K denotes their conjugate quantities.

By using matrix equations, the formulas are expressed in compact form. However, these equations are to mean merely the assembly of the already explained circle equations between two terminals of every one of the connectors which compose the network having many power stations and substations. Therefore, the before described methods may be applied for each case. But, in some cases, the actual network is possible to transform into a simpler form, by eliminating branch points or taking equivalent admittances for loads.

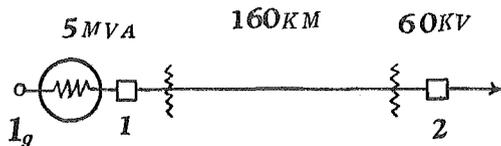


Fig. 5

**Example 1.** A transmission line, 160 Km, 60 KV receiving voltage, has over-current relays at the sending and receiving ends to obtain the tripping area on  $W_{I_0}$  plane with co-ordinate of watt and wattless power, let air-gap voltage  $E_{I_0}$  be taken as reference voltage. The generator 15 MVA, has transient reactance 20 %. The loads are 5 MW pure resistance, 5 MVA, 0.75 PF. induction motor, 5 MVA, 0.5 PF. synchronous machine. The transmission line has total series impedance, including transformer impedance  $Z=25+j100$  ohms, total capacitance,  $Y=j0.0015$  mho.

**Solution:** We may use per unit system, taking capacity 15 MVA as 1 p.u., voltage 60 KV as 1 p.u., current  $15 \text{ MVA} \div \sqrt{3} \text{ 60 KV}$  144.5 A as 1 p.u., impedance  $60 \text{ KV} \div \sqrt{3} \times 144.5$  240 ohms as 1 p.u., admittance  $1 \div 240 = 0.00417$  mho as 1 p.u. Then, the general circuit constants, including the machine transient reactance  $X'd = 0.2$  are

$$A = 0.856 + j 0.0187$$

$$B = 0.0977 + j 0.592$$

$$C = -0.00223 + j0.351$$

$$D = 0.926 + j 0.0185$$

Checking these values,  $AD - BC = 0.99978 - j 0.00038$ .

The receiving current is known from the giving load as

$$I_2 = 0.748 - j 0.508 = 0.904 \angle -34^\circ 10' \text{ p.u.}$$

The air-gap voltage and sending current are

$$E_{I_0} = AE_2 + BI_2 = 0.8688 + j 0.4037 = 0.9525 \angle 25^\circ 23'$$

$$I_{I_0} = CE_2 + DI_2 = I_1 = 0.7734 - j 0.659 = 1.032 \angle -45^\circ 43'$$

The power circle diagram, the circle for relay 1 and the apparent current circle diagram for relay 2 are calculated from equations (1) (2) and (4),

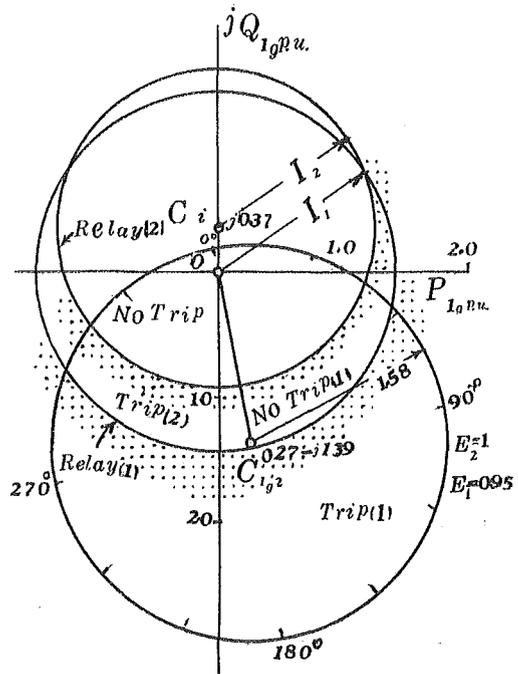


Fig. 6 Trip area of relays in example 1.

$$\begin{aligned}
 W_{1\theta} &= \frac{D}{B} E_{1\theta}^2 + \frac{1}{B} E_{1\theta} E_2 \\
 &= (0.269 - j 1.39) + 1.58 \varepsilon^{j\theta}
 \end{aligned}$$

$$W_{1\theta} = I_1 E_{1\theta} \varepsilon^{j\theta}$$

$$\begin{aligned}
 W_{1\theta} &= \frac{C}{A} E_{1\theta}^2 + \frac{I_2}{A} E_{1\theta} \varepsilon^{j\theta} \\
 &= (0.00549 + j 0.371) + (1.175) \varepsilon^{j\theta}
 \end{aligned}$$

These are shown in fig. 6, (To be continued).

#### References

- (1) K. Ogushi: Circle diagrams for Electrical Eng. Shukyosha, 1941.  
K. Ogushi, G. Miura: Electrical characteristics of interconnected power transmission Systems. On this volume.