Power at Static Stability Limit and Condition of Parallel Running as Generator of Two Machine System with Constant Terminal Voltages.

By

Koji OGUSHI
(Technical Faculty, Hokkaido University)

Preface.

It is known that the static stability limit of transmission power with two synchronous machines is theoretically determined by the following equation,[1]

$$\tan \phi_0 = \frac{M_1+M_2}{M_1-M_2} \tan \phi_B$$

By this equation the phase angle difference ϕ_0 of the air-gap voltages con be easily obtained. But, for practical purposes it is desirable to get limit power rather than the angle at which it lose synchronism. This power depends on the air-gap voltages of machines, which are variable not only with load, but also with method of excitation and with automatic voltage regulator. It may be practical that in the case of the quick response excitation and voltage regulation, transient reactances and their air-gap voltages of the machines should be taken into account, while for ordinary method of excitation and voltage regulation mean values of transient reactance and synchronous reactance with their corresponding internal voltages should be considered. Thus, it is not a simple matter to determine the value of static stability limit power. We may try to get a practical method for determining the limit power by using the circle diagram method. As an extension of this method, it is easy to obtain the condition of parallel running as generator without motor action by using the circle diagram theory.

Notation.

A, B, C, D : General circuit constants.
Z_{1_g}, Z_{2_g} : Impedances of synchronous machines.

Z : Total series impedance of line.
Y : Admittance equivalent to load.
M_1, M_2 : Inertia constant of machines.
ϕ_0 : Phase angle difference between air-gap voltages of machines.
ϕ_B : Angle of the constant B, including machine impedance.
$W=P+jQ$: Watt and wattless power.
$y=g+jb$: Admittance used for Co-ordinate axis, which has relation, $P+jQ=(g+jb)E^2$, taking E as reference vector.
1, 2, 1_g, 2_g, : suffix for sending, receiving end, point corresponding to air-gap voltage.
E : Terminal voltages.
$C_y=\dfrac{D}{B}$: Center of sending power circle diagram on admittance plane.
$R_y=\dfrac{1}{|B|}\dfrac{E}{E_{1_g}}$: Radius of the above circle.
$n=\dfrac{E_1}{E_2}$: Ratio of $|E_1|$ and $|E_2|$ which are kept constant.
$C_{py}=\dfrac{CB_k+DA_k}{AB_k+A_kB}$: Center of zero watt power circle on sending admittance plane.
$R_{py}=\dfrac{1}{AB_k+AB_k}$: Radius of the above.

$$\left.\begin{aligned} &A=1+Z_{1_g}Y+ZY,\ B=Z_{1_g}+Z \\ &C=Y\ ,\qquad\qquad\quad D=1 \end{aligned}\right\}:$$ General circuit constant between 1_g—2 in fig. 1.

$$\left.\begin{aligned} &A=1+Z_{1_g}Y+ZY,\ C=Y \\ &B=Z_{1_g}+Z+Z_{2_g}+Z_{1_g}Z_{2_g}Y+ZZ_{2_g}Y \\ &D=1+Z_{2_g}Y \end{aligned}\right\}:$$ General circuit constant between 1_g—2_g in fig. 1.

Circle diagram of two machine system with constant terminal voltages.

In a transmission line as shown in fig. 1 electric power is transmitted from 1 to 2, keepining the terminal voltages E_1 and E_2 constant through the line and transfomer impedance Z, neglecting leakage and the load admittance Y. Because the air-gap voltages of the machines are unknown, admittance unit is taken as Co-ordinate plane. Taking E_{1_g} the machine 1 air-gap voltage as the reference phase, the sending power circle diagrams between 1_g—1, 1_g—2 and 1_g—2_g terminals are

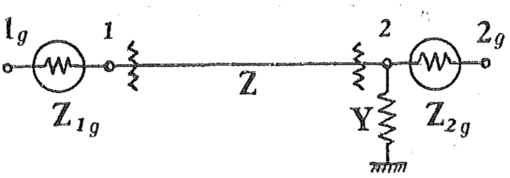

Fig. 1 Transmission line with two Synchronous machines with intermediate load.

as shown in the following formulas on the 1_g admittance plane, $y_{1_g}=g+jb$.

$$y_{1_g} = C_{1_g1}+R_{1_g1} \qquad (1)$$

$$y_{1_g} = C_{1_g2}+R_{1_g2} \qquad (2)$$

$$y_{1_g} = C_{1_g2_g}+R_{1_g2_g} \qquad (3)$$

, impossible

$$\left.\begin{aligned} C_{1_g1} &= \frac{1}{Z_{1_g}}, \quad R_{1_g1} = \frac{1}{Z_{1_g}}\frac{E_1}{E_{1_g}} \\ C_{1_g2} &= \frac{1}{Z+Z_{1_g}}, \quad R_{1_g2} = \frac{1}{Z+Z_{1_g}}\frac{E_2}{E_{1_g}} \\ C_{1_g2_g} &= \frac{1+ZY}{Z_{1_g}+Z+Z_{2_g}+Z_{1_g}Z_{2_g}Y+ZZ_{2_g}Y} \\ R_{1_g2_g} &= \frac{1}{Z_{1_g}+Z+Z+Z_{1_g}Z_{2_g}Y+ZZ_{2_g}Y}\frac{E_{2_g}}{E_{1_g}} \end{aligned}\right\} \qquad (4)$$

Eliminating E in equations (1) and (2), one obtains

$$y_{1_g} = \frac{1-\dfrac{E_1}{E_2}}{Z_{1_g}-(Z+Z_{1_g})\dfrac{E_1}{E_2}} \qquad (5)$$

In the above equation, variable factor is only θ in the value of $\frac{E_1}{E_2} = \frac{E_1}{E_2}\varepsilon^{j\theta}$, showing a locus of circle on y_{1_g} plane. Therefore, the equivalent admittance corresponding to the generator 1_g imput power $(P_{1_g}+jQ_{1_g})$ must be always on this circle (5), if the sending and receiving voltages E_1, E_2 are constant. Transforming equation (5) to show its centre and radius, one gets

$$y_{1_g} = C + R \qquad \cdots\cdots\cdots\cdots \quad (5a)$$

Where

$$C = \frac{Z_{1_gk} - (Z+Z_{1_g})_k n^2}{Z_{1_g} Z_{1_gk} - (Z+Z_{1_g})(Z+Z_{1_g})_k n^2}$$

$$R = \left| \frac{-Zn}{Z_{1_g} Z_{1_gk} - (Z+Z_{1_g})(Z+Z_{1_g})_k n^2} \right| \varepsilon^{j\phi_{12}}$$

$$n = \frac{E_1}{E_2}, \quad \phi_{12}: \text{ variable.}$$

The centers of the above mentioned four circles (1) (2) (3) and (5a) are as shown in fig. 2. The center of circle (5a) is always on the line $\overline{C_{1_g1}C_{1_g2}}$ and on that circle operate the syncheronous machines, if $\frac{E_1}{E_2}$ is kept constant.

Unknown values of air-gap voltages E_{1_g} and E_{2_g} may be obtained from the radius $\overline{C_{1_g1}y_{1_g}}$ and $\overline{C_{1_g2_g}y_{1_g}}$ as shown in equations (1) and (3). The phase difference between the air-gap voltages ϕ_0 is as shown in fig. 2, assuming that E_{2_g} lags to E_{1_g} or that machine 2 acts as motor, while if we take ϕ_0 in opposite direction, machine 1 acts as motor. The angle at stability limit is given by $\tan\phi_0 = \frac{M_1+M_2}{M_1-M_2} \times \tan\phi_B$, where ϕ_B is the angle of the driving impedance $(Z_{1_g}+Z+Z_{2_g}+Z_{1_g}Z_{2_g}Y+ZZ_{2_g}Y)$. If we obtain $y = g_{1_g}+jb_{1_g}$ corresponding to this limit angle on the circle in fig. 2, the required stability limit power of machine 1 is

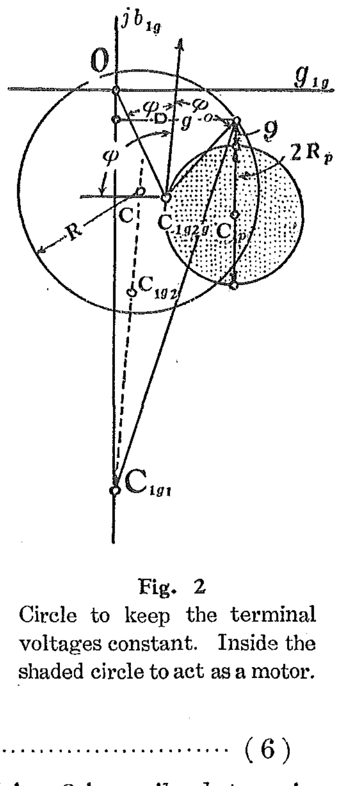

Fig. 2
Circle to keep the terminal voltages constant. Inside the shaded circle to act as a motor.

$$P_{1\max} = g_{1_g} E_{1_g}^2 \qquad \cdots\cdots\cdots\cdots \quad (6)$$

At this stability limit, the limit power of machine 2 is easily determined from the watt power circle diagram on the same Co-ordinate plane y_{1_g}. Its zero circle of watt power diagram is

$$C_{p1_g} = \frac{Y(Z_{1_g}+Z+Z_{2_g}+Z_{1_g}Z_{2_g}Y+ZZ_{2_g}Y)+(1+Z_{2_g}Y)(1+ZY+Z_{1_g}Y)_k}{(1+ZY+Z_{1_g}Y)(Z_{1_g}+Z+Z_{2_g}+Z_{1_g}Z_{2_g}Y+ZZ_{2_g}Y)_k + (1+ZY+Z_{1_g}Y)_k(Z_{1_g}+Z+Z_{2_g}+Z_{1_g}Z_{2_g}Y+ZZ_{2_g}Y)} \quad (7)$$

$$R_{p1_g} = \frac{1}{(1+ZY+Z_{1_g}Y)(Z_{1_g}+Z+Z_{2_g}+Z_{1_g}Z_{2_g}Y+ZZ_{2_g}Y)_k + (1+ZY+Z_{1_g}Y)_k(Z_{1_g}+Z+Z_{2_g}+Z_{1_g}Z_{2_g}Y+ZZ_{2_g}Y)}$$

This circle has a property to pass always the center of power circle diagram $C_{1_g2_g}$. The receivevd power in machine 2 is found as follows.[2]

$$P_2 = \frac{\pm g\,(2R_{p1_g} \pm g)}{2R_{p1_g}} E_{1_g}^2, \qquad (7\,a)$$

Where g is perpendicular distance to C_{p1_g} cirle.

It is known that in the inside of this R_{p1_g} circle on y_{1_g} Co-ordinate plane in fig. 2, machine 2 acts as motor and at the outside as generator. In the case of parallel operation of two power stations, it is important to keep all the machines as generators. As above described, it is clear that the stability limit and the parallel operation of generators are greatly dependent upon the air-gap voltages or machine impedances.

To explain these points calculations are given for the following numerical example.

Problem. A transmission line with two synchironous machines at both ends has line constant, machine impedance and load admittance as shown in fig. 3. The values are given by per unit of 60 MVA, 132 KV base.

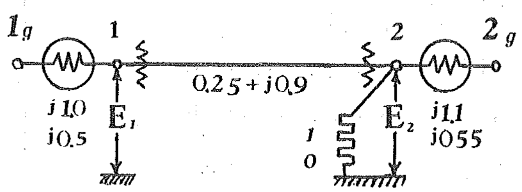

Fig. 3 E_1 E_2 constant. Per unit quantities.

Solution. Power circle diagrams between the terminals $1_g - 1$, $1_g - 2$ and $1_g - 2_g$ on y_{1_g} plane whith reference vector, E_{1_g} and $E_1 = 1$ are given by Table 1, for the next 3 cases.

	Machine impedance		Load
	Z_{1_g}	Z_{2_g}	Y
1	$j\,1$	$j\,1.1$	1
2	$j\,0.5$	$j\,0.55$	1
3	$j\,1$	$j\,1.1$	0

TABLE 1.

	$C_{1_t 1}$	$R_{1_g 1}$	$C_{1_g 2}$	$R_{1_t 2}$	$C_{1_t 2_g}$	$R_{1_g 2_g}$
1	$-j\,1$	$\frac{1}{E_{1_g}}$	$0.0651-j\,0.517$	$\frac{0.52}{E_{1_g}}$	$0.127-j\,0.39$	$0.374\frac{E_{2_g}}{E_{1_g}}$
2	$-j\,2$	$\frac{2}{E_{1_g}}$	$0.125\ -j\,0.7$	$\frac{0.707}{E_{1_g}}$	$0.136-j\,0.515$	$0.465\frac{E_{2_g}}{E_{1_g}}$
3	$-j\,1$	$\frac{1}{E_{1_g}}$	$0.0651-j\,0.517$	$\frac{0.52}{E_{1_g}}$	$0.027-j\,0.33$	$0.331\frac{E_{2_g}}{E_{1_g}}$

The zero watt power circles and the locus of circles for keeping terminal voltages, $\frac{E_1}{E_2}=1=n$ and $\frac{E_1}{E_2}=\frac{1}{0.8}=1.25=n$, are as shown by Table 2.

TABLE 2.

	C_{p1_g}	R_{p1^5}	$n=1.0$		$n=1.25$	
			C	R	C'	R'
1	$0.194-j\,0.517$	0.124	$0.0937-j\,0.337$	0.349	$0.0826-j\,0.394$	0.221
2	$0.31\ -j\,0.56$	0.20	$0.141\ -j\,0.508$	0.52	$0.13\ -j\,0.57$	0.416
3	2.0	2.0	$0.0937-j\,0.337$	0.349	$0.0826-j\,0.394$	0.221

The angles ϕ_B for general circuit constant, $\boldsymbol{B}$ between the terminals 1_g and 2_g and the stability limit angles for $M_1=M_2$, $M_1=\infty$, $M_2=\infty$ are as in Table 3.

TABLE 3.

	ϕ_B	ϕ_0	ϕ_0'	ϕ_0''
		$M_1=M_1$	$M_1=\infty$ 1_g generator ∞Bus	$M_2=\infty$ 2_g Motor ∞Bus
1	$\sim$ 120°	90°	120°	60°
2	$\sim$ 100°	90°	100°	80°
3	$\sim$ 85°	90°	85°	95°

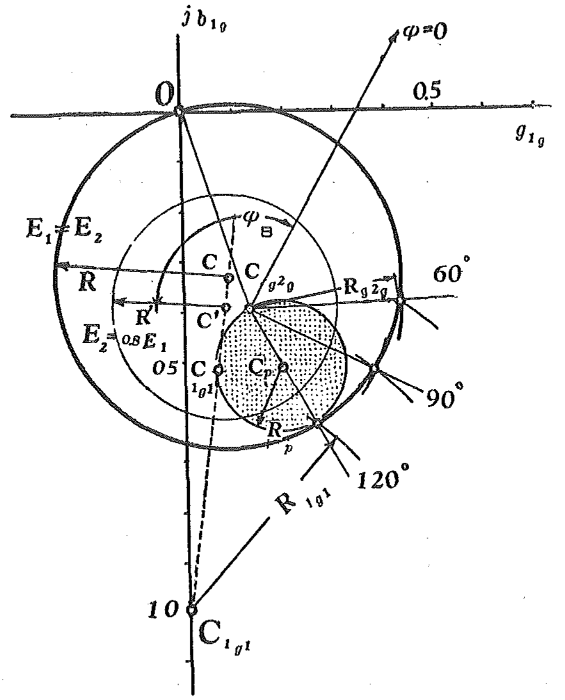

Fig. 4 Case 1. for large machine reactances.

The above results are shown in figs. 4–6. The range of parallel running as generators is clear from C_p circle, because outside of that they are all generators. In the cases of fig. 4 and fig. 5, the two machines operate as generators and synchronism is assuming the governor actions of their prime movers are complete. But, a case may happen when the C_p circle goas outside the C circle by dropping the terminal voltage E_2 as shown in fig. 4, taking C' circle insted of C circle. From fig. 5, it is clear that the stability limit is remarkably improved for transient reactances of machines or by the well-known quick response excitation. Fig. 6 shows the ordinary synchronous motor-generator operation, having no intermediate load.

Table 4 shows the limit powers, obtained from the figures and equations (6), (7a).

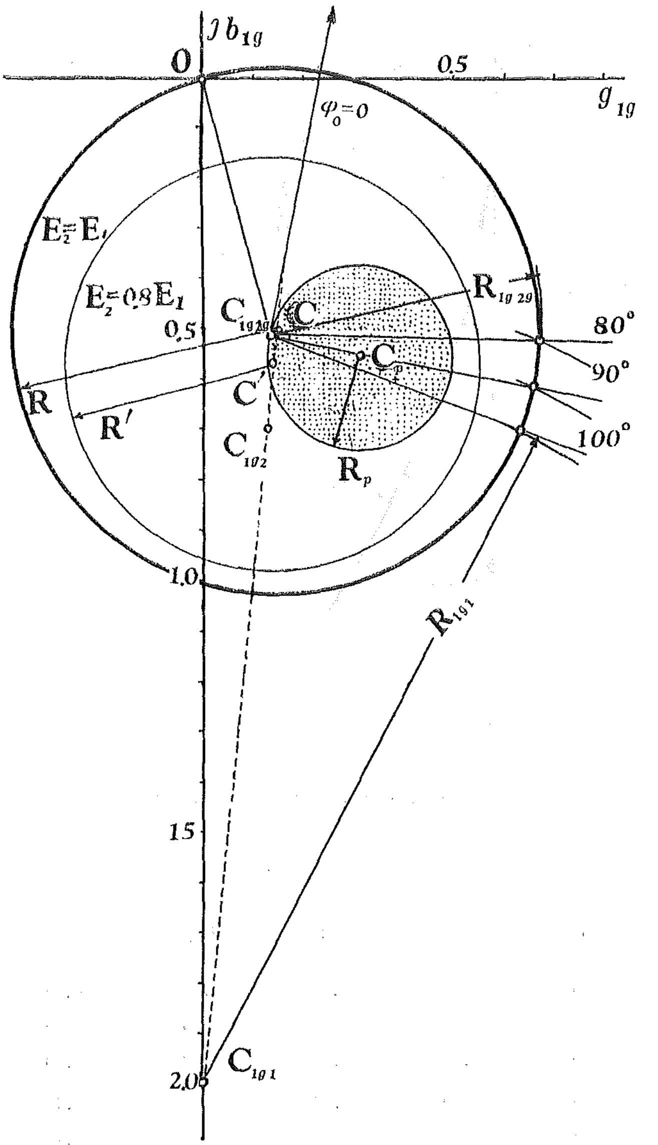

Fig. 5 Case 2. for transient reactances.

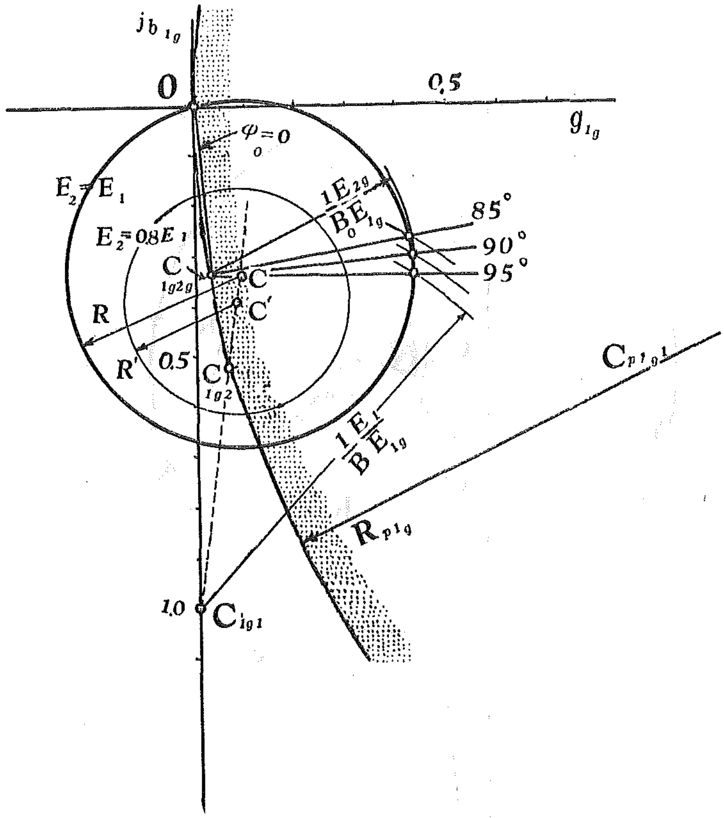

Fig. 6 Case 3. for no intermediate load.

TABLE 4.

	ϕ_0	E_{1_g}	P_1	E_{2_g}	P_2	
1	60	2.22	2.06	1.26	0.35	$E_1=E_2=1$
	90	1.66	1.04	1.26	0.09	
	120	1.33	0.46	1.33	0.00	
2	100	1.39	1.22	1.58	0.61	
	90	1.30	1.10	1.48	0.53	
	80	1.23	1.00	1.40	0.47	
3	95	1.27	0.68	1.53	− 0.58	
	90	1.22	0.64	1.52	− 0.55	
	85	1.18	0.60	1.47	− 0.50	

Conclusion.

Static stability limit of a two machine system with given constant terminal voltages of the machines are solved by circle diagram method, using (1) a locus of circle to satisfy given terminal voltages, (2) two power circle diagram between machine terminals and air-gap voltages, (3) a zero watt circle diagram. As Co-ordinate, conductance and reactance plane may be taken to avoid the air-gap voltages. Line constants, load, inertia constant and machine impedances must be known.

By this method it is possible to determine directly the stability limit angles. The air-gap voltages are obtained from the radius of the power circle diagrams. By using these voltages, the sending and receiving powers are gotten from the Co-ordinate and the zero watt power circle.

It is most usual for two or more power stations to supply power to one substation. In such a case it is important to have knowledge about the range in which one generator acts as a motor. For improvement of stability and parallel running condition, the diagrams are conveniently used to obtain the suitable terminal voltages and excitation method.

References

(1) The original paper of this well known equation :—Wagner and Evans : Static stability limits and the intermediate condensor station. Trans. AIEE. 1928.

(2) K. Ogushi : Circle diagrams for Electrical Eng. Shukyosha. 1941.