Electrical Characteristics of Interconnected Power Transmission Systems.

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Abstract.

A method of analyzing an interconnected transmission system is discussed. Usual transmission circuit constants, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$ are expressed here in matrices and proper calculations are applied. If there is a condition of constant voltages and powers maintained in all stations except very two stations under consideration, the system can be considered as an active network and be represented by ordinary circuit constants. Under such cases, the above calculations are more expanded, leading to many of appropriate circle diagrams be drawn, such as the effective power circle diagrams, the transmission efficiency circle diagrams, etc. Lastly, an example of the numeric calculation is shown on 60 KV transmission lines.

1. Introduction.

As a power system contains ordinarily a good many generating and transforming stations, it is quite desirable to obtain the general transmission circuit constants of this system and to calculate the sending-end operations from the receiving-end conditions, or vice-versa.

It is a well-known fact that on a system composed of one sending-end and one receiving-end terminal the calculations of sending-end operations from receiving-end conditions, and vice-versa, or of the best situation of transmission efficiencies of the system, etc. are executed by the use of circuit constants, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, or by the drawing of circle diagrams under certain changeable loads.

Now, the present authors make such circuit constants to be applicable for a multi-terminal power system by dint of showing them in matrices. Voltages and currents of a multi-terminal power system are shown by one equation with admittance matrices. However, according to kinds of power systems, the number of sending-end terminals and of recieving-end terminals will usually be different, and some of the branch lines between stations may be lacking.

The admittance matrix is always constituted in symmetrical as a whole. A sectional matrix of a sending-end or a recieving-end will be, however, usually an unsymmetrical one, of which the theoretical treatment is considerably difficult. Accordingly as it is the most simple way to express circuit constants of a system by matrices, there must be imagined to exist several severe conditions ; such as that the number of sending-end terminals and of recieving-end terminals are equal, that at least one branch line connecting each sending and receiving station directly, etc.

As it is practically rare to find a system which satisfies all such conditions, so appropriate treatment must be made before starting analyses to obtain actually these constants; such as the exclusion or the modification of lines which are not acceptable, the equalization of the total number of terminals between the two types by presuming sending or receiving ends as receiving or sending ends, etc. There are, however, a few examples in practice which can be applicable in that condition without considering any of the above restriction.

Anyhow, though such superfluous consideration must be needed, this analytic method has a great advantage for catching up the best conditions of voltages or efficiencies of an interconnected power transmission system as if an ordinary one-terminal network were handled. Several kinds of circle diagrams will be studied as the practical applications.

2. General Circuit Constants of Interconnected Transmission Systems.

Fig. 1 shows an equivalent circuit of a multi-terminal interconnected system. Each terminal voltage that is corrected by a standard reference voltage is to be $\boldsymbol{E}_1, \boldsymbol{E}_2, \cdots \boldsymbol{E}_n$; each terminal current to be $\boldsymbol{I}_1, \boldsymbol{I}_2 \cdots, \boldsymbol{I}_r, -\boldsymbol{I}_{r+1}, -\boldsymbol{I}_{r+2} \cdots -\boldsymbol{I}_n$; admittance between terminals, y_{ij}; and a self admittance of a terminal, y_{ii}. Currents with negative sign mean power reception.

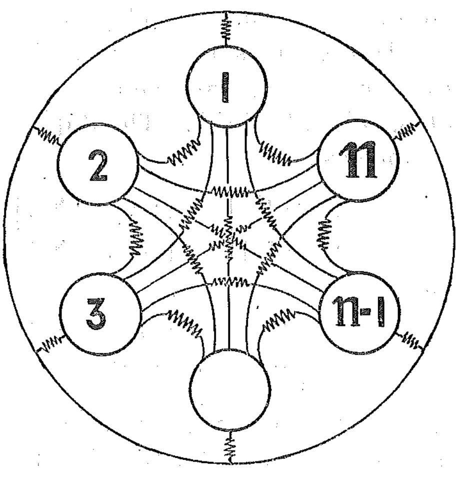


Fig. 1. Connection diagram of a general transmission power network.

y_{ij} is equal practically to a reciprocal of the sum of all series impedances of a line containing transformers too, but accurately equal to $1/\boldsymbol{B}$, that is a driving-point impedance between terminals. Out Likewise, y_{ii} practically is equal to exciting admittances of transformers in both terminals plus half the sum of all capacitive and conductive admittaces in a line, however, accurately to $\sum(A/B-1/B)$ or $\sum D/B-1/B)$, that is the sum of difference of short-circuit admittances and driving-point admittances of a branch line.

Then, the following matrix equation is constructed for the network of Fig. 1.

$$\begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_r \\ \hline -I_{r+1} \\ -I_{r+2} \\ \vdots \\ -I_n \end{pmatrix} = \left(\begin{array}{cccc|ccc} Y_{11} & -y_{12} & -y_{13} & \cdots & -y_{1(r+1)} & -y_{1(r+2)} \cdots & -y_{1n} \\ -y_{21} & Y_{22} & -y_{23} & \cdots & \cdots & \cdots & -y_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -y_{r1} & \cdots & \cdots & Y_{rr} & \cdots & \cdots & -y_{rn} \\ \hline -y_{(r+1)1} & \cdots & \cdots & \cdots & Y_{(r+1)(r+1)} & \cdots & -y_{(r+1)n} \\ -y_{(r+2)1} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -y_{n1} & \cdots & \cdots & \cdots & \cdots & \cdots & Y_{nn} \end{array}\right) \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_r \\ \hline E_{r+1} \\ \vdots \\ E_n \end{pmatrix} \cdots (1)$$

where $Y_{ii} = y_{ii} + y_{i1} + y_{i2} + \cdots\cdots\cdots$

$$= \sum \frac{A}{B} \cdot \text{sum of short-circuit admittances.}$$

This equation is obtained from the fact that the total currents sum of each terminal is zero, for an example, $I_1 = y_{11}E_1 + y_{12}(E_1 - E_2) + y_{13}(E_1 - E_3) + \cdots + y_{1n}(E_1 - E_n)$. In this admittance matrix which is a symmetrical one with rows and columns interchangeable, main diagonal terms are the short-circuit admittances and all other terms are the driving-point admittances, and also $|Y| \neq 0$.

If terminals 1, 2, $\cdots r$ are to be sending-ends and $(r+1)$, $(r+2)$, $\cdots n$ to be receiving-ends, voltages and currents are expressed by matrices $[E_S]$, $[E_R]$, $[I_S]$, $[I_R]$. ThenEq. (1) is shown by use of sectional matrices as follows:

$$\begin{bmatrix} [I_S] \\ -[I_R] \end{bmatrix} = \begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} \begin{bmatrix} [E_S] \\ [E_R] \end{bmatrix}$$

or

$$\left. \begin{aligned} [I_S] &= [S_{11}][E_S] + [S_{12}][E_R] \\ -[I_R] &= [S_{21}][E_S] + [S_{22}][E_R] \end{aligned} \right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (2)$$

where $[S_{11}]$ and $[S_{22}]$ mean the short-circuit admittances of sending-end and of receiving-end terminals of the system. Similarly, $[S_{12}]$ and $[S_{21}]$ mean the drivng admittances.

Compared with Eq. (1), it is clearly seen that $[S_{11}]$ and $[S_{22}]$ are symmetrical matrices and always $|S_{11}| \neq 0$ and $|S_{22}| \neq 0$. While, $[S_{12}]$ and $[S_{21}]$ are generally asymmetrical matrices, and sometimes $|S_{12}|=0$, $|S_{21}|=0$. However, $[S_{12}]$ and $[S_{21}]$ are transposed matrices with each other and

$$[S_{12}]^* = [S_{21}] \quad \cdots\cdots\cdots \quad (3)$$

is always obtained.

From Eq. (2), the voltage and current of the sending-end are obtained as follows:

$$[E_S] = -[S_{21}]^{-1}[S_{22}][E_R] - [S_{21}]^{-1}[I_R]$$
$$[I_S] = \left\{-[S_{11}][S_{21}]^{-1}[S_{22}] + [S_{12}]\right\}[E_R] - [S_{11}][S_{21}]^{-1}[I_R].$$

Now, in order to consider the general circuit constants of the interconnected power system, we put

$$\left.\begin{aligned} [A] &= -[S_{21}]^{-1}[S_{22}] \\ [B] &= -[S_{21}]^{-1} \\ [C] &= -[S_{11}][S_{21}]^{-1}[S_{22}] + [S_{12}] \\ [D] &= -[S_{11}][S_{21}]^{-1} \end{aligned}\right\} \quad \cdots\cdots\cdots \quad (4)$$

Then

$$\left.\begin{aligned} [E_S] &= [A][E_R] + [B][I_R] \\ [I_S] &= [C][E_R] + [D][I_R] \end{aligned}\right\} \quad \cdots\cdots\cdots \quad (5)$$

are obtained in the same manner as in an ordinary transmission line.

Since the above equations contain an inverse matrix $[S_{12}]^{-1}$, the following conditions must be taken into consideration for the formation of Eq. (5).

(a) $|S_{21}| \neq 0$ or $|S_{12}| \neq 0$.

(b) $[S_{11}]$, $[S_{22}]$, $[S_{21}]$ and $[S_{21}]$ are all square matrices of the same order.

Condition (a) means that there is at least one branch line directly connecting a sending-end and a receiving-end, and condition (b) means that the number of sending-ends equals that of the receiving-ends.

From Eq. (4), one also obtains the following relations:

$$\left.\begin{aligned} [D]^*[A] - [B]^*[C] &= [1] \\ [A][D]^* - [B][C]^* &= [1] \\ \text{or} \qquad & \\ [A]^*[D] - [C]^*[B] &= [1] \\ [D][A]^* - [C][B]^* &= [1] \end{aligned}\right\} \quad \cdots\cdots\cdots \quad (4a)$$

$\{[A][B]^*\}$, $\{[C][D]^*\}$, $\{[A]^*[C]\}$ and $\{[B]^*[D]\}$
are symmetrical matrices. .... (4b)

The voltages and currents of the receiving-ends are shown by conditions of the sending-ends as follows:

$$\left.\begin{aligned} [E_R] &= [D]^*[E_S]-[B]^*[I_S] \\ [I_R] &= -[C]^*[E_S]+[A]^*[I_S] \end{aligned}\right\} \quad \cdots\cdots\cdots (6)$$

That is, general circuit constants of a power system must be represented in this case by transposed matrices of Eq. (4).

$$\left.\begin{aligned} [A]^* &= -[S_{22}][S_{12}]^{-1} \\ [B]^* &= -[S_{12}]^{-1} \\ [C]^* &= -[S_{22}][S_{12}]^{-1}[S_{11}]+[S_{21}] \\ [D]^* &= -[S_{12}]^{-1}[S_{11}] \end{aligned}\right\} \quad \cdots\cdots\cdots (4c)$$

As previously shown, the general circuit constants of an interconnected power system can be expressed in the same manner as those of an ordinary, transmission line.

In the following, the applications are to be shown.

Example 1.

As an example of a simplified transmission system, the following schematic diagram is considered. The value of kilo-watts written near stations with arrows means effective powers of those lines, read

TABLE 1.

Generating Stations				Transforming Stations			
No.	Admittance (mho)	Voltage (KV)	Effective power (KW)	No.	Admittance (mho)	Voltage (KV)	Effective power (KW)
1	y_{15} = 0.00186 $-j0.0177$ y_{17} = 0.00246 $-j0.0156$	69.5	$P_{15}=+\sim$ $P_{17}=13\,400$ $P_{11}=+\sim$	5	$y_{57}=y_{15}$ $y_{54}=y_{45}$ y_{56} = 0.00338 $-j\,0.0275$	57.5	$P_{51}=-\sim$ $P_{54}=-\sim$ $P_{56}=-\sim$ $P_{55}=-\sim$
2	y_{23} = 0.00415 $-j0.0093$ y_{26} = 0.00118 $-j0.0135$	65.0	$P_{23}=-\sim$ $P_{26}=+\sim$ $P_{22}=4{,}200$	6	$y_{62}=y_{26}$ $y_{65}=y_{56}$	57.0	$P_{62}=-\sim$ $P_{65}=+\sim$ $P_{66}=-4{,}900$
3	$y_{32}=y_{23}$ y_{37} = 0.000218 $-j0.0125$	63.0	$P_{32}=+\sim$ $P_{37}=-\sim$ $P_{33}=0$	7	$y_{71}=y_{17}$ $y_{73}=y_{37}$	61.2	$P_{71}=-\sim$ $P_{73}=+\sim$ $P_{77}=-9{,}800$
4	y_{45} = 0.0021 $-j0.015$ y_{48} = 0.000420 $-j0.00805$	59.5	$P^{b}_{5}=+\sim$ $P_{48}=+\sim$ $P_{44}=25{,}600$	8	$y_{84}=y_{48}$	59.0	$P_{84}=-3{,}000$ $P_{88}=P_{84}$

usually from the power distribution diagrams of the stations. Each value of admittances, and voltages also are shown in Table 1. Sign $+\sim$ means power sending and $-\sim$ means power reception, of which the values are obtained by drawing circle diagrams.

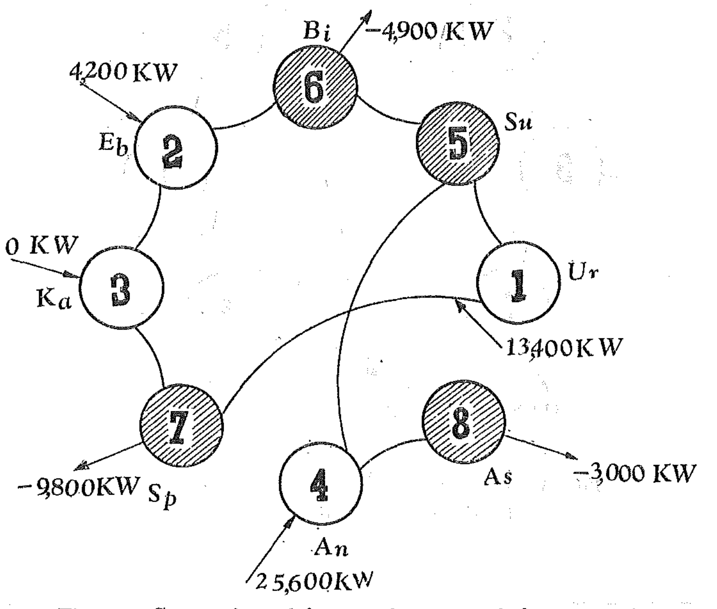


Fig. 2. Connections daigram of a transmission network applicable to example I.

The diagram of Fig. 2 is expressed by actual mhos and kilo-watts. Those values are corrected to the per-unit values with 10 MVa. base power and 60 Kv base voltage, by

$$y_{(p.u.)} = y_{(mho)} \times \frac{(60)^2}{10} . \qquad V_{(p.u.)} = V_{(Kv)} \times \frac{1}{60} .$$

Then, Fig. 2 and Table 1 take the form of Fig. 3 and Table 2.

In Fig. 3 and Table 2, each branch (effective) power is shown together with its reactive power, which is usually obtained by drawing a power circle diagram in each case. Since the details of the methods are of well-known, they may be omitted here. The order, however, of calculation is by starting at station 1 by utilizing $P_{17} = 1.34$, and passing on to stations 3, 2, 6, 5; on the other hand starting from the station 8 passing on 4, 5; and lastly between stations 5 and 1 are completed. As a result, the effective powers are obtained which are shown in Fig. 3 or Table 2.

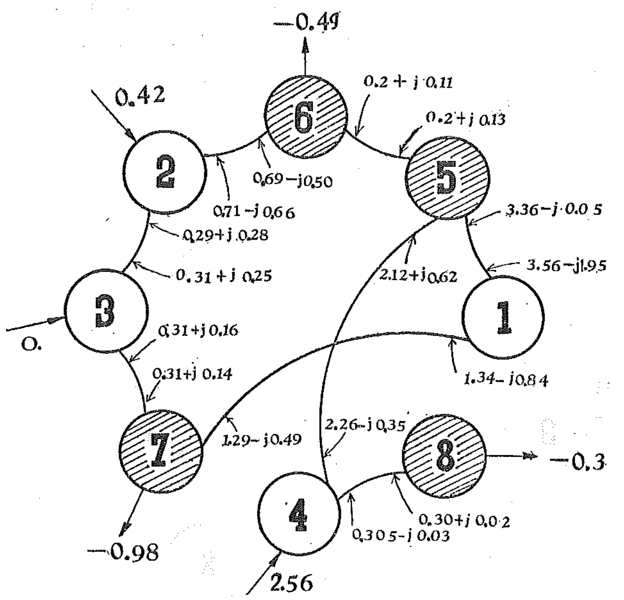


Fig. 3. Diagram of Fig. 2 with powers shown by the per-unit values.

TABLE 2.

Generating Stations				Transforming Stations			
No.	Admittance y (p. u.)	Volt. (p. u.)	Effective power (p. u.)	No.	Admittance y (p. u.)	Volt. (p. u.)	Effective power (p. u.)
1	$y_{15}=6.4\,\underline{\|-84^\circ}$ $y_{17}=5.67\,\underline{\|-81^\circ}$	1.16	$P_{15}=3.56$ $P_{17}=1.34$ $P_{11}=4.90$	5	$y_{51}=y_{15}$ $y_{54}=y_{45}$ $y_{56}=9.98\,\underline{\|-83^\circ}$	0.96	$P_{51}=-3.36$ $P_{54}=-2.12$ $P_{56}=-0.20$ $P_{55}=-5.68$
2	$y_{23}=3.68\,\underline{\|-66^\circ}$ $y_{26}=4.87\,\underline{\|-85^\circ}$	1.08	$P_{23}=-0.29$ $P_{26}=0.71$ $P_{22}=0.42$	6	$y_{62}=y_{26}$ $y_{65}=y_{56}$	0.95	$P_{62}=-0.69$ $P_{65}=0.20$ $P_{66}=-0.49$
3	$y_{32}=y_{23}$ $y_{37}=4.5\,\underline{\|-89^\circ}$	1.05	$P_{32}=0.31$ $P_{37}=-0.31$ $P_{33}=0$	7	$y_{71}=y_{17}$ $y_{73}=y_{37}$	1.02	$P_{71}=-1.29$ $P_{73}=0.31$ $P_{77}=-0.98$
4	$y_{45}=5.45\,\underline{\|-82^\circ}$ $y_{48}=2.9\,\underline{\|-87^\circ}$	0.09	$P_{45}=2.255$ $P_{48}=0.305$ $P_{44}=2.56$	8	$y_{84}=y_{48}$	0.98	$P_{84}=-0.30$ $P_{88}=P_{84}$

The power angles are also obtained by circle diagrams, which result as follows:

$$\begin{cases} \delta_1-\delta_7 = 10^\circ 17' \\ \delta_7-\delta_3 = 3^\circ 40' \\ \delta_3-\delta_2 = 5^\circ 10' \end{cases} \qquad \begin{cases} \delta_2-\delta_6 = 7^\circ 28' \\ \delta_6-\delta_5 = 1^\circ 22' \\ \delta_4-\delta_8 = 6^\circ 10' \end{cases} \qquad \delta_4-\delta_5 = 25^\circ$$

$$\therefore \quad \delta_1-\delta_5 = 27^\circ 57'$$

If $\delta_1 = 0$, assuming the voltage of station 1 to be a reference voltage, then other power angles are determined from the above relations.

$$\left.\begin{aligned} \delta_2 &= -19^\circ 07' & \delta_6 &= -26^\circ 35' \\ \delta_3 &= -13^\circ 57' & \delta_7 &= -10^\circ 17' \\ \delta_4 &= -\ 2^\circ 57' & \delta_8 &= -\ 9^\circ 07' \\ \delta_5 &= -27^\circ 57' & & \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots \text{(E 1)}$$

Magnitudes of voltages in stations, E_1, E_2, $E_3 \cdots E_8$, are shown in Table 2.

As power $W = P + jQ$ at every station is shown in Fig. 3, currents at every part or every branch line are obtained by $I = W/E_k$ where E_k is the conjugated value of E given by Eq. (E 1). That is:

$$\left.\begin{aligned} I_1 &= 4.85\ \angle{-29^\circ 40'} & I_5 &= 5.97\ \angle{-20^\circ 55'} \\ I_2 &= 0.954\ \angle{-85^\circ 07'} & I_6 &= 0.821\ \angle{-77^\circ 50'} \\ I_3 &= 0.086\ \angle{76^\circ 03'} & I_7 &= 1.14\ \angle{-43^\circ 02'} \\ I_4 &= 2.61\ \angle{-11^\circ 25'} & I_7 &= 0.306\ \angle{-5^\circ 17'} \end{aligned}\right\} \quad \cdots\cdots \text{(E 2)}$$

Furthermore, by Eq. (4), circuit constants $[A]$, $[B]$, $[C]$, $[D]$ are obtained as follows:

$$[A] = \begin{pmatrix} 3.40\ \angle{0^\circ 58'} & 1.56\ \angle{181^\circ} & 0 & 0.85\ \angle{-178^\circ} \\ 2.04\ \angle{182^\circ} & 3.06\ \angle{1^\circ 20'} & 0 & 0 \\ 4.30\ \angle{-171^\circ} & 1.97\ \angle{9^\circ} & 2.25\ \angle{4^\circ 28'} & 1.07\ \angle{10^\circ} \\ 0 & 0 & 0 & 1.00\ \angle{0^\circ} \end{pmatrix}$$

$$[B] = \begin{pmatrix} 0.156\ \angle{84^\circ} & 0 & 0 & 0.293\ \angle{-91^\circ} \\ 0 & 0.205\ \angle{85^\circ} & 0 & 0 \\ 0.197\ \angle{-88^\circ} & 0 & 0.222\ \angle{89^\circ} & 0.37\ \angle{97^\circ} \\ 0 & 0 & 0 & 0.345\ \angle{87^\circ} \end{pmatrix}$$

$$[C] = \begin{pmatrix} 34.6\ \angle{-81^\circ 10'} & 18.75\ \angle{98^\circ 25'} & 5.67\ \angle{99^\circ} & 10.24\ \angle{99^\circ 25'} \\ 5.26\ \angle{40^\circ 15'} & 14.15\ \angle{-81^\circ 35'} & 8.28\ \angle{118^\circ 30'} & 3.94\ \angle{124^\circ} \\ 27.1\ \angle{108^\circ 40'} & 4.65\ \angle{-81^\circ 50'} & 13.78\ \angle{-69^\circ 26'} & 8.60\ \angle{-68^\circ 40'} \\ 5.45\ \angle{98^\circ} & 0 & 0 & 5.45\ \angle{-82^\circ} \end{pmatrix}$$

$$[D]=\begin{pmatrix} 1.88\,\underline{|1^\circ25'} & 0 & 0 & 3.53\,\underline{|-173^\circ40'} \\ 0.73\,\underline{|26^\circ} & 1.73\,\underline{|8^\circ10'} & 0.816\,\underline{|203^\circ} & 1.36\,\underline{|211^\circ} \\ 1.58\,\underline{|-166^\circ40'} & 0.76\,\underline{|199^\circ} & 1.78\,\underline{|10^\circ20'} & 2.97\,\underline{|18^\circ20'} \\ 0 & 0 & 0 & 2.88\,\underline{|3^\circ15'} \end{pmatrix} \quad \text{(E 3)}$$

Thus, circuit constants are obtained. If terminal voltages $E_1, E_2, \cdots$ are maintained as constants, voltages behind synchronous impedances need not be considered for the later calculations. However, if constant voltages behind synchronous impedances are assumed, that is, if excitors' currents are maintained constants, these voltages must be calculated together with their power angles too. In those cases, impedance drops due to generators' synchronous impedances and transformers' leakage reactances must be added to the circuit constants of Eq. (E 3). Namely,

$$\left.\begin{aligned} \begin{bmatrix}[E_S]\\ [I_S]\end{bmatrix} &= \begin{bmatrix}[A] & [B]\\ [C] & [D]\end{bmatrix}\begin{bmatrix}[E_R]\\ [I_R]\end{bmatrix} \\ [E_g] &= [E_S]+[X_g][I_S] \\ [E_R] &= [E_t]+[X_t][I_R] \end{aligned}\right\} \quad \text{(E 4a)}$$

where

$$\text{Generator impedances } [X_g]=\begin{pmatrix} jx_1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & jx_r \end{pmatrix}$$

$$\text{Transformer impedances } [X_t]=\begin{pmatrix} jx_{r+1} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & jx_n \end{pmatrix}$$

From those equations, are obtained the following circuit constants including the generator and transformer reactances.

$$\begin{bmatrix}[E_g]\\ [I_S]\end{bmatrix}=\begin{bmatrix}[A]+[X_g][C], & [B]+[X_g][D]+\{[A]+[X_g][C]\}[X_t] \\ [C] & , \quad [D]+[C][X_t]\end{bmatrix}\begin{bmatrix}[E_t]\\ [I_R]\end{bmatrix} \quad \text{(E 4)}$$

We assume, $[X_g]$, $[X_t]$ as the following per unit values with 10 MVa base.

TABLE 3.

Generating stations			Transforming stations		
1.	Ur.	0.157	5.	Su.	0.05
2.	Eb.	0.44	6.	Bi.	0.05
3.	Ka.	1.34	7.	Sp.	0.05
4.	An.	0.333	8.	As.	0.05

Then, circuit constants with constant voltages behind synchronous reactances and with including transformers' reactances at receiving-ends are obtained by Eq. (E 4) as follows:

$$[A]=\begin{pmatrix} 8.80\angle 5^\circ 48' & 3.50\angle 187^\circ 30' & 0.891\angle 189^\circ & 2.46\angle 186^\circ 50' \\ 3.92\angle 154^\circ 20' & 9.26\angle 6^\circ 05' & 3.69\angle 208^\circ 30' & 1.73\angle 213^\circ 55' \\ 40.5\angle 197^\circ 40' & 8.21\angle 8^\circ 22' & 20.65\angle 18^\circ 50' & 12.6\angle 20^\circ 50' \\ 1.81\angle 188^\circ & 0 & 0 & 2.79\angle 5^\circ 10' \end{pmatrix}$$

$$[B]=\begin{pmatrix} 0.89\angle 92^\circ 17' & 0.174\angle -82^\circ 30' & 0.045\angle -81^\circ & 0.966\angle -85^\circ 45' \\ 0.25\angle 153^\circ 40' & 1.42\angle 95^\circ 36' & 0.542\angle -65^\circ & 0.678\angle -59^\circ 30' \\ 4.33\angle -75^\circ 10' & 0.615\angle -64^\circ & 3.64\angle 99^\circ 15' & 4.95\angle 106^\circ 30' \\ 0.09\angle -82^\circ & 0 & 0 & 1.44\angle 91^\circ 57' \end{pmatrix}$$

$$[D]=\begin{pmatrix} 3.60\angle 5^\circ & 0.937\angle 188^\circ 25' & 0.283\angle 189^\circ & 4.05\angle 195^\circ 35' \\ 0.57\angle 65^\circ 40' & 2.44\angle 8^\circ 15' & 1.23\angle 204^\circ 50' & 1.56\angle 211^\circ 20' \\ 2.94\angle 195^\circ 45' & 0.53\angle 203^\circ 45' & 2.46\angle 13^\circ 10' & 3.40\angle 18^\circ 40' \\ 0.272\angle 188^\circ & 0 & 0 & 3.15\angle 3^\circ 40' \end{pmatrix}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad [E\,5]$$

$[C]$ is the same as that of Eq. (E 3).
If there need not be included transformers' reactances, $[X_t]=0$ is used in Eq. (E 4).

As these have been discussed in both cases where terminal voltages are constant and where generator nominal voltages are constant, if analyses are made under constant nominal voltages from the first, then establishment or calculation of Eq. (E 4) is rather more laborious than directly reducing the network by ring-star conversions. In this case, the network is first reduced by ring-star conversions to a general polygonal diagram and is represented only by generator nominal voltages or receiving voltages.

From Eq. (E 4a), generator nominal voltages (induced air-gap voltages) and receiving-end voltages (behind transformer reactance) are calculated using Eq. (E 2), resulting as follows:

$$\left.\begin{array}{ll} \boldsymbol{E}_1 = 1.67\ \underline{|\ 23°20'} & \boldsymbol{E}_5 = 1.04\ \underline{|-44°30'} \\ \boldsymbol{E}_2 = 1.47\ \underline{|-12°25'} & \boldsymbol{E}_6 = 0.92\ \underline{|-28°10'} \\ \boldsymbol{E}_2 = 0.936\underline{|-13°55'} & \boldsymbol{E}_7 = 0.99\ \underline{|-13°05'} \\ \boldsymbol{E}_4 = 1.41\ \underline{|\ 34°40'} & \boldsymbol{E}_8 = 0.98\ \underline{|-10°} \end{array}\right\} \cdots\cdots\cdots \text{(E 6)}$$

In Eq. (E 6), $\boldsymbol{E}_1$, $\boldsymbol{E}_2 \cdots$ should be noted as E_{1g}, E_{2g}, differing from the former values. However, by renewing notations again, $\boldsymbol{E}_1$, $\boldsymbol{E}_2 \cdots$ are used here too. Currents, I_1, I_2, $\cdots$ are held as before.

From the above values, we can also obtain the values of terminal admittances or powers at generators or transformers by such relations as $\boldsymbol{w}_1 = \boldsymbol{I}_1/\boldsymbol{E}_1$, $\boldsymbol{w}_{17} = \boldsymbol{I}_{17}/\boldsymbol{E}_1$, and $\boldsymbol{W}_{11} = \boldsymbol{I}_1\boldsymbol{E}_{1k} = \boldsymbol{w}_1 E_1^2$, $\boldsymbol{W}_{17} = \boldsymbol{I}_{17}\boldsymbol{E}_{1k} = \boldsymbol{w}_{17} E_1^2$, etc.

3. Power Circle Diagrams and Apparent Current Circle Diagrams of Interconnected Transmission Systems.

Power circle diagrams of an ordinary transmission line with constant voltages are generally shown by circles which have centers at points of short-circuit powers and have radii equal to driving powers into a sending power co-ordinate or a receiving power co-ordinate. If this definition is applied to the case of an interconnected power system, the centers of short-circuit powers and the radii of driving powers are shown by matrices for receiving-end circles as follows:

$$\left.\begin{array}{ll} \text{Center:} & -[\boldsymbol{B}]^{-1}[\boldsymbol{A}][\boldsymbol{E}_R][\boldsymbol{E}_R]_k = -[\boldsymbol{S}_{22}][\boldsymbol{E}_R][\boldsymbol{E}_R]_k \\ \text{Radius:} & |[\boldsymbol{B}]^{-1}[\boldsymbol{E}_S][\boldsymbol{E}_R]_k| = |[\boldsymbol{S}_{21}][\boldsymbol{E}_S][\boldsymbol{E}_R]_k| \end{array}\right\} \cdots (7)$$

And for sending-end circles,

$$\left.\begin{array}{ll} \text{Center:} & [\boldsymbol{B}]^{*-1}[\boldsymbol{D}]^*[\boldsymbol{E}_S][\boldsymbol{E}_S]_k = [\boldsymbol{S}_{11}][\boldsymbol{E}_S][\boldsymbol{E}_S]_k \\ \text{Radius:} & |[\boldsymbol{B}]^{*-1}[\boldsymbol{E}_R][\boldsymbol{E}_S]_k| = |[\boldsymbol{S}_{12}][\boldsymbol{E}_R][\boldsymbol{E}_S]_k| \end{array}\right\} \cdots (7a)$$

In like manner, apparent current circle diagrams of an interconnected power system are shown as follows:
For receiving-end circles,

$$\left.\begin{array}{ll} \text{Center:} & -[\boldsymbol{D}]^{-1}[\boldsymbol{C}][\boldsymbol{E}_R][\boldsymbol{E}_R]_k = \{[\boldsymbol{S}_{21}][\boldsymbol{S}_{11}]^{-1}[\boldsymbol{S}_{12}] - [\boldsymbol{S}_{22}]\}[\boldsymbol{E}_R][\boldsymbol{E}_R]_k \\ \text{Radius:} & |[\boldsymbol{D}]^{-1}[\boldsymbol{I}_S][\boldsymbol{E}_R]_k| = |[\boldsymbol{S}_{21}][\boldsymbol{S}_{11}]^{-1}[\boldsymbol{I}_S][\boldsymbol{E}_R]_k| \end{array}\right\}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (8)$$

And for sending-end circles,

$$\text{Center: } [A]^{*-1}[C]^{*}[E_S][E_S]_k=\{[S_{11}]-[S_{12}][S_{22}]^{-1}[S_{21}]\}[E_S][E_R]_k$$
$$\text{Radius: } |[A]^{*-1}[I_R][E_S]_k| = |[S_{12}][S_{22}]^{-1}[I_R][E_S]_k|$$

These results are obtained from Eq. (5) by relatively simple calculations.

The equations represent the total of power circle diagrams or apparent current circle diagrams of the system, between terminal and terminal. Since these equations do not contain an inverse matrix $[S_{12}]^{-1}$ or $[S_{21}]^{-1}$, they can be applied for a system of which the number of sending-ends and recieving-ends differ, on disregarding the condition (a) and (b).

Referring to saturation points, however, the short-circuit power matrix which defines the center contains the driving powers which are transmitted only among sending stations or only among receiving thus constituting a pair of power sending and reception stations. Therefore, there are some circles among the receiving circles group, though some have actually sending characteristics; and likewise some circles are seen among the sending circles group, of which the characteristics are those of power reception.

Accordingly, the center, the short circuit power matrix, does not occupy definite points but contains changeable terms dependent upon loads, resulting somewhat in uneven forms, though the effects is almost cancel each other resulting only in a slight fluctuation or transition of points. The above consideration is applicable to the apparent current circle diagrams too.

As such it must be noted, the circle diagrams are drawn using terms, radii and centers, in the matrix, and they are only a collection of ordinary power circle diagrams between terminal and terminal.

Besides, there are some other methods to draw different kinds of power circle diagrams or apparent current circle diagrams according to the kinds of selected assumptions.

Example 2.

In this example of which the outlines are explained in Example 1, the analyses will be advanced to obtain the power circle diagrams discussed in this section. It is already related in Example 1 that if analyses are started first from considering synchronous reactances and transformer reactances with constant nominal voltages of generators, the network should be reduced to a polygonal diagram of a ring-form by the network reduction. That is to say, in this case the network reduction is less laborious than the former procedures. However, if the reactances of generators or transformers need not be considered, nor the admittance matrices, Eq (1), which is instantly obtained from data, directly assists the drawing of the power circle diagrams by the procedures of Eq. (7) and Eq. (7a).

In this example, the case which includes such reactances will be analyzed. Then, we have two methods of proceeding: one them of is the turning back to draw a new network that includes synchronous and transformer reactances, to reduce it to a ring-form, and to calculate Eq. (1); tne other method is the calculating of the admittance matrix in purely mathematical manner from Eq. (E 5).

Further explanation will be offered in both cases. The first method is simple, the redrawing of Fig. 3 to such as Fig. 4. Each admittance between shaded terminals is listed on Table 2 and each self reactance between the machine itself and the shaded terminal is on Table 3. By the star-ring transformation, it is reduced to the form of Fig. 5.

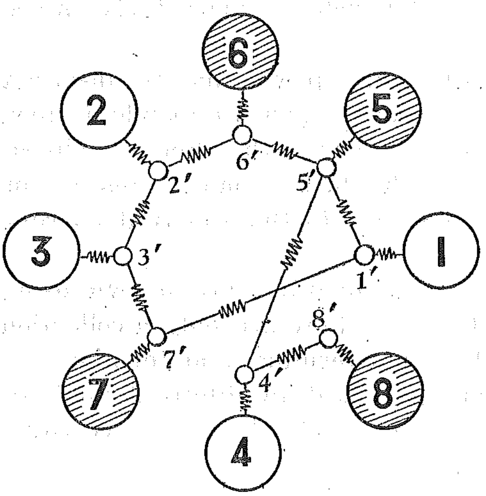


Fig. 4. Connection diagram of the example, including generators' synchronous reactances and transformers' leakage reactances.

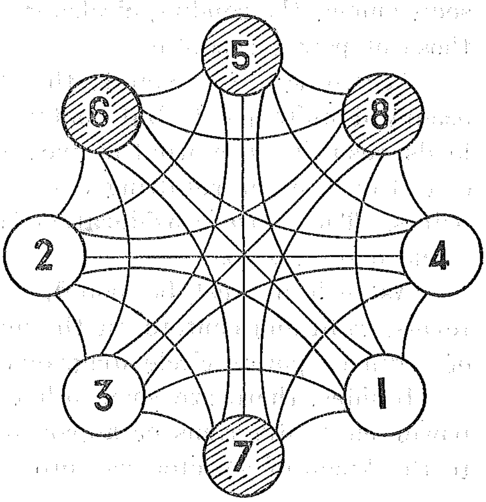


Fig. 5. Diagram of the example reduced by the ring-star reduction.

In Fig. 5, all lines that connect mutual terminals have definite admittance values defined by the reduction. Also, the synchronous reactances and transformer reactances are included in line constants.

The second method is to calculate Eq. (E 5) by the following process.

As the constants $[\boldsymbol{A}]$, $[\boldsymbol{B}]$, $[\boldsymbol{C}]$, $[\boldsymbol{D}]$, of course, include those machine reactances, we only calculate from the constants the original admittance values, that is by an inverese calculation of Eq. (4).

$$\left.\begin{aligned} [\boldsymbol{S}_{11}] &= [\boldsymbol{D}][\boldsymbol{B}]^{-1} \\ [\boldsymbol{S}_{12}] &= [\boldsymbol{C}]-[\boldsymbol{D}][\boldsymbol{B}]^{-1}[\boldsymbol{A}] \\ [\boldsymbol{S}_{21}] &= -[\boldsymbol{B}]^{-1} \\ [\boldsymbol{S}_{22}] &= [\boldsymbol{B}]^{-1}[\boldsymbol{A}] \end{aligned}\right\} \qquad \text{(E 7)}$$

are obtained, which can be used in Eq. (7) or (7a).

Since there need not be carried through two such calculations at the same time, we make it usable by selecting only one, which is more easy, among the two. If we start by the first method, then we must calculate $[A]$, $[B]$, $[C]$, $[D]$ by use of the admittances obtained. If we start by the second, we must calculate the admittance matrices by use of Eq. (E 7) in order to draw the power circle diagrams. Of course, both methods yield the same results.

The results are shown by the next equations.

$$[\boldsymbol{S}_{11}] = \begin{pmatrix} 4.35\,\underline{|-88^\circ32'} & -0.108\,\underline{|-82^\circ50'} & -0.0455\,\underline{|-86^\circ06'} & -0.177\,\underline{|-81^\circ50'} \\ -0.108\,\underline{|-82^\circ50'} & 1.60\,\underline{|-85^\circ26'} & -0.104\,\underline{|-80^\circ50'} & -0.0585\,\underline{|-81^\circ35'} \\ -0.0455\,\underline{|-86^\circ06'} & -0.104\,\underline{|-80^\circ50'} & 0.869\,\underline{|-88^\circ29'} & -0.02\,\underline{|-81^\circ20'} \\ -0.177\,\underline{|-81^\circ50'} & -0.0585\,\underline{|-81^\circ35'} & -0.02\,\underline{|-81^\circ20'} & 2.82\,\underline{|-86^\circ} \end{pmatrix}$$

$$[\boldsymbol{S}_{22}] = \begin{pmatrix} -1.02\,\underline{|-88^\circ52'} & -0.139\,\underline{|-85^\circ40'} & -1.69\,\underline{|-90^\circ} & -0.115\,\underline{|-80^\circ30'} \\ -0.578\,\underline{|-86^\circ18'} & -0.233\,\underline{|-92^\circ40'} & -0.48\,\underline{|-83^\circ15'} & -0.044\,\underline{|-80^\circ10'} \\ -0.106\,\underline{|-77^\circ20'} & -0.036\,\underline{|-80^\circ40'} & -0.356\,\underline{|-95^\circ10'} & -0.008\,\underline{|-70^\circ50'} \\ -1.02\,\underline{|-87^\circ10'} & -0.084\,\underline{|-84^\circ10'} & -0.65\,\underline{|-79^\circ35'} & -0.832\,\underline{|-89^\circ} \end{pmatrix}$$

$$[\boldsymbol{S}_{21}] = \begin{pmatrix} 5.92\,\underline{|-85^\circ04'} & -1.04\,\underline{|-86^\circ40'} & -0.646\,\underline{|-71^\circ30'} & -0.956\,\underline{|-84^\circ} \\ -1.04\,\underline{|-86^\circ40'} & 1.83\,\underline{|-86^\circ22'} & -0.224\,\underline{|-83^\circ10'} & -0.081\,\underline{|-79^\circ50'} \\ -0.646\,\underline{|-71^\circ30'} & -0.224\,\underline{|-83^\circ10'} & 4.09\,\underline{|-84^\circ20'} & -0.101\,\underline{|-75^\circ05'} \\ -0.956\,\underline{|-84^\circ} & -0.081\,\underline{|-79^\circ50'} & -0.101\,\underline{|-75^\circ05'} & 2.13\,\underline{|-85^\circ45'} \end{pmatrix}$$

$$[\boldsymbol{S}_{21}] = [\boldsymbol{S}_{12}]^{*} \qquad \text{(E 8)}$$

where

$$\begin{bmatrix} [S_{11}] & [S_{12}] \\ [S_{21}] & [S_{22}] \end{bmatrix} = \begin{bmatrix} [I_S] \\ -[I_R] \end{bmatrix}' \begin{bmatrix} [E_S] \\ [E_R] \end{bmatrix}^{-1} \quad \cdots\cdots\cdots\cdots \quad \text{(E 9)}$$

As the total generating power of say, terminal 1, is generally represented by

$$\begin{aligned} P_{11}+jQ_{11} &= (P_{12}+jQ_{12})+(P_{13}+jQ_{13})+\cdots+(P_{1n}+jQ_{1n}) \\ &= \left\{(y_{112}+y_{12})\,E_1^2-y_{12}E_2E_{1k}\right\}+\left\{(y_{113}+y_{13})\,E_1^2-y_{13}E_3E_{1k}\right\}+ \\ &\quad \cdots\cdots\cdots\cdots \\ &\quad \left\{(y_{11n}+y_{1n})\,E_1^2-y_{1n}E_nE_{1k}\right\} \\ &= \left(\frac{D_{12}}{B_{12}}E_1^2-\frac{1}{B_{12}}E_2E_{1k}\right)+\left(\frac{D_{13}}{B_{13}}E_1^2-\frac{1}{B_{13}}E_3E_{1k}\right)+\cdots\cdots \\ &\quad \cdots\cdots+\left(\frac{D_{1n}}{B_{1n}}E_1^2-\frac{1}{B_{1n}}E_nE_{nk}\right). \quad \cdots\cdots \quad \text{(E 10)} \end{aligned}$$

the equation itself shows the power circle diagram of the terminal 1, which is a collection of many circle diagrams. As for the first term,

$$(P_{12}+jQ_{12}) = (y_{112}+y_{12})\,E_1^2-y_{12}E_2E_{1k} \quad \cdots\cdots\cdots\cdots \quad \text{(E 11)}$$

it shows the power which is transmitted from terminal 1 to terminal 2. The term $(y_{112}+y_{12})\,E_1^2$ shows the center point and $|y_{12}E_2E_{1k}|$ shows the radius of the circle respectively, which is a well-known sending-

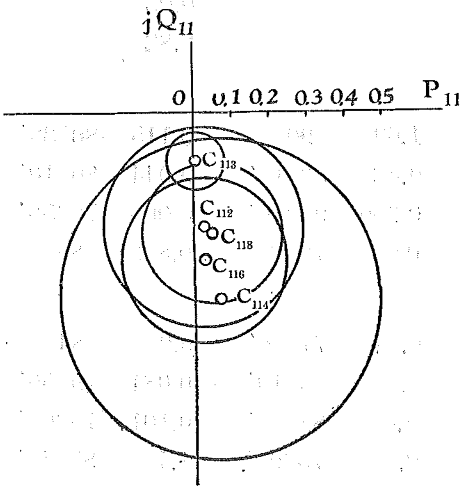


Fig. 6. Power circle diagrams between terminals on the co-ordinate of the sending-end 1, $P_{11}+jQ_{11}$. Circles, C_{15} and C_{17}, are excluded here because of the scale- out.

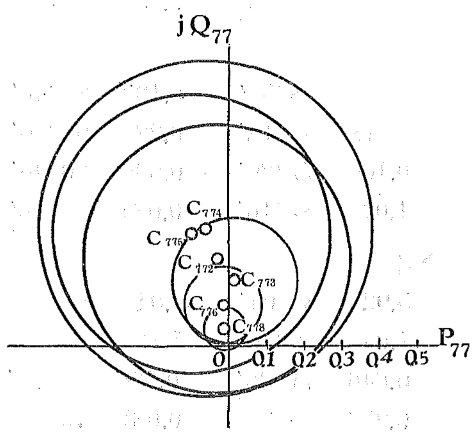


Fig. 7. Power circle diagrams between terminals on the co-ordinate of the receiving-end 7, $P_{77}+jQ_{77}$. One circle, C_{771}, is excluded here because of the scale-out.

end power circle diagram. The same procedures are carried through for many other terms in like manner and many circle diagrams are drawn. In our example, $y_{112}=y_{113}=\cdots=y_{11n}=0$ is taken. The results are shown in Fig. 6 and Fig. 7, for the Stations Ur. and Sp.

4. Mutual Relations of Admittances on Sending-ends and Receiving-ends of Interconnected Transmission Systems.

The ratio of a current to a voltage of each terminal means an admittance of it. Since some terminals have branch lines, the ratio of total currents to the voltage is to be named as the terminal admittance and the ratio of a current in a branch line to the voltage as the branch admittance.

Though the relations of these voltages and currents are precisely shown by Eqs. (5) and (6), some explanations shall be made because of the matrix equations. For an example, the voltage E_1 of the sending terminal 1 is expressed by the sum of terms of the first low of Eq. (5), that is, it is composed of terminal voltages and of total terminal currents at receiving-ends covered from the terminal $(r+1)$ to the terminal n and also of general circuit constants deserved. If it is analyzed to each voltage componentof each recieving-end terminal, as

$$\begin{aligned}\boldsymbol{E}_1 &= \boldsymbol{E}_{1(r+1)}+\boldsymbol{E}_{1(r+2)}+\cdots\cdots\boldsymbol{E}_{1n}\\ &=(\boldsymbol{A}_{1(r+1)}\boldsymbol{E}_{r+1}+\boldsymbol{B}_{1(r+1)}\boldsymbol{I}_{r+1})+(\boldsymbol{A}_{1(r+2)}\boldsymbol{E}_{r+2}+\boldsymbol{B}_{1(r+2)}\boldsymbol{I}_{r+2})+\cdots\\ &\quad\cdots(\boldsymbol{A}_{1n}\boldsymbol{E}_n+\boldsymbol{B}_{1n}\boldsymbol{I}_n)\quad\cdots\cdots\cdots\cdots\quad(5a)\end{aligned}$$

Similarly, the voltage $\boldsymbol{E}_{r+1}$ of the receiving terminal $(r+1)$ is expressed by sending-ends quantities from Eq. (6), as

$$\begin{aligned}\boldsymbol{E}_{r+1} &= \boldsymbol{E}_{(r+1)1}+\boldsymbol{E}_{(r+1)2}+\cdots\cdots\boldsymbol{E}_{(r+1)r}\\ &=(\boldsymbol{D}_{1(r+1)}\boldsymbol{E}_1-\boldsymbol{B}_{1(r+1)}\boldsymbol{I}_1)+(\boldsymbol{D}_{2(r+1)}\boldsymbol{E}_2-\boldsymbol{B}_{2(r+1)}\boldsymbol{I}_2)+\cdots\\ &\quad\cdots(\boldsymbol{D}_{r(r+1)}\boldsymbol{E}_r-\boldsymbol{B}_{r(r+1)}\boldsymbol{I}_r)\quad\cdots\quad(6a)\end{aligned}$$

Also, for terminal total currents,

$$\begin{aligned}\boldsymbol{I}_1 &= \boldsymbol{I}_{1(r+1)}+\boldsymbol{I}_{1(r+2)}+\cdots\cdots+\boldsymbol{I}_{1n}\\ &=(\boldsymbol{C}_{1(r+1)}\boldsymbol{E}_{r+1}+\boldsymbol{D}_{1(r+1)}\boldsymbol{I}_{r+1})+(\boldsymbol{C}_{1(r+2)}\boldsymbol{E}_{r+1}+\boldsymbol{D}_{1(r+1)}\boldsymbol{I}_{r+2})+\cdots\\ &\quad\cdots(\boldsymbol{C}_{1n}\boldsymbol{E}_n+\boldsymbol{D}_{1n}\boldsymbol{I}_n)\quad\cdots\cdots\cdots\cdots\quad(5b)\end{aligned}$$

$$\begin{aligned} I_{r+1} &= I_{(r+1)1} + I_{(r+1)2} + \cdots\cdots I_{(r+1)r} \\ &= (-C_{1(r+1)} E_1 + A_{1(r+1)} I_1) + (-C_{2(r+1)} E_2 + A_{2(r+1)} I_2) + \cdots \\ &\quad \cdots (-C_{r(r+1)} E_r + A_{r(r+1)} I_r) \cdots \end{aligned} \tag{6b}$$

Other voltages and currents, namely, $E_2, E_3, \cdots E_{r+2}, I_2, I_3, \cdots I_{r+2}$, etc., can be estimated in the same way from Eq. (5), (6).

First, relations between sending-ends admittances, $w_1 = \frac{I_1}{E_2}$, $w_2 = \frac{I_2}{E_2}$, $\cdots\cdots$ and receiving-ends admittances, $w_{r+1}, w_{r+2}, \cdots$ are clarified.

For an example, on the sending-end 1, from Eqs. (5a), (5b),

$$w_1 = \frac{I_1}{E_1} = \frac{(C_{1(r+1)} E_{r+1} + D_{1(r+1)} w_{r+1} E_{r+1}) \cdots + (C_{1n} E_n + B_{1n} w_n E_n)}{(A_{1(r+1)} E_{r+1} + B_{1(r+1)} w_{r+1} E_{r+1}) \cdots + (A_{1n} E_n + B_{1n} w_n E_n)} \tag{9a}$$

is derived, where $w_{r+1} = \frac{I_{r+1}}{E_{r+1}}$, $v \cdots\cdots w_n = \frac{I_n}{E_n}$. It is known from the above equation that in the case of an interconnected transmission system a certain admittance on sending-ends is affected by all the admittances on receiving-ends and also by relative values of the voltage on each receiving-end. As already stated, coefficients, A, B, C and D can not be determined unless the number of sending-ends and of receiving-ends is equal. Relations between the above equation and the power or the power factor are shown later.

If all sending-ends admittances are calculated by a matrix,

$$\frac{[I_S]}{[E_S]} = \begin{pmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & w_r \end{pmatrix} = \begin{pmatrix} \frac{I_1}{E_1} & 0 & \cdots & 0 \\ 0 & \frac{I_2}{E_2} & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \frac{I_r}{E_r} \end{pmatrix} \tag{9}$$

of which the elements are easily calculated from Eqs. (5), (6). Second, the admittance on receiving-ends is

$$\frac{[I_R]}{[E_R]} = \begin{pmatrix} w_{r+1} & 0 & \cdots & 0 \\ 0 & w_{r+2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & w_n \end{pmatrix} = \begin{pmatrix} \frac{I_{r+1}}{E_{r+1}} & 0 & \cdots & 0 \\ 0 & \frac{I_{r+2}}{E_{r+2}} & \ddots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \frac{I_n}{E_n} \end{pmatrix} \tag{10}$$

For an example, w_{r+1} is

$$w_{r+1}=\frac{I_{r+1}}{E_{r+1}}=\frac{(-C_{1(r+1)}E_1+A_{1(r+1)}w_1E_1)\cdots+(-C_{r(r+1)}E_r+A_{r(r+1)}w_rE_r)}{(D_{1(r+1)}E_1-B_{1(r+1)}w_1E_1)\cdots+(D_{r(r+1)}E_r-B_{r(r+1)}w_rE_r)} \qquad \text{(10a)}$$

These values of terminal admittances are practically obtained in connection with the power and the power factor of the terminal.

Third, the admittance w_{ij} for the branch current I_{ij} of terminal current I_i is taken into consideration.

$$\frac{[I_S]}{[E_S]}=\begin{pmatrix} w_{1(r+1)} & w_{1(r+2)} & \cdots\cdots \\ w_{2(r+1)} & w_{2(r+2)} & \cdots\cdots \\ \cdots\cdots & & \\ w_{r(r+1)} & w_{r(r+2)}\cdots\cdots & w_{rn} \end{pmatrix}=\begin{pmatrix} \frac{I_{1(r+1)}}{E_1} & \frac{I_{1(r+2)}}{E_1} & \cdots\cdots \\ \frac{I_{2(r+1)}}{E_2} & \frac{I_{2(r+2)}}{E_2} & \cdots\cdots \\ \cdots\cdots & & \\ \cdots\cdots & & \frac{I_{rn}}{E_n} \end{pmatrix}$$

$$w_{1(r+1)}=\frac{C_{1(r+1)}E_{r+1}+D_{1(r+1)}I_{r+1}}{(A_{1(r+1)}E_{r+1}+B_{1(r+1)}I_{r+1})+\cdots(A_{rn}E_n+B_{rn}I_n)}$$

$$\frac{[I_R]}{[E_R]}=\begin{pmatrix} w_{(r+1)1} & w_{(r+1)2} & \cdots\cdots \\ w_{(r+1)1} & w_{(r+2)2} & \cdots\cdots \\ \cdots\cdots & & \\ w_{n1} & w_{n2}\cdots\cdots & w_{nr} \end{pmatrix}=\begin{pmatrix} \frac{I_{(r+1)1}}{E_{r+1}} & \frac{I_{(r+1)2}}{E_{r+1}} & \cdots\cdots \\ \frac{I_{(r+2)1}}{E_{r+2}} & \cdots\cdots & \\ \cdots\cdots & & \\ \frac{I_{n1}}{E_n} & \cdots\cdots & \frac{I_{nr}}{E_n} \end{pmatrix}$$

$$w_{(r+1)1}=\frac{-C_{1(r+1)}E_1+A_{1(r+1)}I_1}{(D_{1(r+1)}E_1-B_{1(r+1)}I_1)\cdots+(D_{r(r+1)}E_r-B_{r(r+1)}I_r)} \qquad \text{(11)}$$

Since voltages in denominators are forms of diagonal matrices, calculations are easy. As the above equations show relations between terminal admittances and branch admittances, branch line currents in the equations are values which are obtained merely by a calculation and not by actual data of the system.

Besides, from branch currents and voltages,

$$\frac{[I_S]}{[E_S]}=\frac{\begin{pmatrix} I_{1\,r+1)} & I_{1(r+2)}\cdots\cdots & I_{1n} \\ I_{2(r+1)} & I_{2(r+2)}\cdots\cdots & \\ \cdots\cdots & & \end{pmatrix}}{\begin{pmatrix} E_{1(r+1)} & E_{1(r+2)}\cdots\cdots & E_{1n} \\ E_{2(r+1)} & E_{2(r+2)}\cdots\cdots & \\ \cdots\cdots & & \end{pmatrix}} \qquad \text{(12)}$$

is derived. From it, an admittance, such as, $\dfrac{I_{ij}}{E_{hk}} = w_{ijhk}$ is taken out, of which the treatment, however, is rather complicated as it is not of a diagonal form.

Example 3.

Terminal admittances are calculated by $w_i = I_i/E_i$ or $w_{ij} = I_{ij}/E_i$ as just described in Example 1). They are as follows as a result of calculations.

$$\begin{cases} w_{15} = 2.1 \;\lvert\underline{-52^\circ} \\ w_{17} = 0.815\lvert\underline{-55^\circ 37'} \\ w_1 = 2.9 \;\lvert\underline{-53^\circ} \end{cases} \qquad \begin{cases} w_{51} = 3.37 \;\lvert\underline{15^\circ 42'} \\ w_{54} = 2.22 \;\lvert\underline{32^\circ 53'} \\ w_{56} = 0.239\lvert\underline{49^\circ 33'} \\ w_5 = 5.74 \;\lvert\underline{23^\circ 35'} \end{cases}$$

$$\begin{cases} w_{23} = 0.254\lvert\underline{217^\circ 20'} \\ w_{26} = 0.61 \;\lvert\underline{-49^\circ 30'} \\ w_2 = 0.65 \;\lvert\underline{-72^\circ 42'} \end{cases} \qquad \begin{cases} w_{62} = 0.972\lvert\underline{-34^\circ 25'} \\ w_{65} = 0.261\lvert\underline{210^\circ 25'} \\ w_6 = 0.894\lvert\underline{-49^\circ 40'} \end{cases}$$

$$\begin{cases} w_{32} = 0.405\lvert\underline{38^\circ 58'} \\ w_{37} = 0.355\lvert\underline{207^\circ 13'} \\ w_3 = 0.092\lvert\underline{89^\circ 58'} \end{cases} \qquad \begin{cases} w_{71} = 1.37 \;\lvert\underline{-18^\circ} \\ w_{73} = 0.34 \;\lvert\underline{207^\circ 13'} \\ w_7 = 1.15 \;\lvert\underline{-29^\circ 57'} \end{cases}$$

$$\begin{cases} w_{45} = 1.64 \;\lvert\underline{-46^\circ 27'} \\ w_{48} = 0.22 \;\lvert\underline{-43^\circ 15'} \\ w_4 = 1.85 \;\lvert\underline{-46^\circ 05'} \end{cases} \qquad w_{84} = w_8 = 0.312\lvert\underline{4^\circ 43'}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots \quad \text{(E 12)}$$

Of course, interrelations such as Eq. (10a) or (11) are maintained among these admittances, under the steady operating phenomena given in the first.

5. Admittance Circle Diagrams and Power factor Circle Diagrams of Interconnected Transmission Systems.

Terminal admittances of an interconnected power system are shown by Eqs. (9a), (10a) as discussed above, however, the number of variables is considerably many. If one terminal in each two sides is taken as a variable and all other terminals as constants, the equations take the form $\dot{v} = (\dot{a} + \dot{b}p)/(\dot{c} + \dot{d}p)$ which are simple vector equations showing general circles.

As sending-end admittances are given by Eq. (7a), the relation between terminal 1 and terminal $(r+1)$, with the assumption that other receiving-end voltages are constants, is shown by the next equation with Eq. (7a) transferred.

$$w_1 = \frac{C' + D\, w_{r+1}}{A' + B\, w_{r+1}} \qquad \cdots\cdots (13)$$

where

$$\left.\begin{aligned} C' &= C_{1(r+1)} + C_{1(r+2)}\, e_{r12} \ \cdots\cdots + C_{1n} e_{r1n} \\ &\quad + D_{1(r+2)} w_{r+2} e_{r12} \ \cdots\cdots + D_{1n} e_{r1n} \end{aligned}\right\}$$

$$\left.\begin{aligned} A' &= A_{1(r+1)} + A_{1(r+2)}\, e_{r12} \ \cdots\cdots + A_{1n} e_{r1n} \\ &\quad + B_{1(r+2)} w_{r+2} e_{r12} \cdots\cdots + B_{1n} w_n e_{r1n}. \\ e_{r12} &= \frac{E_{r+2}}{E_{r+1}}, \ \cdots\cdots\cdots\cdots\ e_{r1n} = \frac{E_n}{E_{r+1}} \end{aligned}\right\} \qquad \cdots\cdots (13a)$$

e_{r1n} is a coefficient of a complex number grown by the difference of voltages among receiving-ends. w_1 thus obtained on terminal 1 has similar relations also with $w_{r+2}, w_{r+3} \cdots w_n$ of other receiving-ends, and moreover $w_2, w_3, \cdots$ on sending terminal 2, 3, $\cdots$ have similarity with $w_{r+1}, w_{r+2}, \cdots$ on receiving-ends.

The circle equation (11) can be treated as quite identical to a case of one terminal transmission line, if C' and A' are respectively assumed as constants. If the circle diagram is drawn in a rectangular admittance co-ordinate, $w_1 = G_1 + jB_1$, the variable becomes a complex number, as $w_{r+1} = G_{r+1} + jB_{r+1} = |w_{r+1}|\varepsilon^{j\theta_{r+1}}$. Therefore, it becomes an admittance circle diagram as a whole, but also becomes a power factor circle diagram if θ is constant, an apparent power circle diagram if $|w_{r+1}|$ constant, and an effective conductance or an effective susceptance circle diagram if G_{r+1} or B_{r+1} is constant. These diagrams are drawn like to cases of an ordinary transmission system line with one terminal on every two sides.

If a power factor circle diagram is developed it is shown by a group of circles, all passing through a short circuit point, $w_{r+1} = \infty$ and an open circuit point, $w_{r+1} = 0$, where

$$w_{1\infty} = \frac{D}{B} \qquad \cdots\cdots (13b)$$

$$w_{10} = \frac{C'}{A'} \qquad \cdots\cdots (13c)$$

The short circuit point defined in Eq. (13b) is quite unrelated to all other receiving-ends, not only in this case but when expressed on a conductance or a susceptance circle diagram too. However, the open circuit point has interrelations with other receiving-end admittances or voltages as shown byEq. (13a), which is one of the most distinctive characters in the case of an interconnected transmission system to be studied now. If an admittance unit is corrected to a power unit in the coordinate, the results are similar.

Centers of the circles are located on a perpendicular bisector of these two points, where the centers show power factors, and angles intersected by circles show the difference of power factors. Standard diagrams among these power factor circle diagrams are the 0. p. f. circle diagram and the 1. p. f. circle diagram of which the centers and radii are analyzed later as they are quite important.

Regarding the constants C' and A', derived up to now, it is an especially important fact that voltages, admittances and line constants of other receiving-ends are all contained in the constants as constants. There is, however, one more method for assuming constants, namely, as will be seen in Eq. (15a) derived below, $w_{r+1}=\dfrac{-C''+Aw_1}{D''-Bw_1}$ contains a sending-end admittance w_1 and a receiving-end admittance w_{r+1}, where C'', and D'' are assumed as constants which contain voltages and admittances and line constants now on sending-ends. From Eq. (15), however, the following equation is induced.

$$w_1=\frac{C''+D''w_{r+1}}{A+Bw_{r+1}} \qquad \cdots\cdots (14)$$

It is equal to Eq. (13) if the system is composed of lines everyone of which has one terminal. As it is in quite similar relation with Eq. (13), a procedure in detail is to be omitted here. C'' and D'' are derived from Eq. (15a). The short circuit and open circuit points are respectively

$$w_{1\infty}=\frac{D''}{B} \qquad \cdots\cdots (14a)$$

$$w_{10}=\frac{C''}{A} \qquad \cdots\cdots (14b)$$

in which, however, both the values are changeable according to loads, dissimilar to the preceding case.

Similarly, a receiving-end admittance has the following interrelation with a sending admittance.

$$w_{r+1}=\frac{-C''+Aw_1}{D''-Bw_1} \qquad \cdots\cdots (15)$$

where

$$C''=-C_{1(r+1)}-C_{2(r+1)}e_{S_{12}} \quad \cdots\cdots \quad -C_{r(r+1)}e_{S1r}$$
$$+A_{2(r+1)}w_2 e_{S_{12}} \quad \cdots\cdots \quad +A_{r(r+1)}w_r e_{S1r}\,.$$
$$D''=D_{1(r+1)}+D_{2(r+1)}e_{S_{12}} \quad \cdots\cdots \quad +D_{r(r+1)}e_{S_1r}$$
$$-B_{2(r+1)}w_2 e_{S_{12}} \cdots\cdots \quad -B_{r(r+1)}w_r e_{S1r}\,.$$
$$e_{S_{12}}=\frac{E_2}{E_1}\,, \quad \cdots\cdots, \quad e_{S1r}=\frac{E_r}{E_1}\,.$$

$$\cdots\cdots (15a)$$

If power circle diagrams are drawn on an admittance co-ordinate, $w_{r+1}=G_{r+1}+jB_{r+1}$, the circles are shown as a group passing upon the following through short circuit and open circuit points.

$$w_{(r+1)\infty}=-\frac{A}{B} \qquad \cdots\cdots (15b)$$

$$w_{(r+1)0}=-\frac{C''}{D''} \qquad \cdots\cdots (15c)$$

Though the short circuit point is a constant one, the open circuit point is changeable according to loads. The 0. p. f. circle diagram and 1. p. f. circle diagram which are standard diagrams here, are discussed later.

In stead of Eq. (15), the following equation can be used too, which is derived from Eq. (13).

$$w_{r+1}=\frac{-C'+A'w_1}{D-Bw_1} \qquad \cdots\cdots (16)$$

In this case, however, an assumption that C' and A' are constant, or in other words, voltages and powers of receiving-ends are constant is presumed, as directly seen in Eq. (13a). Like the case of Eq. (15), we get

$$w_{(r+1)\infty}=-\frac{A'}{B} \qquad \cdots\cdots (16a)$$

$$w_{(r+1)0}=-\frac{C'}{D} \qquad \cdots\cdots (16b)$$

Next, a branch admittance, $w_{1(r+1)}$ is taken for a branch line between the sending-end 1 to the receiving-end $(r+1)$, and the interrelation of w_{r+1} to a receiving-end admittance is examined.

$$w_{1(r+1)} = \frac{C+Dw_{r+1}}{A'+Bw_{r+1}} \qquad (17)$$

Power factor circle diagrams are shown by circles passing the following one each short circuit and open circuit points.

$$w_{1(r+1)\infty} = \frac{D}{B} \qquad (17a)$$

$$w_{1(r+1)0} = \frac{C}{A'} \qquad (17b)$$

Though the short circuit point is a constant one, the open circuit point varies with loads.

On the contrary, if a receiving-end admittance is shown from sending conditions as a function of one of sending-end admittance,

$$w_{(r+1)1} = \frac{-C+Aw_1}{D''-Bw_1} \qquad (18)$$

where, circles passing both as short circuit and an open circuit point

$$w_{(r+1)\infty} = -\frac{A}{B} \qquad (18a)$$

$$w_{(r+1)0} = -\frac{C}{D''} \qquad (18b)$$

are power factor circle diagrams. 0. p. f. and 1. p. f. circle diagrams are explained later.

While, the relations between sending or receiving admittances and branch admittances are shown also as follows, on the contrary to the above expression:

$$w_1 = \frac{C+D''w_{(r+1)1}}{A+Bw_{(r+1)1}} \qquad (19)$$

and

$$w_{r+1} = \frac{-C+A'w_{1(r+1)}}{D-Bw_{1(r+1)}} \qquad (20)$$

Short circuit and open circuit points are respectively

$$\left.\begin{aligned} w_{1\infty} &= \frac{D''}{B} \\ w_{10} &= \frac{C}{A} \end{aligned}\right\} \qquad (19a)$$

and

$$\left.\begin{aligned} w_{(r+1)\infty} &= -\frac{A'}{B} \\ w_{(r+1)0} &= -\frac{C}{D} \end{aligned}\right\} \quad \text{(20a)}$$

resulting in constant values of the open circuit points.

The correcton of coordinate units to the power $(P+jQ)$ will be made merely by multiplication of the square of every terminal voltage to the original admittances; such as $|E_1|^2$ for w_1 of Eq. (13), $|E_{r+1}|^2$ for w_{r+1} of Eq. (15), etc. On the other hand, the correction to the current units may be made by multiplication of voltages. If power output is expressed by matrices, transposed matrices are used such as $[P+jQ]=[E]_k[I]=[E]_k[[W][E]]^*$. In Eqs. (9) and (10), every term element is to be multiplied by the square of voltage if the power units are desired.

Example 4.

The examples of admittance circle diagram will be explained by reference to the Ur. Generating Station for the Sp. and the Su. Transforming stations. Terminal admittances are already shown in Example 3). Selecting every one terminal as a parameter and assuming other terminals as constants, circuit constants are obtained by Eqs. (13a) and (15a).

a.)

$$w_1 = \frac{C' + Dw_5}{A' + Bw_5}$$

Calculations show circuit constants as follows:

$$\left.\begin{aligned} A' &= 4.20 \ \underline{|-47°40'} \\ B &= 0.89 \ \underline{|\ 92°17'} \\ C' &= 16.13 \ \underline{|212°27'} \\ D &= 3.60 \ \underline{|\ 5°} \end{aligned}\right\} \quad \text{(E 13)}$$

Accordingly,

$$\left.\begin{aligned} w_{1\infty}E_1^2 &= \frac{D}{B}E_1^2 = 11.30 \ \underline{|-87°17'} \\ w_{10}E_1^2 &= \frac{C'}{A'}E_1^2 = 10.72 \ \underline{|-99°53'} \end{aligned}\right\} \quad \text{(E 14)}$$

b.)

$$w_1 = \frac{C' + D w_7}{A + B w_7}$$

Circuit constants are

$$\left.\begin{array}{l} A' = 1.73 \ \underline{|37°17'} \\ B = 0.0446 \ \underline{|-81°} \\ C' = 5.22 \ \underline{|-16°52'} \\ D = 0.283 \ \underline{|189°} \end{array}\right\} \quad \cdots\cdots\cdots\cdots\cdots (E\,15)$$

And

$$\left.\begin{array}{l} w_{1\infty} E_1^2 = \dfrac{D}{B} E_1^2 = 17.7 \ \underline{|-90°} \\ w_{10} E_1^2 = \dfrac{C'}{A'} E_1^2 = 8.42 \ \underline{|-54°09'} \end{array}\right\} \quad \cdots\cdots\cdots\cdots\cdots (E\,16)$$

c.)

$$w_5 = \frac{-C'' + A w_1}{D'' - B w_1}$$

where

$$\left.\begin{array}{l} A = 8.80 \ \underline{|5°48'} \\ B = 0.89 \ \underline{|92°17'} \\ C'' = 21.9 \ \underline{|-47°40'} \\ D'' = 2.46 \ \underline{|25°20'} \end{array}\right\} \quad \cdots\cdots\cdots\cdots\cdots (E\,17)$$

$$\left.\begin{array}{l} w_{5\infty} E_5^2 = -\dfrac{A}{B} E_5^2 = 10.7 \ \underline{|93°31'} \\ w_{50} E_5^2 = -\dfrac{C''}{D''} E_5^2 = 9.60 \ \underline{|107°} \end{array}\right\} \quad \cdots\cdots\cdots\cdots\cdots (E\,18)$$

d.)

$$w_7 = \frac{-C'' + A w_1}{D'' - B w_1}$$

where

$$\left.\begin{array}{l} A = 0.891 \ \underline{|189°} \\ B = 0.0446 \ \underline{|-81°} \\ C'' = 3.22 \ \underline{|131°30'} \\ D'' = 0.59 \ \underline{|-49°} \end{array}\right\} \quad \cdots\cdots\cdots\cdots\cdots (E\,19)$$

$$\left.\begin{aligned} w_{7\infty}E_7^2 &= 19.6 \angle 90^\circ \\ w_{70}E_7^2 &= 5.36 \angle 0^\circ 30' \end{aligned}\right\} \quad \cdots\cdots \quad (E20)$$

e.)

$$w_1 = \frac{C'' + D''w_5}{A + Bw_5}$$

Since this equation is derived from that of c.) by transforming inversely, the circuit constants are quite equal to Eq. (E 17).
And

$$\left.\begin{aligned} w_{1\infty}E_1^2 &= \frac{D''}{B}E_1^2 = 7.72 \angle -66^\circ 57' \\ w_{10}E_1^2 &= \frac{C''}{A}E_1^2 = 6.95 \angle -53^\circ 28' \end{aligned}\right\} \quad \cdots\cdots \quad (E\,21)$$

f.)

$$w_1 = \frac{C'' + D''w_7}{A + Bw_7}$$

The circuit constants are equal to Eq. (E19), for the same reason as e.

$$\left.\begin{aligned} w_{1\infty}E_1^2 &= \frac{D''}{B}E_1^2 = 36.9 \angle 32^\circ \\ w_{10}E_1^2 &= \frac{C''}{A}E_1^2 = 10.08 \angle -59^\circ 30' \end{aligned}\right\} \quad \cdots\cdots \quad (E\,22)$$

g.)

$$w_5 = \frac{-C' + A'w_1}{D - Bw_1}$$

This is transformed from a.), and circuit constants are shown by Eq. (E 13).

$$\left.\begin{aligned} w_{5\infty}E_5^2 &= -\frac{A'}{B}E_5^2 = 5.10 \angle 40^\circ 03' \\ w_{50}E_5^2 &= -\frac{C'}{D}E_5^2 = 4.85 \angle 27^\circ 27' \end{aligned}\right\} \quad \cdots\cdots \quad (E\,23)$$

h.)

$$w_7 = \frac{-C' + A'w_1}{D - Bw_1}$$

Circuit constants are shown by Eq. (E 15).

$$\left.\begin{aligned} w_{7\infty}E_7^2 &= -\frac{A'}{B}E_7^2 = 38.1 \underline{|-61°43'} \\ w_{70}E_7^2 &= -\frac{C'}{D}E_7^2 = 18.1 \underline{|-25°52'} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 24)}$$

Next, branch admittances are to be studied, for examples, only with the Sp. Station (receiving-end) and the Ur. Station (sending-end).

i.)

$$w_{17} = \frac{C+Dw_7}{A'+Bw_7} = \frac{I_{17}}{E_1}$$

$$\left.\begin{aligned} A' &= 1.73 \underline{|37°17'} \\ B &= 0.0446\underline{|-81°} \\ C &= 5.67 \underline{|99°} \\ D &= 0.283 \underline{|189°} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 25)}$$

$$\left.\begin{aligned} w_{17\infty}E_1^2 &= \frac{D}{B}E_1^2 = 17.7 \underline{|-90°} \\ w_{170}E_1^2 &= \frac{C}{A'}E_1^2 = 9.15 \underline{|61°43'} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 26)}$$

j.)

$$w_{71} = \frac{-C+Aw_1}{D''-Bw_1} = \frac{I_{71}}{E_7}$$

$$\left.\begin{aligned} A &= 0.891 \underline{|189°} \\ B &= 0.0446\underline{|-81°} \\ C &= 5.67 \underline{|99°} \\ D'' &= 0.59 \underline{|-49°} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 27)}$$

$$\left.\begin{aligned} w_{71\infty}E_7^2 &= -\frac{A}{B}E_7^2 = 19.6 \underline{|90°} \\ w_{710}E_7^2 &= -\frac{C}{D''}E_7^2 = 9.42\underline{|-32°} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 28)}$$

k.)

$$w_1 = \frac{C+D''w_{71}}{A+Bw_{71}}$$

This is transformed from. Circuit constants are shown by Eq. (E 27).

$$\left.\begin{aligned} w_{1\infty}E_1^2 &= \frac{D''}{B}E_1^2 = 36.9 \underline{|32°} \\ w_{10}E_1^2 &= \frac{C}{A}E_1^2 = 17.7 \underline{|-90°} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 29)}$$

1.)

$$w_7 = \frac{-C + A' w_{17}}{D - B w_{17}}$$

Circuit constants are given by Eq. (E 25).

$$\left.\begin{aligned} w_{7\infty} E_7^2 &= -\frac{A'}{B} E_7^2 = 38.1 \underline{|-61^\circ 43'} \\ w_{70} E_7^2 &= -\frac{C}{D} E_7^2 = 19.6 \underline{|\ 90^\circ} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (E\ 30)$$

From the previous data, admittance circle diagrams are drawn which pass through the short circuit and the open circuit points. Those points are shown on Fig. 8 to Fig. 19 together with fundamental circle diagrams shown below.

6. Fundamental Circuit Diagrams, Namely, O. p. f. and fl. p. f. Circle Diagrams of Interconnected Transmission Systems.

These are two circles intersecting at right angle each other and are standard circles on which many sorts of circle diagrams are based. These are similar to the circle in a case of an ordinary transmission line with one terminal on each of the two ends. Namely, points of intersection of the two circles are one each short circuit and open circuit point, which are respectively a center of power circle diagrams and a center of current circle diagrams. The centers of the two circles are the maximum effective power points. Furthermore, these two circles are the axes which are the projected axes of a sending or a receiving rectangular coordinate by the method of equiangular projection.

If W_{11}, the power of the sending-end 1, is shown by receiving-end conditions with an admittance of the terminal $(r+1)$ to be a parameter assuming all other receiving-end admittances as constants,

$$\begin{aligned} W_{11} = P_{11} + jQ_{11} = w_1 |E_1|^2 &= \frac{C' + D w_{r+1}}{A' + B w_{r+1}} |E_1|^2 \\ &= \frac{C' + D(G_{r+1} + jB_{r+1})}{A' + B(G_{r+1} + jB_{r+1})} |E_1|^2 \quad \cdots\cdots\cdots\cdots (21) \end{aligned}$$

If power factor is zero here, $G_{r+1} = 0$ is put used.

$$W_{11} = \frac{C' + jDB_{r+1}}{A' + jBB_{r+1}} |E_1|^2$$

Therefore, when B_{r+1} is changed from 0 to ∞, a locus thus obtained indicates the 0. p. f. circle diagram. Its center is derived by substituting a conjugated value of $\boldsymbol{B}_{r+1}=-\boldsymbol{A}'/j\boldsymbol{B}$ which makes $\boldsymbol{W}_{11}=\infty$, that is $(B_{r+1})_k=-jA'/B$, into the above equation, resulting in

$$\boldsymbol{C}_{p11}=\frac{\boldsymbol{C}'\boldsymbol{B}_k+\boldsymbol{D}\boldsymbol{A}'_k}{\boldsymbol{A}'\boldsymbol{B}_k+\boldsymbol{A}'_k\boldsymbol{B}}|\boldsymbol{E}_1|^2 \quad \cdots\cdots (22)$$

Its radius is derived by the difference of the length from a center point to an open circuit point, that is by $\left|\boldsymbol{C}_{p11}-\frac{\boldsymbol{C}'}{\boldsymbol{A}'}|\boldsymbol{E}_1|^2\right|$,

$$R_{p11}=\left|\frac{\boldsymbol{A}'_k('\boldsymbol{A}\boldsymbol{D}-\boldsymbol{B}\boldsymbol{C}')}{\boldsymbol{A}'(\boldsymbol{A}'\boldsymbol{B}_k+\boldsymbol{A}'_k\boldsymbol{B}}E_1^2\right| \quad \cdots\cdots (22a)$$

Since it can be precisely verified that

$$\left|\frac{M}{\overline{M}}\cdot\frac{X+jY}{Z+jU}\right|\equiv\left|\frac{X+jY}{Z+jU}\right|$$

where $M=x+jy$ and x, y, X, Y, Z and U are respectivery real numbers, therefore Eq. (22a) is equal to

$$R_{p11}=\left|\frac{\boldsymbol{A}'\boldsymbol{D}-\boldsymbol{B}\boldsymbol{C}'}{\boldsymbol{A}'\boldsymbol{B}_k+\boldsymbol{A}'_k\boldsymbol{B}}E_1^2\right| \quad \cdots\cdots (22b)$$

Since $\boldsymbol{A}'\boldsymbol{D}-\boldsymbol{B}\boldsymbol{C}'\neq 1$ on such an interconnected transmission system, calculations become somewhat complicated compared to that of an ordinary line with one terminal on every end.

Nextly, a center and a radius of the 1. p. f. circle diagram is derived from Eq. (21) putting $jB_{r+1}=0$, as follows:

$$\left.\begin{aligned}\boldsymbol{C}_{q11}&=\frac{\boldsymbol{C}'\boldsymbol{B}_k-\boldsymbol{D}\boldsymbol{A}'_k}{\boldsymbol{A}'\boldsymbol{B}_k-\boldsymbol{A}'_k\boldsymbol{B}}E_1^2\\ R_{q11}&=\left|\frac{-\boldsymbol{A}'\boldsymbol{D}+\boldsymbol{B}\boldsymbol{C}'}{\boldsymbol{A}'\boldsymbol{B}_k-\boldsymbol{A}'_k\boldsymbol{B}}E_1^2\right|\end{aligned}\right\} \quad \cdots\cdots (23)$$

These fundamental circles, C_{p11}, and C_{q11}, are the projection of rectangular coordinate axes of $W_{r+1}=w_{r+1}|E_{r+1}|^2$, the power of the receiving-end $(r+1)$ shown on the coordinate W_{11}.

On the other hand, we can obtain a different expression where voltages, powers, and line constants on sending-ends are assumed to be constant, that is, from Eq. (14),

$$\boldsymbol{W}_{11}=\boldsymbol{w}_1|\boldsymbol{E}_1|^2=\frac{\boldsymbol{C}''+\boldsymbol{D}''\boldsymbol{w}_{r+1}}{\boldsymbol{A}+\boldsymbol{B}\boldsymbol{w}_{r+1}}|\boldsymbol{E}_1|^2 \quad \cdots\cdots (24)$$

Then in this case,

$$\left.\begin{aligned} C_{p11} &= \frac{C''B_k + D''A_k}{AB_k + A_kB} |E_1|^2 \\ R_{p11} &= \left| \frac{AD'' - BC''}{AB_k + A_kB} E_1^2 \right| \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (25)$$

$$\left.\begin{aligned} C_{q11} &= \frac{C''B_k - D''A_k}{AB_k - A_kB} E_1^2 \\ R_{q11} &= \left| \frac{-AD'' + BC''}{AB_k - A_kB} E_1^2 \right| \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (26)$$

where constants in equations include conditions of all sending-end terminals.

Next, if coordinate axes of the sending-end 1 are projected into the coordinate of the receiving-end $(r+1)$, fundamental circle diagrams on the sending-end 1 are obtained by Eq. (15).

$$\begin{aligned} W_{(r+1)(r+1)} &= P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)} = w_{r+1} |E_{r+1}|^2 \\ &= \frac{-C'' + A(G_1 + jB_1)}{D'' - B(G_1 + jB_1)} |E_{r+1}|^2 \end{aligned} \quad \cdots\cdots\cdots\cdots (27)$$

If $G_1 = 0$, the center and radius of 0. p. f. circle diagram are derived as follows:

$$\left.\begin{aligned} C_{(r+1)(r+1)} &= -\frac{C''B_k + AD_k''}{D''B_k + D_k''B} E_{r+1}^2 \\ R_{p(r+1)(r+1)} &= \left| \frac{-AD'' + BC''}{D''B_k + D_k''B} E_{r+1}^2 \right| \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (28)$$

And if $jB_{r+1} = 0$, those of 1. p. f. circle diagram are derived:

$$\left.\begin{aligned} C_{q(r+1)(r+1)} &= \frac{C''B_k - AD_k''}{-D''B_k + D_k'B} E_{r+1}^2 \\ R_{q(r+1)(r+1)} &= \left| \frac{-AD'' + BC''}{-D''B_k + D_k'B} E_{r+1}^2 \right| \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (29)$$

Instead of Eq. (27), if Eq. (16) is used where voltages and powers on receiving-ends are assumed constant,

$$W_{(r+1)(r+1)} = \frac{-C' + A'(G_1 + jB_1)}{D - B(G_1 + jB_1)} E_{r+1}^2 \quad \cdots\cdots\cdots\cdots (30)$$

Centers and radii of 0. p. f. and 1. p. f. circle diagrams are respectively

$$\left.\begin{aligned} C_{p(r+1)(r+1)} &= -\frac{C'B_k+A'D_k}{DB_k+D_kB}E_{r+1}^2 \\ R_{p(r+1)(r+1)} &= \left|\frac{-A'D+BC'}{DB_k+D_kB}E_{r+1}^2\right| \end{aligned}\right\} \qquad \cdots\cdots (31)$$

and

$$\left.\begin{aligned} C_{q(r+1)(r+1)} &= \frac{C'B_k-A'D_k}{-DB_k+D_kB}E_{r+1}^2 \\ R_{q(r+1)(r+1)} &= \left|\frac{-A'D+BC'}{-DB_k+D_kB}E_{r+1}^2\right| \end{aligned}\right\} \qquad \cdots\cdots (32)$$

In the next, for sending branch capacity, the following equations are derived from Eq. (17).

$$W_{1(r+1)} = P_{1(r+1)} + jQ_{1(r+1)} = \frac{C+Dw_{r+1}}{A'+Bw_{r+1}}E_1^2 \qquad \cdots\cdots (33)$$

$$\left.\begin{aligned} C_{p1(r+1)} &= \frac{CB_k+DA'_k}{A'B_k+A'_kB}E_1^2 \\ R_{p(r+1)} &= \left|\frac{A'D-BC}{A'B_k+A'_kB}E_1^2\right| \end{aligned}\right\} \qquad \cdots\cdots (34)$$

$$\left.\begin{aligned} C_{q1(r+1)} &= \frac{CB_k-DA'_k}{A'B_k-A'_kB}E_1^2 \\ R_{q1(r+1)} &= \left|\frac{-A'D+BC}{A'B_k-A'_kB}E_1^2\right| \end{aligned}\right\} \qquad \cdots\cdots (35)$$

Receiving branch power is shown from Eq. (18) as follows:

$$W_{(r+1)1} = P_{(r+1)1} + jQ_{(r+1)1} = \frac{-C+A(G_1+jB_1)}{D''-B(G_1+jB_1)}E_{r+1}^2 \qquad \cdots\cdots (36)$$

$$\left.\begin{aligned} C_{p(r+1)1} &= -\frac{CB_k+AD''_k}{D''B_k+D''_kB}E_{r+1}^2 \\ R_{p(r+1)1} &= \left|\frac{-AD''+BC}{D''B_k+D''_kB}E_{r+1}^2\right| \end{aligned}\right\} \qquad \cdots\cdots (37)$$

$$\left.\begin{aligned} C_{q(r+1)1} &= \frac{CB_k-AD''_k}{-D''B_k+D''_kB}E_{r+1}^2 \\ R_{q(r+1)1} &= \left|\frac{-D''A+BC}{-D''B_k+D''_kB}E_{r+1}^2\right| \end{aligned}\right\} \qquad \cdots\cdots (38)$$

On the contrary if the terminal capacity is expressed by the branch capacity, from Eqs. (19), (20),

$$W_1 = \frac{C + D'' w_{(r+1)1}}{A + B w_{(r+1)1}} E_1'^2 \quad \cdots\cdots (39)$$

$$W_{r+1} = \frac{-C + A' w_{1(r+1)}}{D - B w_{1(r+1)}} E_{r+1}^2 \quad \cdots\cdots (40)$$

The 0. p. f. circle diagram is defined as

$$\left.\begin{aligned} C_{p11} &= \frac{CB_k + D''A_k}{AB_k + A_kB} E_1^2 \\ R_{p11} &= \left| \frac{AD'' - BC}{AB_k + A_kB} E_1'^2 \right| \end{aligned}\right\} \quad \cdots\cdots (41)$$

$$\left.\begin{aligned} C_{p(r+1)(r+1)} &= -\frac{CB_k + A'D_k}{DB_k + D_kB} E_{r+1}^2 \\ R_{p(r+1)(r+1)} &= \left| \frac{-A'D + BC}{DB_k + D_kB} E_{r+1}^2 \right| \end{aligned}\right\} \quad \cdots\cdots (42)$$

The 1. p. f. circle diagrams are also

$$\left.\begin{aligned} C_{q11} &= \frac{CB_k - D''A_k}{AB_k - A_kB} E_1^2 \\ R_{q11} &= \left| \frac{-AD'' + BC}{AB_k - A_kB} E_1^2 \right| \end{aligned}\right\} \quad \cdots\cdots (43)$$

$$\left.\begin{aligned} C_{q(r+1)(r+1)} &= \frac{CB_k - A'D_k}{-DB_k + D_kB} E_{r+1}^2 \\ R_{q(r+1)(r+1)} &= \left| \frac{-DA' + BC}{-DB_k + D_kB} E_{r+1}^2 \right| \end{aligned}\right\} \quad \cdots\cdots (44)$$

Example 5.

Fundamental circle diagrams will be calculated in this Example by the data given under Example 4). The heading letter in the following corresponds in each case to that of Example 4).

a.)

$$W_{11} = \frac{C' + Dw_5}{A' + Bw_5} E_1^2 .$$

Circuit constants are shown in a). Example 4), that is by Eq. (E 13). The 0. p. f. circle diagram is given by

$$C_{p11} = \frac{C'B_k + DA'_k}{A'B_k + A'_kB} E_1^2 = -0.942 - j\,11.9 \quad \cdots\cdots (E\,31)$$

$$R_{p11} = \left| \frac{A'D - BC'}{A'B_k + A'_kB} E_1^2 \right| = 1.63$$

The 1. p. f. circle diagram is by

$$\left.\begin{aligned} C_{q11} &= \frac{C'B_k - DA'_k}{A'B_k - AB_{kB}} E_1^2 = -0.237 - j\,9.48 \\ R_{q11} &= \left| \frac{-A'D + BC'}{A'B_k - A'_kB} E_1^2 \right| = 1.93 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots \text{(E 32)}$$

b.)

$$W_{11} = \frac{C' + Dw_7}{A' + Bw_7} E_1^2$$

Circuit constants are shown under b.) Example 4), by Eq. (E 15). These comments are same.

The 0. p. f. circle diagram is:

$$\left.\begin{aligned} C_{p11} &= \frac{C'B_k - DA'_k}{A'B_k + A'_kB} E_1^2 = 12.6 - j16.9 \\ R_{p55} &= \left| \frac{A'D - BC'}{A'B_k + A'_kB} E_1^2 \right| = 12.6 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots \text{(E 33)}$$

The 1. p. f. circle diagram is:

$$\left.\begin{aligned} C_{q11} &= \frac{C'B_k - DA'_k}{A'B_k - A'_kB} E_1^2 = -0.451 - j\,10.95 \\ R_{q55} &= \left| \frac{-A'D + BC'}{A'B_k - A'_kB} E_1^2 \right| = 6.76 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots \text{(E 34)}$$

c.)

$$W_{55} = \frac{-C'' + Aw_1}{D'' - Bw_1} E_5^2$$

The 0. p. f. circle diagram:

$$\left.\begin{aligned} C_{p55} &= -\frac{C''B_k + AD''_k}{D''B_k + D''_kB} E_5^2 = -3.57 + j\,12.5 \\ R_{p55} &= \left| \frac{-AD'' + BC''}{D''B_k + D''_kB} E_5^2 \right| = 3.37 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots \text{(E 35)}$$

The 1. p. f. circle diagram:

$$C_{q55} = \left| \frac{C''B_k + AD''_k}{-D''B_k + D''_kB} \right| E_5^2 = -1.41 + j\,9.5 \quad \Big\} \qquad \cdots\cdots\cdots\cdots \text{(E 36)}$$

$$R_{q55} = \left| \frac{-AD''+BC''}{-D''B_k+D_k''B} E_5^2 \right| = 1.43$$

d.)

$$W_{77} = \frac{-C''+Aw_1}{D''-Bw_1} E_7^2$$

The 0. p. f. circle diagram:

$$\left. \begin{aligned} C_{p77} &= -\frac{C''B_k+AD_k''}{D''B_k+D_k''B} E_7^2 = 8.78+j\,11.5 \\ R_{p77} &= \left| \frac{-AD''+BC''}{D''B_k+D_k''B} E_7^2 \right| = 11.9 \end{aligned} \right\} \quad \cdots\cdots\cdots\cdots \text{(E 37)}$$

The 1. p. f. circle diagram:

$$\left. \begin{aligned} C_{q77} &= \frac{C''B_k-AD_k''}{-D''B_k+D_k''B} E_7^2 = -12.9+j\,5.50 \\ R_{q77} &= \left| \frac{-AD''+BC''}{-D''B_k+D_k''B} E_7^2 \right| = 19.1 \end{aligned} \right\} \quad \cdots\cdots\cdots\cdots \text{(E 38)}$$

e.)

$$W_{11} = \frac{C''+D''w_5}{A+Bw_5} E_1^2$$

The 0. p. f. circle diagram:

$$\left. \begin{aligned} C_{p11} &= \frac{C''B_k+D''A_k}{AB_k+A_kB} E_1^2 = 15.95-j\,15.5 \\ R_{p11} &= \left| \frac{AD''-BC''}{AB_k+A_kB} E_1^2 \right| = 15.3 \end{aligned} \right\} \quad \cdots\cdots\cdots\cdots \text{(E 39)}$$

The 1. p. f. circle diagram:

$$\left. \begin{aligned} C_{q11} &= \frac{C''B_k-D''A_k}{AB_k-A_kB} E_1^2 = 3.53-j\,6.30 \\ R_{q11} &= \left| \frac{-AD''+BC''}{AB_k-A_kB} E_1^2 \right| = 0.94 \end{aligned} \right\} \quad \cdots\cdots\cdots\cdots \text{(E 40)}$$

f.)

$$W_{11} = \frac{C''+D''w_7}{A+Bw_7} E_1^2$$

The 0. p. f. circle diagram:

$$\left.\begin{aligned} C_{p11} &= \frac{C''B_k + D''A_k}{AB_k + A_kB} E_1^2 = \infty \\ R_{p11} &= \left| \frac{AD'' - BC''}{AB_k + A_kB} E_1^2 \right| = \infty \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 41)}$$

The 1. p. f. circle diagram:

$$\left.\begin{aligned} C_{q11} &= \frac{C''B_k - D''A_k}{AB_k - A_kB} E_1^2 = 18.4 + j\,5.51 \\ R_{q11} &= \left| \frac{-AD'' + BC''}{AB_k - A_kB} E_1^2 \right| = 19.1 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 42)}$$

g.)

$$W_{55} = \frac{-C' + A'w_1}{D - Bw_1} E_5^2$$

The 0. p. f. circle diagram:

$$\left.\begin{aligned} C_{p55} &= -\frac{C'B_k + A'D_k}{DB_k + D_kB} E_5^2 = -6.90 - j\,1.42 \\ R_{p55} &= \left| \frac{-A'D + BC'}{DB_k + D_kB} E_5^2 \right| = 11.9 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 43)}$$

The 1. p. f. circle diagram:

$$\left.\begin{aligned} C_{q55} &= \frac{C'B_k - A'D_k}{-DB_k + D_kB} E_5^2 = 4.13 + j\,2.76 \\ R_{q55} &= \left| \frac{-A'D + BC'}{-DB_k + D_kB} E_5^2 \right| = 0.564 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 44)}$$

h.)

$$W_{77} = \left| \frac{-C' + A'w_1}{D - Bw_1} E_7^2 \right.$$

The 0. p. f. circle diagram:

$$\left.\begin{aligned} C_{p77} &= -\frac{C'B_k + A'D_k}{DB_k + D_kB} E_7^2 = \infty \\ R_{p77} &= \left| \frac{-A'D + BC'}{DB_k + D_kB} E_7^2 \right| = \infty \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 45)}$$

The 1. p. f. circle diagram:

$$\left.\begin{aligned} C_{q77} &= \frac{C'B_k - A'D_k}{-DB_k + D_kB} E_7^2 = 17.2 - j\,20.8 \\ R_{q77} &= \left| \frac{-A'D + BC'}{-DB_k + D_kB} E_7^2 \right| = 12.9 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots \text{(E 46)}$$

i.)

Next, branch power is handled the same as in i.) Example 4).

$$W_{17} = \frac{C + Dw_7}{A' + Bw_7} E_1^2$$

The 0. p. f. circle diagram:

$$\left.\begin{aligned} C_{p17} &= \frac{CB_k + DA'_k}{A'B_k + A'_kB} E_1^2 = 26.1 - j\,8.86 \\ R_{p17} &= \left| \frac{A'D - BC}{A'B_k + A'_kB} E_1^2 \right| = 27.6 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots \text{(E 47)}$$

The 1. p. f. circle diagram:

$$\left.\begin{aligned} C_{q17} &= \frac{CB_k - DA'_k}{A'B_k - A'_kB} E_1^2 = -4.76 - j\,3.65 \\ R_{q17} &= \left| \frac{-A'D + BC'}{A'B_k - A'_kB} E_1^2 \right| = 14.8 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots \text{(E 48)}$$

j.)

$$W_{71} = \frac{-C + Aw_1}{D'' - Bw_1} E_7^2$$

The 0. p. f. circle diagram:

$$\left.\begin{aligned} C_{p71} &= -\frac{CB_k + AD''_k}{D''B_k + D''_kB} E_7^2 = 11.7 + j\,9.81 \\ R_{p71} &= \left| \frac{-AD'' + BC}{D''B_k + D''_kB} E_7^2 \right| = 15.2 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots \text{(E 49)}$$

The 1. p. f. circle diagram:

$$\left.\begin{aligned} C_{q71} &= \frac{CB_k - AD''_k}{-D''B_k + D''_kB} E_7^2 = -15.7 + j\,0.88 \\ R_{q71} &= \left| \frac{-D''A + BC}{-D''B_k + D''_kB} E_7^2 \right| = 24.7 \end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots \text{(E 50)}$$

k.)

$$W_{11}=\frac{C+D''w_{71}}{A+Bw_{71}}E_1^2$$

The 0. p. f. circle diagram :

$$\left.\begin{aligned} C_{p11}&=\frac{CB_k+D''A_k}{AB_k+A_kB}E_1^2=\infty \\ R_{p11}&=\left|\frac{AD''-BC}{AB_k+A_kB}E_1^2\right|=\infty \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 51)}$$

The 1. p. f. circle diagram :

$$\left.\begin{aligned} C_{q11}&=\frac{CB_k-D''A_k}{AB_k-A_kB}E_1^2=15.7+j\,0.88 \\ R_{q11}&=\left|\frac{-AD''+BC}{AB_k-A_kB}E_1^2\right|=24.7 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 52)}$$

l.)

$$W_{77}=\frac{-C'+A'w_{17}}{D-Bw_{17}}E_7^2$$

The 0. p. f. circle diagram :

$$\left.\begin{aligned} C_{p77}&=-\frac{CB_k+A'D_k}{DB_k+D_kB}E_7^2=\infty \\ R_{p77}&=\left|\frac{-A'D+BC}{DB_k+D_kB}E_7^2\right|=\infty \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 53)}$$

The 1. p. f. circle diagram :

$$\left.\begin{aligned} C_{q77}&=\frac{CB_k-A'D_k}{-DB_k+D_kB}E_7^2=9.04-j\,6.93 \\ R_{q77}&=\left|\frac{-DA'+BC}{-DB_k+D_kB}E_7^2\right|=28.1 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 54)}$$

Regarding branch power obtained now, if some explanations are made circles i) are expressed on the branch coordinate, $P_{17}+jQ_{17}$; circles j) are on $P_{71}+jQ_{71}$; circles k) are concerned with branch admittance w_{71} expressed on $P_{11}+jQ_{11}$; circles l) are concerned with w_{17} on $P_{77}+jQ_{77}$. These fundamental circle diagrams are shown in Fig. 8 to Fig. 19 together with short circuit points, open circuit points, and with effective power circle diagrams and others.

7. Effective Power Circle Diagrams of Interconnected Transmission Systems.

First, the effective power on a sending-end will be expressed by quantities on receiving-ends. The effective power on terminal $\boldsymbol{I}$ is

$$2P_{11} = \boldsymbol{E}_{1k}\boldsymbol{I}_1 + \boldsymbol{E}_1\boldsymbol{I}_{1k}$$

where $\boldsymbol{E}_1$ and $\boldsymbol{I}_1$ are given by Eqs. (5a) and (5b). As is clearly seen from Eqs. (5a) and (5b), $\boldsymbol{E}_1$ or $\boldsymbol{I}_1$ varies with the load voltages of many receiving-ends. Therfore, it is assumed that only a load on the receiving-end $(r+1)$ varies with other load voltages remaining constant, since the consideration of every voltage at any given time is obviously impossible.

Then, this effective power can be shown in the power coordinates by methods similar to power factor circle diagrams previously discussed.

$$\begin{aligned}2P_{11} = &(\boldsymbol{A}'_k\boldsymbol{E}_{(r+1)k} + \boldsymbol{B}_k\boldsymbol{I}_{(r+1)k})(\boldsymbol{C}'\boldsymbol{E}_{r+1} + \boldsymbol{D}\boldsymbol{I}_{r+1}) \\ &+ (\boldsymbol{A}'\boldsymbol{E}_{r+1} + \boldsymbol{B}\boldsymbol{I}_{r+1})(\boldsymbol{C}'_k\boldsymbol{E}_{(r+1)k} + \boldsymbol{D}_k\boldsymbol{I}_{(r+1)k}) \quad \cdots\cdots\cdots (45)\end{aligned}$$

where A' and C' are constants shown in Eq. (13a). If we put $\boldsymbol{E}_{(r+1)k}\boldsymbol{I}_{(r+1)} = P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)}$, $\boldsymbol{I}_{(r+1)}\boldsymbol{I}_{(r+1)k} = (P^2_{(r+1)(r+1)} + Q^2_{(r+1)(r+1)})/|E_{r+1}|^2$, $\boldsymbol{E}_{r+1}\boldsymbol{E}_{(r+1)k} = |\boldsymbol{E}_{r+1}|^2$ substituted into Eq. (45), then

$$\begin{aligned}2P_{11} = &(\boldsymbol{A}'_k\boldsymbol{D} + \boldsymbol{A}'\boldsymbol{D}_k + \boldsymbol{B}'_k\boldsymbol{C}' + \boldsymbol{B}\boldsymbol{C}'_k)P_{(r+1)(r+1)} \\ &+ (\boldsymbol{A}'_k\boldsymbol{D} - \boldsymbol{A}'\boldsymbol{D}_k - \boldsymbol{B}_k\boldsymbol{C}' + \boldsymbol{B}\boldsymbol{C}'_k)jQ_{(r+1)(r+1)} \\ &+ \frac{P^2_{(r+1)(r+1)} + Q^2_{(r+1)(r+1)}}{|E_{r+1}|^2}(\boldsymbol{B}_k\boldsymbol{D} + \boldsymbol{B}\boldsymbol{D}_k) + (\boldsymbol{A}'_k\boldsymbol{C}' - \boldsymbol{A}'\boldsymbol{C}'_k)|E_{r+1}|^2.\end{aligned}$$

If it is defined

$$R_{pr} = \left|\frac{E^2_{r+1}}{\boldsymbol{B}_k\boldsymbol{D} + \boldsymbol{B}\boldsymbol{D}_k}\right| \quad \cdots\cdots\cdots\cdots\cdots\cdots (46)$$

$$\left.\begin{aligned} R_{p(r+1)(r+1)} &= \left|\frac{A'D - BC'}{B_kD + BD_k}E^2_{r+1}\right| \\ \boldsymbol{C}_{p(r+1)(r+1)} &= -\frac{\boldsymbol{C}'\boldsymbol{B}_k + \boldsymbol{A}'\boldsymbol{D}_k}{\boldsymbol{B}_k\boldsymbol{D} + \boldsymbol{B}\boldsymbol{D}_k}|\boldsymbol{E}_{r+1}|^2 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (31)$$

it then follows that

$$\begin{aligned}2P_{11}R_{pr} + R^2_{p(r+1)(r+1)} = &P^2_{(r+1)(r+1)} + Q^2_{(r+1)(r+1)} \\ &+ P_{(r+1)(r+1)}(\boldsymbol{A}'_k\boldsymbol{D} + \boldsymbol{A}'\boldsymbol{D}_k + \boldsymbol{B}_k\boldsymbol{C}' + \boldsymbol{B}\boldsymbol{C}'_k)R_{pr} \\ &+ jQ_{(r+1)(r+1)}(\boldsymbol{A}'_k\boldsymbol{D} - \boldsymbol{A}'\boldsymbol{D}_k - \boldsymbol{B}_k\boldsymbol{C}' + \boldsymbol{B}\boldsymbol{C}'_k)R_{pr}\end{aligned}$$

$$= \left\{ \frac{C'B_k + A'D_k}{B_kD + BD_k} |E_{r+1}|^2 + P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)} \right\}$$

$$\times \left\{ \frac{C'_kB + A'_kD}{BD_k + B_kD} |E_{r+1}|^2 + P_{(r+1)(r+1)} - jQ_{(r+1)(r+1)} \right\}$$

If scalars are converted to vectors,

$$r_{pr}\varepsilon^{j\phi_r} = -C_{p(r+1)(r+1)} + (P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)}) \quad \cdots\cdots\cdots\cdots\cdots (47)$$

Eq. (47) is the effective power circle diagram drawn on a coordinate of the receiving-end $(r+1)$, where $C_{p(r+1)(r+1)}$ is a definite point obtained as before and $|r_{pr}|$ is changeable only by an angle ϕ_r under the premise of the effective power P_{11} being constant.

Radius $|r_{pr}|$ is

$$|r_{pr}| = \sqrt{2P_{11}R_{pr} + R^2_{p(r+1)(r+1)}} \quad \cdots\cdots\cdots\cdots\cdots (48a)$$

Then, the effective power P_{11} becomes

$$P_{11} = \frac{(r_{pr} + R_{p(r+1)(r+1)})(r_{pr} - R_{p(r+1)(r+1)})}{2R_{pr}} \quad \cdots\cdots\cdots\cdots\cdots (48)$$

When $r_{pr} = R_{p(r+1)(r+)}$, then $P_{11}=0$, that is to say, the sending power becomes zero, and the circle coincides to the 0. p. f. circle diagram discussed in Eq. (31). In the area outside the 0. p. f. circle, P_{11} is positive, and in the inside P_{11} is negative. It is a maximum value when $r_{pr}=0$, when the value is

$$P_{11\max} = \frac{R^2_{p(r+1)(r+1)}}{2R_{pr}} = \frac{(-A'D + BC')(-A'_kD_k + B_kC_k)}{2(DB_k + D_kB)} |E_{r+1}|^2 \quad (49)$$

In the former case voltages or powers of receiving-ends are assumed constant. If those of sending-ends except end 1 are assumed constant, then from Eqs. (6a), (6b),

$$I_{r+1} = -C''E_1 + AI_1$$

$$E_{r+1} = D''E_1 - BI_1$$

$$\left. \begin{aligned} E_1 &= \frac{AE_{r+1} + BI_{r+1}}{AD'' - BC''} = \frac{1}{\Delta}(AE_{r+1} + BI_{r+1}) \\ I_1 &= \frac{C''E_{r+1} + D''I_{r+1}}{AD'' - BC''} = \frac{1}{\Delta}(C''E_{r+1} + D''I_{r+1}) \end{aligned} \right\} \quad \cdots\cdots\cdots (50)$$

are derived just as in the former case. If these equations are used for calculating the effective power, P_{11}, corresponding to Eq. (45) becomes as follows:

$$2P_{11}\Delta\Delta_k = (\boldsymbol{A}_k\boldsymbol{E}_{(r+1)k} + \boldsymbol{B}_k\boldsymbol{I}_{(r+1)k})(\boldsymbol{C}''\boldsymbol{E}_{r+1} + \boldsymbol{D}''\boldsymbol{I}_{r+1})$$
$$+ (\boldsymbol{A}\boldsymbol{E}_{r+1} + \boldsymbol{B}\boldsymbol{I}_{r+1})(\boldsymbol{C}_k'\boldsymbol{E}_{(r+1)k} + \boldsymbol{D}''\boldsymbol{I}_{(r+1)k} \quad \cdots\cdots (51)$$

As an intermediate calculation is the same as before, it is here neglected. Then the effective power circle diagram of the terminal $(r+1)$ drawn on the coordinate $P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)}$ with assumption that P_{11} is constant, is

$$r'_{pr}\varepsilon^{j\phi'_r} = -\boldsymbol{C}'_{p(r+1)(r+1)} + (P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)}) \quad \cdots\cdots (52)$$

The center is

$$\boldsymbol{C}'_{p(r+1)(r+1)} = -\frac{\boldsymbol{C}''\boldsymbol{B}_k + \boldsymbol{A}\boldsymbol{D}_k''}{\boldsymbol{B}_k\boldsymbol{D}'' + \boldsymbol{B}\boldsymbol{D}_k''}|E_{r+1}|^2 \quad \cdots\cdots (28)$$

and the radius is

$$r'_{pr} = \sqrt{2P_{11}\Delta\Delta_k R'_{pr} + R^2_{p(r+1)(r+1)}} \quad \cdots\cdots (53)$$

where

$$R'_{pr} = \left|\frac{E^2_{r+1}}{\boldsymbol{B}_k\boldsymbol{D}'' + \boldsymbol{B}\boldsymbol{D}_k''}\right| \quad \cdots\cdots (54)$$

and

$$R_{p(r+1)(r+1)} = \left|\frac{\boldsymbol{A}\boldsymbol{D}'' - \boldsymbol{B}\boldsymbol{C}''}{\boldsymbol{B}_k\boldsymbol{D}'' + \boldsymbol{B}\boldsymbol{D}_k''}E^2_{r+1}\right| \quad \cdots\cdots (28)$$

The effective power of terminal 1 is then

$$P_{11} = \frac{(r'_{pr} + R_{p(r+1)(r+1)})(r'_{pr} - R_{p(r+1)(r+1)})}{2\Delta\Delta_k R'_{pr}} \quad \cdots\cdots (55)$$

When $P_{11}=0$, the radius becomes $r_{pr} = R_{p(r+1)(r+1)}$, and the circle coincides with the 0. p. f. circle diagram discussed in Eq. (28). The maximum sending power is acquired when $r_{pr}=0$, as

$$P_{11\,max} = \frac{R^2_{p(r+1)(r+1)}}{2\Delta\Delta_k R'_{pr}} = \frac{|E_{r+1}|^2}{2(\boldsymbol{B}_k\boldsymbol{D}'' + \boldsymbol{B}\boldsymbol{D}_k'')} = \frac{R'_{pr}}{2} \quad \cdots\cdots (56)$$

The method will be applicable for all other terminals of sending-ends.

Next, an effective power of receiving-ends will be drawn on power coordinates of sending-ends. For an example, $P_{(r+1)(r+1)}$ of the receiving-end $(r+1)$ is drawn on the coordinate $P_{11} + jQ_{11}$ of the sending-end 1.

$$2P_{(r+1)(r+1)} = \boldsymbol{E}_{(r+1)k}\boldsymbol{I}_{(r+1)} + \boldsymbol{E}_{(r+1)}\boldsymbol{I}_{(r+1)k}$$

$\boldsymbol{E}_{(r+1)}$ and $\boldsymbol{I}_{(r+1)}$ are derived from Eqs. (6a), (6b) and (15a) under the premise of assuming each sending voltage or power as cocstant except the sending-end 1, as shown in Eq. (15a).

$$2P_{(r+1)(r+1)} = (\boldsymbol{D}_k''\boldsymbol{E}_{1k} - \boldsymbol{B}_k\boldsymbol{I}_{1k})(-\boldsymbol{C}''\boldsymbol{E}_1 + \boldsymbol{A}\boldsymbol{I}_1)$$
$$+(\boldsymbol{D}''\boldsymbol{E}_1 - \boldsymbol{B}\boldsymbol{I}_1)(-\boldsymbol{C}_k''\boldsymbol{E}_{1k} + \boldsymbol{A}_k\boldsymbol{I}_{1k}) \quad \cdots\cdots (57)$$

While, $P_{(r+1)(r+1)}$ is expressed too by assuming receiving powers or voltages as constants except the receiving-end 1, namely by use of Eqs. (5a) and (5b), $E_1 = A'E_{r+1} + BI_{r+1}$, $I_1 = C'E_{r+1} + DI_{r+1}$, which are modified as follows:

$$\left.\begin{aligned} \boldsymbol{E}_{r+1} &= \frac{\boldsymbol{D}\boldsymbol{E}_1 - \boldsymbol{B}\boldsymbol{I}_1}{\boldsymbol{A}'\boldsymbol{D} - \boldsymbol{B}\boldsymbol{C}'} = \frac{1}{\Delta'}(\boldsymbol{D}\boldsymbol{E}_1 - \boldsymbol{B}\boldsymbol{I}_1) \\ \boldsymbol{I}_{r+1} &= \frac{-\boldsymbol{C}'\boldsymbol{E}_1 + \boldsymbol{A}'\boldsymbol{I}_1}{\boldsymbol{A}'\boldsymbol{D} - \boldsymbol{B}\boldsymbol{C}'} = \frac{1}{\Delta'}(-\boldsymbol{C}'\boldsymbol{E}_1 + \boldsymbol{A}'\boldsymbol{I}_1) \end{aligned}\right\} \quad \cdots\cdots (58)$$

whence consequently,

$$2P_{(r+1)(r+1)} = \frac{1}{\Delta'\Delta_k'}\Big[(\boldsymbol{D}_k\boldsymbol{E}_{1k} - \boldsymbol{B}_k\boldsymbol{I}_{1k})(-\boldsymbol{C}'\boldsymbol{E}_1 + \boldsymbol{A}'\boldsymbol{I}_1)$$
$$+(\boldsymbol{D}\boldsymbol{E}_1 - \boldsymbol{B}\boldsymbol{I}_1)(-\boldsymbol{C}_k'\boldsymbol{E}_{1k} + \boldsymbol{A}_k'\boldsymbol{I}_{1k})\Big] \quad \cdots\cdots (59)$$

Therfore the effective power circle diagram on the receiving coordinate is obtained like the above on the sending coordinate. From Eq. (57), obtained

$$r_{ps}\varepsilon^{j\phi_s} = -\boldsymbol{C}_{p11} + (P_{11} + jQ_{11}) \quad \cdots\cdots (60)$$

From Eq. (59), obtained

$$r_{ps}'\varepsilon^{j\phi_s'} = -\boldsymbol{C}_{p11}' + (P_{11} + jQ_{11}) \quad \cdots\cdots (61)$$

whence

$$r_{ps} = \sqrt{R_{p11}^2 - 2P_{(r+1)(r+1)}R_{ps}} \quad \cdots\cdots (62)$$

$$R_{ps} = \left|\frac{E_1^2}{\boldsymbol{A}\boldsymbol{B}_k + \boldsymbol{A}_k\boldsymbol{B}}\right| \quad \cdots\cdots (63)$$

$$\left.\begin{aligned} R_{p11} &= \left|\frac{\boldsymbol{A}\boldsymbol{D}'' - \boldsymbol{B}\boldsymbol{C}''}{\boldsymbol{A}\boldsymbol{B}_k + \boldsymbol{A}_k\boldsymbol{B}}E_1^2\right| \\ \boldsymbol{C}_{p11} &= \frac{\boldsymbol{C}''\boldsymbol{B}_k + \boldsymbol{D}''\boldsymbol{A}_k}{\boldsymbol{A}\boldsymbol{B}_k + \boldsymbol{A}_k\boldsymbol{B}}E_1^2 \end{aligned}\right\} \quad \cdots\cdots (25)$$

$$r_{ps}' = \sqrt{R_{p11}'^2 - 2P_{(r+1)(r+1)}\Delta'\Delta_k'R_{ps}'} \quad \cdots\cdots (64)$$

$$R_{ps}' = \left|\frac{E_1^2}{\boldsymbol{A}'\boldsymbol{B}_k + \boldsymbol{A}_k'\boldsymbol{B}}\right| \quad \cdots\cdots (65)$$

$$\left.\begin{aligned} R'_{p11} &= \left| \frac{A'D - BC'}{A'B_k + A'_k B} \right| E_1^2 \\ C'_{p11} &= \frac{C'B_k + DA'_k}{A'B_k + A'_k B} E_1^2 \end{aligned}\right\} \quad \cdots\cdots \text{(22)}$$

From the radius of the circle of an effective power on the receiving-end $(r+1)$,

$$P_{(r+1)(r+1)} = \frac{(r_{ps} - R_{p11})(r_{ps} + R_{p11})}{2R_{ps}} \quad \cdots\cdots \text{(66)}$$

or

$$P_{(r+1)(r+1)} = \frac{(r''_{ps} - R'_{p11})(r'_{ps} + R'_{p11})}{2\Delta'\Delta'_k R'_{ps}} \quad \cdots\cdots \text{(67)}$$

If $P_{(r+1)(r+1)}=0$, it coincides to the 0. p. f. circle diagram which has the radius of R'_{p11} or R_{p11} shown in Eq. (22) or Eq. (25), and in the inside of the circle, the power is positive, and on the outside, the power is negative. Of these the maximum value is gained when $r_{ps}=r'_{ps}=0$.

$$P_{(r+1)(r+1)\max} = \frac{R^2_{p11}}{2R_{ps}} = \frac{(AD''-BC'')(A_kD''_k - B_kC''_k)}{2(AB_k + A_kB)} E_1^2 \quad \text{(68)}$$

$$P_{(r+1)(r+1)\max} = \frac{R'^2_{p11}}{2\Delta'\Delta'_k R'_{ps}} = \frac{E_1^2}{2(A'B_k + A'_kB)} = \frac{R'_{ps}}{2} \quad \cdots\cdots \text{(69)}$$

Next, the power, $P_{1(r+1)}$, which is transmitted out from total effective power P_{11} of the sending-end 1 to the receiving-end $(r+1)$, is drawn on the coordinate of the total power of the receiving-end $(r+1)$, $P_{(r+1)(r+1)}+jQ_{(r+1)(r+1)}$. From Eq. (11), it is obtained

$$\begin{aligned} 2P_{1(r+1)} &= (A'_kE_{(r+1)k} + B_kI_{(r+1)k})(CE_{r+1} + DI_{r+1}) + \\ &\quad (A'E_{r+1} + BI_{r+1})(C_kE_{(r+1)k} + D_kI_{(r+1)k}) \quad \cdots\cdots \text{(70)} \end{aligned}$$

In a similar way by which Eq. (47) is derived from Eq. (45),

$$r'_{pr}\varepsilon^{j\theta''_r} = -C''_{p(r+1)(r+1)} + (P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)}) \quad \cdots\cdots \text{(71)}$$

is obtained as the effective power circle diagram.

While, on the contrary, the power $P_{(r+1)1}$ which is a part of the total power of the receiving-end $(r+1)$ and is the one transmitted from the sending-end 1, is drawn on the coordinate 1. From Eq. (11),

$$\begin{aligned} 2P_{(r+1)1} &= (D''_kE_{1k} - B_kI_{1k})(-CE_1 + AI_1) \\ &\quad + (D''E_1 - BI_1 - C_kE_{1k} + A_kI_{1k}) \quad \cdots\cdots \text{(72)} \end{aligned}$$

is obtained, and then the effective power circle diagram is shown by

$$r''_{ps}\varepsilon^{j\phi''_s} = -C''_p + (P_{11} + jQ_{11}) \quad \cdots\cdots (73)$$

where

$$r''_{pr} = \sqrt{2P_{1(r+1)}R''_{pr} + R''^2_{p(r+1)(r+1)}} \quad \cdots\cdots (74)$$

$$R''_{pr} = \left| \frac{E^2_{r+1}}{B_k D + B D_k} \right| \quad \cdots\cdots (46)$$

$$\left.\begin{aligned} R''_{p(r+1)(r+1)} &= \left| \frac{A'D - BC}{DB_k + D_k B} E^2_{r+1} \right| \\ C''_{p(r+1)(r+1)} &= -\frac{CB_k + A'D_k}{DB_k + D_k B} E^2_{r+1} \end{aligned}\right\} \quad \cdots\cdots (42)$$

$$r''_{ps} = \sqrt{R''^2_{p1} - 2P_{(r+1)1}R''_{ps}} \quad \cdots\cdots (75)$$

$$R''_{ps} = \left| \frac{E^2_1}{AB_k + A_k B} \right| \quad \cdots\cdots (63)$$

$$\left.\begin{aligned} R''_{p1} &= \left| \frac{AD'' - BC}{A_k B + A_k B} E^2_1 \right| \\ C''_{p11} &= \left| \frac{CB_k + D''A_k}{AB_k + A_k B} E^2_1 \right| \end{aligned}\right\} \quad \cdots\cdots (41)$$

If $P_{1(r+1)}$ or $P_{(r+1)1}$ becomes zero, the above coincides with the 0. p. f. circle diagram of Eqs. (41) or (42).

Viewing from another point, the effective power P_{11} of the sending-end 1 will be drawn on the coordinate of $P_{(r+1)1} + jQ_{(r+1)1}$, which is the branch power of the receiving-end $(r+1)$. As from Eq. (11), is obtained $E_{r+1} = D''E_1 - BI_1$, $I_{(r+1)1} = -CE_1 + AI_1$, the expression of P_{11} becomes

$$2P_{11} = \frac{(A_k E_{(r+1)k} + B_k I_{(r+1)1k})(CE_{r+1} + D''I_{(r+1)1}) + (AE_{r+1} + BI_{(r+1)1})(C_k E_{(r+1)k} + D''_k I_{(r+1)1k})}{(AD'' - BC)(A_k D''_k - B_k C_k)} \quad \cdots (76)$$

where D'' is a constant containing receiving-end powers or voltages to be assumed as constants.

In the same way, also $P_{(r+1)(r+1)}$, the effective power of the receiving-end $(r+1)$ can be shown on the coordinate of $P_{1(r+1)} + jQ_{1(r+1)}$ which is the branch power transmitted from the sending-end 1. Using from Eq. (11) that $E_1 = A'E_{r+1} + BI_{r+1}$, and $I_{1(r+1)} = CE_{r+1} + DI_{r+1}$,

$$2P_{(r+1)(r+1)} = \frac{(D_k E_{1k} - B_k I_{1(r+1)k})(-CE_1 + A'I_{1(r+1)}) + (DE_1 - BI_{1(r+1)})(-C_k E_{1k} + A'_k I_{1(r+1)k})}{(A'D - BC)(A'_k D_k - B_k C_k)} \quad \cdots (77)$$

is obtained. Circle diagram equations are derived in the same way as before.

$$r'''_{pr}\varepsilon^{j\phi'''_r} = -C_{p(r+1)1} + (P_{(r+1)1} + jQ_{(r+1)1}) \quad \cdots\cdots (78)$$

or

$$r'''_{ps}\varepsilon^{j\phi'''_s} = -C_{p1(r+1)} + (P_{1(r+1)} + jQ_{1(r+1)}) \quad \cdots\cdots (79)$$

where

$$r'''_{pr} = \sqrt{2P_{11}(AD''-BC)(A_kD'_k-B_kC_k)R'''_{pr}+R^2_{p(r+1)}} \quad \cdots\cdots (80)$$

$$\left.\begin{aligned} C_{p(r+1)1} &= -\frac{CB_k+AD''_k}{D''B_k+D''_kB}E^2_{r+1} \\ R'''_{pr} &= \left|\frac{E^2_{r+1}}{D''B_k+D''_kB}\right| \\ R_{p(r+1)1} &= \left|\frac{AD''-BC}{D''B_k+D''_kB}E^2_{r+1}\right| \end{aligned}\right\} \quad \cdots\cdots (37)$$

$$r'''_{ps} = \sqrt{R^2_{p1(r+1)}-2P_{(r+1)(r+1)}(A'D-BC)(A'_kD_k-B_kC_k)R'''_{ps}} \quad (81)$$

$$\left.\begin{aligned} C_{p1(r+1)} &= \frac{CB_k+DA'_k}{A'B_k+A'_kB}E^2_1 \\ R''_{ps} &= \left|\frac{E^2_1}{A'B_k+A^*_kB}\right| \\ R_{p1(r+1)} &= \left|\frac{A'D-BC}{A'B_k+A'_kB}E^2_1\right| \end{aligned}\right\} \quad \cdots\cdots (34)$$

The circle of Eq. (78) concerns the 0. p. f. circle diagram of Eq. (37), and the circle of Eq. (79) concerns that of Eq. (34).

Lastly, each branch effective power of terminals can be shown on a coordinate of each branch effective power of sending or receiving terminals. However, it is the same as the case of drawing an ordinary circle diagram of an one-terminal transmission line, so explanations are omitted here. As power circle diagrams discussed in Section 3 are drawn on each branch power coordinate, it may be convenient to draw branch effective power circle diagrams or efficiency circle diagrams on these coordinates simultaneously. Those are nothing but the methods concerned with ordinary transmission lines. Namely, a fundamental circle on the receiving-end coordinate is with a radius, $R_{pr} = \left|\frac{E^2_{r+1}}{DB_k+D_kB}\right|$

and a center, $C_{pr} = -\frac{(CB_k + AD_k)}{DB_k + D_kB} E^2_{r+1}$; and that on the sending-end coordinate is with $R_{ps} = \left| \frac{E^2_1}{AB_k + A_kB} \right|$ and $C_{ps} = \frac{CB_k + DA_k}{AB_k + A_kB} E^2_1$; from which an effective power or a transmission efficiency are derivid out. In this case, however, it is quite noteworthy to see that the voltage on terminals which are interconnected by branch lines with each other, should coincide to a certain value obtained from certain mutual relationships. This is not an arbitrary one on effective power circle diagrams but is restricted by all other terminal voltages of the system.

Example 6.

From the data previously derived for the main power transmission system in Hokkaido, now effective circle diagrams will be shown. As the calculation is made only be equations is this Section, it is done by a routine manner as in the former case. The heading letters of the following also correspond to those of Examples 5) and 4). The value of effective power, P, which is to be maintained as a constant in the diagram, is assumed as a value under the normal operation which is obtained from wE^2 or IE_k by use of Eqs, (E1), (E2) or (E12), as is just mentioned in the last of Example 1).

a.)

$$r'_{p1}\,\varepsilon^{j\phi'} = -\boldsymbol{C}'_{p11} + P_{11} + jQ_{11}$$

where

$$R'_{p1} = \left| \frac{E^2_1}{\boldsymbol{A}'\boldsymbol{B}_k + \boldsymbol{A}'_k\boldsymbol{B}} \right| = 0.488 \quad \cdots\cdots \quad \text{(E 55)}$$

$$\boldsymbol{C}'_{p11} = -0.942 - j\,11.9, \quad R'_{p11} = 1.63 \quad \cdots\cdots \quad \text{(E 31)}$$

$$P_{55} = 5.68$$

$$\therefore \quad |r'_{p1}| = \sqrt{R'^2_{p11} + 2P_{55}\varDelta'\varDelta'_kR'_{p1}} = 8.02$$

and

$$|\varDelta'| = |\boldsymbol{A}'\boldsymbol{D} - \boldsymbol{B}\boldsymbol{C}''| = 3.33$$

$$P_{55\,\max} = \frac{R'_{p1}}{2} = 0.244,$$ which shows the minimum power.

b.)

$$r'_{p1}\,\varepsilon^{j\phi'} = -\boldsymbol{C}'_{p11} + P_{11} + jQ_{11}$$

where

$$R'_{p1} = \left| \frac{E_1^2}{A'B_k + A'_k B} \right| = 38.2 \quad \cdots\cdots\cdots \quad \text{(E 56)}$$

$$C'_{p11} = 12.6 - j\,16.9, \quad R'_{p11} = 12.6 \quad \cdots\cdots\cdots \quad \text{(E 33)}$$

$$P_{77} = 0.985$$

$$\therefore \quad |r'_{p1}| = \sqrt{R'^2_{p11} - 2P_{77}\Delta'\Delta'_k R'_{p1}} = 12.9$$

$$|\Delta'| = 0.33$$

and

$$P_{77\,\max} = \frac{R'_{p1}}{2} = 19.1 \; .$$

c.)

$$r'_{p5}\varepsilon^{j\delta'} = -C'_{p55} + P_{55} + jQ_{55}$$

where

$$R'_{p5} = \left| \frac{E_5^2}{B_k D'' + B D''_k} \right| = 0.63 \quad \cdots\cdots\cdots \quad \text{(E 57)}$$

$$C'_{p55} = -3.57 + j\,12.5, \quad R'_{p55} = 3.37 \quad \cdots\cdots\cdots \quad \text{(E 35)}$$

$$P_{11} = 4.88$$

$$\therefore \quad |r'_{p5}| = \sqrt{2P_{11}\Delta\Delta_k R'_{p5} + R'^2_{p55}} = 13.7$$

$$|\Delta| = 5.35$$

and

$$P_{11\,\max} = \frac{R'_{p5}}{2} = 0.315$$ which shows the minimum power.

d.)

$$r'_{p7}\varepsilon^{j\phi'} = -C'_{p77} + P_{77} + jQ_{77}$$

where

$$R'_{p7} = \left| \frac{E_7^2}{B_k D'' + B D''_k} \right| = 22.0 \quad \cdots\cdots\cdots \quad \text{(E 58)}$$

$$C'_{p77} = 8.78 + j\,11.5, \quad R'_{p7} = 11.9 \quad \cdots\cdots\cdots \quad \text{(E 37)}$$

$$P_{11} = 4.88$$

$$\therefore \quad |r'_{p7}| = \sqrt{2P_{11}\Delta\Delta_k R'_{p7} + R'^2_{p77}} = 14.3$$

$$|\Delta| = 0.542$$

and

$$P_{11\,max} = \frac{R'_{p7}}{2} = 11.0$$

e.)

$$r_{p1}\varepsilon^{j\phi} = -\boldsymbol{C}_{p11} + P_{11} + jQ_{11}$$

where

$$R_{p1} = \left| \frac{E_1^2}{\boldsymbol{A}\boldsymbol{B}_k + \boldsymbol{A}_k\boldsymbol{B}} \right| = 2.90 \quad \cdots\cdots \text{(E 59)}$$

$$\boldsymbol{C}_{p11} = 15.95 - j\,15.5, \quad R_{p\,11} = 15.3 \quad \cdots\cdots \text{(E 39)}$$

$$P_{55} = 5.68$$

$$\therefore \quad |r_{p1}| = \sqrt{R_{p11}^2 - 2P_{55}R_{p1}} = 14.2$$

and

$$P_{55\,max} = \frac{R_{p11}^2}{2R_{p1}} = 40.4$$

f.)

$$r_{p1}\varepsilon^{j\phi} = -\boldsymbol{C}_{p11} + P_{11} + jQ_{11}$$

where

$$R_{p1} = \left| \frac{E_1^2}{AB_k + A_kB} \right| = \infty \quad \cdots\cdots \text{(E 60)}$$

$$\boldsymbol{C}_{p11} = R_{p11} = \infty \quad \cdots\cdots \text{(E 41)}$$

$$P_{77} = 0.985$$

$$\therefore \quad |r_{p1}| = \sqrt{R_{p11}^2 - 2P_{77}R_{p1}} = \infty$$

and

$$P_{77\,max} = \frac{R_{p11}^2}{2R_{p1}} = \infty$$

g.)

$$r_{p5}\varepsilon^{j\phi} = -\boldsymbol{C}_{p55} + P_{55} + jQ_{55}$$

where

$$R_{p5} = \left| \frac{E_5^2}{\boldsymbol{B}_k\boldsymbol{D} + \boldsymbol{B}\boldsymbol{D}_k} \right| = 3.55 \quad \cdots\cdots \text{(E 61)}$$

$$\boldsymbol{C}_{p55} = -6.90 - j\,1.42, \quad R_{p55} = 11.85 \quad \cdots\cdots \text{(E 43)}$$

$$P_{11} = 4.88$$

$$\therefore \quad |r_{p5}| = \sqrt{2P_{11}R_{p5}+R_{p55}^2} = 13.2$$

and

$$P_{11\,max} = \frac{R_{p55}^2}{2R_{p5}} = 19.7$$

h.)

$$r_{p7}\varepsilon^{j\phi} = -C_{p77}+P_{77}+jQ_{77}$$

where

$$R_{p7} = \left|\frac{E_7^2}{B_k D + B D_k}\right| = \infty \quad \cdots\cdots \quad (E\ 62)$$

$$C_{p77} = R_{p77} = \infty \quad \cdots\cdots \quad (E\ 45)$$

$$\therefore \quad |r_{p7}| = \infty$$

and

$$P_{11\,max} = \infty$$

i.)

$$r_{p1}'''\varepsilon^{j\phi'''} = -C_{p17}+P_{17}+jQ_{17}$$

where

$$R_{p1}''' = \left|\frac{E_1^2}{A'B_k + A_k'B}\right| = 38.2 \quad \cdots\cdots \quad (E\ 63)$$

$$C_{p17} = 26.1-j\,8.86, \quad R_{p17} = 27.6 \quad \cdots\cdots \quad (E\ 47)$$

$$P_{77} = 0.985$$

$$\therefore \quad |r_{p1}'''| = \sqrt{R_{p17}^2-2P_{77}\Delta'''\Delta_k''R_{p1}'''} = 26.8$$

$$|\Delta'''| = 0.722$$

j.)

$$r_{p7}'''\varepsilon^{j\phi'''} = -C_{p71}+P_{71}+jQ_{71}$$

where

$$R_{p7}''' = \left|\frac{E_7^2}{D''B_k + D_k''B}\right| = 22.0 \quad \cdots\cdots \quad (E\ 64)$$

$$C_{p71} = 11.7+j\,9.81, \quad R_{p71} = 15.2 \quad \cdots\cdots \quad (E\ 49)$$

$$P_{11} = 4.88$$

$$\therefore \quad |r'_{p7}| = \sqrt{2P_{11}\Delta'''\Delta''_{k}R'''_{p7} + R^2_{p71}} = 18.3$$

$$|\Delta'''| = 0.694$$

k.)

$$r''_{p1}\varepsilon^{j\phi''} = -C''_{p11} + P_{11} + jQ_{11}$$

where

$$R''_{p1} = \infty \quad \cdots\cdots (65)$$

$$C''_{p11} = R''_{p11} = \infty \quad \cdots\cdots (51)$$

$$\therefore \quad |r''_{p1}| = \infty$$

l.)

$$r''_{p7}\varepsilon^{j\phi''} = -C''_{p} + P_{77} + jQ_{77}$$

where

$$R''_{p7} = \infty \quad \cdots\cdots (66)$$

$$C''_{p77} = R''_{p7} = \infty \quad \cdots\cdots (53)$$

$$\therefore \quad |r''_{p7}| = \infty$$

Those circle diagrams are shown in Fig. 8 to Fig. 19. It is clearly seen that all the circles pass through their operating points of power on their co-ordinates, the reason for which is that the effective powers used in calculations are of the normal steady values given in Fig. 3 by drawing ordinary power circle diagrams.

8. Transmission Power Ratio Circle Diagrams Between Terminals, Synthetic Transmission Efficiency Circle Diagrams, and Apposite Terminal Voltages of Interconnected Transmission Systems.

Effective power circle diagrams discussed formerly can easily be transformed into circle diagrams which show the ratio of a sending effective power to a receiving effective power. This diagram becomes the efficiency circle diagram when a system is composed of an ordinary transmission lines each with two terminals. In the case of an interconnected transmission system, it, of course, does not show the very transmission efficiencies unless the total sending or receiving power is not handled.

However, it can be completed in the same way as that of an ordinary transmission line. Namely, on a certain 0. p. f. circle diagram discussed before, a group of circles that intersect in rectangular with the circle which intersects the fundamental circles in rectangular and which centers at an intersecting point of a vertical axis of the co-ordinate with a horizontal line passing a center of the 0. p. f. circle diagram, and that have centers on this horizontal line, are all supplying a relation that ratios of the effective powers between terminals are constant. These circle diagrams on sending power co-ordinate are differed from those on receiving power co-ordinates with only the directions of signs.

If an example is shown with the sending power ratio circle diagram, which is derived from the effective power circle diagram, upon a sending power co-ordinate; such as, if Eqs. (47), (52), (71), (78) and (80) are represented by the following equation,

$$\left.\begin{aligned} &\sqrt{2P_{11}\Delta\Delta_k R_{pr}+R^2_{p(r+1)(r+1)}}=-\boldsymbol{C}_{p(r+1)(r+1)}+(P_{(r+1)(r+1)}+jQ_{(r+1)(r+1)}) \\ &\text{where it is assumed} \\ &\qquad \boldsymbol{C}_{p(r+1)(r+1)}=-\alpha_{p(r+1)}-j\beta_{p(r+1)} \end{aligned}\right\} \quad (82)$$

If also a ratio of power is

$$\eta=\frac{P_{(r+1)(r+1)}}{P_{11}} \quad \cdots\cdots (83)$$

then we get

$$\frac{2P_{(r+1)(r+1)}}{\eta}\Delta\Delta_k R_{pr}+R^2_{p(r+1)(r+1)}=(P_{(r+1)(r+1)}+\alpha_{p(r+1)})^2+(Q_{(r+1)(r+1)}+\beta_{p(r+1)})^2$$

As it is modified to the form

$$R^2_{p(r+1)(r+1)}-\alpha^2_{p(r+1)}+\left\{\frac{\Delta\Delta_k R_{pr}}{\eta}-\alpha_{p(r+1)(r+1)}\right\}^2$$
$$=\left\{P_{(r+1)(r+1)}-\left(\frac{\Delta\Delta_k R_{pr}}{\eta}-\alpha_{p(r+1)(r+1)}\right)\right\}^2+(Q_{(r+1)(r+1)}+\beta_{p(r+1)})^2 \quad (84)$$

a center and a radius of the equation become as follows:

$$\left.\begin{aligned} r_{\eta r}&=\sqrt{R^2_{p(r+1)(r+1)}-\alpha^2_{p(r+1)}+\left(\frac{\Delta\Delta_k R_{pr}}{\eta}-\alpha_{p(r+1)}\right)^2}=\sqrt{-\alpha^2_{br}+\alpha^2_{\eta(r+1)}} \\ \boldsymbol{C}_{\eta r}&=\left(\frac{\Delta\Delta_k R_{pr}}{\eta}-\alpha_{p(r+1)}\right)-j\beta_{p(r+1)}=\alpha_{\eta(p+1)}-j\beta_{p(r+1)} \end{aligned}\right\} \quad (85)$$

If this equation is expressed in vectors,

$$r_{\eta r}=\boldsymbol{C}_{\eta r}+(P_{(r+1)(r+1)}+jQ_{(r+1)(r+1)}) \quad \cdots\cdots (84a)$$

On the contrary, let us express a sending power ratio circle diagram on a sending-end coordinate from Eqs. (58), (59), (70), (77) etc, which are the effective power circle diagrams on sending-end co-ordinates. For example, if an effective power circle diagram is shown by the following equations,

$$\left.\begin{aligned} \sqrt{R^2_{p11}+2P_{(r+1)(r+1)}\Delta'\Delta'_k R_{ps}} &= -C_{p11}+(P_{11}+jQ_{11}) \\ C_{p11} &= \alpha_{p1}+j\beta_{p1} \end{aligned}\right\} \quad \cdots\cdots\cdots (86)$$

then, if power ratio, Eq. (83) is substituted

$$R^2_{p11}+2\eta P_{11}\Delta'\Delta_k R_{ps} = (P_{11}-\alpha_{p1})^2+(Q_{11}-\beta_{p1})^2 .$$

By modification it will be as follows:

$$R^2_{p11}-\alpha^2_{p1}+(\alpha_{p1}+\eta R_{ps}\Delta'\Delta'_k)^2 = \left\{P_{11}-(\alpha_{p1}+\eta R_{ps}\Delta'\Delta'_k)\right\}+(Q_{11}-\beta_{p1})^2 \quad (87)$$

or

$$r^2_{\eta s} = (P_{11}-\alpha_{\eta s})^2+(Q_{11}-\beta_{p1})^2 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (87a)$$

where

$$\left.\begin{aligned} r_{\eta s} &= \sqrt{R^2_{p11}-\alpha^2_{p1}+(\alpha_{p1}+\eta R_{ps}\Delta'\Delta'_k)^2} = \sqrt{-\alpha^2_{bs}+\alpha^2_{\eta 1}} \\ C_{\eta s} &= (\alpha_{p1}+\eta R_{ps}\Delta'\Delta'_k)+j\beta_{p1} = \alpha_{\eta 1}+j\beta_{p1} \end{aligned}\right\} \quad \cdots\cdots (88)$$

Accordigly, the sending efficiency circle diagram is represented by a group of circles which have centers on a line passing a point, C_{p11} that is the center of a fundamental circle diagram, R_{p11}.

Example 7.

This is the drawing of the sending power ratio ciacle diagrams in Fig. 8 to Fig. 19, to show their synthetic transmission efficiencies. Calculations are due to the equations given in this Section; the efficiency η is to be taken as that of the normal steady values obtained from the ratio of the receiving to the sending power.

a.)

$$r_{\eta 1} = C_{\eta 1}+P_{11}+jQ_{11}$$
$$\eta = P_{55}/P_{11} = 116\,\%$$

where

$$\left.\begin{aligned} C'_{p11} &= \alpha'_{p1}+j\beta'_{p1} = -0.942-j\,11.9 \\ R'_{p11} &= 1.63 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (E\,31)$$

$$R'_{p1} = 0.488 \quad \cdots\cdots \quad (E\,55)$$

$$|\Delta'| = 3.33$$

$$\therefore \quad C_{\eta 1} = (\alpha'_{p1} + \eta R'_{p1}\Delta'\Delta'_k) + j\beta'_{p1} = 5.34 - j\,11.9$$

$$|r_{\eta 1}| = \sqrt{R'^2_{p11} - \alpha'^2_{p1} + (\alpha'_{p1} + \eta R'_{p1}\Delta'\Delta'_k)^2} = 5.50 \quad \cdots\cdots \quad (E\,67)$$

b.)

$$r_{\eta 1} = C_{\eta 1} + P_{11} + jQ_{11}$$

$$\eta = P_{77}/P_{11} = 20.2\%$$

where

$$\left.\begin{aligned} C'_{p11} &= \alpha'_{p1} + j\beta'_{p1} = 12.6 - j\,16.9 \\ R'_{p11} &= 12.6 \end{aligned}\right\} \quad \cdots\cdots \quad (E\,33)$$

$$R'_{p1} = 38.2 \quad \cdots\cdots \quad (E\,56)$$

$$|\Delta'| = 0.33$$

$$\therefore \quad C_{\eta 1} = (\alpha'_{p1} + \eta R'_{p1}\Delta'\Delta'_k) + j\beta'_{p1} = 13.44 - j\,16.9$$

$$|r_{\eta 1}| = \sqrt{R'^2_{p11} - \alpha'^2_{p1} + (\alpha'_{p1} + \eta R'_{p1}\Delta'\Delta'_k)^2} = 13.44 \quad \cdots\cdots \quad (E\,68)$$

c.)

$$r_{\eta 5} = C_{\eta 5} + P_{55} + jQ_{55}$$

$$\eta = P_{55}/P_{11} = 116\%$$

where

$$\left.\begin{aligned} C'_{p55} &= -\alpha'_{p5} - j\beta'_{p5} = -3.57 + j12.5 \\ R'_{p55} &= 3.37 \end{aligned}\right\} \quad \cdots\cdots \quad (E\,35)$$

$$R'_{p5} = 0.63 \quad \cdots\cdots \quad (E\,57)$$

$$|\Delta| = 5.35$$

$$\therefore \quad C_{\eta 5} = \left(\frac{\Delta\Delta_k R'_{p5}}{\eta} - \alpha'_{p5}\right) - j\beta'_{p5} = 11.93 + j\,12.5$$

$$|r_{\eta 5}| = \sqrt{R'^2_{p55} - \alpha'^2_{p5} + \left(\frac{\Delta\Delta_k R'_{p5}}{\eta} - \alpha'_{p5}\right)^2} = 11.85 \quad \cdots\cdots \quad (E\,69)$$

Also we can obtain $\alpha_{65} = 1.18$ which is the radius of a common intersected circle in rectangular with the circle C_{p55} and with all the sending rower ratio circle diagrams.

d.)

$$r_{\eta 7} = C_{\eta 7} + P_{77} + jQ_{77}$$
$$\eta = P_{77}/P_{11} = 20.2\%$$

where

$$\left.\begin{aligned} C'_{p77} &= -\alpha'_{p7} - j\beta'_{p7} = 8.78 + j\,11.5 \\ R'_{p77} &= 11.9 \end{aligned}\right\} \quad \cdots\cdots \text{(E 37)}$$

$$R'_{p7} = 22.0 \quad \cdots\cdots \text{(E 58)}$$

$$|\varDelta| = 0.542$$

$$\therefore \quad C_{\eta 7} = \left(\frac{\varDelta \varDelta_k R'_{p7}}{\eta} - \alpha'_{p7}\right) - j\beta'_{p7} = 40.68 + j\,11.5$$

$$|r_{\eta 7}| = \sqrt{R'^{\,2}_{p77} - \alpha'^{\,2}_{p7} + \left(\frac{\varDelta \varDelta_k R'_{p7}}{\eta} - \alpha'_{p7}\right)^2} = 41.4 \quad \cdots\cdots \text{(E 70)}$$

e.)

$$r_{\eta 1} = C_{\eta 1} + P_{11} + jQ_{11}$$
$$\eta = P_{55}/P_{11} = 116\%$$

where

$$\left.\begin{aligned} C_{p11} &= \alpha_{p1} + j\beta_{p1} = 15.95 - j\,15.5 \\ R_{p11} &= 15.3 \end{aligned}\right\} \quad \cdots\cdots \text{(E 39)}$$

$$R_{p1} = 2.90 \quad \cdots\cdots \text{(E 59)}$$

$$\therefore \quad C_{\eta 1} = (\alpha_{p1} - \eta R_{p1}) + j\beta_{p1} = 12.58 - j\,15.5$$

$$|r_{\eta 1}| = \sqrt{R^2_{p11} - \alpha^2_{p1} + (\alpha_{p1} - \eta R_{p1})^2} = 11.7 \quad \cdots\cdots \text{(E 71)}$$

f.)

$$r_{\eta 1} = C_{\eta 1} + P_{11} + jQ_{11}$$
$$\eta = P_{77}/P_{11} = 20.2\%$$

where

$$\left.\begin{aligned} C_{p11} &= R_{p11} = R_{p1} = \infty \\ \therefore \quad C_{\eta 1} &= |r_{\eta 1}| = \infty \end{aligned}\right\} \quad \cdots\cdots \text{(E 72)}$$

Accordingly, the circle coincides with the effective power circle diagram.

g.)

$$r_{\eta 5} = C_{\eta 5} + P_{55} + jQ_{55}$$
$$\eta = P_{55}/P_{11} = 116\%$$

where

$$\left.\begin{aligned} C_{p55} &= -\alpha_{p5} - j\beta_{p5} = -6.90 - j\,1.42 \\ R_{p55} &= 11.85 \end{aligned}\right\} \quad \cdots\cdots \text{(E 43)}$$

$$R_{p5} = 3.55 \quad \cdots\cdots \text{(E 61)}$$

$$\therefore \quad r_{\eta 5} = \sqrt{R_{p55}^2 - \alpha_{p5}^2 + \left(\frac{R_{p5}}{\eta} - \alpha_{p5}\right)^2} = 10.4 \quad \cdots\cdots \text{(E 73)}$$

h.)

$$r_{\eta 7} = C_{\eta 7} + P_{77} + jQ_{77}$$
$$\eta = P_{77}/P_{11} = 20.2\%$$

where

$$C_{p77} = R_{p77} = R_{p7} = \infty \quad \cdots\cdots \text{(E 45), (E 62)}$$

$$\therefore \quad C_{\eta 7} = |r_{\eta 7}| = \infty \quad \cdots\cdots \text{(E 74)}$$

Eq. (E 74) coincides with the effective power circle diagram.

i.)

$$r_{\eta 17} = C_{\eta 17} + P_{17} + jQ_{17}$$
$$\eta = P_{77}/P_{17} = 77\%$$

where

$$\left.\begin{aligned} C_{p17} &= \alpha_{p17} + j\beta_{p17} = 26.1 - j\,8.86 \\ R_{p17} &= 27.6 \end{aligned}\right\} \quad \cdots\cdots \text{(E 47)}$$

$$R_{p1}''' = 38.2 \quad \cdots\cdots \text{(E 63)}$$

$$\therefore \quad C_{\eta 17} = (\alpha_{p17} - \eta R_{p1}''' \Delta''' \Delta_k''') + j\beta_{p17} = 10.8 - j\,8.86$$

$$|r_{\eta 17}| = \sqrt{R_{p17}^2 - \alpha_{p17}^2 + (\alpha_{p17} - \eta R_{p1}''' \Delta''' \Delta_k''')^2} = 12.0 \quad \cdots\cdots \text{(E 75)}$$

j.)

$$r_{\eta 17} = C_{\eta 71} + P_{71} + jQ_{71}$$
$$\eta = P_{71}/P_{11} = 26.3\%$$

where

$$\left.\begin{aligned} C_{p71} &= -\alpha_{p71} - j\beta_{p71} = 11.7 + j\,9.81 \\ R_{p71} &= 15.2 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 49)}$$

$$R'''_{p7} = 22.0$$

$$\therefore \quad C_{\eta 77} = \left(\frac{\Delta'''\Delta'''_k R'''_{p7}}{\eta} - \alpha_{p71}\right) - j\beta_{p71} = 51.8 + j\,9.81$$

$$|r_{\eta 71}| = \sqrt{R^2_{p71} - \alpha^2_{p71} + \left(\frac{\Delta'''\Delta'''_k R'''_{p7}}{\eta} - \alpha_{p71}\right)^2} = 51.3 \quad \cdots\cdots\cdots \text{(E 76)}$$

k.)

$$r_{\eta 71} = C_{\eta 71} + P_{11} + jQ_{11}$$

$$\eta = P_{71}/P_{11} = 26.3\%$$

where

$$\left.\begin{aligned} C''_{p11} &= R''_{p11} = R''_{p1} = \infty \\ \therefore \quad C_{\eta 71} &= |r_{\eta 71}| = \infty \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \text{(E 77)}$$

It coincides with the effective power circle diagram.

l.)

$$r_{\eta 7} = C_{\eta 7} + P_{77} + jQ_{77}$$

$$\eta = P_{77}/P_{11} = 77\%$$

where

$$C''_{p77} = R''_{p77} = R''_{p7} = \infty \quad \cdots\cdots\cdots\cdots \text{(E 53), (E 66)}$$

$$\therefore \quad C_{\eta 7} = |r_{\eta 7}| = \infty \quad \cdots\cdots\cdots\cdots \text{(E 78)}$$

It coincides with the effective power circle diagram.

These sending power ratio circle diagrams just considered are shown in Fig. 8 to Fig. 19, together with the previous diagrams. Of course, any other diagram is drawn under a certain value of efficiency η, by the same procedure as in this example.

9. Reactive Power Circle Diagrams of Interconnected Transmission Systems.

The method of procedure is quite similar to that for effective power circle diagrams, from which the only difference is that of treating reactive power in this case. Therefore, successive equations will be derived here as in the former cases.

If, first, a load of the receiving-end $(r+1)$ is only changeable with other receiving-end voltages or powers to be assumed constants, the reactive power of the sending-end 1 is expressed in the following way.

$$2Q_{11} = (\boldsymbol{I}_1\boldsymbol{E}_{1k} - \boldsymbol{I}_{1k}\boldsymbol{E}_1) = (\boldsymbol{C}'\boldsymbol{E}_{r+1} + \boldsymbol{D}\boldsymbol{I}_{r+1})(\boldsymbol{A}'_k\boldsymbol{E}_{(r+1)k} + \boldsymbol{B}_k\boldsymbol{I}_{(r+1)k}) - (\boldsymbol{C}'_k\boldsymbol{E}_{(r+1)k} + \boldsymbol{D}_k\boldsymbol{I}_{(r+1)k})(\boldsymbol{A}'\boldsymbol{E}_{r+1} + \boldsymbol{B}\boldsymbol{I}_{r+1}) \quad \cdots\cdots \quad (89)$$

A' and C' are constants defined by Eq. (13a).

Substituting relations $\boldsymbol{E}_{(r+1)k}\boldsymbol{I}_{r+1} = P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)}$, $\boldsymbol{I}_{(r+1)}\boldsymbol{I}_{(r+1)k} = (P^2_{(r+1)(r+1)} + Q^2_{(r+1)(r+1)})/|E_{r+1}|^2$, and $\boldsymbol{E}_{(r+1)k}\boldsymbol{E}_{(r+1)} = |E_{r+1}|^2$,

$$\begin{aligned} \boldsymbol{E}_q.\ 89 = & (\boldsymbol{A}'_k\boldsymbol{D} - \boldsymbol{A}'\boldsymbol{D}_k - \boldsymbol{B}\boldsymbol{C}'_k + \boldsymbol{B}_k\boldsymbol{C}')P_{(r+1)(r+1)} \\ & + (\boldsymbol{A}'_k\boldsymbol{D} + \boldsymbol{A}'\boldsymbol{D}_k - \boldsymbol{B}\boldsymbol{C}'_k - \boldsymbol{B}_k\boldsymbol{C}')jQ_{(r+1)(r+1)} \\ & + \frac{P^2_{(r+1)(r+1)} + Q^2_{(r+1)(r+1)}}{|E_{r+1}|^2}(\boldsymbol{B}_k\boldsymbol{D} - \boldsymbol{B}\boldsymbol{D}_k) + |E_{r+1}|^2(\boldsymbol{A}'_k\boldsymbol{C}' - \boldsymbol{A}'\boldsymbol{C}'_k). \end{aligned}$$

If it is defined

$$R_{qr} = \left| \frac{E^2_{r+1}}{-\boldsymbol{D}\boldsymbol{B}_k + \boldsymbol{D}_k\boldsymbol{B}} \right| \quad \cdots\cdots \quad (90)$$

$$\left. \begin{aligned} R_{q(r+1)(r+1)} &= \left| \frac{-\boldsymbol{A}'\boldsymbol{D} + \boldsymbol{B}\boldsymbol{C}'}{-\boldsymbol{D}\boldsymbol{B}_k + \boldsymbol{D}_k\boldsymbol{B}} E^2_{r+1} \right| \\ C_{q(r+1)(r+1)} &= \frac{\boldsymbol{C}'\boldsymbol{B}_k - \boldsymbol{A}'\boldsymbol{D}_k}{-\boldsymbol{D}\boldsymbol{B}_k + \boldsymbol{D}_k\boldsymbol{B}} E^2_{r+1} \end{aligned} \right\} \quad \cdots\cdots \quad (32)$$

also with attention to $(R_{qr})_k = -R_{qr}$, it followes that

$$\begin{aligned} 2Q_{11}R_{qr} + R^2_{q(r+1)(r+1)} = & (P^2_{(r+1)(r+1)} + Q^2_{(r+1)(r+1)}) \\ & + P_{(r+1)(r+1)}(\boldsymbol{A}'_k\boldsymbol{D} - \boldsymbol{A}'\boldsymbol{D}_k - \boldsymbol{B}\boldsymbol{C}'_k + \boldsymbol{B}_k\boldsymbol{C}')R_{qr} \\ & + jQ_{(r+1)(r+1)}(\boldsymbol{A}'_k\boldsymbol{D} + \boldsymbol{A}'\boldsymbol{D}_k - \boldsymbol{B}\boldsymbol{C}'_k - \boldsymbol{B}_k\boldsymbol{C}')R_{qr} \\ & + \left\{(\boldsymbol{D}\boldsymbol{B}_k - \boldsymbol{D}_k\boldsymbol{B})(\boldsymbol{A}'_k\boldsymbol{C}' - \boldsymbol{A}'\boldsymbol{C}'_k) - (-\boldsymbol{A}'\boldsymbol{D} + \boldsymbol{B}\boldsymbol{C}')(-\boldsymbol{A}'_k\boldsymbol{D}_k + \boldsymbol{B}_k\boldsymbol{C}_k)\right\} R^2_{qr} \\ = & \left\{ \frac{\boldsymbol{A}'\boldsymbol{D}_k - \boldsymbol{B}_k\boldsymbol{C}'}{-\boldsymbol{D}\boldsymbol{B}_k + \boldsymbol{D}_k\boldsymbol{B}} E^2_{r+1} + (P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)}) \right\} \left\{ \frac{\boldsymbol{A}'_k\boldsymbol{D} - \boldsymbol{B}\boldsymbol{C}'_k}{\boldsymbol{D}\boldsymbol{B}_k - \boldsymbol{D}_k\boldsymbol{B}} E^2_{r+1} \right. \\ & \left. + (P_{(r+1)(r+1)} - jQ_{(r+1)(r+1)}) \right\} \end{aligned}$$

If schalars are converted to vectors, a vector equation is

$$r_{qr}\varepsilon^{j\psi_r} = -C_{q(r+1)(r+1)} + (P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)}) \quad \cdots\cdots \quad (91)$$

where

$$|r_{qr}| = \sqrt{2Q_{11}R_{qr} + R^2_{q(r+1)(r+1)}} \quad \cdots\cdots (91a)$$

The center point of Eq. (91) is equal to the center of a 1. p. f. circle diagram of a receiving-end coordinate. The equation expresses the reactive power circle diagram studied now. Also from Eq. (91a), the reactive power of terminal 1 is

$$Q_{11} = -\frac{(r_{qr} + R_{q(r+1)(r+1)})(r_{qr} - R_{q(r+1)(r+1)})}{2R_{qr}} \quad \cdots\cdots (92)$$

Inside of the circle, $R_{q(r+1)(r+1)}$ represents positive (capacitive) reactive power, and outside represents negative (inductive) reactive power. The maximum sending power is obtained when $r_{qr}=0$ as

$$Q_{11\max} = \frac{R^2_{q(r+1)(r+1)}}{2R_{qr}} = \frac{(-A'D+BC')(-A'_kD_k+B_kC'_k)}{2(DB_k-D_kB)}|E_{r+1}|^2 \quad (93)$$

Though the discussion up till now assumes receiving-end voltages and powers as constants except the terminal $(r+1)$, nextly the case which assumes sending-end voltages or powers except the terminal 1 as constants is to be treated. In this case from $I_{r+1} = -C''E_1 + AI_1$, $E_{r+1} = D''E_1 - BI_1$, we get, as previously derived

$$\left.\begin{aligned} E_1 &= \frac{AE_{r+1}+BI_{r+1}}{AD''-BC} = \frac{1}{\Delta}AE_{r+1}+BI_{r+1}) \\ I_1 &= \frac{C''E_{r+1}+D''I_{r+1}}{AD''-BC} = \frac{1}{\Delta}(C''E_{r+1}+D''I_{r+1} \end{aligned}\right\} \quad \cdots\cdots (50)$$

By using those,

$$\begin{aligned} 2Q_{11}\Delta\Delta_k &= (C''E_{r+1}+D''I_{r+1})(A_kE_{(r+1)k}+B_kI_{(r+1)k}) \\ &\quad -(C''_kE_{(r+1)k}+D''_kI_{(r+1)k})(AE_{r+1}+BI_{r+1}) \quad \cdots\cdots (94) \end{aligned}$$

Therefore, the reactive power circle diagram on the coordinate of the terminal $(r+1)$, $P_{(r+1)(r+1)}+jQ_{(r+1)(r+1)}$, with assuming P_{11} as a constant, is shown by the following vector equation like as before.

$$r'_{qr}\varepsilon^{j\phi'_r} = -C'_{q(r+1)(r+1)} + (P_{(r+1)(r+1)} + jQ_{(r+1)(r+1)}) \quad \cdots\cdots (95)$$

where

$$C'_{q(r+1)(r+1)} = \frac{C''B_k - AD''_k}{-D''B_k + D'_kB}E^2_{r+1} \quad \cdots\cdots (29)$$

$$r'_{qr} = \sqrt{2Q_{11}\Delta\Delta_kR'_{qr} + R'^2_{q(r+1)(r+1)}} \quad \cdots\cdots (96)$$

and

$$R'_{qr}=\left|\frac{E^2_{r+1}}{-D''B_k+D''_kB}\right| \quad \cdots\cdots (97)$$

$$R'_{q(r+1)(r+1)}=\left|\frac{-AD''+BC''}{-D''B_k+D''_kB}E^2_{r+1}\right| \quad \cdots\cdots (29)$$

The reactive power of the terminal 1 is then

$$Q_{11}=-\frac{(r'_{qr}+R'_{q(r+1)(r+1)})\,(r'_{qr}-R'_{q(r+1)(r+1)})}{2\Delta\Delta_kR'_{qr}} \quad \cdots\cdots (98)$$

If $Q_{11}=0$, we get $r_{pr}=R'_{p(r+1)(r+1)}$, and the above circle coincides with the 1. p. f. circle diagram. The maximum sending power is reached when $r_{pr}=0$, that is on the center of the circle,

$$Q_{11\,\max}=\frac{R'^2_{q(r+1)(r+1)}}{2\Delta\Delta_kR'_{qr}}=\frac{|E_{r+1}|^2}{2(D''B_k-D'_kB)}=\frac{R'_{pr}}{2} \quad \cdots\cdots (99)$$

The above analyses are applicable to every terminal of the interconnected power system.

Second, a receiving-end reactive power will be shown on a power coordinate of a sending-end. For an example, a reactive power $Q_{(r+1)(r+1)}$ of the receiving-end $(r+1)$ is to be shown on a power coordinate of the sending-end 1, $P_{11}+jQ_{11}$.

$$2Q_{(r+1)(r+1)}=I_{(r+1)}E_{(r+1)k}-I_{(r+1)k}E_{(r+1)}$$

If $E_{(r+1)}$ and $I_{(r+1)}$ are so expressed that all sending voltages or powers except the terminal 1 are assumed to be constant, being contained to their line constants, we get

$$\begin{aligned}2Q_{(r+1)(r+1)}&=(-C''E_1+AI_1)\,(D''_kE_{1k}-B_kI_{1k})\\&\quad-(-C''_kE_{1k}+A_kI_{1k})\,(D''E_1-BI_1) \quad \cdots\cdots (100)\end{aligned}$$

Or if $E_{(r+1)}$ and $I_{(r+1)}$ are such that all the receiving-end voltages or powers except the terminal $(r+1)$ are assumed to be constant, then using $E_1=A'E_{r+1}+BI_{r+1}$, $I_1=C'E_{r+1}+DI_{r+1}$,

$$\left.\begin{aligned}E_{r+1}&=\frac{DE_1-BI_1}{A'D-BC'}=\frac{1}{\Delta'}(DE_1-BI_1)\\I_{r+1}&=\frac{-C'E_1+A'I_1}{A'D-BC'}=\frac{1}{\Delta'}(-C'E_1+A'I_1)\end{aligned}\right\} \quad \cdots\cdots (58)$$

we get

$$\begin{aligned}2Q_{(r+1)(r+1)}&=\frac{1}{\Delta'\Delta'_k}\{(-C'E_1+A'I_1)\,(D_kE_{1k}-B_kI_{1k})\\&\quad-(-C'_kE_{1k}+A'_kI_{1k})\,(DE_1-BI_1)\} \quad \cdots\cdots (101)\end{aligned}$$

From the first, Eq. (100)

$$r_{qs}\varepsilon^{j\varphi_s} = -C_{q11} + (P_{11} + jQ_{11}) \quad \cdots\cdots (102)$$

and from the second, Eq. (101)

$$r'_{qs}\varepsilon^{j\varphi'_s} = -C'_{q11} + (P_{11} + jQ_{11}) \quad \cdots\cdots (103)$$

are respectively obtained, where

$$r_{qs} = \sqrt{R_{q11}^2 - 2Q_{(r+1)(r+1)} R_{qs}} \quad \cdots\cdots (104)$$

$$R_{qs} = \left| \frac{E_1^2}{AB_k - A_kB} \right| \quad \cdots\cdots (105)$$

$$\left.\begin{aligned} R_{q11} &= \left| \frac{-AD'' + BC''}{AB_k - A_kB} E_1^2 \right| \\ C_{q11} &= \frac{C''B_k - D''A_k}{AB_k - A_kB} E_1^2 \end{aligned}\right\} \quad \cdots\cdots (26)$$

$$r'_{qs} = \sqrt{R'^2_{q11} - 2Q_{(r+1)(r+1)}\Delta'\Delta'_k R'_{qs}} \quad \cdots\cdots (106)$$

$$R'_{qs} = \left| \frac{E_1^2}{A'B_k - A'_kB} \right| \quad \cdots\cdots (107)$$

$$\left.\begin{aligned} R'_{q11} &= \left| \frac{-A'D + BC'}{A'B_k - A'_kB} E_1^2 \right| \\ C'_{q11} &= \frac{C'B_k - DA'_k}{A'B_k - A'_kB} E_1^2 \end{aligned}\right\} \quad \cdots\cdots (23)$$

From the radius of the circles, the reactive power on the receiving-end coordinate $(r+1)$, is

$$Q_{(r+1)(r+1)} = -\frac{(r_{qs} + R_{q11})(r_{qs} - R_{q11})}{2R_{qs}} \quad \cdots\cdots (108)$$

or

$$Q_{(r+1)(r+1)} = -\frac{(r'_{qs} + R'_{q11})(r'_{qs} - R'_{q11})}{2\Delta'\Delta'_k R'_{qs}} \quad \cdots\cdots (109)$$

If $Q_{(r+1)(r+1)}=0$, it respectively coincides with the 1. p. f. circle diagram which has a radius, R_{q11} or R'_{q11}, and inside of the cicrle shows the possitive (capacitive) power and outside shows the negative (inductive) power. The maximum power is reached when $r_{qs}=r'_{qs}=0$, of which the value is

$$Q_{(r+1)(r+1,\max)} = \frac{R_{q11}^2}{2R_{qs}} \quad \cdots\cdots (110)$$

or

$$Q_{(r+1)(r+1)\max} = \frac{R'^2_{q11}}{2\Delta'\Delta'_k R'_{qs}} = \frac{R'_{qs}}{2} \quad \cdots\cdots (111)$$

Third, the reactive branch power $Q_{1(r+1)}$ which is transmitted from the total reactive power Q_{11} of terminal 1 to the receiving-end $(r+1)$, will be shown on a coordinate of the total receiving power $P_{(r+1)(r+1)}+jQ_{(r+1)(r+1)}$ of the terminal $(r+1)$.

$$\begin{aligned} 2Q_{1(r+1)} &= (\boldsymbol{A}'_k\boldsymbol{E}_{(r+1)k}+\boldsymbol{B}_k\boldsymbol{I}_{(r+1)k})(\boldsymbol{C}\boldsymbol{E}_{r+1}+\boldsymbol{D}\boldsymbol{I}_{r+1}) \\ &\quad -(\boldsymbol{A}'\boldsymbol{E}_{(r+1)}+\boldsymbol{B}\boldsymbol{I}_{(r+1)})(\boldsymbol{C}_k\boldsymbol{E}_{(r+1)k}+\boldsymbol{D}_k\boldsymbol{I}_{(r+1)k}) \quad \cdots\cdots (112) \end{aligned}$$

whence

$$r''_{qr}\varepsilon^{j\psi''_r} = -\boldsymbol{C}''_{q(r+1)(r+1)}+(P_{(r+1)(r+1)}+jQ_{(r+1)(r+1)}) \quad \cdots\cdots (113)$$

To the contrary, the branch power $Q_{(r+1)1}$ which is transmitted from terminal 1 and is actually a part of the total receiving reactive power, $Q_{(r+1)(r+1)}$, of the terminal $(r+1)$, is likewise shown on the coordinate, $P_{11}+jQ_{11}$, of the terminal 1 power.

$$\begin{aligned} 2Q_{(r+1)1} &= (\boldsymbol{D}''_k\boldsymbol{E}_{1k}-\boldsymbol{B}_k\boldsymbol{I}_{1k})(-\boldsymbol{C}\boldsymbol{E}_1+\boldsymbol{A}\boldsymbol{I}_1) \\ &\quad -(\boldsymbol{D}''\boldsymbol{E}_1-\boldsymbol{B}\boldsymbol{I}_1)(-\boldsymbol{C}_k\boldsymbol{E}_{1k}+\boldsymbol{A}_k\boldsymbol{I}_{1k}) \quad \cdots\cdots (114) \end{aligned}$$

whence

$$r''_{qs}\varepsilon^{j\psi''_s} = -\boldsymbol{C}''_{q11}+(P_{11}+jQ_{11}) \quad \cdots\cdots (115)$$

where on both

$$r''_{qr} = \sqrt{2Q_{11}R''_{qr}+R''^2_{q(r+1)(r+1)}} \quad \cdots\cdots (116)$$

$$R''_{qr} = \left|\frac{E^2_{r+1}}{-\boldsymbol{D}\boldsymbol{B}_k+\boldsymbol{D}_k\boldsymbol{B}}\right| \quad \cdots\cdots (90)$$

$$\left.\begin{aligned} R''_{q(r+1)(r+1)} &= \left|\frac{-\boldsymbol{A}'\boldsymbol{D}+\boldsymbol{B}\boldsymbol{C}}{-\boldsymbol{D}\boldsymbol{B}_k+\boldsymbol{D}_k\boldsymbol{B}}E^2_{r+1}\right| \\ \boldsymbol{C}''_{q(r+1)(r+1)} &= \frac{\boldsymbol{C}\boldsymbol{B}_k-\boldsymbol{A}'\boldsymbol{D}_k}{-\boldsymbol{D}\boldsymbol{B}_k+\boldsymbol{D}_k\boldsymbol{B}}E^2_{r+1} \end{aligned}\right\} \quad \cdots\cdots (44)$$

and

$$r''_{qs} = \sqrt{R''^2_{q11}-2Q_{11}R''_{qs}} \quad \cdots\cdots (117)$$

$$R''_{qs} = \left|\frac{E^2_1}{\boldsymbol{A}\boldsymbol{B}_k-\boldsymbol{A}_k\boldsymbol{B}}\right| = R_{qs} \quad \cdots\cdots (105)$$

$$\left.\begin{aligned} R''_{q11} &= \left| \frac{-AD''+BC}{AB_k - A_k B} E_1^2 \right| \\ C''_{q11} &= \frac{CB_k - D''A_k}{AB_k - A_k B} E_1^2 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (43)$$

Moreover, similarly to the process of effective power circle diagrams discussed above in Section 7, Q_{11}, the reactive power of the sending-end 1 can be shown on a coordinate of $P_{(r+1)1}+jQ_{(r+1)1}$ which is a branch power transmitted from terminal 1 to terminal $(r+1)$ and which is a part of the total receiving power of the terminal $(r+1)$; or the reactive power $Q_{(r+1)(r+1)}$ of the terminal $(r+1)$ can be shown on a coordinate of $P_{1(r+1)}+jQ_{1(r+1)}$ which is a part of $P_{11}+jQ_{11}$ and is a component transmitted from the terminal $(r+1)$; etc,

Since these theoretical treatments are quite identical to those of Section 7, they will be omitted with details here.

10. Effective Conductance and Effective Susceptance Power Circle Diagrams of Interconnected Transmission Systems.

As in the previous section, admittance circle diagrams are discussed assuming the power factor θ to be constant in a formula of $w=G+j\beta = |w|\,\varepsilon^{j\theta}$. In this section, however, cases of assuming G or B to be constants are taken up.

First, G will be constant, calling the effective conductance circle diagrams. If sending-end voltages or powers are assumed constant, and an admittance of the sending-end 1 is expressed by receiving terminal conditions with taking only the terminal $(r+1)$ among receiving-ends as a parameter,

$$W_{11} = \frac{C'+Dw_{r+1}}{A'+Bw_{r+1}} E_1^2 = \frac{C'+D\,(G_{r+1}+jB_{r+1})}{A'+B\,(G_{r+1}+jB_{r+1})} E_1^2 \quad \cdots\cdots\cdots \quad (118)$$

If G_{r+1} is constant

$$W_{11} = \frac{(C'+DG_{r+1})+jDB_{r+1}}{(A'+BG_{r+1})+jBB_{r+1}} E_1^2 \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (119)$$

whence the center and radius of a circle are derived instantly.

$$\left.\begin{aligned} C_{g11} &= \frac{C'B_k + DA'_k + 2DB_kG_{r+1}}{A'B_k + A'_kB + 2BB_kG_{r+1}} E_1^2 \\ r_{g11} &= \left| \frac{A'D - BC'}{A'B_k + A'_kB + 2BB_kG_{r+1}} E_1^2 \right| = \frac{E_1^2 R_{p11}}{2B^2R'_{ps}G_{r+1} + E_1^2} \end{aligned}\right\} \quad (120)$$

where R'_{ps} is given by Eq. (65) and R_{p11} by Eq. (22b). While, the effective conductance G_{r+1} is given from the above equations as

$$G_{r+1} = \frac{E_1^2}{B^2}\left(\frac{1}{2R'_{ps}} \mp \frac{R_{p11}}{2r_{g11}R'_{ps}}\right) \qquad (121)$$

In the like manner, if sending-end admittances are assumed as constants instead of receiving-end admittances, from Eq. (14)

$$W_{11} = \frac{C'' + D''w_{r+1}}{A + Bw_{r+1}} E_1^2$$

may be used, from which

$$\left.\begin{aligned} C'_{g11} &= \frac{C''B_k + D''A_k + 2D''B_kG_{r+1}}{AB_k + A_kB + 2BB_kG_{r+1}} E_1^2 \\ r'_{g11} &= \left| \frac{AD'' - BC''}{AB_k + A_kB + 2BB_kG_{r+1}} E_1^2 \right| \end{aligned}\right\} \qquad (122)$$

and

$$G_{r+1} = \frac{E_1^2}{B^2}\left(\frac{1}{2R_{ps}} \mp \frac{R_{p11}}{2r_{g11}R_{ps}}\right) \qquad (123)$$

where R_{ps} is given by Eq. (63) and R_{p11} by Eq. (25).

Next these will be shown on receiving-end coordinates. According to wheather one assumes sending admittances as constants

$$W_{(r+1)(r+1)} = \frac{-C'' + Aw_1}{D'' - Bw_1} E_{r+1}^2$$

or receiving admittances as constants

$$W_{(r+1)(r+1)} = \frac{-C' + A'w_1}{D - Bw_1} E_{r+1}^2$$

there are two ways of response. Namely, from the former, the center or a radius of the effective conductance circle diagram is given by

$$\left.\begin{aligned} C_{g(r+1)(r+1)} &= \frac{-C''B_k - AD''_k + 2AB_kG_1}{D''B_k + D''_kB - 2BB_kG_1} E_{r+1}^2 \\ r_{g(r+1)(r+1)} &= \left| \frac{-AD'' + BC''}{D''B_k + D''_kB - 2BB_kG_1} E_{r+1}^2 \right| \end{aligned}\right\} \qquad (124)$$

$$G_1 = \frac{E_{r+1}^2}{B^2}\left(\frac{1}{2R'_{pr}} \mp \frac{R_{p(r+1)(r+1)}}{2r_{g(r+1)(r+1)}R'_{pr}}\right) \qquad (125)$$

where R'_{pr} and $R_{p(r+1)(r+1)}$ are given by Eq. (54) and Eq. (28). Also from the later

$$\left.\begin{aligned} C_{g(r+1)(r+1)} &= \frac{-C'B_k - A'D_k + 2A'B_kG_1}{DB_k + D_kB - 2BB_kG_1} E_{r+1}^2 \\ r_{g(r+1)(r+1)} &= \left| \frac{-A'D + BC'}{DB_k + D_kB - 2BB_kG_1} E_{r+1}^2 \right| \end{aligned}\right\} \cdots\cdots\cdots (126)$$

$$G_1 = \frac{E_{r+1}^2}{B^2}\left(\frac{1}{2R_{pr}} \pm \frac{R_{p(r+1)(r+1)}}{2r_{g(r+1)(r+1)}R_{pr}}\right) \cdots\cdots\cdots (127)$$

where R_{pr} and $R_{p(r+1)(r+1)}$ are given by Eq. (46) and Eq. (31).
and

$$\left.\begin{aligned} C_{g(r+1)1} &= \frac{-CB_k - AD_k'' + 2AB_kG_1}{D''B_k + D_k''B - 2BB_kG_1} E_{r+1}^2 \\ r_{g(r+1)1} &= \left| \frac{-AD'' + BC}{D''B_k + D_k''B - 2BB_kG_1} E_{r+1}^2 \right| \end{aligned}\right\} \cdots\cdots\cdots (130)$$

$$G_1 = \frac{E_{r+1}^2}{B^2}\left(\frac{1}{2R_{pr}'} \pm \frac{R_{p(r+1)1}}{2r_{g(r+1)1}R_{pr}'}\right) \cdots\cdots\cdots (131)$$

$R_{p(r+1)1}$, R_{pr}' are given by Eq. (37) and (54).

On the other hand, for a sending-end branch admittance, from Eq. (33)

$$W_{1(r+1)} = \frac{C + Dw_{r+1}}{A' + Bw_{r+1}} E_1^2 \cdots\cdots\cdots (33)$$

and for a receiving-end branch admittance,

$$W_{(r+1)1} = \frac{-C + Aw_1}{D'' - Bw_1} E_1^2 \cdots\cdots\cdots (36)$$

are obtained respectively. Starting from those equations, their conductance circle diagrams are obtained in the following way.

$$\left.\begin{aligned} C_{g1(r+1)} &= \frac{CB_k + DA_k' + 2DB_kG_{r+1}}{A'B_k + A_k'B + 2BB_kG_{r+1}} E_1^2 \\ r_{g1(r+1)} &= \left| \frac{A'D - BC}{A'B_k + A_k'B + 2BB_kG_{r+1}} E_1^2 \right| \end{aligned}\right\} \cdots\cdots\cdots (128)$$

$$G_{r+1} = \frac{E_1^2}{B^2}\left(\frac{1}{2R_{ps}'} \mp \frac{R_{p1(r+1)}}{2r_{g1(r+1)}R_{ps}'}\right) \cdots\cdots\cdots (129)$$

$R_{p1(r+1)}$, R_{bs}' are given by Eq. (34) and (65).

Also on the contrary, even if terminal powers are expressed by branch power as in Eq. (39) or (40), conductance circle diagrams about both cases can be induced in similar ways.

Second, an effective susceptance circle diagram is taken. It is quite identical to that of the effective conductance circle diagram just discussed except for treating B as a constant instead of G. For an example, that on a sending-end coordinate and a receiving-end coordinate is expressed as follows:

$$\left.\begin{aligned} C_{b11} &= \frac{C'B_k - DA'_k + 2jDB_kB_{r+1}}{A'B_k - A'_kB + 2jBB_kB_{r+1}} E_1^2 \\ r_{b11} &= \left| \frac{A'D - BC'}{A'B_k - A'_kB + 2jBB_kB_{r+1}} E_1^2 \right| \end{aligned}\right\} \qquad (132)$$

$$B_{r+1} = \frac{E_1^2}{B^2}\left(\frac{1}{2R'_{qs}} \mp \frac{R_{q11}}{2r_{b11}R'_{qs}}\right) \qquad (133)$$

where R_{q11} and R'_{qs} are obtained by Eqs. (23) and (107) respectively.

And

$$\left.\begin{aligned} C_{b(r+1)(r+1)} &= \frac{C''B_k - AD''_k - 2jAB_kB_1}{-D''B_k + D''_kB + 2jBB_kB_1} E_{r+1}^2 \\ r_{b(r+1)(r+1)} &= \frac{-AD'' + BC''}{-D''B_k + D''_kB + 2jBB_kB_1} E_{r+1}^2 \end{aligned}\right\} \qquad (134)$$

$$B_1 = \frac{E_{r+1}^2}{B^2}\left(\frac{1}{2R'_{qr}} \pm \frac{R_{q(r+1)(r+1)}}{2r_{b(r+1)(r+1)}R'_{qr}}\right) \qquad (135)$$

where $R_{q(r+1)(r+1)}$ and R'_{qr} are derived from Eq. (29) and (97) respectively.

11. Resistance Circle Diagrams and Reactance Circle Diagrams of Interconnected Transmission Systems.

In this case a reciprocal $1/w$ is used instead of an admittance w. First, if receiving-end resistance is expressed on a sending-end coordinate with assuming only the resistance of the terminal $(r+1)$ as a parameter,

$$\begin{aligned} W_{11} &= \frac{C' + Dw_{r+1}}{A' + Bw_{r+1}} E_1^2 = \frac{C'/w_{r+1} + D}{A'/w_{r+1} + B} E_1^2 \\ &= \frac{C'(R_{r+1} + jX_{r+1}) + D}{A'(R_{r+1} + jX_{r+1}) + B} E_1^2 = \frac{(D + C'R_{r+1}) + jX_{r+1}C'}{(B + A'R_{r+1}) + jX_{r+1}A'} E_1^2 \end{aligned} \qquad (136)$$

The resistance circle diagram which is drawn by assuming R_{r+1} as a constant and X_{r+1} as changeable, is defined therefore by the following center and radius.

$$\left.\begin{aligned} C_{r\cdot 1} &= \frac{C'B_k + DA'_k + 2C'A'_kR_{r+1}}{A'B_k + A'_kB + 2A'A'_kR_{r+1}} E_1^2 \\ r_{r11} &= \left| \frac{A'D - BC'}{A'B_k + A'_kB + 2A'A'_kR_{r+1}} E_1^2 \right| = \frac{E_1^2 R_{p11}}{2R_{r+1}A'^2R'_{ps} + E_1^2} \end{aligned}\right\} \cdots (137)$$

whence the resistance of the terminal $(r+1)$, R_{r+1} is calculated by

$$R_{r+1} = \frac{E_1^2}{A'^2}\left(\frac{1}{2R'_{ps}} \mp \frac{R_{p11}}{2r_{r11}R'_{ps}}\right) \quad \cdots\cdots (138)$$

where R_{p11} and R'_{ps} are given by Eqs. (22b) and (65). As it is now assumed receiving-end voltages or powers are constant, if the next sending-end voltages or powers are assumed constant,

$$W_{11} = \frac{C'' + D''w_{r+1}}{A + Bw_{r+1}} E_1^2 = \frac{C''(R_{r+1} + jX_{r+1}) + D''}{A(R_{r+1} + jX_{r+1}) + B} E_1^2 \cdots (139)$$

is used. The center and radius of a resistance circle diagram is obtained in the same manner as before.

$$\left.\begin{aligned} C'_{r11} &= \frac{C''B_k + D''A_k + 2C''A_kR_{r+1}}{AB_k + A_kB + 2AA_kR_{r+1}} E_1^2 \\ r'_{r11} &= \left| \frac{AD'' - BC''}{AB_k + A_kB + 2AA_kR_{r+1}} E_1^2 \right| \end{aligned}\right\} \cdots\cdots (140)$$

$$R_{r+1} = \frac{E_1^2}{A^2}\left(\frac{1}{2R_{ps}} \mp \frac{R_{p11}}{2r'_{r11}R_{ps}}\right) \quad \cdots\cdots (141)$$

where R_{p11} and R_{ps} are given by Eqs. (25) and (63) respectively.

Then, that like to the previovs case now on receiving-end coordinates are as follows:

$$\begin{aligned} W_{(r+1)(r+1)} &= \frac{-C''(R_1 + jX_1) + A}{D''(R_1 + jX_1) - B} E_{r+1}^2 \\ &= \frac{(A - C''R_1) - jC''X_1}{(-B + D''R_1) + jD''X_1} E_1^2 \quad \cdots\cdots (142) \end{aligned}$$

or

$$\begin{aligned} W_{(r+1)(r+1)} &= \frac{-C'(R_1 + jX_1) + A'}{D(R_1 + jX_1) - B} E_{r+1}^2 \\ &= \frac{(A' - C'R_1) - jC'X}{(-B + DR_1) + jDX_1} E_{r+1}^2 \quad \cdots\cdots (143) \end{aligned}$$

The centers and radii of Eqs. (142) and (143) are respectively,

$$\left.\begin{aligned} C_{r(r+1)(r+1)} &= \frac{-C''B_k - AD''_k + 2C''D''_k R_1}{D''B_k + D''_k B - 2D''D''_k R_1} E^2_{r+1} \\ r_{r(r+1)(r+1)} &= \left| \frac{-AD'' + BC''}{D''B_k + D''_k B - 2D''D''_r R_1} E^2_{r+1} \right| \end{aligned}\right\} \cdots\cdots (144)$$

or

$$\left.\begin{aligned} C'_{r(r+1)(r+1)} &= \frac{-C'B_k - A'D_k + 2C'D_k R_1}{DB_k + D_k B - 2DD_k R_1} E^2_{r+1} \\ r'_{r(r+1)(r+1)} &= \left| \frac{A'D + BC'}{DB_k + D_k B - 2DD_k R_1} E^2_{r+1} \right| \end{aligned}\right\} \cdots\cdots (145)$$

Resistance, R_1 is accordingly,

$$R_1 = \frac{E^2_{r+1}}{D^2}\left(\frac{1}{2R'_{pr}} \pm \frac{R_{p(r+1)(r+1)}}{2r_{r(r+1)(r+1)} R'_{pr}}\right) \cdots\cdots\cdots (146)$$

or

$$R_1 = \frac{E^2_{r+1}}{D^2}\left(\frac{1}{2R_{p1}} \pm \frac{R_{p(r+1)(r+1)}}{2r'_{r(r+1)(r+1)} R_{pr}}\right) \cdots\cdots\cdots (147)$$

In Eq. (146), $R_{p(r+1)(r+1)}$ and R'_{pr} are given by Eqs. (28) and (54); in Eq. (147), $R_{p(r+1)(r+1)}$ and R_{pr} are given by Eqs. (31) and (46) respectively. With regard to sending stet and a receiving-end branch line resistances, the following equations are respectively obtained from Eqs. (33) and (36).

$$W_{1(r+1)} = \frac{C + Dw_{r+1}}{A' + Bw_{r+1}} E^2_1 = \frac{C(R_{r+1} + jX_{r+1}) + D}{A'(R_{r+1} + jX_{r+1}) + B} E^2_1 \cdots\cdots (148)$$

$$W_{(r+1)1} = \frac{-C + Aw_1}{D'' - Bw_1} E^2_1 = \frac{-C(R_1 + jX_1) + A}{D''(R_1 + jX_1) - B} E^2_1 \cdots\cdots (149)$$

For these, centers or radii are as follows:

$$\left.\begin{aligned} C_{r1(r+1)} &= \frac{CB_k + DA'_k + 2CA'_k R_{r+1}}{A'B_k + A'_k B + 2A'A'_k R_{r+1}} E^2_1 \\ r_{r1(r+1)} &= \left| \frac{A'D - BC}{A'B_k + A'_k B + 2A'A'_k R_{r+1}} E^2_1 \right| \end{aligned}\right\} \cdots\cdots (150)$$

$$R_{r+1} = \frac{E^2_1}{A'^2}\left(\frac{1}{2R'_{ps}} \mp \frac{R_{p1(r+1)}}{2r_{r1(r+1)} R'_{ps}}\right) \cdots\cdots\cdots (151)$$

where $R_{p1(r+1)}$, R'_{ps} are given by Eqs. (34) and (65).

$$\left.\begin{aligned} C_{r(r+1)1} &= \frac{-CB_k - AD''_k + 2CD''_k R_1}{D''B_k + D''_k B - 2D''D''_k R_1} E^2_{r+1} \\ r_{r(r+1)1} &= \left| \frac{-AD'' + BC}{D''B_k + D''_k B - 2D''D''_k R_1} E^2_{r+1} \right| \end{aligned}\right\} \cdots\cdots (152)$$

$$R_{r+1} = \frac{E_{r+1}^2}{D^2}\left(\frac{1}{2R'_{pr}} \pm \frac{R_{p(r+1)1}}{2r_{r(r+1)}R'_{pr}}\right) \quad \cdots\cdots (153)$$

where $R_{p(r+1)}$, R'_{pr} are given by Eqs. (37) and (54).

This time, a sending and a receiving-end of the power will be shown by the branch power by use of Eqs. (19) and (20).

$$W_{11} = \frac{C(R_{(r+1)1}+jX_{(r+1)1})+D''}{A(R_{(r+1)1}+jX_{(r+1)1})+B}E_1^2 \quad \cdots\cdots (154)$$

and

$$W_{(r+1)(r+1)} = \frac{-C(R_{1(r+1)}+jX_{1(r+1)})+A'}{D(R_{1(r+1)}+jX_{1(r+1)})-B}E_{r+1}^2 \quad \cdots\cdots (155)$$

$R_{(r+1)1}$ and $X_{(r+1)1}$ are respectively a resistance and a reactance of a branch impedance $1/w_{(r+1)1}$, and so forth. If a result regarding Eq. (154) is shown

$$\left.\begin{aligned} C''_{r11} &= \frac{CB_k+D''A_k+2CA_kR_{1(r+1)}}{AB_k+A_kB+2AA_kR_{1(r+1)}}E_1^2 \\ r''_{r11} &= \left|\frac{AD''-BC}{AB_k+A_kB+2AA_kR_{1(r+1)}}E_1^2\right| \end{aligned}\right\} \quad \cdots\cdots (156)$$

$$R_{1(r+1)} = \frac{E_1^2}{A^2}\left(\frac{1}{2R''_{ps}} \mp \frac{R''_{p11}}{2r''_{r11}R_{ps}}\right) \quad \cdots\cdots (157)$$

where R''_{ps} and R''_{p11} are given by Eqs. (63) and 41. That regarding Eq. (155) is obtained in like manner.

The same courses of analyses are applied also for reactance circle diagrams, in the cases of which the variables are resistances instead of reactances. For examples, the expression on the sending-end 1 coordinate with assuming receiving voltages or powers as constants, is

$$\left.\begin{aligned} W_{11} &= \frac{(D+jX_{r+1}C')+R_{r+1}C'}{(B+jX_{r+1}A')+R_{r+1}A'}E_1^2 \\ C'_{l11} &= \frac{C'B_k-DA'_k-2jC'A'_kX_{r+1}}{A'B_k-A'_kB-2jA'A_kX_{r+1}}E_1^2 \\ r_{l11} &= \left|\frac{-A'D+BC'}{A'B_k-A'_kB-2jA'A'_kX_{r+1}}E_1^2\right| \end{aligned}\right\} \quad \cdots\cdots (158)$$

and with assuming sending voltages or powers as constants except the end 1, it is

$$W_{11} = \frac{(D''+jX_{r+1}C'')+R_{r+1}C''}{(B+jX_{r+1}A)+R_{r+1}A}E_1^2 \quad \cdots\cdots (159)$$

$$\left.\begin{aligned} C'_{l11} &= \frac{C''B_k - D''A_k - 2jC''A_kX_{r+1}}{AB_k - A_kB - 2jAA_kX_{r+1}} E_1^2 \\ r'_{l11} &= \left| \frac{-AD'' + BC''}{AB_k - A_kB - 2jAA_kX_{r+1}} E_1^2 \right| \end{aligned}\right\} \cdots\cdots (160)$$

Also, the expressions on the receiving-end $(r+1)$ coordinate with assuming sending voltages or powers and receiving voltages or powers as constants, are respectively as follows:

$$W_{(r+1)(r+1)} = \frac{(A - jX_1C'') - R_1C''}{(-B + jX_1D'') + R_1D''} E_{r+1}^2 \cdots\cdots\cdots\cdots (161)$$

$$\left.\begin{aligned} C_{l(r+1)(r+1)} &= \frac{C''B_k - AD''_k + 2jC''D''_kX_1}{-D''B_k + D''_kB - 2jD''D''_kX_1} E_{r+1}^2 \\ r_{l(r+1)(k+1)} &= \left| \frac{AD'' - BC''}{-D''B_k + D''_kB - 2jD''D''_kX_1} E_{r+1}^2 \right| \end{aligned}\right\} \cdots\cdots (162)$$

and

$$W_{(r+1)(r+1)} = \frac{(A' - jX_1C') - R_1C'}{(-B + jX_1D) + R_1D} E_{r+1}^2 \cdots\cdots\cdots\cdots (163)$$

$$\left.\begin{aligned} C'_{l(r+1)(r+1)} &= \frac{C'B_k - A'D_k + 2jC'D_kX_1}{-DB_k + D_kB - 2jDD_kX_1} E_{r+1}^2 \\ r'_{l(r+1)(r+1)} &= \left| \frac{A'D - BC'}{-DB_k + D_kB - 2jDD_kX_1} E_{r+1}^2 \right| \end{aligned}\right\} \cdots\cdots (164)$$

Next, regarding branch powers, the reactance circle diagrams for sending and receiving branch lines shown on sending and receiving coordinates respectively, are as follows:

$$W_{1(r+1)} = \frac{(D + jX_{r+1}C) + R_{r+1}C}{(B + jX_{r+1}A') + R_{r+1}A'} E_1^2 \cdots\cdots\cdots\cdots (165)$$

$$W_{(r+1)1} = \frac{(A - jX_1C) - R_1C}{(-B + jX_1D'') + R_1D''} E_{r+1}^2 \cdots\cdots\cdots\cdots (166)$$

whence results are:

$$\left.\begin{aligned} C_{l1(r+1)} &= \frac{CB_k - DA'_k - 2jCA'_kX_{r+1}}{A'B_k - A'_kB - 2jA'A'_kX_{r+1}} E_1^2 \\ r_{l1(r+1)} &= \left| \frac{-A'D + BC}{A'B_k - A'_kB - 2jA'A'_kX_{r+1}} E_1^2 \right| \end{aligned}\right\} \cdots\cdots (167)$$

$$\left.\begin{aligned} C_{l(r+1)1} &= \frac{CB_k - AD''_k + 2jCD''_kX_1}{-D''B_k + D''_kB - 2jD''D''_kX_1} E_{r+1}^2 \\ r_{l(r+1)1} &= \left| \frac{AD'' - BC}{-D''B_k + D''_kB - 2jD''D''_kX_1} E_{r+1}^2 \right| \end{aligned}\right\} \cdots\cdots (168)$$

On the contrary, sending and receiving-end reactance circle diagrams can be expressed on each branching coordinate respectively, like the resistance circle diagrams, so any discussion will be omitted here.

12. Apparent Admittance and Apparent Impedance Circle Diagrams of Interconnected Transmission Systems.

First, apparent admittance circle diagrams on the receiving-end coordinate, $P_{(r+1)(r+1)}+jQ_{(r+1)(r+1)}$, will be considered. (From Eq. 13),

$$w_1 = \frac{I_1}{E_1} = \frac{C'+Dw_{r+1}}{A'+Bw_{r+1}} = \frac{\left(w_{r+1}+\dfrac{C'}{D}\right)}{\left(w_{r+1}+\dfrac{A'}{B}\right)} \times \frac{D}{B} \quad \cdots\cdots\cdots \ (169)$$

While, from Eqs. (16a) and (16b),

Short circuit point $w_{(r+1)\infty} = -\dfrac{A'}{B}$.

Open circuit point $w_{(r+1)1} = -\dfrac{C'}{D}$.

are substituted in Eq. (169), resulting

$$|w| = \frac{\overline{w_{r+1}w_{(r+1)0}}}{\overline{w_{r+1}w_{(r+1)\infty}}} \times \left|\frac{D}{B}\right| \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \ (170)$$

If $|w_1|$ is a constant value, then a ratio of $\overline{w_{r+1}w_{(r+1)0}}$ (the length from w_{r+1} to an open circuit point) to $\overline{w_{r+1}w_{(r+1)\infty}}$ (the length from w_{r+1} to a short circuit point) is constant, of which the fact shows that it is represented by a circle centering on a line, $\overline{w_{(r+1)\infty}w_{(r+1)0}}$. (On a power cordinate, E^2_{r+1} is multiplied, making a point of w_{r+1} as $W_{(r+1)(r+1)}$)

Also if there is a condition $|w_1| = \left|\dfrac{D}{B}\right|$, this ratio becomes 1 and the circle becomes a perpendicular bisector of a line, $\overline{w_{(r+1)\infty}w_{(r+1)0}}$.

Generally, a value of $|w_1|$ is calculated by the following may in quite identical to that of an ordinary one-terminal transmission line. Namely, a circle which passes through points $w_{(r+1)\infty}$ and $w_{(r+1)0}$ and which centers at the bisecting point of a line $\overline{w_{(r+1)\infty}w_{(r+1)0}}$, is considered first, and the radius of this, say a_{r+1}, is obtained as follows:

$$a_{r+1} = \frac{1}{2}\left|\frac{A'}{B} - \frac{C'}{D}\right| = \frac{1}{2}\left|\frac{A'D-BC'}{BD}\right| \quad \cdots\cdots\cdots\cdots\cdots \ (171)$$

Now, let a new coordinate be considered of which the two axes are such that the bisector obtained now is $P'_{(r+1)(r+1)}$ axis and a line perpendicular to it is $Q'_{(r+1)(r+1)}$ axis. And an apparent circle diagram freely selected which is located on this new coordinate is to has a radius of $r_{y(r+1)}$ and has a center point of which the abscissa with $Q'_{(r+1)(r+1)}$ axis is $\beta_{y(r+1)}$. Then, we get

$$\beta^2_{y(r+1)} = a^2_{(r+1)} + r^2_{y(r+1)} \quad \cdots\cdots (172)$$

and

$$\frac{\overline{w_{r+1} w_{(r+1)0}}}{\overline{w_{r+1} w_{(r+1)\infty}}} = \frac{\beta_{y(r+1)} - a_{r+1}}{r_{y(r+1)}} \quad \cdots\cdots (173)$$

Accordingly,

$$|w_1| = \frac{\beta_{y(r+1)} - a_{r+1}}{r_{y(r+1)}} \cdot \frac{|\boldsymbol{D}|}{|\boldsymbol{B}|} \quad \cdots\cdots (174)$$

From Eqs. (172) and (174) also

$$\left.\begin{aligned} \beta_{y(r+1)} &= \frac{\dfrac{\boldsymbol{D}^2}{\boldsymbol{B}^2} + w_1^2}{\dfrac{\boldsymbol{D}^2}{\boldsymbol{B}^2} - w_1^2}\, a_{r+1} \\ r_{y(r+1)} &= \frac{2w_1 \left|\dfrac{\boldsymbol{D}}{\boldsymbol{B}}\right| a_{r+1}}{\dfrac{\boldsymbol{D}^2}{\boldsymbol{B}^2} - w_1^2} \end{aligned}\right\} \quad \cdots\cdots (175)$$

are obtained.

Since receiving-end voltages or powers have been handled as constants up till now, the same process is repeated in a case of handling sending-end voltages or powers as constants, the only difference being to enploy Eq. (14) instead of Eq. (13).

In the next, the diagram projected on the receiving-end 1 coordinate will be obtained. From Eq. (13),

$$w_{r+1} = \frac{w_1 - \dfrac{\boldsymbol{C}''}{\boldsymbol{A}}}{-w_1 + \dfrac{\boldsymbol{D}''}{\boldsymbol{B}}} \times \frac{\boldsymbol{A}}{\boldsymbol{B}} \quad \cdots\cdots (176)$$

Since a magnitude of w_{r+1}, $|w_{r+1}|$ is a constant,

$$|w_{r+1}| = \left| \frac{w_1 - \frac{C''}{A}}{w_1 - \frac{D''}{B}} \times \frac{A}{B} \right| = \frac{\overline{w_1 w_0}}{\overline{w_1 w_\infty}} \times \left| \frac{A}{B} \right| \qquad (177)$$

where $w_{1\infty}$ and w_{10} are respectively a short circuit and an open circuit point of the coordinate 1 as given by Eqs. (14a) and (14b).

If a line $\overline{w_{1\infty} w_{10}}$ and its perpendicular bisector are taken respectivly as the Q'_{11} axis and the P'_{11} axis of a new coördinate, and if the radius and the center of every apparent admittance circle diagram are r_{y1} and $(0, \beta_{y1})$ respectively, then we get:

$$|w_{r+1}| = \frac{\beta_{y1} - a_1}{r_{y1}} \left| \frac{A}{B} \right| \qquad (178)$$

where a_1 is the radius of a circle intersected with perpendicular to all circle diagrams.

$$a_1 = \frac{\overline{w_{1\infty} w_{10}}}{2} = \frac{1}{2} \left| \frac{AD'' - BC''}{AB} \right| \qquad (179)$$

and

$$\beta_{y1} = \frac{\frac{A^2}{B^2} + w^2_{r+1}}{\frac{A^2}{B^2} - w^2_{r+1}} a_1 \qquad (180)$$

$$r_{y1} = \frac{2w_{r+1} \left| \frac{A}{B} \right| a_1}{\frac{A^2}{B^2} - w^2_{r+1}} \qquad (181)$$

As those values are derived by assuming sending powers or voltages to be constant and by including them into coefficients, similar results are derived with constant receiving powers or voltages. Moreover, regarding to branch line admittances, $w_{1(r+1)}$ or $w_{(r+1)1}$, similar apparent admittance circle diagrams can be drawn.

Since, nextly, the apparent impedance circle diagrams must be drawn by use of only the reciprocals of quantities of the apparent admittance circle diagrams, the former can be represented by the apparent admittance circle diagrams themselves. As in Eq. (174) or (178), apparent impedances are obtained from the following equations.

$$|Z_1| = \frac{r_{y(r+1)}}{\beta_{y(r+1)} - a_{r+1}} \left| \frac{B}{D} \right| \qquad (182)$$

$$|Z_{r+1}| = \frac{r_{y1}}{\beta_{y1}-\alpha_1}\left|\frac{B}{D}\right| \qquad (183)$$

13. Current Angle and Voltage Angle Circle Diagrams of Interconnected Transmission Systems.

The ratio of a current I_1 of the sending-end 1 to a current I_{r+1} of the receiving-end $(r+1)$ is

$$\frac{I_{r+1}}{I_1} = \frac{|I_{r+1}|}{|I_1|}\varepsilon^{j\theta_{r+1}-j\theta_1} = p\varepsilon^{j\delta}$$

where

$$p = \frac{|I_{r+1}|}{|I_1|} \quad \text{and} \quad \delta = \theta_{r+1}-\theta_1$$

If δ is constant. Eq. (184) shows a circle, which is the current's power angle circle diagram as follows:

$$p\varepsilon^{j\delta} = \frac{AI_1 - C''E_1}{I_1} = \frac{AW_{11}-C''E_1^2}{W_{11}} = \frac{W_{11}-\frac{C''}{A}E_1^2}{W_{11}} \times A$$

$$\therefore \; W_{11} = \frac{C''E_1^2}{A-p\varepsilon^{j\delta}} \qquad (185)$$

that is, showing a group of circles which pass upon both an original point and an open circuit point, $\frac{C''}{A}E_1^2$.

When coefficients, C' and A' are used instead of C'' and D'' they are circles which pass through both an original point and an open circuit point, $\frac{C'}{A'}E_1^2$.

Next, a current I_1 and branch current $I_{(r+1)1}$ introduce, as in the preceding, the following circles,

$$\frac{I_{(r+1)1}}{I_1} = \frac{|I_{(r+1)1}|}{|I_1|}\varepsilon^{j\theta_{(r+1)1}-j\theta_1} = p'\varepsilon^{j\delta'}$$

$$p'\varepsilon^{j\delta'} = \frac{AI_1-CE_1}{I_1} = \frac{W_{(r+1)1}-\frac{C}{A}E_1^2}{W_{(r+1)1}} \times A \qquad (186)$$

passing an original point and an open circuit point of $\frac{C}{A}$.

In the next, voltages on both terminals are taken instead of currents,

$$\frac{\boldsymbol{E}_{r+1}}{\boldsymbol{E}_1} = \frac{|\boldsymbol{E}_{r+1}|}{|\boldsymbol{E}_1|} \varepsilon^{j\varphi_{r+1} - j\varphi_1} = q\varepsilon^{j\xi}$$

where

$$q = \frac{|\boldsymbol{E}_{r+1}|}{|\boldsymbol{E}_1|} \quad \text{and} \quad \xi = \varphi_{r+1} - \varphi_1 .$$

and if ξ is assumed as a constant, it shows a group of circles, called the voltage's power angle circle diagram,

$$q\varepsilon^{j\xi} = \frac{\boldsymbol{D}''\boldsymbol{E}_1 - \boldsymbol{B}\boldsymbol{I}_1}{\boldsymbol{E}_1} = \frac{\frac{\boldsymbol{D}''}{\boldsymbol{B}} E_1^2 + \boldsymbol{W}_{11}}{\boldsymbol{E}_1^2} \times \boldsymbol{B} \quad \cdots\cdots\cdots\cdots\cdots (188)$$

having a center on a short circuit point, $\frac{\boldsymbol{D}''}{\boldsymbol{B}} \boldsymbol{E}_1^2$. If coefficients $\boldsymbol{C}'$ and $\boldsymbol{A}'$ are used, a short circuit point becomes $\frac{\boldsymbol{D}}{\boldsymbol{B}} \boldsymbol{E}_1^2$.

Next, the relation between voltages $\boldsymbol{E}_1$ and $\boldsymbol{E}_{(r+1)1}$,

$$\frac{\boldsymbol{E}_{(r+1)1}}{\boldsymbol{E}_1} = q'\varepsilon^{j\delta'} = \frac{\boldsymbol{D}\boldsymbol{E}_1 - \boldsymbol{B}\boldsymbol{I}_1}{\boldsymbol{E}_1} = \frac{\frac{\boldsymbol{D}}{\boldsymbol{B}} E_1^2 - \boldsymbol{W}_{(r+1)1}}{E_1^2} \times \boldsymbol{B} \quad (189)$$

shows a group of circles centering at a srhot circuit point, $\frac{\boldsymbol{D}}{\boldsymbol{B}} E_1^2$. As those discussed till now are all shown on the sending coordinate 1, those are also shown on any sending coordinate, however, they need not be given on receiving coordinates so far as the power angles themselves are inverstigated.

14. Calculation of Voltages and Currents for Unbalanced Faults of Interconnected Transmission Systems.

Unbalanced alternating three phase current is quite precisely studied usually by the symmetrical coordinates. In many cases unbalanced current usually occurred during the faults, and it effects considerably remarkable results regarding such things as relay operations, apparatus heatings, mechanical stress, etc.

Generally, circuit constants of an interconnected transmission system are represented by four constants $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, if every two ends are selected among receiving-ends and sending-ends respectively with assuming other sending or receiving voltages or powers to be constant according to assumptions or conditions given. The detailed discussions about these points have been thoroughly worked out in previous sections. These constants now are transformed respectively to symmetrical components by means of the symmetrical coordinates ordinarily handled.

Furthermore, unbalanced voltages and currents of a sending-end, say $\boldsymbol{E}_{0a}$, $\boldsymbol{E}_{0b}$, $\boldsymbol{E}_{0c}$, and $\boldsymbol{I}_{0a}$, $\boldsymbol{I}_{0b}$, $\boldsymbol{I}_{0c}$, are transformed to their symmetrical connponents, say, $\boldsymbol{E}_{00}$, $\boldsymbol{E}_{01}$, $\boldsymbol{E}_{02}$, and $\boldsymbol{I}_{00}$, $\boldsymbol{I}_{01}$, $\boldsymbol{I}_{02}$. Similarly, those of a receiving-end $\boldsymbol{E}_{1a}$, $\boldsymbol{E}_{1b}$, E_{1c}, and $\boldsymbol{I}_{1a}$, $\boldsymbol{I}_{1b}$, I_{1c}, are transformed to $\boldsymbol{E}_{10}$, $\boldsymbol{E}_{11}$, $\boldsymbol{E}_{12}$, and $\boldsymbol{I}_{10}$, $\boldsymbol{I}_{11}$, $\boldsymbol{I}_{12}$. A suffix 0 refers to a sending-end and suffix 1 to a receiving-end.

It can be considered that each individual component exists separately, and the results are superposed as a whole after each case calculated. First, for the positive sequence components:

$$\left.\begin{aligned}\boldsymbol{E}_{01} &= \boldsymbol{A}_1\boldsymbol{E}_{11}+\boldsymbol{B}_1\boldsymbol{I}_{11}\\ \boldsymbol{I}_{01} &= \boldsymbol{D}_1\boldsymbol{I}_{11}+\boldsymbol{C}_1\boldsymbol{E}_{11}\end{aligned}\right\} \qquad (190)$$

Positive sequence current is quite the same as the ordinary three phase alternating current, therefore in Eq. (190) are nothing but the fundamental equations of a transmission system previously discussed. If $\boldsymbol{E}_{01}$ is taken as a nominal voltage of a generator, the synchronous impedance of a generator must be added in series to the coefficients, $\boldsymbol{A}_1$, $\boldsymbol{B}_1$, $\boldsymbol{C}_1$, $\boldsymbol{D}_1$.

Seconder, for the negative phase sequence components:

$$\left.\begin{aligned}\boldsymbol{E}_{02} &= \boldsymbol{A}_2\boldsymbol{E}_{12}+\boldsymbol{B}_2\boldsymbol{I}_{12}\\ \boldsymbol{I}_{02} &= \boldsymbol{D}_2 I_{12}+\boldsymbol{C}_2\boldsymbol{E}_{12}\end{aligned}\right\} \qquad (191)$$

If a sending voltage $\boldsymbol{E}_{02}$ is taken as a nominal generator voltage, $\boldsymbol{E}_{02}$ is equal to 0. In this case, the negative-sequence impedance of the generator must be added in series to the coefficients, $\boldsymbol{A}_2$, $\boldsymbol{B}_2$, $\boldsymbol{C}_2$, $\boldsymbol{D}_2$, that is, the negative sequence impedances of a transmission system, are quite equal to $\boldsymbol{A}_1$, $\boldsymbol{B}_1$, $\boldsymbol{C}_1$, $\boldsymbol{D}_1$ respectively, unless the system itself does not contain rotating machines.

Lastly, for the zero sequence components:

$$\left.\begin{aligned}\boldsymbol{E}_{00} &= \boldsymbol{A}_0 E_{10}+\boldsymbol{B}_0\boldsymbol{I}_{10}\\ \boldsymbol{I}_{00} &= \boldsymbol{D}_0 I_{10}+\boldsymbol{C}_0\boldsymbol{E}_{10}\end{aligned}\right\} \qquad (192)$$

If $\boldsymbol{E}_{00}$ is taken as a nominal voltage of a generator, then $E_{00}=0$.

(a) A Fault with a line to earth.

External conditions, $\boldsymbol{E}_{1a}=0$, $\boldsymbol{I}_{1b}=\boldsymbol{I}_{1c}=0$ directly follow the next relationships among sequence components.

$$E_{10}+E_{11}+E_{12} = E_{1a} = 0$$

$$I_{10} = I_{11} = I_{12}$$

From Eqs. (190) to (192), the following equations are obtained.

$$I_{11} = I_{10} = I_{12} = \frac{E_{01}}{A_1\left(\dfrac{B_0}{A_0}+\dfrac{B_1}{A_1}+\dfrac{B_2}{A_2}\right)} = \frac{E_{01}}{A_1(Z_0+Z_1+Z_2)} \quad \cdots (193)$$

$$\left.\begin{aligned} E_{11} &= \frac{(Z_0+Z_2)\,E_{01}}{A_1(Z_0+Z_1+Z_2)} \\ E_{10} &= \frac{-Z_0E_{01}}{A_1(Z_0+Z_1+Z_2)} \\ E_{12} &= \frac{-Z_2E_{01}}{A_1(Z_0+Z_1+Z_2)} \end{aligned}\right\} \quad \cdots\cdots (194)$$

$$\left.\begin{aligned} I_{01} &= \frac{\{D_1+C_1(Z_0+Z_2)\}\,E_{01}}{A_1(Z_0+Z_1+Z_2)} \\ I_{00} &= \frac{E_{01}}{A_0A_1(Z_0+Z_1+Z_2)} \\ I_{02} &= \frac{E_{01}}{A_2A_1(Z_0+Z_1+Z_2)} \end{aligned}\right\} \quad \cdots\cdots (195)$$

where

$$Z_0 = \frac{B_0}{A_0}, \quad Z_1 = \frac{B_1}{A_1}, \quad Z_2 = \frac{B_2}{A_2} \quad \cdots\cdots (196)$$

(b) A Fault with two lines to earth.

From terminal conditions, $E_{1b}=E_{1c}=0$, $I_{1a}=0$ there are obtained the following relations of symmetrical components.

$$E_{10} = E_{11} = E_{12}$$

$$I_{10}+I_{11}+I_{12} = I_{1a} = 0$$

By substituting the conditions in Eqs. (190), (191), and (192) with assuming $E_{00}=E_{02}=0$ just as before, the following results are obtained.

$$\left.\begin{aligned} I_{11} &= \frac{(y_0+y_2)\,E_{01}}{B_1(y_0+y_1+y_2)} \\ I_{10} &= \frac{-y_0E_{01}}{B_1(y_0+y_1+y_2)} \end{aligned}\right\} \quad \cdots\cdots (197)$$

$$\left.\begin{aligned} I_{12} &= \frac{-y_2 E_{01}}{B_1(y_0+y_1+y_2)} \\ I_{01} &= \frac{\{C_1+D_1(y_0+y_2)\}\,E_{01}}{B_1(y_0+y_1+y_2)} \\ I_{0\,1} &= \frac{-E_{01}}{B_0 B_1(y_0+y_1+y_2)} \\ I_{02} &= \frac{-E_{01}}{B_1 B_2(y_0+y_1+y_2)} \end{aligned}\right\} \quad \cdots\cdots (198)$$

where

$$\left.\begin{aligned} y_0 &= \frac{1}{Z_0} = \frac{A_0}{B_0} \\ y_1 &= \frac{1}{Z_1} = \frac{A_1}{B_1} \\ y_2 &= \frac{1}{Z_2} = \frac{A_2}{B_2} \end{aligned}\right\} \quad \cdots\cdots (199)$$

(c) A Fault among two lines.

External conditions are $E_{1b}=E_{1c}$, $I_{1b}=-I_{1c}$ and $I_{1a}=0$. Conditions converted to the symmetrical components, are and $I_{10}=0$, $E_{10}=0$, $I_{11}=-I_{12}$ and $E_{11}=E_{12}$.

Accordingly, the following results are obtained in the same way as before.

$$\left.\begin{aligned} I_{11} &= -I_{12} = \frac{y_2 E_{01}}{A_1(Z_1+Z_2)} \\ I_{10} &= 0 \end{aligned}\right\} \quad \cdots\cdots (200)$$

$$\left.\begin{aligned} I_{01} &= \frac{(C_1+D_1 y_2)\,E_{01}}{A_1(Z_1+Z_2)} \\ I_{02} &= \frac{(C_2-D_2 y_2)\,E_{01}}{A_1(Z_1+Z_2)} \\ I_{00} &= 0 \end{aligned}\right\} \quad \cdots\cdots (201)$$

Up to this present point, three kinds of faults are dealt with. These faults are assumed to have occurred directly, without such an occurrence as an arc, etc. If arcs are attended with the faults, arcs are replaced in their equivalent resistances.

The above discussions assume voltages or currents at a fault point being known, and calculate unbalanced voltages or currents of a sending-end. If unbalanced voltages or currents in other points, such as a load point, are to be calculated, they are easily obtained as voltages or currents at a fault point are already known. Namely, first, coefficients, A, B, C, D of lines from a fault point to any other points, such as a load point, are calculated, and second, operations are calculated as in the previous procedures.

15. Conclusion.

The electrical characteristics of an interconnected transmission system are analyzed by use of characterized circuit constants $[\boldsymbol{A}]$, $[\boldsymbol{B}]$, $[\boldsymbol{C}]$ and $[\boldsymbol{D}]$. These matrices constants must be all of square and of same order, as is discussed in the paper. This assumption, however, is relatively easily accomplished by modifying or correcting the system or by presuming appropriate stations to be certain definite stations.

Examples show the direct details of the theories. According to whether assuming sending-ends to be constant or assuming receiving-ends constant, there are considerable differences in the results, as clearly seen in Figs. 8 to 19. This assumption must be duely executed by inspecting or examining the distribution, the operation and the capacities of actual generators, loads and characteristics of the system, in order to obtain the most suitable and correct approximation of results.

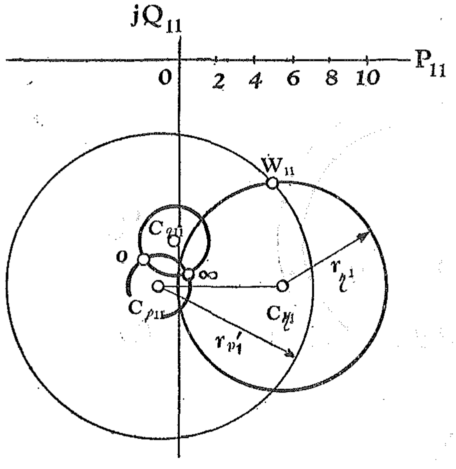


Fig. 8.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficiency circle, between stations Ur. and Su.

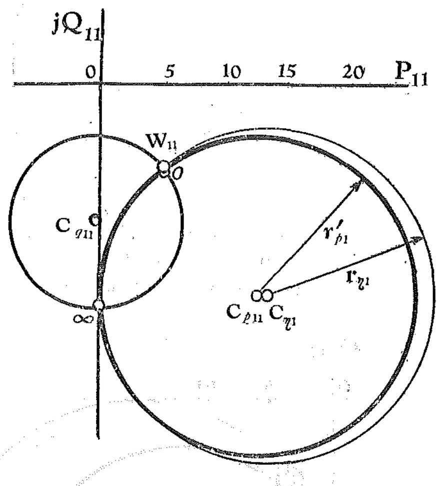


Fig. 9.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficiency circle, between stations Ur. and Sp.

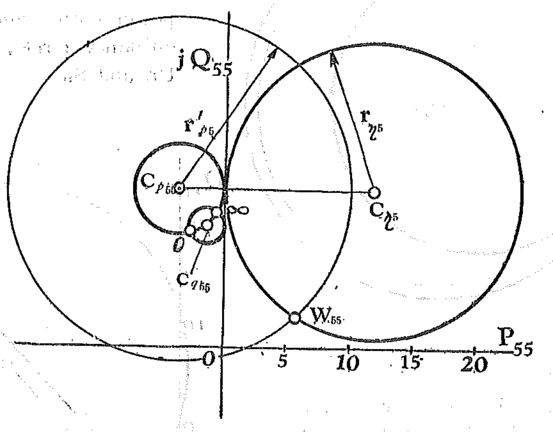


Fig. 10.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficiency circle, between stations Su. and Ur.

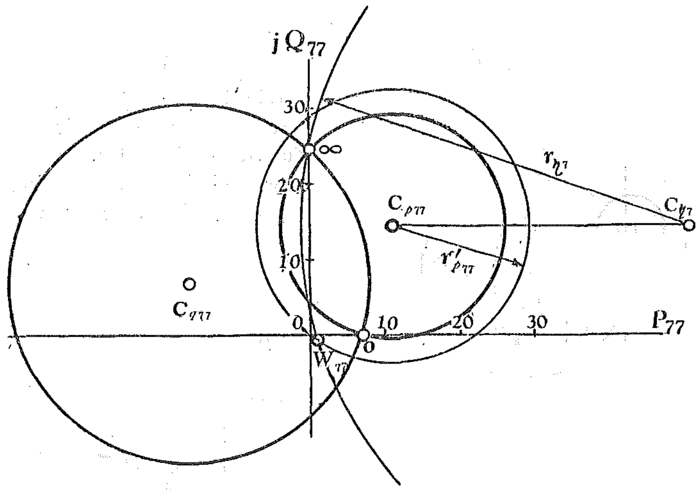


Fig. 11.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficiency circle, between stations Sp. and Ur.

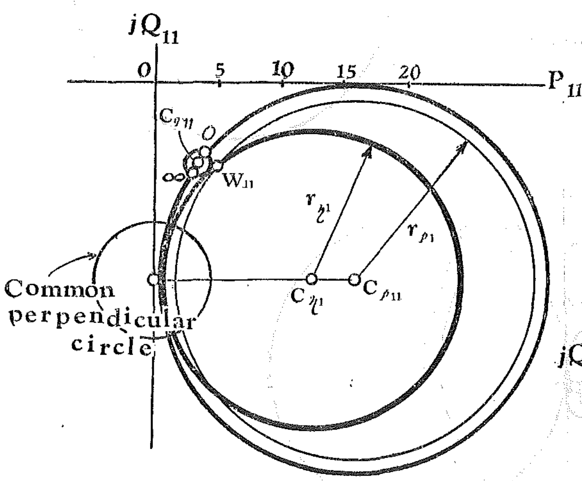
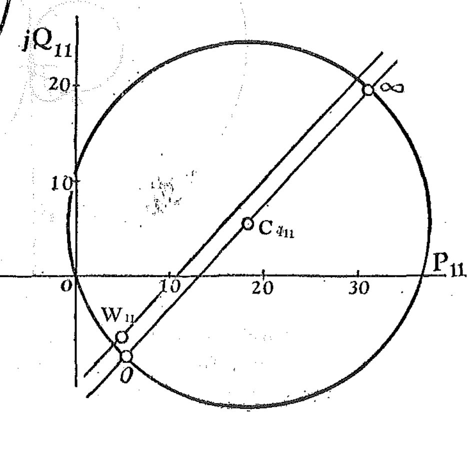


Fig. 13.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficiency circle, between stations Ur. and Sp.

Fig. 12.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficiency circle, between stations Ur. and Su.



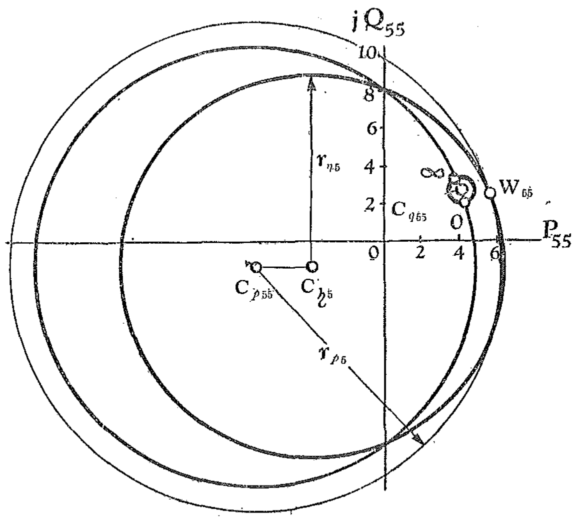


Fig. 14.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficiency circle, between stations Su. and Ur.

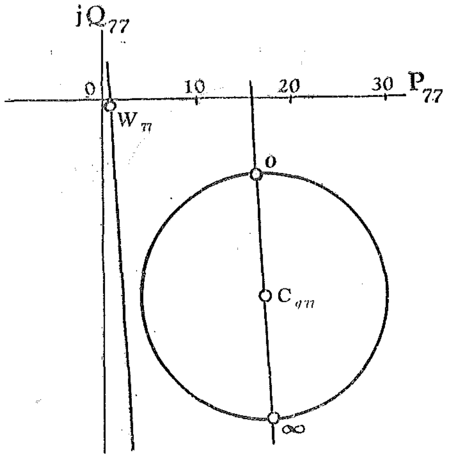


Fig. 15.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficiency circle, between stations Sp. and Ur.

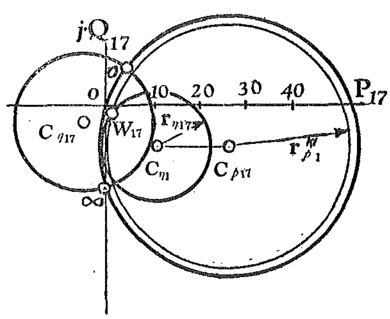


Fig. 16.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficiency circle, between stations Ur. and Sp.

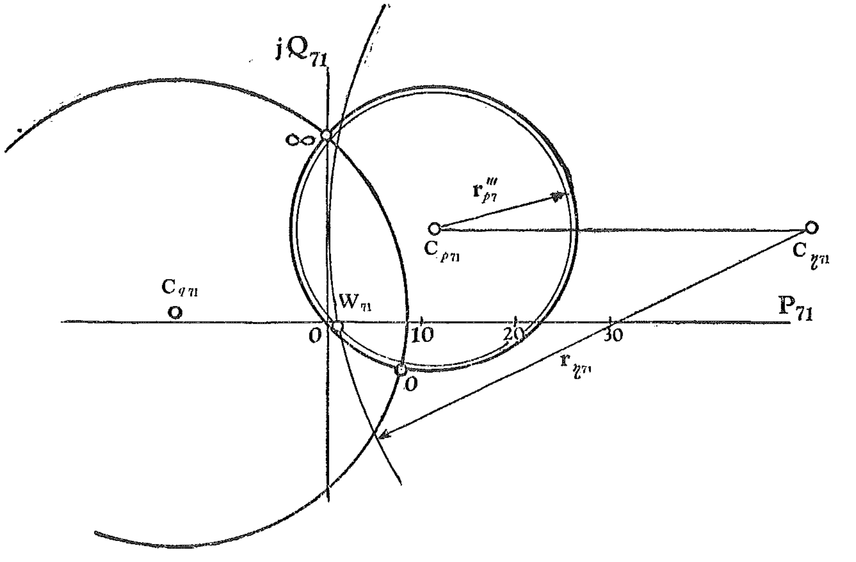


Fig. 17.

Circle diagram showing one short circuit and one open circuit points, fundamental circles, effective power circle, and a sending ratio efficiency circle, between stations Sp. and Ur.

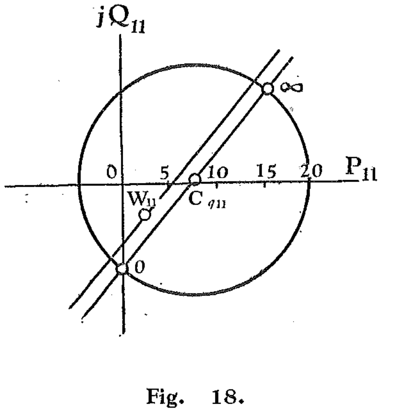


Fig. 18.

Circle diagrams showing one short circuit and one open circuit points, fundamental circles an effective power circle, and a sending ratio efficiency circle, between stations Ur. and Sp.

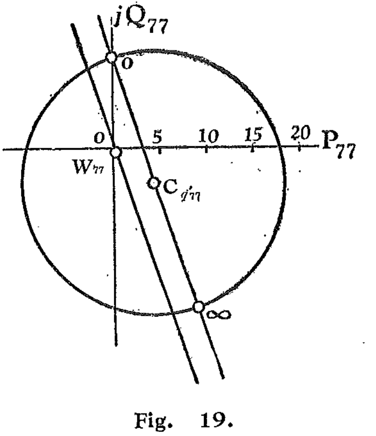


Fig. 19.

Circle diagrams showing one short circuit and one open circuit point, fundamental circles, an effective power circle, and a sending ratio efficinncy circle, between stations Sp. and Ur.