Carbon Piles for the Automatic Voltage Regulator.

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Abstract.

This paper deals with the characteristics of the carbon piles and a graphical solution of the constant voltage apparatus, and with the theoretical considerations of the procedure in design.

1. Introduction.

The carbon pile is now widely used as a component part of constant voltage apparatus. Its characteristic merits are that it is very simple in mechanism and durable against mechanical vibrations. But on the other hand, the demerits are the relatively large voltage regulation and hysteresis. Moreover there are not yet available reasonable specifications for the carbon pile and this is the great obstacle to designing and using it.

This paper deals with the characteristics and the graphical soltion of the operating characteristics of the constant voltage apparatus.

2. General characteristics of carbon pile.

Among several characteristics of the c. p. the relations between resistance, compressive force and the contraction thereby, are important. These characteristics are subject generally to hysteresis but that can be largely eliminated by shocking. Fig. 2 & 3 are obtained by shocking. Fig. 2 shows the properties of the KS type c. p. of a train generator and Fig. 3 shows that of a lamp voltage regulator. The KS type is now used by the Japanese National Railway. It is difficult to obtain consistent

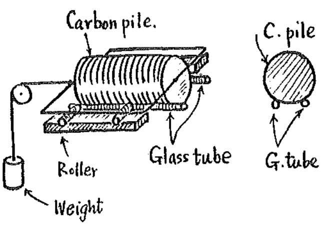

Fig. 1 Measurement of Resistance.

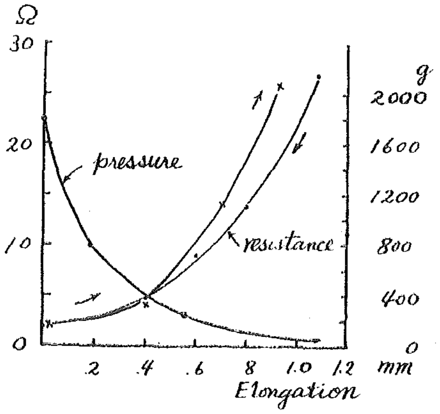

Fig. 2 Characteristics of the carbon pile for the train generator.

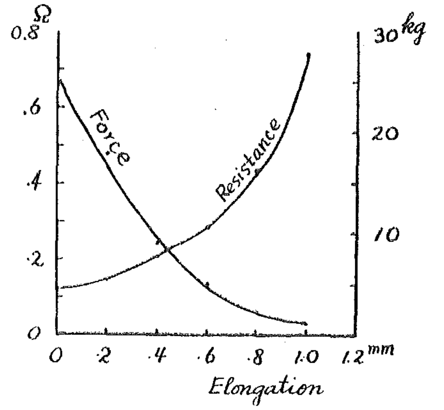

Fig. 3 Characteristics of the c. p. for the lamp voltage regulator.

results at all times. The characteristic curve is favorable to be straight within operating ranges, and the inclination must not be too steep as mentioned later. The resistance is mainly upon the contact resistance. As an example the K S type c. p. is considered, of which the dimensions are as follows:

Diameter :—48 mm.

Thickness :—2.0 mm and 0.8 mm alternately.

Total number of discs :—46 (2.0 mm × 23, 0.8 mm × 23).

If the resistance be assumed to be one Ohm, the number of contacts per disc to be three, the contact resistances to be all equal, and the shape of contact point to be a circle with its radius r, then r can be calculated as follows:

R : contact resistance
ρ : conductivity
r : radius of the contact circle

according to R. Holm we obtain

$$R = \rho/(2r) \qquad (1)$$

Therefore assuming the resistivity ρ to be 0.004 Ω–cm, we obtain;

$$r = \rho/(2R) = 0.31 \text{ mm}.$$

Thus the contact area is very small compared to the dimensions. In accordance with the increase of contact resistance, the radius of contact decreases inverse proportionally.

The resistance distribution in the carbon pile is not uniform as shown in the following Table 1.

TABLE 1. Resistance distribution per each contact.

Compressive force gr.	Thickness of pile mm.	Contact resistance per each contact mΩ											
760	0.8	48	61	39	56	37	36	38	33	35	21	36	29
	2.0	15	21	18	12	11	18	23	18	17	31	10	19
160	0.8	60	51	66	83	68	83	65	62	48	62	64	88
	2.0	35	35	24	20	18	21	50	33	38	21	42	30

3. Temperature Rise.

Electrical heat is generated in the c. p. and therefore the temperature rises. If the temperature is too high, then the device will soon deteriorate.

(1) **measurement of temperature rise**

The temperature rise at the contact points consists of two, elements one of them is the surface temperature rise of the discs observed as a solid block and the other is that temperature which lies between the contact point and the surface of the block. Thus the temperature rise at the contact points or the maximum temperature rise is the sum of the two. In the experiments, the surface temperature was measured by a copper-costantan thermocouple (Figs. 4 & 5) and on the other hand a mercury thermometer was used by placing the bulb in contact with the surface of the pile. The temperature distribution was measured by the thermocouple

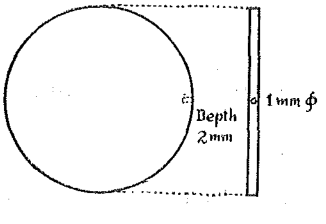

Fig. 4 Hole containing a thermocouple.

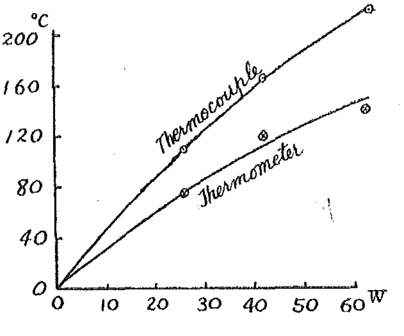

Fig. 5 Temperature rise of the c. p. for the train generator.

(Fig. 6); the temperature is higher in the middle range than at either end. The unequal distribution of temperature is probably due to the unequal distribution of resistance. The temperature-rise time curve is given in Fig. 7; the pile is 48 mm in diameter and 65 mm in length. The thermal time constant is 12 min., from the figure.

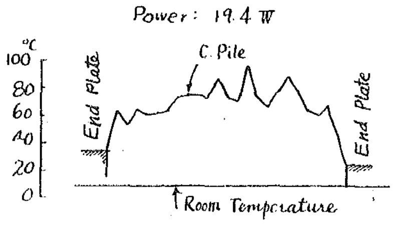

Fig. 6 Temperature distribution of the c. p. for the train generator.

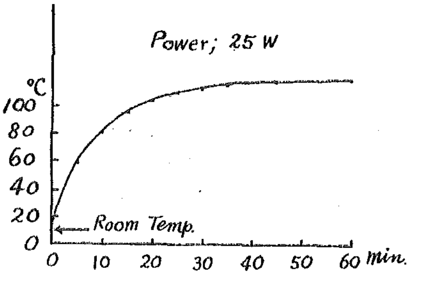

Fig. 7 Temperature rise-Time curve of the c. p. for the train generator.

(2) **The temperature rise of the contact point (calculation).**

If it be assumed that R. Holm's formula[3] is applicable to this problem, the temperature rise is given by the formula

$$\theta = V^2/(8k\rho) \quad \cdots\cdots (2)$$

In this expression V is the potential difference between two bodies, k is the thermal conductivity (W/°C-cm^2) and ρ is the resistivity (Ω-cm). By the way, if the voltage at the c. p. terminal is 25 volts, then we obtain

$$V = 0.55 \text{ volts.}$$

Assuming $k=0.2$ and $\rho=0.005$, we obtain $\theta=38$°C. If the temperature at the surface of the pile observed as a solid block is 119°C then the temperature at the contact point or the max. temperature will be 157°C. These numerical values are applicable to the K S II type train generator at 1800 r. p. m.

In order to decrease θ in expression (2), it will be effective to limit V per one contact.

K S type lamp voltage regulating

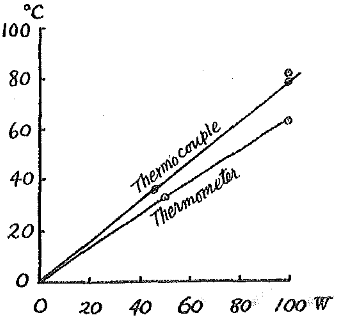

Fig. 8 Temperature rise of the c.p. for the lamp voltage regulator.

carbon pile is 68 mm in diameter and 115 mm in length. Its thermal time constant is about 25 min. according to Figs. 8, 9 & 10.

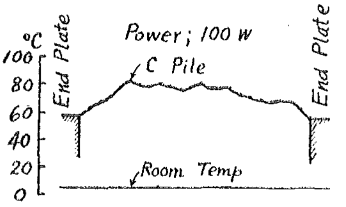

Fig. 9 Temperature distribution of c. p. for the lamp voltage regulator.

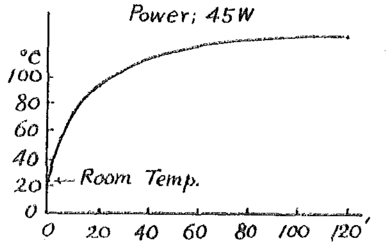

Fig. 10 Temperature rise-Time curve of the c. p. lamp regulator.

In these above experiments, the temperature was rather uniformly distributed, but it was by chance observed that the temperature at one contact was extremely high and red hot points were observed.

4. On the vertical pile type.

The writer studied the relations between the vertical pile form and the horizontal type.

(1) **Preliminary test.**

By the device illustrated in Fig. 12 the resistance was measured with a measuring current of 1 Amp.

The upper electrode was made of copper, 32 mm in diameter and 4.8 gr. in weight and the surface was cleaned with sand paper. The c. p. consisted of discs of two thicknesses, 2 mm and 0.8 mm alternately. to a total number of 46.

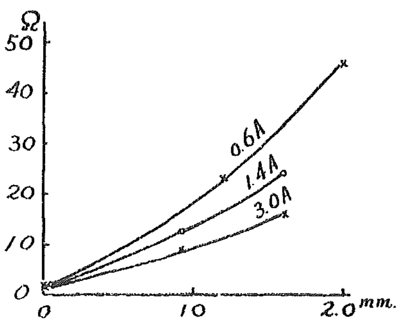

Fig. 11 Effect of currents upon the resistance of c. p.

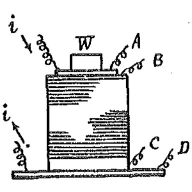

Fig. 12
Vertical method.

The lower electrode *C* was 48 mm in diameter, 5 mm in thickness and made of carbon. *D* was made of copper 4 mm in thickness. The effects of the weight of the upper electrode upon the contact resistance are known by Table 2.

The weight *W* of the upper electrode has great effects upon the resistance.

TABLE 2. Effect of metal electrode upon contact resistance.

W gr.	2.0 mm × 23			0.8 mm × 23		
	R_{AC}	R_{BC}	R_{AD}	R_{AC}	R_{BC}	R_{AD}
0	11.5	9.8	12.0	19.0	18.0	20.0
5	10.0	9.1	10.4	16.2	15.0	16.2
10	9.5	8.5	9.6	14.8	13.4	14.7
20	8.2	7.3	8.5	12.7	11.8	12.8
60	5.9	5.4	6.5	8.7	7.6	8.9
100	5.0	4.7	5.5	7.2	6.4	7.3
200	3.5	3.3	4.0	4.8	4.1	5.1

(2) **Resistance by the vertical pile type.**

The resistance R_{bc} was measured and given in Table 3. In this table column II shows the resistance when the uppermost disc in column I was replaced at the lowermost position while the others remained unchanged.

TABLE 3. Resistance after shocks.

W gr.	2.0 mm × 23					0.8 mm × 23				
	I	II	III	IV	V	I′	II′	III′	VI′	V′
0	7.5 Ω	8.1	8.4	8.5	9.0	13.9	15.0	15.2	15.5	15.8
5	6.7	6.8	7.4	7.5	8.2	12.5	13.2	13.1	13.4	13.2
10	6.8	6.4	7.1	7.1	7.4	11.3	12.1	12.0	12.1	11.9
20	6.2	5.9	6.3	6.1	6.6	9.4	10.3	10.8	10.4	10.3
60	4.2	4.1	4.4	4.2	4.6	6.9	7.5	7.6	7.0	7.0
100	3.1	2.9	3.3	3.3	3.6	4.8	5.8	5.5	5.4	5.4

TABLE 4. Resistance by no shocks.

W gr.	2.0 mm × 23					0.8 mm × 23				
	I	II	III	IV	V	I′	II′	III′	IV′	V′
0	9.5 Ω	9.7	9.3	10.4	8.5	15.2	15.6	16.2	16.6	16.2
5	8.4	9.1	8.7	9.6	8.0	14.0	13.4	14.1	14.2	14.2
10	7.9	8.6	8.3	8.7	7.4	13.0	12.4	12.4	12.8	12.6
20	6.6	7.1	7.1	7.4	6.5	11.7	10.3	11.0	11.6	11.3
60	4.4	4.8	4.8	5.0	4.6	7.1	6.7	7.4	7.1	7.2
100	3.5	3.7	3.9	3.9	3.7	6.2	5.2	5.6	6.2	5.6

Columns III, IV& V are similar. The resistances shown in Table 3 were obtained after giving shocks to the table on which the apparatus was set up, and the ones in Table 4 were obtained when no shocks were given. Comparing these two tables it is noticeable that the resistance will become smaller by giving shocks. Table 5 also shows the effect of shocks upon the c. p. resistance.

TABLE 5. Effect of vibration on the Resistance, R_{BC} Ohm.

First set	Vertical shock	Horizontal shock	re-set	Vertical shock	Horizontal shock	re-set	Vertical shock
9.8	9.1	8.4	9.4	9.1	8.3	9.4	8.2

(3) **Resistance by the horizontal type carbon pile and relations between the horizontal and the vertical types.**

Table 6 gives the resistance R_{bc} in Fig. 12 by the horizontal type pile.

TABLE 6. Resistance by the horizontal method.

Resistance / Force	2.0 mm × 23				0.8 mm × 23				
	I	II	III	Mean	I	II	III	IV	Mean
5 g.	33	29	29.2	30.4 Ω	29	32	29	29	29.8 Ω
10	21.2	22.4	21.5	21.7	23.5	25.5	24	24.8	24.4
15	15.1	16.2	16.5	15.9	20.5	21.4	19.8	20.0	20.4
20	13.9	13.4	15.0	14.1	18.2	19.4	17.4	18.0	18.3
35	9.4	9.4	9.6	9.5	13.0	13.5	12.5	14.0	13.3
60	6.8	7.1	7.0	6.9	9.3	9.4	8.7	9.8	9.3
100	4.3	4.55	4.7	4.52	6.95	—	6.0	7.2	6.7
160	3.1	3.5	3.2	3.27	5.1	—	4.65	4.65	4.8
200	2.7	2.82	2.7	2.74	4.1	—	4.05	4.00	4.05
260	2.1	2.1	—	2.10	3.45	—	3.25	3.25	3.32
2000	0.38	—	—	0.38	0.61	—	—	—	0.61

Now we will find the relation between R_v and R_h, where R_v is the resistance by the vertical type and R_h is the resistance by the horizontal type.

Fig. 13 gives the relation between R_h and the pressure ; now, in Fig. 14 the compressive force between the uppermost disc and the second is

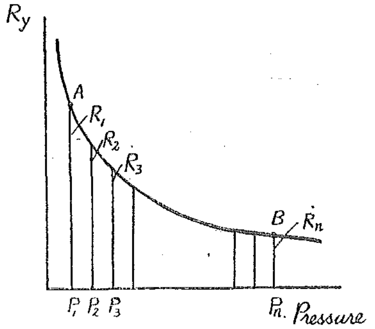

Fig. 13 a Resistance-pressure curve of the c. p. at the horizontal position.

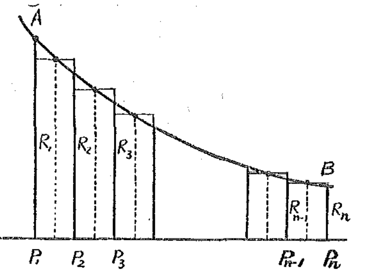

Fig. 13 b. Method of approximate calculation.

$$P_1 = P_0 + w$$

where P_0 is the sum of the weight W and the weight of the upper electrode, and w is the weight of one element disc of the pile. The compressive force between the second and the third discs is

$$P_2 = P_0 + 2w$$

Similarly we obtain

$$P_n = P_0 + nw$$

If the discs are all homogeneous, then the resistance between the first and the second disc in Fig. 14 is $1/n$ times as large as R_1 corresponding to P_1 in Fig. 13.

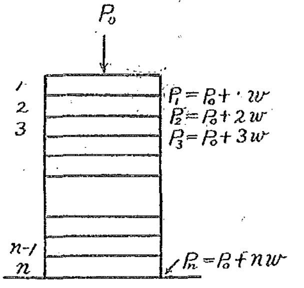

Fig. 14 Pressure distribution by the vertical type c. p.

Thus

$$R_v = (R_1 + R_2 + R_3 + \cdots \quad + R_n)/n \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (3)$$

To simplify the above calculation (3), an approximate method is proposed:

$$
\begin{aligned}
&R_1+R_2+R_3+R_4+\cdots\cdots+R_{n-1}+R_n \\
&=\frac{R_1}{2}+\frac{R_1+R_2}{2}+\cdots+\frac{R_{n-1}+R_n}{2}+\frac{R_n}{2} \\
&=\left(\frac{R_1+R_n}{2}\right)+\frac{1}{w}\left\{\frac{R_1+R_2}{2}w+\cdots+\frac{R_{n-1}+R_n}{2}w\right\} \\
&=\frac{R_1+R_n}{2}+\frac{1}{w}\text{ area }(P_1ABP_n) \qquad \cdots\cdots\cdots (4)
\end{aligned}
$$

From (3) and (4), we obtain

$$
R_v=\frac{R_1+R_n}{2\times\text{No. of piles}}+\frac{\text{Area }(P_1ABP_n)}{\text{Total weight of pile}} \qquad \cdots\cdots\cdots (5)
$$

Table (7) was caluclated from Table (6) by means of expression (5). By comparing this table with Table (3) we can recognize the correctness of expression (5).

TABLE 7. Calculated value by the expression (5)

W gr.	R_v (Ω)	
	2.0 mm	0.8 mm
0	7.92 Ω	16.4 Ω
5	7.23	14.6
10	6.70	13.3
20	5.86	11.2
60	4.13	7.34
100	3.23	5.72

(4) **KS type lamp voltage regulator c. p. (Vertical type)**

The measurement was made by the method illustrated in Fig. 12. As for the effect of the surface film, there were no very distinguishable differences between the clean copper electrode and the one which had been exposed to the air for several days.

The lower electrode was made of brass. Table 8 gives the resistance by the vertical type carbon pile, of which the measuring current was 1 ampere.

In this table "average" expresses the mean value of I, II, ··· and VI, and the "calculated ualue" expresses the value determined by formula (5). The value used in the above calculations is adopted in the following Table 10.

TABLE 8. Measured value of resistance by the horizontal type carbon pile.

W gr.	R_{AD}	R_{BD}	R_{BC}	W gr.	R_{AD}	R_{BD}	R_{BC}
	Ω	Ω	Ω		Ω	Ω	Ω
0	3.3	1.75	1.70	1,000	0.62	0.41	0.38
20	2.4	1.54	1.48	2,000	0.35	0.29	0.26
50	1.80	1.25	1.22	4,500	0.21	0.16	0.145
100	1.41	1.02	0.96	9,000	0.12	0.080	0.070
200	1.12	0.82	0.76	20,000	0.07	0.045	0.041
500	0.78	0.56	0.52				

Diameter 68 mm Thickness 5 mm
No. of discs 23 Weight of each disc 23.5 gr.

TABLE 9.

W gr.	I	II	III	IV	V	VI	Average	Calcu. value (5)
	Ω	Ω	Ω	Ω	Ω	Ω	Ω	Ω
0	1.70	1.32	1.44	1.80	1.80	1.72	1.63	1.80
20	1.48	1.21	1.39	1.58	1.62	1.55	1.47	1.48
50	1.22	1.02	1.19	1.38	1.47	1.28	1.29	1.24
100	0.96	0.95	1.10	1.18	1.21	1.17	1.09	1.05
200	0.76	0.80	0.79	0.85	0.79	0.78	0.80	0.77

TABLE 10. Resistance by horizontal type carbon pile.

Force gr.	Resistance (Ω)			Force gr.	Resistance (Ω)		
	I	II	III		I	II	III
20	11.0	8.6	—	2,250	0.20	0.20	0.285
50	4.3	4.4	—	4,250	0.121	0.115	0.160
100	2.8	2.4	—	4.500	0.111	0.109	—
200	1.42	1.40	—	9.000	0.067	0.060	0.075
300	1.08	1.05	—	18,000	—	—	0.036
500	0.72	0.77	0.56	19,000	0.039	—	—
1,000	0.46	0.45	0.36	20,000	0.036	0.33	0.033

(5) **Summary**

(1) In the horizontal type carbon pile every element of the carbon pile has about an equal resistance, but in the vertical type the element at the topmost position has a larger resistance than the

bottom one.

(2) The upper electrode has large effects on the resistance in the vertical type.

(3) The metal electrode such as copper or brass which easily rusts has large contact resistances.

(4) The contact resistance will become smaller after shockings.

(5) The resistance by the vertical type carbon pile is calculated by the formula from the values secured with the horizontal type.

Note,

The relation between the compressive force and the contraction x_v in the vertical type carbon pile will be calculated from the values in the horizontal type. Similarly to equation (5), we can obtain

$$x_v = \frac{x_1 + x_n}{2n} + \frac{\text{area}\,(P_1ABP_n)}{nw}$$

5. Constant voltage apparatus with a carbon pile.

The constant voltage apparatus with a carbon pile has been adopted in the train lighting equipment by the Japanese National Railway (Figs. 15 & 16). In Fig. 15 G is the shunt generator, F is the field winding with the carbon CP in series. The electromagnetic force acting on CP is produced by the electromagnet M, which is connected to the terminal of the generator G with the resistance r_0 in series. If the voltage is small and M does not act, then the spring S compresses CP.

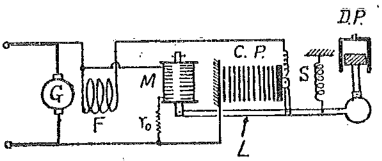

Fig. 15 KS type automatic voltage regulator for the train generator.

Now, if the generator speeds up and the voltage begins to increase, then electromagnet M will act, the compressive force decrease and the current following through F will decrease while the voltage will decrease, too, Owing to the self-inductance and the eddy current, flux change of F has time lag therefore the voltage of G does not yet decrease to a favorable

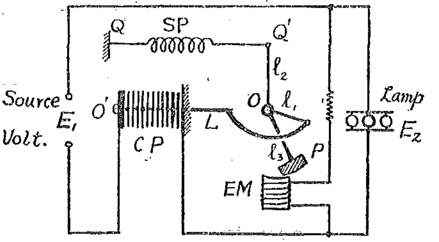

Fig. 16 KS type automatic voltage regulator for train lighting.

value in spite of the resistance of CP having a favorable value.

After the resistance of CP increases above the favorable value, the voltage of G decreases and some time later the voltage will decrease to the value determined by the resistance of CP and the voltage will be less than the favorable one. Next, thus decreased voltage has to recover its value contrary to the former phenomena.

This is the hunting of the carbon pile voltage regulator. To suppress this the dash pot DP is adopted. Fig. 16 shows the connection of the lamp regulator and the apparatus keeps E_2 constant in spite of E_1 being variable. In order to realize this, CP is controlled by the electromagnet EM operated with the load voltage E_2. In case of smaller E_1, CP will be compressed by the spring SP, by means of 1_2 1_1 and L. If E_1 increases, then E_2 will also increase and the electromagnet EM will make the resistance of CP lager, and therefore the voltage drop in it will become larger, and thus E_2 will be kept constant. As well as in Fig. 15 a dash pot has to be used to suppress hunting.

(1) **Theory of the operation of Fig. 15.**

The forces CP, M and S in Fig. 15 will be converted at the position of the plunger indicated in Fig. 17.

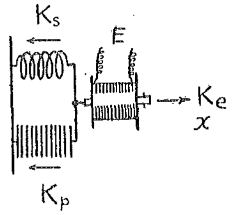

Fig. 17
Simplified equivaleht voltage regulator.

Fig. 18 explains the operation: the curve K_s shows the relation between the force of S and the displacement x of the plunger. K_p shows the relation between the force of CP and x, and K_e shows the reration between the force of the plunger and its displacement. When the three forces are in equilibrium, the plunger will be at a standstill.

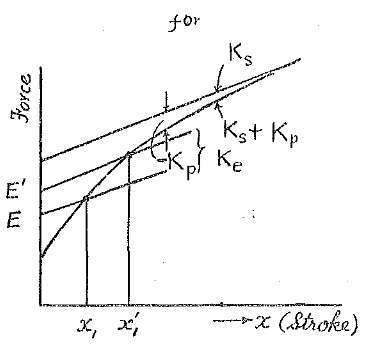

Fig. 18
Static characteristics of the c. p. voltage regulator.

$$K_s + K_p - K_e = 0 \cdots\cdots\cdots\cdots (7)$$

In Fig. 18 x_1 is the plunger position when the voltage is E and x_1', in case of E'.

In Fig. 19, if the inclination of K_e is not larger than that of $K_s + K_p$, then there will be only one intersection, but if the inclination of K_e is larger, then there will

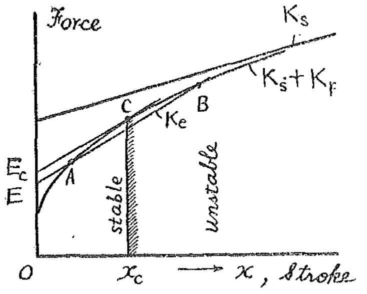

Fig. 19 Stability of the static characteristics.

be two intersections. In this case A is stable, and B is unstable. If the voltage E rises gradually, then the displacement x will become larger and arrive at the point x_c. Beyond this voltage, x will become infinity. Thus in the static characteristics of the plunger the region to the right-hand of x_c is unstable and the left-hand region is stable. In general, it is favorable that the curves K_c and K_s+K_p coincide[1].

The operating characteristics of the constant voltage apparatus shown in Fig. 15 are treated graphically in Fig. 20. In this Fig. the first quadrant is the same as in Figure 18. In the second quadrant, the curve gives the relation between the generator voltage E and the plunger pull at $x=0$; in the theird quadrant the curves give the relation between the resistance and the generator emf. at several speeds; in the fourth quadrant the curve gives the relation between the plunger displacement and the c. p. resistance.

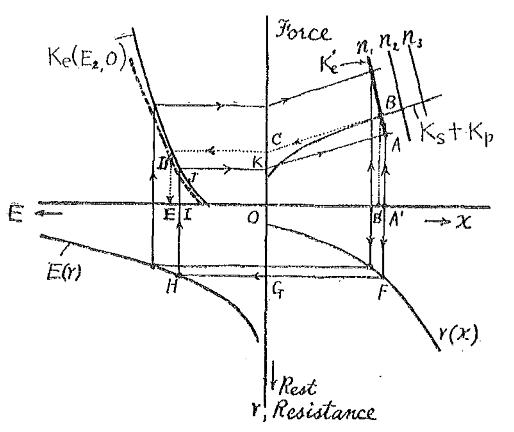

Fig. 20 Method of obtaining the donamical characteristics.

At first, if we take the plunger displacement x arbitrarily, such as the point A', then the resistance of the pile r will be decided, and the generator voltage E and the force of the plunger K_c will also be decided. As to the force of plunger, the value at the position $x=0$ will be decided at first, and next the point A is decided by the static characteristics. The point A is on the operating characteristic K_c'. Repeating the above operation we can obtain the operating characteristics K_c'. Of course, for some other value of the speed, we have another curve for K_c'. At one speed n_1, the inter-section B of K_c' and K_s+K_p, of which x–coordinate gives the plunger position, the generator voltage E will be given in the order of B, C, D and E. Thus the generator voltages at several speeds are obtained and the voltage-speed relations are obtainable Figs. 21, 22 & 23).

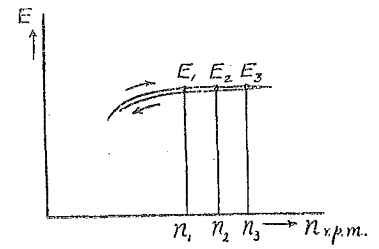

Fig. 21 a The voltage-speed curve.

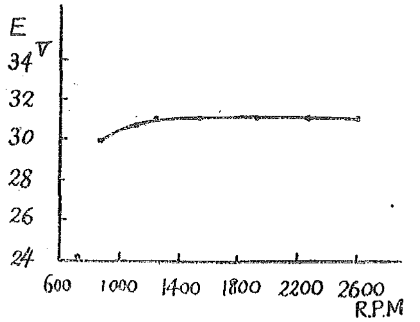

Fig. 21 b Voltage speed curve of train generator (KS type).

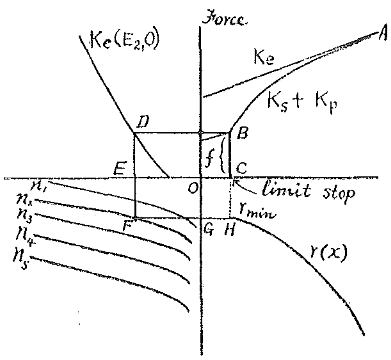

Fig. 22 The minimum speed n_2 at which the plunger begins to operate.

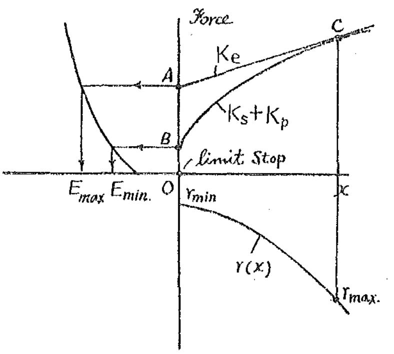

Fig. 23 The maximum and the minimum output voltage.

I. The effect of the resistance of the electromagnet.

As the pull of the electromagnet is decided by the current flowing through it, even if the terminal voltage of the electromagnet is constant, the pull will change according to the resistance change. The generator voltage will rise in proportion to the resistance increase. For this reason, we have to use the coil, of which the temperature coefficient of resistance is negligibly small. In Fig. 19, we used the voltage corresponding to the point C for the sake of convenience of explanation, but we had to use the current for the sake of correctness. Denoting this current i_c, the maximum voltage E_{max} in the Fig. 15.

$$E_{max} = i_c r_f \qquad \cdots\cdots (8)$$

where r_f is the total resistance of the electromagnet.

II. Theory of operation (Fig. 16.)

The graphical solution of the apparatus in Fig. 16 is given in Fig. 24, in which the first quadrant gives the characteristic of the plunger, the carbon pile and the spring. K_e is the force of plunger and K_s+K_p is the resultant force of the carbon pile and the spring. The second quadrant gives the relation between the voltage E_2 and the plunger force at $x=0$. The third quadrant gives the relation between E_2 and $1+R/r$ at the constant value of source voltage E_1.

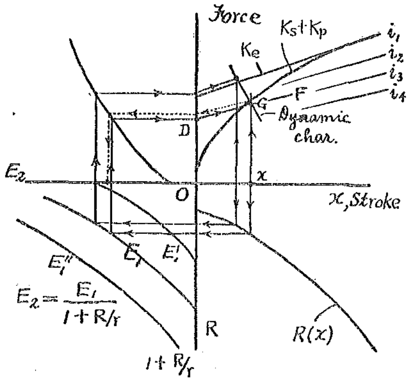

Fig. 24 The method of obtaining the dynamic characteristics.

R is the resistance of c. p. and r is that of the load, then

$$E_2=E_1/(1+R/r) \cdots\cdots\cdots (9)$$

The expression (9) was used in drawing the curve.

The fourth quadrant gives the measured curves between the resistance of the carbon pile and the displacement of plunger x. Changing the scale, the curves $R(x)$ represent the curve, $1+Rr$; and by using this method the operating characteristics were obtained. That is to say, at first selecting the position of plunger at the arbitrary point x, then the pile resistance $R(x)$ is decided thereby, then the plunger force Gx will be obtained in the order represented by the dotted line. The point G is on the operating characteristics, which will be obtained by representing the above process. The inclination of the operating characteristic is not so steep as that of the constant voltage generator in Fig. 15 so that it is necessary to minimize the frictions of the moving parts. According to the variation of the source voltage E_1 the operating characteristic will be a different one and the plunger position will be obtained thereby. Fig. 25 is obtained graphically from the

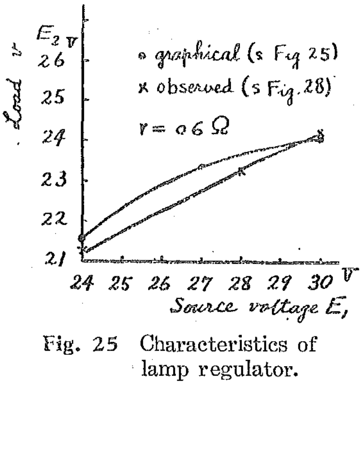

Fig. 25 Characteristics of lamp regulator.

above descrived drawings. In order to learn the change of the load voltage E_2 due to the charge of the load current at the constant source voltge E_1, we have to select the cerresponding curve $1+R/r$ according the load resistance r in the fourth quadrant and then E_2 will be obtained.

III. The hysteresis.

In the constant voltage apparatus the electromagnet and the carbon pile have the hysteresis phenomena which disturb the operations. Fig. 21–b (cf. Fig. 15) shows the voltage speed curve of the KS type generator at 780 ~2680 r.p.m. The voltage at accelerating speed is about one volt higher than that at the lowering speed. Fig. 26 (cf. Fig. 16) shows the hysteresis of the load voltage E_2 when the source voltage E_1 rises from 16 volts to 4o volts and lowers itself again. The hysteresis depends upon the magnetic hysteresis, greatly. Next a description will be given of the hysteresis phenomena of the apparatus shown in the Fig. 16; use was made of the lamp voltage regulating apparatus of the KS type generator (30 V, 70 A.). The source voltage E_1 was raised from 19.2 volts to 40.6 volts and then lowered to 15.4 volts with the results as given in Table 11 and Figs. 26, 27 and 28.

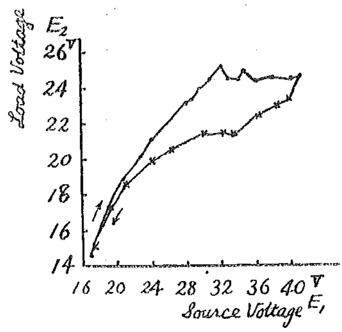

Fig. 26 Relations between the source voltage and the load voltage (observed).

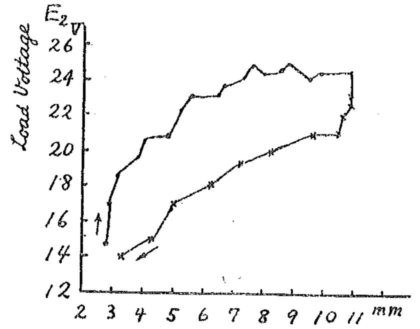

Fig. 27 Relation between the load voltage and the displacement of plunger (observed).

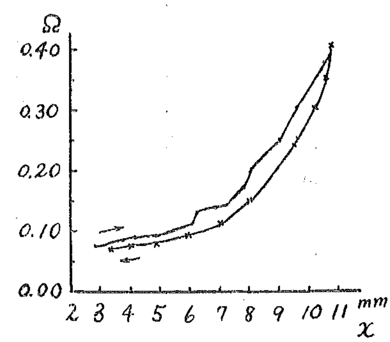

Fig. 28 Relation between the resistance of c. p. for the lamp regulator and the displacement of plunger.

TABLE 11.

E_1 Source voltage	E_2 Load voltage	ΔE $=E_1-E_2$	x Contraction:	I Load current	R $=\frac{\Delta E}{I}$
V	V	V	mm	A	Ω
19.2	17.0	2.15	2.9	28.2	0.0763
20.2	18.0	2.2	3.1	29.3	0.0751
21.0	18.7	2.3	3.2	30.0	0.0767
22.5	19.8	2.7	3.8	32.5	0.0831
23.7	20.8	2.85	4.1	33.7	0.0846
24.3	21.1	3.2	4.7	35.0	0.0915
26.1	22.6	34.5	5.0	36.0	0.0958
26.8	23.2	3.6	5.2	36.0	0.100
27.7	23.4	4.3	6.2	37.5	0.115
28.5	24.0	4.5	6.4	37.5	0.133
29.7	24.4	5.3	7.0	38.5	0.138
31.1	25.4	5.7	7.3	38.6	0.148
31.7	24.7	7.0	7.8	39.0	0.180
32.7	24.8	7.9	8.2	39.0	0.202
33.6	25.3	8.3	8.5	39.5	0.210
35.0	24.5	10.5	9.2	39.5	0.266
37.0	24.9	12.1	9.6	39.5	0.306
39.7	25.0	14.7	10.4	39.2	0.375
40.6	25.2	15.4	10.6	39.0	0.395
39.7	24.0	15.7	10.6	38.0	0.413
38.2	23.5	14.7	10.6	37.2	0.395
35.8	22.8	13.0	10.5	36.2	0.359
32.5	21.6	10.9	10.2	35.5	0.307
29.6	21.7	7.9	9.4	35.0	0.226
26.0	20.9	5.1	8.2	33.5	0.152
23.5	19.9	3.6	7.2	31.5	0.114
21.2	18.5	2.7	6.1	29.5	0.091
19.5	17.3	2.2	5.0	26.5	0.083
17.0	15.1	1.9	4.2	24.0	0.079
15.4	13.9	1.5	3.4	22.0	0.068

Analysis of the data.

Figs. 29, 30 and 31 are for explaining principles; curves *I* and *II* are the load voltages E_2 when increasing E_1 and I' and II' are E_2 when decreasing E_1. If there is no hysteresis and the relation between

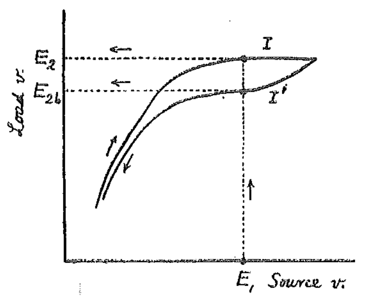

Fig. 29 Explanation on the hysteresis effects of the Fig. 26.

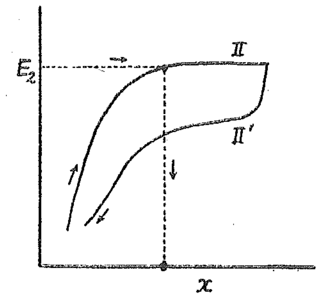

Fig. 30 Explanation on the hysteresis effects of the Fig. 27.

TABLE 12.

E_1	E_2	x	R_I	R_I+r	E_{20}	R_I'	$R_I'+r$	E_{2a}	E_{2b}	$E_{2a}-E_{20}$	$E_{2b}-E_{2a}$
V	V	mm	Ω	Ω	V	Ω	Ω	V	V	V	V
39.7	25.1	10.4	0.365	0.990	25.0	0.364	0.989	25.1	24.0	+ 0.1	− 1.1
38.2	25.1	9.9	0.33	0.955	25.0	0.282	0.907	26.3	23.5	+ 1.3	− 2.8
35.8	25.0	9.35	0.282	0.902	24.8	0.224	0.849	26.4	22.8	+ 1.6	− 3.6
32.5	24.9	8.2	0.202	0.827	24.6	0.152	0.777	26.1	21.6	+ 1.5	− 4.5
29.6	24.5	7.0	0.144	0.769	24.1	0.112	0.737	25.1	21.7	+ 1.0	− 3.4
26.0	22.6	5.0	0.097	0.722	22.5	0.083	0.708	23.0	20.9	+ 0.5	− 2.1
23.5	20.6	4.0	0.085	0.710	20.7	0.074	0.699	21.0	19.9	+ 0.3	− 1.1
21.2	18.9	3.2	0.078	0.703	18.9	0.069	0.694	19.2	18.5	+ 0.3	− 0.7
19.5	17.3	3.0	0.076	0.701	17.4	0.068	0.693	17.6	17.3	+ 0.2	− 0.3
17.0	15.1	2.9	0.075	0.700	15.2	0.068	0.693	15.3	15.1	+ 0.1	− 0.2

where $E_{20} = E_1 \dfrac{r}{R_I + r}$, $E_{2a} = E_1 \dfrac{r}{R_I' + r}$

E_2 and E_1 is given by I in the Fig. 29, then one can obtain E_2 corresponding to E_1 by means of the curve in Fig. 26. Table 12 gives such values. In the table x is the plunger displacement obtained by Fig. 27 by means of the method represented in Fig. 30; R_I & R_I' are the resistance of c. p. obtained by Fig. 28 by means of the method represented in Fig. 31; R_I is the resistance when raising the voltage and R_I' is the one when lowering the voltage; r is the load resistance. E_{20} is the value of E_2 when there is no hysteresis and

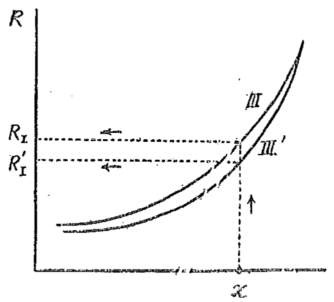

Fig. 31
Explanation on the hysteresis effects of the Fig. 28.

E_{2a} is the value of E_2 when the resistance of c. p. has hysteresis. E_{2b} is the measured value of E_2.

Then $E_{2a}-E_{20}$ = due to the hysteresis of the pile.

$E_{2b}-E_{2a}$ = due to the hysteresis of the plunger.

In this way it is possible to analyse the hysteresis phenomena of the apparatus. Table 12 has thus been obtained which explains that the hysteresis of the plunger operates to reduce E_2 and that of the pile operates to enlarge E_2 and the former predominates as the result of it, as shown in Fig. 26. The apparatus shown in Fig. 16 has an electromagnet with a yoke, which is the main cause of the hysteresis in its operation.

6. On the Design.

I. The theory of constancy of E_2 (Load Voltage).

In order to keep constancy of the load voltage E_2, it is necessary that favorable relations be kept between the plunger displacement x and that of the carbon pile z.

$$dE_2 = \frac{\partial E_2}{\partial E_1} dE_1 + \frac{\partial E_2}{\partial R} \cdot \frac{dR}{dE_2} \cdot \frac{dE_2}{dE_1} dE_1 \qquad \cdots\cdots (1)$$

$$dE_2/dE_1 = \frac{\partial E_2/\partial E_1}{1 - \dfrac{\partial E_2}{\partial R} \cdot \dfrac{dR}{dE_2}} \qquad \cdots\cdots (2)$$

and

$$dR/dE_2 = \frac{dR}{dz}\frac{dz}{dx}\frac{dx}{dE_2} \qquad \cdots\cdots (3)$$

on the other hand

$$+K_e(x, E_2)+K_s(x)+K_p(z)/m = 0 \qquad \cdots\cdots (4)$$

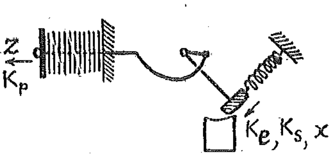

Fig. 32

Explanation of Symbols.

where

$$m = dx/dz \qquad \cdots\cdots (5)$$

and R is the pile resistance, E_1 is the source voltage and E_2 is the output voltage.

From expression (4) we have

$$\frac{\partial}{\partial x}\left(K_e+K_s+K_p/m\right)\frac{dx}{dE_2} + \frac{\partial K_e}{\partial E_2} = 0 \qquad \cdots\cdots (6)$$

$$\therefore \quad dx/dE_2 = \frac{-\partial K_e/\partial E_2}{\frac{\partial}{\partial x}(K_e+K_s)+\frac{1}{m^2}\frac{\partial K_p}{\partial z}} \quad \cdots\cdots (7)$$

Substituting (3) and (7) into (2), we have

$$dE_2/dE_1 = \frac{\partial E_2/\partial E_1}{1+\frac{\partial E_2}{\partial R}\cdot\frac{dR}{dz}\cdot\frac{1}{m}\cdot\frac{\partial K_e/\partial E_2}{\frac{\partial}{\partial x}(K_e+K_s)+\frac{1}{m^2}\cdot\frac{\partial K_p}{\partial z}}} \quad \cdots (2)'$$

To keep E_2 constant in spite of the change of E_1 the denominator has to be infinitely large, namely

$$\frac{\partial}{\partial x}(K_e+K_s)+\frac{1}{m^2}\frac{\partial K_p}{\partial z}=0 \quad \cdots\cdots (8)$$

$$\therefore \quad m=\frac{dx}{dz}=\sqrt{\frac{|\partial K_p/\partial z|}{k_e-k_s}} \quad \cdots\cdots (9)$$

where

$k_e=\partial K_e/\partial x$ and $k_s=-\partial K_s/\partial x$. From (9) we have

$$k_s=k_e-\frac{1}{m^2}\left|\frac{\partial K_p}{\partial z}\right| \quad \cdots\cdots (10)$$

And also from (9) we have,

$$x=\frac{1}{\sqrt{k_e-k_s}}\int_{z_0}^{z}\sqrt{|\partial K_p/\partial z|}\cdot dz \quad \cdots\cdots (11)$$

If $E_1=E_{10}, x=0,\ z=z_0$ and $m=m_0$; then we have from expression (4)

$$K_e(0,E_2)+K_s(0)+\frac{1}{m_0}K_p(z_0)=0 \quad \cdots\cdots (12)$$

and

$$k_s=k_e-\frac{1}{m_0^2}\,|(\partial K_p/\partial z)_0| \quad \cdots\cdots (10)'$$

From (10)′ we have

$$m_0=\left(\frac{dx}{dz}\right)_0=\frac{1}{\sqrt{k_e-k_s}}\sqrt{-(\partial K_p/\partial z)_0} \quad \cdots\cdots (9)'$$

Equation (11) will be transformed into the following:—

$$x=\frac{m_0}{\sqrt{-(\partial K_p/\partial z)_0}}\int_{z_0}^{z}\sqrt{-\partial K_p/\partial z}\cdot dz \quad \cdots\cdots (11)'$$

The maximum value of x or x_m corresponds to the maximum value of z or z_m when R becomes the maximum value R_m and

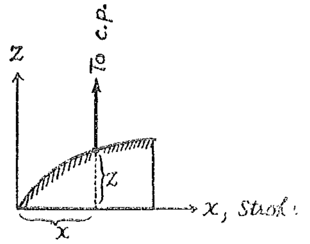

Fig. 33 One method to determine the curve of the cam.

$$x_m = \frac{1}{\sqrt{k_e - k_s}} \int_{z_0}^{z_m} \sqrt{-\frac{\partial K_p}{\partial z}} \cdot dz \quad \cdots\cdots (11)''$$

and we have

$$x = x_m \frac{\int_{z_0}^{z} \sqrt{-\frac{\partial K_p}{\partial z}} \cdot dz}{\int_{z_0}^{z_m} \sqrt{-\frac{\partial K_p}{\partial z}} \cdot dz} \quad \cdots\cdots (11)'''$$

If the relation between x and z which is given by the above expressions is maintained by a suitable mechanism, then the output voltage E_2 will be kept constant in spite of the change of E_1 or the source voltage. of course, it is necessary to minimize the mechanical friction and the magnetic hysteresis.

As to Fig. 15, we have

$$dE = \frac{\partial E}{\partial n} dn + \frac{\partial E}{\partial R} \cdot \frac{dR}{dE} \cdot \frac{dE}{dn} dn$$

$$dE/dn = \frac{\partial E / \partial n}{1 - \frac{\partial E}{\partial R} \cdot \frac{dR}{dE}}$$

$$dR/dE = \frac{dR}{dz} \cdot \frac{dz}{dx} \cdot \frac{dx}{dE}$$

where E is the terminal voltage of the generator

n; the r. p. m. of the generator.

R; the resistance of the carbon pile.

As for the balance of the force we have the same expression as (4) and the value of dx/aE will be the same as (7).

Therefore we have

$$\frac{dE}{dn}=\frac{\dfrac{\partial E}{\partial n}}{1+\dfrac{\partial E}{\partial R}\cdot\dfrac{dR}{dz}\cdot\dfrac{1}{m}\cdot\dfrac{\partial K_e/\partial E}{\dfrac{\partial}{\partial x}(K_e+K_s)+\dfrac{1}{m^2}\dfrac{\partial K_p}{\partial z}}}\ .$$

The above expression is the same as the result obtained by inserting n and E instead of E_1 and E_2 respectively into equation (2)′.

II Procedure in Desing.

The procedure in designing the constant voltage apparatus will properly be as follows :—

(1) Assuming the upper and lower limits of the source voltage E_{max} and E_{min}, the upper and lower limits of resistance R_{max} and R_{min} are decided.

(2) The dimensions and number of discs of carbon are determined by the maximum consumed power and the maximum voltage drop.

(3) The elongation of pile z_m will be determined by the above two.

(4) Assuming the plunger stroke to be a suitable value x_m, we have the value of m_0 by (11)′.

(5) Assuming the value of k_e, the spring constant k_s is obtained by (10)′.

(6) The stability[2] of operation has to be discriminated and if it is unstable then it is necessary to change the value x_m and the kind of the carbon pile.

(7) The relation between x and z will be obtained by (11)′.

(8) $K_p(z_0)$ is determined by R_{min}, and from the expression (12) we have

$$K_s(0)\fallingdotseq-\frac{1}{m_0}K_p(z_0)$$

and

$$K_e(x_m,E_2)+K_s(x_m)=0$$

Thus the forces of the spring and the electromagnet are decided.

7. Conclusion.

The present paper describes the measured values of the two important characteristics namely resistance and contraction affected by compressive forces.

As to the temperature rise, experimental and calculated values are given. The significance of the vertical type carbon pile was made clear. The graphical solution of the operating characteristics of the constant voltage apparatus was described.

A procedure for designing such apparatus was proposed from the point of view of theoretical consideration.

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