On the Thick Plate Problem.

(Second Report)

By

Kin-ichi HATA

(Received June 25, 1954)

CONTENTS

§ I. Introduction.

The author wishes, first of all, to make some introductory remarks on the application of the method of Michell and Love to the various kinds of boundary value problems. Now that particular solutions for the problems of thick plate under variable pressure or tangential load have been published in the present writer's first report (I)[1)]—that report will hereafter be called "(I)"—and in which needed features have been stated of generalized plane stress solutions which may correpond to complementary solutions so that preliminary works on the whole has been completed, it remains to describe some manners in which the boundary conditions of boundary value problems are to be fulfilled. In this report illustrative descriptions chiefly of problems concerning the rectangular thick plate will be given. Incidentally, the term "particular solutions" may be inappropriate but not sound singular and so may be permitted to be used, the author thinks. Mathematical computations involved in this report will be lengthy but may be said to be essentially simple owing to the ingenious method proposed by Michell and Love. As mentioned in the introduction and Sec. III of (I) regarding the order of accuracy of this method, which is principally dependent on the assumption made concerning the derivative of σ_z with respect to z at the plane surfaces of the plate and, further, on the approximate forms, in a strict sense, of τ_{xz}, τ_{yz} in the generalized plane stress solution, though needless to say, Prof. Love says ambiguously in his book that the special forms of τ_{xz}, τ_{yz} and σ_z are assumed in a proper way, conditions on the bounding cylindrical surfaces of the plate, which are varied with respect to z coordinate, could not be satisfied generally by the use of this method. Therefore, the writer cannot but adopt the reduced boundary conditions represented by the resultant forces and couples and, if necessary, apply Kirchhoff's theorem regarding torsional couple and vertical tangential force on the cylindrical surface. Certainly due to this deficiency of accuracy it is necessary to apply Love's method to the moderately thick plate, but in an obvious way the above mentioned inevitable process of calculation in regard to boundary conditions may have a correspondingly slight improper effect which is merely of local perturbation. So the method will serve to enable one to arrive readily at the solutions with sufficient accuracy, if the thickness of the plate is properly taken. In view of the above stated properties of this method it will be pertinent to indicate the forms of solutions in the higher degree of accuracy furnished by the infinitesimal theory of elasticity or of solutions which are to be obtained under general boundary conditions for sufficiently thick plate. In a later report the present writer proposes to show this complete solution, though it seems not necessarily satisfactory. What follows will deal chiefly with the cases in which loading function satisfies the equation of Helmboltz, i. e., it is a v-function, and tractions are applied to the upper plane surface of the plate for the sake of simplification, since, as described in Sec. IV and VI of (I), the cases of general loading function and lower plane surface load can be treated easily. In Sec. II there will be offered some additional remarks on the feature of solutions for generalized plane stress furnished by Prof. Love and Southwell, which were not discussed in the first report. Therein is shown the verification of the fact that solutions by Love may be quite the same as those by Southwell. It is presumed that this verification is needed from the viewpoint of consistency. In section V brief mention of mixed boundary value problems is made and it is shown that these problems can be solved correspondingly easily with the aid of the procedure of Love but it is to the author's regret that the descriptions may be too sketchy on account of limitation of space.

§ II. Additional Remarks on the Generalized Plane Stress Solutions.

Generalized plane stress solutions are so indispensable to solve the thick plate problem by the method adopted in our paper that we could not pass over the fact that there exist two, at first sight, different-seeming kinds of generalized plane stress solutions, viz., solutions by A. E. H. Love[2] and by R. V. Southwell[3]. It can be shown as in the following that these two kinds of solutions are identical with each other to the order of accuracy, whereby the so-called generalized plane stress solutions are constructed.

For reference, solutions by Southwell are quoted as follows:

$$\left.\begin{aligned} \sigma_x &= \frac{\partial^2 \bar{\chi}}{\partial y^2} + \frac{1}{1+\nu}\frac{\partial^2}{\partial x \partial y}\Gamma(x,y,z) , \\ \sigma_y &= \frac{\partial^2 \bar{\chi}}{\partial x^2} - \frac{1}{1+\nu}\frac{\partial^2}{\partial x \partial y}\Gamma(x,y,z) , \\ \tau_{xy} &= -\frac{\partial^2 \bar{\chi}}{\partial x \partial y} + \frac{1}{2(1+\nu)}\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right)\Gamma(x,y,z) , \end{aligned}\right\} \quad \cdots\cdots \quad (2.1)$$

where

$$\left.\begin{aligned} \bar{\chi} &= \chi - \frac{1}{2}\frac{\nu}{1+\nu}z^2 F_0 + \frac{1}{6}\frac{2-\nu}{1+\nu}z^3 F_1 , \\ \chi &= \chi_0 + z\chi_1 , \ \begin{pmatrix} \nabla^2\chi_0 = F_0 , & \nabla^2\chi_1 = F_1 , \\ \nabla^2 F_0 = \nabla^2 F_1 = 0 . & \end{pmatrix} \\ \Gamma(x,y,z) &= \frac{\partial \Phi}{\partial z} , \quad \Phi = H(x,y,z) - \frac{1}{2}z^2 f(x,y) \\ & (\nabla^2 H = 0 , \quad \nabla_1^2 f(x,y) = -2G_1) , \end{aligned}\right\} \quad \cdots\cdots \quad (2.2)$$

G_1 is a conjugate plane-harmonic function of F_1 and it is clear that we can take F_0, F_1 equal to Θ_0, Θ_1 respectively, the last two functions being found in the solutions by Love.

Taking account of only the restraint which is to be imposed on the tangential stresses τ_{xz}, τ_{yz} the following forms for them are seen to be desirable,

$$\left.\begin{aligned} \tau_{xz} &= \frac{-1}{2(1+\nu)}\frac{\partial}{\partial y}\nabla_1^2\Phi = \frac{1}{2(1+\nu)}(h^2-z^2)\frac{\partial\Theta_1}{\partial x}\,, \\ \tau_{yz} &= \frac{1}{2(1+\nu)}\frac{\partial}{\partial x}\nabla_1^2\Phi = \frac{1}{2(1+\nu)}(h^2-z^2)\frac{\partial\Theta_1}{\partial y}\,, \end{aligned}\right\} \cdots\cdots\cdots\cdots \quad (2.3)$$

putting $H(x,y,z)$ in the form

$$H(x,y,z) = H_0 + zH_1 + \frac{1}{2}z^2H_2\,,$$

$$(\nabla_1^2 H_1 = \nabla_1^2 H_2 = 0\,,\quad \nabla_1^2 H_0 + H_2 = 0)$$

in which $H_2(x,y)$ is taken to be $h^2G_1(x,y)$. The right-hand sides of Eqs. (2.3) are the forms obtained by Love and this agreement is shown in the paper by Southwell. Generalized plane stress solutions by Love are indicated on pages 432–433 of (I).

Now we shall show that the forms of stresses (2.1) could be transformed into those (2.7a) of (I) by rearranging and converting the second terms of the right-hand sides of Eqs. (2.1). It is noticeable that the function $\Gamma(x,y,z) = \dfrac{\partial\Phi}{\partial z} = H_1 + zh^2G_1 - zf$ is not a plane-harmonic function, though Southwell implies it is so, and hence the second terms in (2.1) cannot be merged in the first terms. Though $\bar{\chi}$ in (2.1) considerably resembles χ (2.1c) plus χ' (2.8) of (I), they are certainly different and $\bar{\chi}$ in (2.2) can be rewritten into the following form, using the symbols adopted by Love,

$$\bar{\chi} = \left(\chi_0 - \frac{1}{2}\frac{\nu}{1+\nu}z^2\Theta_0\right) + \left(z\chi_1' + \frac{1}{6}\frac{2-\nu}{1+\nu}z^3\Theta_1\right) - \frac{2}{1-\nu}z\chi_1'\,,\quad (2.4)$$

because, it being possible to neglect the indetermination regarding plane harmonic function in this event, formulae

$$\nabla_1^2\chi_1 = \Theta_1\,,\quad \nabla_1^2\chi_1' = \frac{-(1-\nu)}{(1+\nu)}\Theta_1$$

yield the relation between χ_1 and χ_1'.

$$\chi_1 = \frac{-(1+\nu)}{(1-\nu)}\chi_1' = \chi_1' + \frac{-2}{1-\nu}\chi_1'\,. \quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (2.5)$$

H_1 and h^2G_1 in $\Gamma(x,y,z)$ can, doubtless, be merged in χ_0 and χ_1' in the first and second parentheses of the expression (2.4) respectively. When this annexation is performed, we can in the sequel put $\Gamma(x,y,z)$ in (2.1) in the form

$$\Gamma(x,y,z) \equiv \Gamma'(x,y,z) = -zf(x,y)\,, \quad \cdots\cdots \quad (2.6)$$

and also put for convenience' sake the expression (2.4) in the form

$$\bar{\chi} = \chi^1 + \chi^2 - \frac{2}{1-\nu} z\chi_1' = \bar{\chi}' - \frac{2}{1-\nu} z\chi_1'\,, \quad \cdots\cdots \quad (2.7)$$

in which $\chi^1 = \left(\chi_0 - \frac{1}{2}\frac{\nu}{1+\nu} z^2\Theta_0\right)$, $\chi^2 = \left(z\chi_1' + \frac{1}{6}\frac{2-\nu}{1+\nu} z^3\Theta_1\right)$. Thus we have for τ_{xy}

$$\tau_{xy} = \frac{-\partial^2\bar{\chi}}{\partial x\,\partial y} + \frac{1}{2(1+\nu)}\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right)\Gamma'(x,y,z)$$

$$= \frac{-\partial^2\bar{\chi}'}{\partial x\,\partial y} + \frac{2z}{(1-\nu)}\frac{\partial^2}{\partial x\,\partial y}\chi_1' + \frac{z}{2(1+\nu)}\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right)\left(-f(x,y)\right) \quad (2.8)$$

and, if we perform the operation $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ on the right-hand side of (2.8) except the first term, it vanishes, as is readily proved. So we can write

$$\tau_{xy} = \frac{-\partial^2\bar{\chi}'}{\partial x\,\partial y} = \frac{-\partial^2}{\partial x\,\partial y}(\chi^1 + \chi^2)\,, \quad \cdots\cdots \quad (2.9a)$$

$$\frac{2z}{(1-\nu)}\frac{\partial^2}{\partial x\,\partial y}\chi_1' + \frac{z}{2(1+\nu)}\left(\frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial x^2}\right)(-f(x,y) = 0\,, \quad (2.9b)$$

because restriction imposed on two arbitrary plane-harmonic functions contained in χ_1' and $f(x,y)$ caused by the formula (2.9b) would never undermine the generality.

Next we apply the operations $\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ to the formula (2.9b) and then we obtain the relations.

$$\left.\begin{aligned} &\frac{\partial}{\partial x}\left\{\frac{2z}{(1-\nu)}\frac{\partial^2}{\partial y^2}\chi_1' + \frac{z}{(1+\nu)}\Theta_1 + \frac{z}{(1+\nu)}\frac{\partial^2 f}{\partial x\,\partial y}\right\} = 0\,, \\ &\frac{\partial}{\partial y}\left\{\frac{2z}{(1-\nu)}\frac{\partial^2}{\partial x^2}\chi_1' + \frac{z}{(1+\nu)}\Theta_1 - \frac{z}{(1+\nu)}\frac{\partial^2 f}{\partial x\,\partial y}\right\} = 0 \end{aligned}\right\} \quad (2.10)$$

It is evident that the expression in each brace of (2.10) can be equated to zero within the theory of elasticity. Hence we may write from formula (2.10)

$$\frac{-2z}{(1-\nu)}\frac{\partial^2}{\partial y^2}\chi_1' - \frac{z}{(1+\nu)}\frac{\partial^2 f}{\partial x\,\partial y} = \frac{z}{(1+\nu)}\Theta_1\,.$$

Using this relation normal stress σ_x in (2.1) can readily lead to the form of solution found by Love:—

$$\sigma_x = \frac{\partial^2}{\partial y^2}(\chi^1+\chi^2) + \frac{z}{(1+\nu)}\Theta_1 \,.$$

We have likewise for σ_y $\qquad \sigma_y = \frac{\partial}{\partial x^2}(\chi^1+\chi^2) + \frac{z}{(1+\nu)}\Theta_1 \,.$

Thus two kinds of solutions prove to be entirely equal within an irrelevant, arbitrary additive plane-harmonic function and hence we can apply the generalized plane stress solutions of Love without any apprehension to solve boundary value problems. Southwell sought, indeed, solutions in such manner that in the earlier part of his calculation he applied only the condition for σ_z, setting conditions for τ_{xz} and τ_{yz} aside. He thus obtained first general solutions under the former condition and, therefore, solutions due to him seem at first glance more general than Love's solutions. Neverthless, it may be said to be desirable to consider all the conditions for σ_z, τ_{xz} and τ_{yz} from the outset.

§ III. Rectangular Thick Plate Simply Supported at the Boundary under Normal Upper Surface Load, the Loading Function of Which is a v-Function.

We shall use the term "v-function" as defined on page 448 in Sec. IV of (I). Most notations to be used in this paper are found in Sec. II and IV of (I) or self-explanatory. In the following, if necessary, suffixes 1, 2, and 3 may be attached to the right sides of labels in order to distinguish between quantities which correspond to the solutions of three kinds, namely, plane stress, basic parts of generalized plane stress and particular solutions respectively. Particular solutions needed in this case are found in Sec. IV of (I) and the required boundary conditions will be as follows:

$$\left.\begin{array}{l} T=0\,,\quad S=0\,,\quad G=0\,,\quad w_0=w(x,y,0)=0\,, \\ \text{namely} \\ T_{1,1}+T_{1,3}=0\,,\ S_{1,1}+S_{1,3}=0\,,\ G_{1,2}+G_{1,3}=0\,,\quad \text{at}\ \ x=\pm a\,, \\ T_{2,1}+T_{2,3}=0\,,\ S_{2,1}+S_{2,3}=0\,,\ G_{2,2}+G_{2,3}=0\,,\quad \text{at}\ \ y=\pm b\,, \\ w_{0,2}+w_{0,3}=0 \quad \text{on the boundary,} \\ \text{since} \\ T_{1,2}=T_{2,2}=S_{1,2}=S_{2,2}=w_{0,1}=0\,. \end{array}\right\} \quad (3.1)$$

Cartesian coodinate system (x, y, z) is taken as shown in Fig. 1 and the z-axis is drawn upwards as usual. This coordinate system is adopted for symmetry. Let a given loading function be of the following form :

$$\sigma_z = -p_{mn} \sin \alpha_m(x+a) \sin \beta_n(y+b) \equiv -pv(x,y) \equiv -pv, \text{ at } z=h, \quad \cdots\cdots (3.2)$$

where $\alpha_m = \dfrac{m\pi}{2a}$, $\beta_n = \dfrac{n\pi}{2b}$ and m, n are positive integers. In this paper we shall solve boundary value problems by the use of the method similar to Hencky's related to thin plate problem. From particular solutions in Sec. IV of (I) we have on the boundary

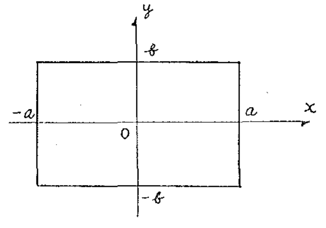


Fig. 1

$$T_3 = 0\,, \quad G_3 = 0\,, \quad w_{0,3} = w_3(x,y,0) = 0\,, \quad \cdots\cdots (3.3)$$

$$S_{1,3} = -S_{2,3} = \frac{K}{k^3} 2\nu(1-\cosh 2kh)(\sinh 2kh - 2kh) \frac{\partial^2 v(x,y)}{\partial x \partial y}$$
$$= Q \frac{\partial^2 v(x,y)}{\partial x \partial y}\bigg|_{x=\pm a, \text{ or } y=\pm b} \quad (3.4)$$

and, hence, in order to satisfy the boundary conditions (3.1) we only have to take into account the plane stress solutions (2.1) of (I), as is easily seen from the forms of the resultant forces and couples due to generalized plane stresses (2.10) of (I). Now the following four cases are conceivable.

case I $(m=2m', n=2n')$, case II $(m=2m'+1, n=2n'+1)$,

case III $(m=2m', n=2n'+1)$, case IV $(m=2m'+1, n=2n')$, $\cdots\cdots$ (3.5)

in which m' and n' are, of course, positive integers.

Firstly we take case I and, considering the forms of $S_{1,3}$ and $S_{2,3}$ and the boundary conditions concerning T, we may write

$$\chi'' = 2h\chi_0 - \frac{1}{3}\frac{\nu}{1+\nu} h^3\Theta_0 \quad \cdots\cdots (3.6)$$

$$= \sum_r A_r Y_r(y) \sin \alpha_r x + \sum_s B_s X_s(x) \sin \beta_s y\,, \quad \cdots\cdots (3.7)$$

with $X_s(x) = a \operatorname{cash} \beta_s a \sinh \beta_s x - x \sinh \beta_s a \cosh \beta_s x$,

$Y_r(y) = b \cosh \alpha_r b \sinh \alpha_r y - y \sinh \alpha_r b \cosh \alpha_r y$, in which $\alpha_r = \frac{r\pi}{2a}$, $\beta_s = \frac{s\pi}{2b}$, $r=2r'$, $s=2s'$. We shall delete suffixes r and s of α_r and β_s always in the sequel. It will be obvious that $T_{1,1} = \frac{\partial^2 \chi''}{\partial y^2}$ and $T_{2,1} = \frac{\partial^2 \chi''}{\partial x^2}$ derived from (3.7) vanish at the edges $x = \pm a$ and $y = \pm b$ respectively. Then the following relation must hold at the edges $x = \pm a$:

$$S_{1,1} = \frac{-\partial^2 \chi''}{\partial x \partial y} = -\Big[\sum_r A_r \big\{(ab \cosh \alpha b - \sinh \alpha b) \cosh \alpha y + \\ - \alpha \sinh \alpha b \cdot y \sinh \alpha y\big\} \alpha (-1)^{r'} + \sum_s B_s \beta (\beta a - \sinh \beta a \cosh \beta a) \times \\ \times \cos \beta y \Big] = -S_{1,3} = -(-1)^{n'} Q \alpha_m \beta_n \cos \beta_n y . \quad \cdots\cdots\cdots (3.8)$$

Then, if we substitute the formulae

$$\left.\begin{aligned} \cosh \alpha y &= \frac{1}{b} \sum_{s'=0}^{\infty} \varepsilon_{s'} \cos \beta y \int_0^b \cosh \frac{r\pi}{2a} y \cos \frac{s'\pi}{b} y \, dy , \\ y \sinh \alpha y &= \frac{1}{b} \sum_{s'=0}^{\infty} \varepsilon_{s'} \cos \beta y \int_0^b y \sinh \frac{r\pi}{2a} y \cos \frac{s'\pi}{b} y \, dy , \end{aligned}\right\} \quad (3.9)$$

in which $\varepsilon_0 = 1$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \cdots = 2$, into Eq. (3.8) and equate coefficients of similar terms on the two sides of the equation, we obtain the relation:

$$\sum_{\substack{r \\ (r=2r')}} A_r \frac{(-1)^{s'+r'+1} \varepsilon_{s'} 2a^2 b r'^2 s'^2}{(r'^2 b^2 + a^2 s'^2)^2} \sinh^2 \alpha b + B_s (\beta a - \sinh \beta a \cosh \beta a) \frac{s'\pi}{b} \\ \left.\begin{aligned} &= (-1)^{n'} Q \alpha_m \beta_n , \quad \text{for} \quad s' = n' , \\ &= 0 , \quad \text{for} \quad n' \neq s' . \end{aligned}\right\} \quad \cdots\cdots\cdots (3.10)$$

By the condition in (3.1)

$$S_2 = S_{2,1} + S_{2,3} = 0 \quad \text{at} \quad y = \pm b ,$$

we find another similar relation in the same way as stated above.

$$A_r (ab - \sinh \alpha b \cosh \alpha b) \alpha + \sum_s B_s \frac{(-1)^{s'+r'+1} \varepsilon_{r'} 2ab^2 r'^2 s'^2 \sinh^2 \beta a}{(b^2 r'^2 + a^2 s'^2)^2} \\ \left.\begin{aligned} &= (-1)^{m'} Q \alpha_m \beta_n , \quad \text{for} \quad r' = m' , \\ &= 0 , \quad \text{for} \quad r' \neq m' . \end{aligned}\right\} \quad \cdots\cdots (3.11)$$

Then two sequences of coefficients $\{A_r\}$, $\{B_s\}$ can be determined from these formulae (3.10), (3.11), applying the method of successive approximation. As in the theory of thin plate, a problem of thick plate with simply supported edges can be solved readily in comparison with that of thick plate with clamped edges.

Next we shall determine the form of χ'' for case II ($m=2m'+1$, $n=2n'+1$). We may put for χ'' (3.6)

$$\chi'' = \sum_r A_r Y_r(y) \cos \alpha x + \sum_s B_s X_s(x) \cos \beta y , \qquad \cdots\cdots\cdots\cdots\cdots (3.12)$$

in which

$$\left.\begin{aligned} X_s(x) &= a \sinh \beta a \cosh \beta x - x \cosh \beta a \sinh \beta x , \\ Y_r(y) &= b \sinh \alpha b \cosh \alpha y - y \cosh \alpha b \sinh \alpha y , \end{aligned}\right\} \qquad \cdots\cdots (3.13)$$

$$\left(\alpha = \frac{r\pi}{2a} , \ \beta = \frac{s\pi}{2b} ; \ r=2r'+1 , \ s=2s'+1) \right) .$$

It is easily seen that the conditions $T_1=0$ at $x=\pm a$ and $T_2=0$ at $y=\pm b$ are fulfilled by (3.12), (3.13).

By the condition

$$S_{1,1} = \frac{-\partial^2\chi''}{\partial x\, \partial y} = -S_{1,3} = (-1)^{n'+1} Q \alpha_m \beta_n \sin \beta_n y \qquad \cdots\cdots\cdots (3.14)$$

at the edge $x=a$, utilizing formulae

$$\left.\begin{aligned} \sinh \alpha y &= \frac{2}{b} \sum_{s'=0}^{\infty} \sin \frac{(2s'+1)}{2b} \pi y \int_0^b \sinh \frac{(2r'+1)}{2a} \pi y \sin \frac{(2s'+1)}{2b} \pi y dy, \\ y \cosh \alpha y &= \frac{2}{b} \sum_{s'=0}^{\infty} \sin \frac{(2s'+1)}{2b} \pi y \int_0^b y \cosh \frac{(2r'+1)}{2a} \pi y \sin \frac{(2s'+1)}{2b} \pi y dy, \end{aligned}\right\}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3.15)$$

which obviously hold in the rage $-b<y<b$, we obtain the relation:—

$$\begin{aligned} &\sum_r A_r \frac{(-1)^{s'+r'} 4a^2 b r^2 s^2}{(b^2r^2+a^2s^2)^2} \cosh^2 \alpha b + B_s (\beta a + \cosh \beta a \sinh \beta a) \beta \\ &\qquad\qquad \left.\begin{aligned} &= (-1)^{n'} Q \alpha_m \beta_n, && \text{for } s=n , \\ &= 0 , && \text{for } s \neq n . \end{aligned}\right\} \quad (3.16) \end{aligned}$$

The condition at the edge $x=-a$ similar to (3.14) evidently leads to the same relation as (3.16) because of symmetry. Then by the similar condition pertaining to shearing stress resultant at the edges $y=\pm b$, we get the relation.

$$A_r(ab+\cosh ab\cdot\sinh ab)\,\alpha+\sum_s B_s(-1)^{s'+r'}\frac{4ab^2r^2s^2\cosh^2\beta a}{(b^2r^2+a^2s^2)^2}$$
$$\left.\begin{aligned}&=(-1)^{m'}Q\alpha_m\beta_n, &&\text{for}\quad m=r\,,\\ &=0\,, &&\text{for}\quad m\neq r\,.\end{aligned}\right\}\quad(3.17)$$

From Eqs. (3.16), (3.17) two sequences $\{A_r\}$, $\{B_s\}$ may be determined in the same manner as before.

For case III ($m=2m'$, $n=2n'+1$) we may write χ'' (3.6) in the following form:

$$\chi''=\sum_r A_rY_r(y)\sin\alpha x+\sum_s B_sX_s(x)\cos\beta y\,,\qquad\cdots\cdots\cdots\cdots(3.18\text{a})$$

in which

$$\left.\begin{aligned}Y_r(y)&=b\sinh\alpha b\cosh\alpha y-y\cosh\alpha b\sinh\alpha y\,,\\ X_s(x)&=a\cosh\beta a\sinh\beta x-x\sinh\beta a\cosh\beta x\,,\end{aligned}\right\}\quad\cdots\cdots(3.18\text{b})$$
$$\left(\alpha=\frac{r\pi}{2a}\,,\ \beta=\frac{s\pi}{2b}\,;\ r=2r',\ s=2s'+1\right).$$

For the conditions regarding $S_{1,1}$, $S_{2,1}$ we have from (3.4)

$$S_{1,1}=-S_{1,3}=\pm(-1)^{n'}Q\alpha_m\beta_n\sin\beta_n y\qquad\text{at } x=\pm a\ \cdots(3.19\text{a})$$
$$S_{2,1}=-S_{2,3}=\mp(-1)^{m'}Q\alpha_m\beta_n\cos\alpha_m x\qquad\text{at } y=\pm b\ \cdots(3.19\text{b})$$

Observing the right-hand sides of equations (3.19), χ'' can be taken to be an odd function of x and an even function of y and, hence, the form of χ'' (3.18a) has been taken in consideration of the conditions that $T_{1,1}=0$ at $x=\pm a$; $T_{2,1}=0$, at $y=\pm b$.

Using the formulae (3.15), we get the relation from (3.19a).

$$\sum_r A_r\frac{(-1)^{s'+r'+1}16a^2bs^2r'^2}{(b^2r^2+a^2s^2)^2}\cosh^2\alpha b+B_s(\beta a-\sinh\beta a\cosh\beta a)(-\beta)$$
$$\left.\begin{aligned}&=(-1)^{n'+1}Q\alpha_m\beta_n\,, &&\text{for}\quad n=s\,,\\ &=0\,, &&\text{for}\quad n\neq s\,,\end{aligned}\right\}\quad(3.20)$$

which is obtained by the condition at $x=a$, while the condition at $x=-a$ only results in the same relation as (3.20).

From (3.19b) we have similarly the relation:—

$$A_r(\alpha b+\sinh\alpha b\cosh\alpha b)\,\alpha+\sum_s B_s\frac{\varepsilon_{r'}(-1)^{r'+s'+1}2ab^2r^2s^2\sinh^2\beta a}{(b^2r^2+a^2s^2)^2}$$
$$\left.\begin{aligned}&=(-1)^{m'}Q\alpha_m\beta_n\,, &&\text{for}\quad m=r\,,\\ &=0\,, &&\text{for}\quad m\neq r\,.\end{aligned}\right\}\quad(3.21)$$

Accordingly coefficients $\{A_r\}$, $\{B_s\}$ are readily determined by successive approximation.

We shall discuss case IV only for reference.
For case IV ($m=2m'+1$, $n=2n'$) by the conditions

$$S_{1,1} = -S_{1,3} = -Q\frac{\partial^2 v(x,y)}{\partial x\,\partial y} = \pm(-1)^{n'} Q\,\alpha_m\beta_n \cos\beta_n y, \quad \text{at } x=\pm a, \quad \cdots\cdots\cdots\cdots (3.22a)$$

$$S_{2,1} = -S_{2,3} = Q\frac{\partial^2 v(x,y)}{\partial x\,\partial y} = \mp(-1)^{m'} Q\alpha_m\beta_n \sin\alpha_m x, \quad \text{at } y=\pm b, \quad \cdots\cdots\cdots\cdots (3.22b)$$

$$T_{1,1}=0 \quad \text{at} \quad x=\pm a, \qquad T_{2,1}=0 \quad \text{at} \quad y=\pm b,$$

utilizing formulae (3.9), we can derive the following relation from the condition (3.22a):

$$\sum_r A_r \frac{\varepsilon_{s'}(-1)^{s'+r'+1}2a^2br^2s^2}{(b^2r^2+a^2s^2)^2}\sinh^2 ab + B_s(\beta a+\cosh\beta a\sinh\beta a)\beta$$
$$\left.\begin{aligned} &= (-1)^{n'}Q\,\alpha_m\beta_m, && \text{for } s=n, \\ &= 0, && \text{for } s\neq n. \end{aligned}\right\} \quad (3.23)$$

$$\left(\alpha=\frac{r\pi}{2a},\quad \beta=\frac{s\pi}{2b};\quad r=2r'+1,\quad s=2s'\right)$$

And by the use of formulae similar to (3.15) the following formula is obtainable from the condition (3.22b) :—

$$A_r(\alpha b-\sinh\alpha b\cosh\alpha b)\alpha+\sum_s B_s(-1)^{r'+s'}\frac{4ab^2r^2s^2\cosh^2\beta a}{(b^2r^2+a^2s^2)^2}$$
$$\left.\begin{aligned} &= (-1)^{m'}Q\,\alpha_m\beta_n && (r=m), \\ &= 0 && (r\neq m). \end{aligned}\right\} \quad \cdots (3.24)$$

Hence, two sequences of coefficients $\{A_r\}$, $\{B_s\}$ can be determined from two sequences of equations of types (3.23) and (3.24).

Thus we have found that the forms of χ'', χ_0 and Θ_0 for four cases can be obtained from them. Therefore, all plane stress solutions can be found immediately with the aid of formulae (2.1), (2.5) of (I) and then we can determine the solutions to the whole problem by the superposition of these and particular solutions in Sec. IV of (I). It is noteworthy that in this case of rectangular thick plate simply supported at the edges the solutions to the problem are obtained without the aid of generalized plane stress solutions. Now we shall indicate the forms of solutions in case I.

We have from (3.7) by means of the equation $\nabla_1^2\chi''=2h\nabla_1^2\chi_0=2h\Theta_0$

$$\chi_0=\frac{1}{2h}\left(\chi''+\frac{1}{3}\frac{\nu}{1+\nu}h^3\Theta_0\right)=\frac{1}{2h}\Big[\sum_r A_r\Big\{\Big(b\cosh\alpha b+ -\frac{1}{3}\frac{\nu}{1+\nu}h^2\alpha\sinh\alpha b\Big)\sinh\alpha y-y\sinh\alpha b\cosh\alpha y\Big\}\sin\alpha x+ +\sum_s B_s\Big\{\Big(a\cosh\beta a-\frac{1}{3}\frac{\nu}{1+\nu}h^2\beta\sinh\beta a\Big)\sinh\beta x+ -x\sinh\beta a\cosh\beta x\Big\}\sin\beta y\Big], \quad\cdots\cdots (3.25\text{a})$$

$$\Theta_0=\frac{-1}{h}\Big(\sum_r A_r\alpha\sinh\alpha b\sinh\alpha y\sin\alpha x+\sum_s B_s\beta\sinh\beta a\sinh\beta x\sin\beta y\Big). \quad\cdots\cdots (3.25\text{b})$$

According to (2.1) of (I) we have for stress components

$$\sigma_{x,1}=\frac{\partial^2}{\partial y^2}\chi=\frac{\partial}{\partial y^2}\Big(\chi_0-\frac{1}{2}\frac{\nu}{1+\nu}z^2\Theta_0\Big)=\sum_r A_r\frac{\alpha}{2h}\Big[\Big\{\alpha b\cosh\alpha b-2\sinh\alpha b+ +\frac{\nu}{1+\nu}\alpha^2\sinh\alpha b\cdot\Big(z^2-\frac{1}{3}h^2\Big)\Big\}\sinh\alpha y-\alpha\sinh\alpha b\cdot y\cosh\alpha y\Big]\sin\alpha x+ -\sum_s B_s\frac{\beta^2}{2h}\Big[\Big\{a\cosh\beta a+\frac{\nu}{1+\nu}\beta\sinh\beta a\cdot\Big(z^2-\frac{1}{3}h^2\Big)\Big\}\sinh\beta x+ -\sinh\beta a\cdot x\cosh\beta x\Big]\sin\beta y, \quad\cdots\cdots (3.26\text{a})$$

$$\sigma_{y,1}=\frac{\partial^2}{\partial x^2}\chi=-\sum A_r\frac{\alpha^2}{2h}\Big[\Big\{b\cosh\alpha b+\frac{\nu}{1+\nu}\alpha\sinh\alpha b\cdot\Big(z^2-\frac{1}{3}h^2\Big)\Big\}\times \times\sinh\alpha y-y\sinh\alpha b\cdot\cosh\alpha y\Big]\sin\alpha x+\sum_s B_s\frac{\beta}{2h}\Big[\Big\{\beta a\cosh\beta a+ -2\sinh\beta a+\frac{\nu}{1+\nu}\beta^2\sinh\beta a\cdot\Big(z^2-\frac{1}{3}h^2\Big)\Big\}\sinh\beta x+ -\beta\sinh\beta a\cdot x\cosh\beta x\Big]\sin\beta y, \quad\cdots\cdots (3.26\text{b})$$

$$\tau_{xy,1}=\frac{-\partial^2\chi}{\partial x\partial y}=-\sum_r A_r\frac{\alpha}{2h}\Big[\Big\{\alpha b\cosh\alpha b+\frac{\nu}{1+\nu}\alpha^2\sinh\alpha b\cdot\Big(z^2-\frac{1}{3}h^2\Big)+$$

$$-\sinh ab\Big\}\cosh ay-a\sin ab\cdot y\sinh ay\Big]\cos ax+$$

$$-\sum_s B_s\frac{\beta}{2h}\Big[\Big\{\beta a\cosh\beta a+\frac{\nu}{1+\nu}\beta^2\sinh\beta a\cdot\Big(z^2-\frac{1}{3}h^2\Big)-\sinh\beta a\Big\}\times$$

$$\times\cosh\beta x-\beta\sinh\beta a\cdot x\sinh\beta x\Big]\cos\beta y. \qquad\cdots\cdots\cdots\cdots (3.26c)$$

By (2.5) of (I) we obtain for displacements

$$u_1=\frac{1}{2hE}\Big[\sum_r A_r\Big\{\Big(2\sinh ab-\nu z^2a^2\sinh ab-(1+\nu)ab\cosh ab+$$

$$+\frac{1}{3}\nu h^2a^2\sinh ab\Big)\sinh ay+(1+\nu)a\sinh ab\cdot y\cosh ay\Big\}\cos ax+$$

$$+\sum_s B_s\Big\{\Big(-2\sinh\beta a-\nu z^2\beta^2\sinh\beta a-(1+\nu)\beta a\cosh\beta a+\frac{1}{3}\nu h^2\beta^2\sinh\beta a+$$

$$+(1+\nu)\sinh\beta a\Big)\cosh\beta x+(1+\nu)\beta\sinh\beta a\cdot x\cosh\beta x\Big\}\sin\beta y\Big], \quad (3.27a)$$

$$v_1=\frac{1}{2hE}\Big[\sum_r A_r\Big\{\Big(-2\sinh ab-\nu z^2a^2\sinh ab-(1+\nu)ab\cosh ab+$$

$$+\frac{1}{3}\nu h^2a^2\sinh ab+(1+\nu)\sinh ab\Big)\cosh ay+(1+\nu)a\sinh ab\cdot y\sinh ay\Big\}\times$$

$$\times\sin ax+\sum_s B_s\Big\{\Big(2\sinh\beta a-\nu z^2\beta^2\sinh\beta a-(1+\nu)\beta a\cosh\beta a+$$

$$+\frac{1}{3}\nu h^2\beta^2\sinh\beta a\Big)\sinh\beta x+(1+\nu)\beta\sinh\beta a\cdot x\cosh\beta x\Big\}\cos\beta y\Big], \quad (3.27b)$$

$$w_r=\frac{\nu z}{hE}\Big\{\sum_r A_r a\sinh ab\cdot\sinh ay\sin ax+\sum_s B_s\beta\sinh\beta a\sinh\beta x\sin\beta y\Big\}.$$

$$\cdots\cdots\cdots\cdots (3.27c)$$

So the solutions to the problem under consideration can easily be obtained by composing the above written solutions and particular solutions in Sec. IV of (I). For examples, we have for σ_x

$$\sigma_x=\sigma_{x,1}+\sigma_{x,3}=\sum_r A_r\frac{a}{2h}\Big[\Big\{ab\cosh ab-2\sinh ab+\frac{\nu}{1+\nu}\times$$

$$\times\sinh ab\cdot a^2\Big(z^2-\frac{1}{3}h^2\Big)\Big\}\sinh ay-a\sinh ab\cdot y\cosh ay\Big]\sin ax+$$

$$-\sum B_s\frac{\beta}{2h}\Big[\Big\{\beta a\cosh\beta a+\frac{\nu}{1+\nu}\sinh\beta a\cdot\beta^2\Big(z^2-\frac{1}{3}h^2\Big)\Big\}\sinh\beta x+$$

$$-\beta\sinh\beta a\cdot x\cosh\beta x\Big]\sin\beta y+\frac{K}{k^2}\Big[\Big(2\nu\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}\Big)\times$$

$$\times\sinh 2kh\sinh k(z+h)+\Big(4\nu\frac{\partial^2 v}{\partial y^2}+3\frac{\partial^2 v}{\partial x^2}\Big)kh\sinh k(z-h)+$$

$$+\frac{\partial^2 v}{\partial x^2}\Big\{-kh\cosh 2kh\sinh k(z+h)+2k^3h^2\cosh k(z-h)+$$

$$+kz\Big(\sinh 2kh\cosh k(z+h)+2kh\cosh k(z-h)\Big)\Big\}\Big], \quad\cdots\cdots (3.28a)$$

and for displacement u

$$u=u_1+u_3=(3.27a)+\frac{-(1+\nu)}{E}\frac{K}{k^2}\frac{\partial v}{\partial x}\Big[(2\nu-1)\sinh 2kh\sinh k(z+h)+$$

$$+kh\cosh 2kh\sinh k(z+h)+(4\nu-3)kh\sinh k(z-h)-2k^3h^2\cosh k(z-h)+$$

$$-kz\Big\{\sinh 2kh\cosh k(z+h)+2kh\cosh k(z-h)\Big\}\Big]. \quad\cdots\cdots\cdots (3.28b)$$

$$\Big(K=p\Big/(\sinh^2 2kh-4k^2h^2),\quad k^2=\alpha_m^2+\beta_n^2\Big).$$

Here we write only the forms of Θ_0, χ_0 and χ in the other three cases II, III, IV, although for these cases expressions similar to the above must be presented, since they are so lengthy.

case II.:

$$\Theta_0=\frac{-1}{h}\Big(\sum_r A_r\alpha\cosh ab\cosh\alpha y\cos\alpha x+\sum_s B_s\beta\cosh\beta a\cosh\beta x\cos\beta y\Big),$$

$$\chi_0=\chi|_{z=0},$$

$$\chi=\chi_0-\frac{1}{2}\frac{\nu}{1+\nu}z^2\Theta_0=\frac{1}{2h}\Big[\sum_r A_r\Big\{\Big(b\sinh\alpha b+\frac{\nu}{1+\nu}\alpha\cosh\alpha b\times$$

$$\times\Big(z^2-\frac{1}{3}h^2\Big)\Big)\cosh\alpha y-\cosh\alpha b\cdot y\sinh\alpha y\Big\}\cos\alpha x+$$

$$+\sum_s B_s\Big\{\Big(a\sinh\beta a+\frac{\nu}{1+\nu}\beta\cosh\beta a\cdot\Big(z^2-\frac{1}{3}h^2\Big)\Big)\cosh\beta x+$$

$$-\cosh\beta a\cdot x\sinh\beta x\Big\}\cos\beta y\Big], \quad\cdots\cdots\cdots (3.29)$$

$(r=2r'+1,\ s=2s'+1)$, $\{A_r\}$, $\{B_s\}$ from (3.16), (3.17),

case III.:

$$\Theta_0=\frac{-1}{h}\Big(\sum_r A_r\alpha\cosh\alpha b\cosh\alpha y\sin\alpha x+\sum_s B_s\beta\sinh\beta a\sinh\beta x\cos\beta y\Big),$$

$$\chi_0 = \chi|_{z=0} \ ,$$

$$\chi = \frac{1}{2h}\Big[\sum_r A_r\Big\{\Big(b\sinh ab + \frac{\nu}{1+\nu}a\cosh ab\cdot\Big(z^2 - \frac{1}{3}h^2\Big)\cosh ay +$$

$$-\cosh ab\cdot y\sinh ay\Big\}\sin ax + \sum_s B_s\Big\{\Big(a\cosh\beta a + \frac{\nu}{1+\nu}\beta\sinh\beta a\cdot\times$$

$$\times\Big(z^2 - \frac{1}{3}h^2\Big)\Big)\sinh\beta x - \sinh\beta a\cdot x\cosh\beta x\Big\}\cos\beta y\Big], \quad \cdots (3.30)$$

$(r = 2r'\ ,\ \ s = 2s'+1)$. $\{A_r\}$, $\{B_s\}$ from (3.20), (3.21).

case IV.:

$$\Theta_0 = \frac{-1}{h}\Big[\sum_r A_r a\sinh ab\cdot\sinh ay\cos ax + \sum_s B_s\beta\cosh\beta a\cosh\beta x\sin\beta y\Big] \ ,$$

$$\chi_0 = \chi|_{z=0} \ ,$$

$$\chi = \frac{1}{2h}\Big[\sum_r A_r\Big\{\Big(b\cosh ab + \frac{\nu}{1+\nu}a\sinh ab\cdot\Big(z^2 - \frac{1}{3}h^2\Big)\Big)\sinh ay +$$

$$-\sinh ab\cdot y\cosh ay\Big\}\cos ax + \sum_s B_s\Big\{\Big(a\sinh\beta a + \frac{\nu}{1+\nu}\beta\cosh\beta a\cdot\times$$

$$\times\Big(z^2 - \frac{1}{3}h^2\Big)\cosh\beta x - \cosh\beta a\cdot x\sinh\beta x\Big\}\sin\beta y\Big], \quad \cdots\cdots (3.31)$$

$(r = 2r'+1\ ,\ s = 2s')$, $\{A_r\}$, $\{B_s\}$ from (3.23), (3.24).

When a simply supported rectangular plate is bent by general pressure applied to the upper surface, we can get the solutions according to Sec. VI of (I), if a given intensity of pressure $f(x,y) = F(x+a, y+b)$ can be expanded in a double trigonometric series of the form

$$f(x,y) = F(x+a, y+b) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} p_{mn}\sin\alpha_m(x+a)\sin\beta_n(y+b),$$

$$= \sum_m\sum_n p_{mn}v_{mn}(x,y),$$

in which

$$\alpha_m = \frac{m\pi}{2a}\ ,\ \beta_n = \frac{n\pi}{2b}\ ,\ p_{mn} = \frac{1}{ab}\int_0^{2b}\int_0^{2a}F(x,y)\sin\alpha_m x\sin\beta_n y\,dx\,dy\ ,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (3.32)$$

The forms of solutions which correspond to a single term of the series (3.32) or a sinusoidal load, both so-called particular and complementary, were obtained in the foregoing and, of course, labels which represent these solutions ought to have suffixes m, n originally, but they have been dropped for the sake of brevity. As a consequence, solution σ_x, for example, to this problem will be of the form

$$\sigma_x = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sigma_{x,mn} , \qquad (3.33)$$

and the expression (3.28a) is a solution $\sigma_{x,mn}$ in case I, where m and n are both even positive integers.

§ IV. Rectangular Thick Plate Clamped at the Boundary Stressed by Normal Surface Load, the Loading Function of Which is a v-Function.

We shall take a coordinate system as indicated in Fig. 1 and let loading function be of the type of (3.2). The origin of the coordinates lies at the middle point of the plate and, hence, the plane (x,y) is a middle plane and thickness is $2h$, needless to say. As the boundary conditions for the clamped plate we may take

$$u_0 = 0 , \quad v_0 = 0 , \quad w_0 = 0 , \quad \frac{\partial w_0}{\partial \nu} = 0 , \qquad (4.1)$$

in which the system (u_0, v_0, w_0) denotes displacement of any point $(x, y, 0)$ of the middle plane of plate and ν is the direction of the outer normal to the bounding curve. In the first place we shall determine the expressions for χ_0 and Θ_0 contained in the plane stress solutions in Sec. II of (I). Considering the fact that $u_{0,2}$, $v_{0,2}$ in (2.7b) of (I) vanish on the middle plane $z=0$, we may put by (2.5), (4.29) of (I) as follows:

$$u_0 = u_{0,1} + u_{0,3} = \frac{1}{E}\delta_1 = \frac{1}{E}\left(\xi - (1+\nu)\frac{\partial \chi_0}{\partial x} + J\frac{\partial v}{\partial x}\right) , \qquad (4.2a)$$

$$v_0 = v_{0,1} + v_{0,3} = \frac{1}{E}\delta_2 = \frac{1}{E}\left(\eta - (1+\nu)\frac{\partial \chi_0}{\partial y} + J\frac{\partial v}{\partial y}\right) , \qquad (4.2b)$$

in which

$$J = -(1+\nu)\frac{K}{k^2}\Big[\left\{(2\nu-1)\sinh 2kh + kh\cosh 2kh + (3-4\nu)kh\right\} \times$$
$$\times \sinh kh - 2k^2h^2\cosh kh\Big] , \qquad (4.3)$$

where

$$K = p\Big/(\sinh^2 2kh - 4k^2h^2) , \quad k^2 \equiv k_{mn}^2 = \alpha_m^2 + \beta_n^2 = \frac{\pi^2}{4}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right) ,$$

$$p \equiv p_{mn} , \quad v \equiv v_{mn}(x, y) = \sin \alpha_m(x+a) \sin \beta_n(y+b) .$$

Then, if we put

$$\delta_1 = \bar{\chi}_1 + J\frac{\partial v}{\partial x}, \quad \delta_2 = \bar{\chi}_2 + J\frac{\partial v}{\partial y}, \qquad \cdots\cdots (4.4)$$

$\bar{\chi}_1$, $\bar{\chi}_2$ may be taken to be plane biharmonic functions in this case, considering the property of $u_{0,1}$ and $v_{0,1}$.

First we shall treat case I ($m = 2m'$, $n = 2n'$). Observing the forms of $J\frac{\partial v}{\partial x}$, $J\frac{\partial v}{\partial y}$, or

$$\left.\begin{aligned} J\frac{\partial v}{\partial x} &= (-1)^{m'+n'} J\alpha_m \cos\alpha_m x \sin\beta_n y\ , \\ J\frac{\partial v}{\partial y} &= (-1)^{m'+n'} J\beta_n \sin\alpha_m x \cos\beta_n y\ , \end{aligned}\right\} \qquad \cdots\cdots (4.5)$$

with $\alpha_m = \frac{\pi m}{2a}$, $\beta_n = \frac{\pi n}{2b}$, we may write for $\bar{\chi}_1$

$$\begin{aligned} \bar{\chi}_1 &= \sum_r A_r (b\cosh\alpha b \sinh\alpha y - y\sinh\alpha b\cosh\alpha y)\cos\alpha x + \\ &+ \sum_s (B_s \cosh\beta x + C_s x\sinh\beta x)\sin\beta y\ . \qquad \cdots\cdots (4.6) \\ &\left(\alpha = \frac{r\pi}{2a},\quad \beta = \frac{s\pi}{2b},\quad r = 2r',\quad s = 2s'\right). \end{aligned}$$

Accordingly we get by (4.2a), (4.6)

$$\begin{aligned} \nabla_1^2 \bar{\chi}_1 &= -(1+\nu)\frac{\partial\Theta_0}{\partial x} = -\sum_r A_r 2\alpha \sinh\alpha b \sinh\alpha y\cos\alpha x + \\ &+ \sum_s C_s 2\beta\cosh\beta x\sin\beta y\ . \qquad (\nabla_1^2\xi = 0) \qquad \cdots\cdots (4.7) \end{aligned}$$

By integrating this equation we find for Θ_0.

$$\Theta_0 = \frac{2}{(1+\nu)}\left(\sum_r A_r \sinh\alpha b\sinh\alpha y\sin\alpha x - \sum_s C_s \sinh\beta x\sin\beta y\right), \qquad \cdots\cdots (4.8)$$

and, hence, ξ will be of the form

$$\begin{aligned} \xi &= \int^x \Theta_0\, dx = \frac{2}{(1+\nu)}\left(-\sum_r A_r \frac{1}{\alpha}\sinh\alpha b\sinh\alpha y\cos\alpha x + \right. \\ &\left. - \sum_s C_s \frac{1}{\beta}\cosh\beta x\sin\beta y\right) . \qquad \cdots\cdots (4.9) \end{aligned}$$

Now we can obtain the expression for $\frac{\partial\chi_0}{\partial y}$ from $\frac{\partial\chi_0}{\partial x} = \frac{-1}{(1+\nu)}(\bar{\chi}_1 - \xi)$

and write $\bar{\chi}_2$ as

$$\bar{\chi}_2 = \sum_r \frac{A_r}{(1+\nu)\alpha}\Big[\Big\{(1+\nu)\,\alpha b\cosh\alpha b+(3-\nu)\sinh\alpha b\Big\}\cosh\alpha y +$$
$$-(1+\nu)\,\alpha\sinh\alpha b\cdot y\sinh\alpha y\Big]\sin\alpha x+\sum_r\frac{1}{(1+\nu)\,\beta}\Big[\Big\{(1+\nu)\beta B_s +$$
$$+C_s(3-\nu)\Big\}\sinh\beta x+\beta(1+\nu)C_s\cdot x\cosh\beta x\Big]\cos\beta y\,. \qquad (4.10)$$

Then the second term of (4.10) may be expected to have the following form in view of the boundary condition:

$$\sum_s D_s(a\cosh\beta a\sinh\beta x - x\sinh\beta a\cdot\cosh\beta x)\cos\beta y\,, \quad \cdots\cdots \quad (4.11)$$

and, whence, it is found that B_s, C_s are to be of the types

$$B_s = D_s\Big(a\cosh\beta a+\frac{(3-\nu)}{\beta(1+\nu)}\sinh\beta a\Big)\,, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (4.12)$$

$$C_s = -D_s\sinh\beta a\,.$$

Putting for A_r, D_s

$$A_r = A'_r(1+\nu)\alpha\,, \quad D_s = B'_s(1+\nu)\beta \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (4.13)$$

and deleting dashes of A'_r, B'_s, we finally obtain the forms of $\bar{\chi}_1$, $\bar{\chi}_2$ and, thus, of δ_1, δ_2.

$$\delta_1 = Eu_0 = \sum_r A_r(1+\nu)\,\alpha(b\cosh\alpha b\sinh\alpha y - y\sinh\alpha b\cosh\alpha y)\times$$
$$\times\cos\alpha x+\sum_s B_s\Big[\Big\{(1+\nu)\beta a\cosh\beta a+(3-\nu)\sinh\beta a\Big\}\cosh\beta x +$$
$$-(1+\nu)\beta\sinh\beta a\cdot x\sinh\beta x\Big]\sin\beta y+J\frac{\partial v}{\partial x}\,, \quad \cdots\cdots\cdots \quad (4.14a)$$

$$\delta_2 = Ev_0 = \sum_r A_r\Big[\Big\{(1+\nu)\,\alpha b\cosh\alpha b+(3-\nu)\sinh\alpha b\Big\}\cosh\alpha y +$$
$$-(1+\nu)\,\alpha\sinh\alpha b\cdot y\sinh\alpha y\Big]\sin\alpha x+\sum_s B_s(1+\nu)\beta\Big(a\cosh\beta a\sinh\beta x +$$
$$-x\sinh\beta a\cosh\beta x\Big)\cos\beta y+J\frac{\partial v}{\partial y}\,. \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad (4.14b)$$

Formulae (4.14) obviously satisfy the conditions

$\delta_1 = 0$ at $y \pm b$, $\delta_2 = 0$ at $x = \pm a$ respectively and, therefore, the boundary conditions (4.1) only require that

$$\delta_1 = 0 \text{ at } x = \pm a \text{ and } \delta_2 = 0 \text{ at } y = \pm b\,, \quad \cdots\cdots\cdots\cdots \quad (4.15)$$

that is to say,

$$\sum_{r} A_r (1+\nu)\,\alpha\,(b\cosh\alpha b\sinh\alpha y - y\sinh\alpha b\cosh\alpha y)(-1)^{r'} +$$

$$+\sum_{s} B_s \Big\{(1+\nu)\beta a + (3-\nu)\sinh\beta a\cdot\cosh\beta a\Big\}\sin\beta y =$$

$$=(-1)^{n'+1} J\alpha_m \sin\beta_n y\,, \qquad \cdots\cdots (4.16\text{a})$$

$$\sum_{r} A_r \Big\{(1+\nu)\,\alpha b + (3-\nu)\sinh\alpha b\cosh\alpha b\Big\}\sin\alpha x + \sum_{s} B_s (1+\nu)\,\times$$

$$\times\,\beta(a\cosh\beta a\sinh\beta x - \sinh\beta a\cdot x\cosh\beta x)(-1)^{s'} = (-1)^{m'+1} J\beta_n \sin\alpha_m x\,.$$

$$\cdots\cdots (4.16\text{b})$$

By means of the following expansions

$$\left.\begin{aligned} \sinh\alpha y &= \frac{2}{b}\sum_{s'=1}^{\infty}\sin\frac{s'\pi}{b}y\int_0^b \sinh\frac{r'\pi}{a}y\sin\frac{s'\pi}{b}y\,dy\,, \\ y\cosh\alpha y &= \frac{2}{b}\sum_{s'=1}^{\infty}\sin\frac{s'\pi}{b}y\int_0^b y\cosh\frac{r'\pi}{a}y\sin\frac{s'\pi}{b}y\,dy\,, \end{aligned}\right\} \cdots (4.17)$$

we obtain from (4.16a)

$$\sum_{r} A_r \frac{(1+\nu)(-1)^{r'+s'+1}4a^2b^2r'^2s'}{\pi(b^2r'^2+a^2s'^2)^2}\sinh^2\alpha b + B_s\Big\{(1+\nu)\beta a + (3-\nu)\sinh\beta a\cosh\beta a\Big\}$$

$$\left.\begin{aligned} &= (-1)^{n'+1} J\alpha_m\,, && \text{for} \quad s=n\,, \\ &= 0\,, && \text{for} \quad s\neq n\,. \end{aligned}\right\} \quad (4.18)$$

Similarly we have from (4.16b)

$$A_r\Big\{(1+\nu)\,\alpha b + (3-\nu)\sinh\alpha b\cosh\alpha b\Big\} + \sum_{s} B_s \frac{(1+\nu)}{\pi}\frac{(-1)^{s'+r'+1}4a^2b^2r's'^2\sinh^2\beta a}{(b^2r'^2+a^2s'^2)^2}$$

$$\left.\begin{aligned} &= (-1)^{m'+1} J\beta_n\,, && \text{for} \quad m=r\,, \\ &= 0\,, && \text{for} \quad m\neq r\,. \end{aligned}\right\} \quad (4.19)$$

Solutions of Eqs. (4.18), (4.19) can easily be obtained.

Next we shall take case II ($m=2m'+1$, $n=2n'+1$). In the same way as in case I we get for $\bar{\chi}_1$ and $\bar{\chi}_2$ the expressions

$$\bar{\chi}_1 = -\sum_{r} A_r (1+\nu)\,\alpha\,(b\sinh\alpha b\cosh\alpha y - y\cosh\alpha b\sinh\alpha y)\sin\alpha x +$$

$$+\sum_{s} B_s\Big[\Big\{(3-\nu)\cosh\beta a + (1+\nu)\beta a\sinh\beta a\Big\}\sinh\beta x - (1+\nu)\beta\cosh\beta a\,\times$$

$$\times\, x\cosh\beta x\Big]\cos\beta y\,, \qquad \cdots\cdots (4.20\text{a})$$

$$\bar{\chi}_2 = \sum_{r} A_r\Big[\Big\{(3-\nu)\cosh\alpha b + (1+\nu)\,\alpha b\sinh\alpha b\Big]\sinh\alpha y +$$

$$-(1+\nu)\alpha\cosh\alpha b\cdot y\cosh\alpha y\Big]\cos\alpha x-\sum_s B_s(a\sinh\beta a\cosh\beta x+$$

$$-x\cosh\beta a\cdot\sinh\beta x)(1+\nu)\beta\cdot\sin\beta y\,, \qquad (4.20\text{b})$$

in which $\alpha=\frac{r\pi}{2a}$, $\beta=\frac{s\pi}{2b}$, $r=2r'+1$, $s=2s'+1$.

Considering the forms of $J\frac{\partial v}{\partial x}$, $J\frac{\partial v}{\partial y}$

$$J\frac{\partial v}{\partial x}=(-1)^{m'+n'+1}J\alpha_m\sin\alpha_m x\cos\beta_n y\,,$$

$$J\frac{\partial v}{\partial y}=(-1)^{m'+n'+1}J\beta_n\sin\alpha_m x\sin\beta_n y\,,$$

it is found by the help of expansions of the types of (3.15) that

$$-\sum_r A_r\frac{(1+\nu)(-1)^{s'+r'}8a^2b^2sr^2\cosh^2\alpha b}{\pi(b^2r^2+a^2s^2)^2}+B_s\Big\{(3-\nu)\sinh\beta a\cosh\beta a-(1+\nu)\beta a\Big\}$$

$$\left.\begin{aligned}&=(-1)^{n'}J\alpha_m, &&\text{for}\quad n=s\,,\\ &=0\,, &&\text{for}\quad n\neq s\,,\end{aligned}\right\}\quad(4.21\text{a})$$

$$A_r\Big\{(3-\nu)\sinh\alpha b\cosh\alpha b-(1+\nu)\alpha b\Big\}-\sum B_s\frac{(1+\nu)(-1)^{s'+r'}8a^2b^2rs^2\cosh^2\beta a}{\pi(b^2r^2+a^2s^2)^2}$$

$$\left.\begin{aligned}&=(-1)^{m'}J\beta_n, &&\text{for}\quad m=r\,,\\ &=0\,, &&\text{for}\quad m\neq r\,.\end{aligned}\right\}\quad(4.21\text{b})$$

For case III ($m=2m'$, $n=2n'+1$) in consequence of the forms $J\frac{\partial v}{\partial x}$, $J\frac{\partial v}{\partial y}$ and of the boundary conditions we may write $\bar{\chi}_1$ and $\bar{\chi}_2$ as

$$\bar{\chi}_1=\sum_r A_r(1+\nu)\alpha(b\sinh\alpha b\cosh\alpha y-y\cosh\alpha b\cdot\sinh\alpha y)\cos\alpha x+$$

$$+\sum_s B_s\Big[\Big\{(1+\nu\beta a\cosh\beta a+(3-\nu)\sinh\beta a\Big\}\cosh\beta x-(1+\nu)\beta\times$$

$$\times\sinh\beta a\cdot x\sinh\beta x\Big]\cos\beta y\,, \qquad (4.22\text{a})$$

$$\bar{\chi}_2=\sum_r A_r\Big[\Big\{(1+\nu)\alpha b\sinh\alpha b+(3-\nu)\cosh\alpha b\Big\}\sinh\alpha y-(1+\nu)\alpha\times$$

$$\times\cosh\alpha b\cdot y\cosh\alpha y\Big]\sin\alpha x-\sum_s B_s(1+\nu)\beta(a\cosh\beta a\sinh\beta x+$$

$$-x\sinh\beta a\cosh\beta x)\sin\beta y, \qquad (4.22\text{b})$$

where $\alpha=\frac{r\pi}{2a}$, $\beta=\frac{s\pi}{2b}$, $r=2r'$, $s=2s'+1$.

Then by the aid of the developments

$$\left.\begin{aligned}\cosh \alpha y &= \frac{2}{b}\sum_{s'=0}^{\infty}\cos\frac{(2s'+1)\pi}{2b}y\int_0^b \cosh\frac{r\pi}{2a}y\cos\frac{(2s'+1)\pi}{2b}ydy\ ,\\ y\sinh\alpha y &= \frac{2}{b}\sum_{s'=0}^{\infty}\cos\beta y\int_0^b y\sinh\alpha y\cos\beta y dy\ ,\end{aligned}\right\}\quad(4.23)$$

we have the relation by (4.22a)

$$\begin{aligned}&\sum_r A_r\frac{(1+\nu)(-1)^{s'+r'}8a^2b^2r^2s\cosh^2\alpha b}{\pi(b^2r^2+a^2s^2)^2}+B_s\Big\{(1+\nu)\beta a+(3-\nu)\times\\ &\times\sinh\beta a\cosh\beta a\Big\}=(-1)^{n'+1}J\alpha_m \qquad \text{for}\quad s=n\ ,\\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for}\quad s\neq n\ ,\end{aligned}\quad(4.24a)$$

and, using the developments of the types (4.17), we obtain from (4.22b) the relation.

$$\begin{aligned}&A_r\Big\{(3-\nu)\sinh\alpha b\cosh\alpha b-(1+\nu)\alpha b\Big\}+\sum_s B_s\frac{(1+\nu)(-1)^{r'+s'}8a^2b^2rs^2\sinh^2\beta a}{\pi(b^2r^2+a^2s^2)^2}\\ &\qquad=(-1)^{m'}J\beta_n\quad \text{for}\quad r=m\ ,\\ &\qquad=0\ ,\qquad\quad\ \ \text{for}\quad r\neq m\ .\end{aligned}\quad(4.24b)$$

For reference we indicate the forms of $\bar{\chi}_1$ and $\bar{\chi}_2$ and the required relations in case IV. It will be readily seen that these are also obtained by the proper interchange of labels and suffixes in the results of case III in this section. We may write

$$\begin{aligned}\bar{\chi}_1=&-\sum_r A_r(1+\nu)\alpha(b\cosh\alpha b\sinh\alpha y-y\sinh\alpha b\cosh\alpha y)\sin\alpha x+\\ &+\sum_s B_s\Big[\Big\{(3-\nu)\cosh\beta a+(1+\nu)\beta a\sinh\beta a\Big\}\sinh\beta x-(1+\nu)\beta\cosh\beta a\ \times\\ &\times x\cosh\beta x\Big]\sin\beta y\ ,\end{aligned}\quad(4.25a)$$

$$\begin{aligned}\bar{\chi}_2=&A_r\Big[\Big\{(1+\nu)\alpha b\cosh\alpha b+(3-\nu)\sinh\alpha b\Big\}\cosh\alpha y-(1+\nu)\alpha\ \times\\ &\times\sinh\alpha b\cdot y\sinh\alpha y\Big\}\cos\alpha x+\sum_s B_s(1+\nu)\beta(a\sinh\beta a\cosh\beta x-\\ &x\cosh\beta a\sinh\beta x)\cos\beta y\ ,\end{aligned}\quad(4.25b)$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=2r'+1$, $s=2s'$,

and

$$+\sum_r A_r \frac{(1+\nu)(-1)^{s'+s'}8a^2b^2r^2s\sinh^2 ab}{\pi(b^2r^2+a^2s^2)^2}+B_s\{(3-\nu)\sinh\beta a\cosh\beta a+$$

$$\left.\begin{aligned}-(1+\nu)\beta a\} &= (-1)^{n'}J\alpha_m, \qquad \text{for } n=s,\\ &=0, \qquad \text{for } n\neq s,\end{aligned}\right\} \cdots (4.26\text{a})$$

$$A_r\{(3-\nu)\sinh ab\cosh ab+(1+\nu)ab\}+\sum B_s\frac{(1+\nu)(-1)^{s'+r'}8a^2b^2rs^2\cosh^2\beta a}{\pi(b^2r^2+a^2s^2)^2}$$

$$\left.\begin{aligned}&=(-1)^{m+'1}J\beta_n, \qquad \text{for } r=m,\\ &=0, \qquad \text{for } r\neq m.\end{aligned}\right\} \quad (4.26\text{b})$$

Next we shall indicate the forms of Θ_0 and χ_0 definitely. From (4.8), (4.12) and (4.13) we have in case I

$$\Theta_0=\sum_r A_r 2\alpha\sinh ab\sinh\alpha y\sin\alpha x+\sum_s B_s 2\beta\sinh\beta a\sinh\beta x\sin\beta y, \quad (4.27\text{a})$$

and from (4.2), (4.6), (4.9), (4.12) and (4.13)

$$\chi_0=\frac{1}{(1+\nu)}\int^x\{\xi-\bar{\chi}_1\}dx=\frac{-1}{(1+\nu)}\Big[\sum_r A_r\frac{1}{\alpha}\{(2\sinh ab+(1+\nu)ab\cosh ab)\times$$

$$\times\sinh\alpha y-(1+\nu)\alpha\sinh ab\cdot y\cosh\alpha y\}\sin\alpha x+\sum_s B_s\frac{1}{\beta}\{(2\sinh\beta a+$$

$$+(1+\nu)\beta a\cosh\beta a)\sinh\beta x-(1+\nu)\beta\sinh\beta a\cdot x\cosh\beta x\}\sin\beta y\Big], \quad (4.27\text{ b})$$

From these basic functions (4.27) we can readily find the solutions of the first kind according to (2.1) and (2.5) of (I). For example, we have

$$\sigma_{x,1}=\frac{\partial^2\chi}{\partial y^2}=\frac{-1}{(1+\nu)}\Big[\sum_r A_r\cdot\alpha\Big\{\Big(\nu(-2+\alpha^2z^2)\sinh ab+(1+\nu)ab\cosh ab\Big)\times$$

$$\times\sinh\alpha y-(1+\nu)\alpha\sinh ab\cdot y\cosh\alpha y\Big\}\sin\alpha x+\sum_s B_s(-\beta)\times$$

$$\times\Big\{\Big((2+\nu\beta^2z^2)\sinh\beta a+(1+\nu)\beta a\cosh\beta a\Big)\sinh\beta x-(1+\nu)\beta\times$$

$$\times\sinh\beta a\cdot x\cosh\beta x\Big\}\sin\beta y\Big], \qquad \cdots\cdots (4.28\text{a})$$

$$Eu_1=\sum_r A_r\Big[\{\nu\alpha^2z^2\sinh ab+(1+\nu)ab\cosh ab\}\sinh\alpha y-(1+\nu)\alpha\sinh ab\times$$

$$\times y\cosh\alpha y\Big]\cos\alpha x+\sum_s B_s\Big[\{(3-\nu+\nu\beta^2z^2)\sinh\beta a+(1+\nu)\beta a\cosh\beta a\}\times$$

$$\times\cosh\beta x-(1+\nu)\beta\sinh\beta a\cdot x\sinh\beta x\Big]\sin\beta y. \qquad \cdots\cdots (4.28\text{b})$$

Here we write the forms of Θ_0, χ_0 and χ.

for case II ($m=2m'+1,\ n=2n'+1$):

$$\Theta_0 = \sum_r A_r 2\alpha \cosh \alpha b \cosh \alpha y \cos \alpha x + \sum_s B_s 2\beta \cosh \beta a \cosh \beta x \cos \beta y \ ,$$

$$\chi_0 = \chi|_{z=0} \ ,$$

$$\chi = \frac{-1}{(1+\nu)}\Big[\sum_r A_r \frac{1}{\alpha}\Big\{\Big((1+\nu)\alpha b \sinh \alpha b + (2+\nu\alpha^2 z^2)\cosh \alpha b\Big)\cosh \alpha y + - (1+\nu)\alpha \cosh \alpha b \cdot y \sinh \alpha y\Big\}\cos \alpha x + \sum_s B_s \frac{1}{\beta}\Big\{\Big((1+\nu)\beta a \sinh \beta a + + (2+\nu\beta^2 z^2)\cosh \beta a\Big)\cosh \beta x - (1+\nu)\beta \cosh \beta a \cdot x \sinh \beta x\Big\}\cos \beta y\Big] \ , \qquad (4.29)$$

$\Big(r=2r'+1,\ \ s=2s'+1\Big)$, $\{A_r\}$, $\{B_r\}$ from (4.21)

for case III ($m=2m'\ \ n=2n'+1$):

$$\Theta_0 = \sum_r A_r 2\alpha \cosh \alpha b \cosh \alpha y \sin \alpha x + \sum_s B_s 2\beta \sinh \beta a \sinh \beta x \cos \beta y \ ,$$

$$\chi_0 = \chi|_{z=0} \ ,$$

$$\chi = \frac{-1}{(1+\nu)}\Big[\sum_r A_r \frac{1}{\alpha}\Big\{\Big((1+\nu)\alpha b \sinh \alpha b + (2+\nu\alpha^2 z^2)\cosh \alpha b\Big)\cosh \alpha y + - (1+\nu)\alpha \cosh \alpha b \cdot y \sinh \alpha y\Big\}\sin \alpha x + \sum_s B_s \frac{1}{\beta}\Big\{\Big((1+\nu)\beta a \cosh \beta a + + (2+\nu\beta^2 z^2)\sinh \beta a\Big)\sinh \beta x - (1+\nu)\beta \sinh \beta a \cdot x \cosh \beta x\Big\}\cos \beta y\Big] \ , \qquad (4.30)$$

$(r=2r'\ \ s=2s'+1)$, $\{A_r\}$, $\{B_s\}$ from (4.24),

for case IV ($m=2m'+1,\ n=2n'$):

$$\Theta_0 = \sum_r A_r 2\alpha \sinh \alpha b \sinh \alpha y \cos \alpha x + \sum_s B_s 2\beta \cosh \beta a \cosh \beta x \sin \beta y \ ,$$

$$\chi_0 = \chi|_{z=0} \ ,$$

$$\chi = \frac{-1}{(1+\nu)}\Big[\sum_r A_r \frac{1}{\alpha}\Big\{\Big((1+\nu)\alpha b \cosh \alpha b + (2+\nu\alpha^2 z^2)\sinh \alpha b\Big)\sinh \alpha y + - (1+\nu)\alpha \sinh \alpha b \cdot y \cosh \alpha y\Big\}\cos \alpha x + \sum_s B_s \frac{1}{\beta}\Big\{\Big((1+\nu)\beta a \sinh \beta a + + (2+\nu\beta^2 z^2)\cosh \beta a\Big)\cosh \beta x - (1+\nu)\beta \cosh \beta a \cdot x \sinh \beta x\Big\}\sin \beta y\Big] \ , \qquad (4.31)$$

$(r=2r'+1,\ \ s=2s')$, $\{A_r\}$, $\{B_s\}$ from (4.26).

In the next place we shall seek solutions regarding basic parts of generalized plane stress for the clamped plate. First we shall have to determine the form of w_0 by means of particular solutions and the boundary conditions for w_0.

By (2.5), (2.7b) and (4.31) of (I) we have

$$w_0 = w_{0,1}+w_{0,2}+w_{0,3} = w_{0,2}+w_{0,3} = \frac{1}{E}\left\{(1+\nu(\chi_1'+h^2\Theta_1\right\} + \\ + \frac{(1+\nu)}{E}\frac{K}{k}\Big[\left\{2(\nu-1)\sinh 2kh - kh\cosh 2kh\right\}\cosh kh + \\ + (4\nu-3)kh\cosh kh - 2k^2h^2\sinh kh\Big]\,v(x,y)\,, \quad \cdots\cdots\cdots\cdots\cdots (4.32)$$

in which

$$\nabla_1^2\chi_1' = -\frac{(1-\nu)}{(1+\nu)}\Theta_1\,, \quad \nabla_1^2\Theta_1 = 0$$

and by performing the operation $\nabla_1^4 = \nabla_1^2\cdot\nabla_1^2$ on $w_0(x,y)$ (4.32) we obtain the differential equation

$$\nabla_1^4 w_0 = -\bar{J}\,v(x,y)\,, \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4.33)$$

in which $\bar{J} = \dfrac{(1+\nu)}{E}\dfrac{pk^3}{\sinh 2kh - 2kh}\left\{2(1-\nu)\cosh kh + kh\sinh 2kh\right\}$.

It is to be noticed that according as kh approaches to zero this differential equation becomes reduced to the well-known equation $D\nabla_1^4 w_0 = Z'$, in which $Z' = \int_{-h}^{h}\rho Z dz + \sigma_z|_{z=h} - \sigma_z|_{z=-h}$, and Z is volume force per unit of mass in the direction of the z-axis and ρ is the density of mass of material. However, Prof. Love in his book says this differential equation is correct, whether the formulae defining the curvature of the middle surface of the plate is exactly or only approximately correct and, hence, this remark of his may be said to be not entirely reliable. In this case equation (4.33) reduces to $\nabla_1^4 w_0(x,y) = -pv(x,y)$, as in the theory of thin plate, since $Z=0$, $\sigma_z|_{z=h} = -pv(x,y)$, $\sigma_z|_{z=-h}=0$, $D = \dfrac{2}{3}\dfrac{Eh^3}{(1-\nu^2)}$. Now we come upon the following boundary value problem:

differential equation $\nabla_1^4 w_0(x,y) = -\bar{J}\,v(x,y)$ (4.33) valid at any point of the middle surface of the rectangular thick plate, and boundary conditions

$$w_0 = 0\,, \quad \frac{\partial w_0}{\partial \nu} = 0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (4.34)$$

along the boundary and ν has been defined before.

Accordingly general solution of the differential equation (4.33) is to be given as the superposition of complementary solution of the homogeneous equation $\nabla_1^4 w_0=0$ and particular solution of (4.33), which by inspection can be shown to be of the form

$$w_{0,3} = -\frac{\bar{J}}{k^4} v(x,y) \quad \cdots\cdots \quad (4.35)$$

and complementary solution is obviously a plane biharmenic function. Therefore, we can put

$$w_0(x,y) = -\frac{\bar{J}}{k^4}\left\{\bar{\chi}(xy)+v(x,y)\right\} . \quad \cdots\cdots \quad (4.36)$$

Substituting this expression into the second condition of (4.34), we have

$$\left.\begin{aligned} \frac{\partial\bar{\chi}}{\partial x} &= -\alpha_m \sin\beta_n(y+b)\cos\alpha_m(x+a)|_{x=\pm a}, \quad \text{at } x=\pm a , \\ \frac{\partial\bar{\chi}}{\partial y} &= -\beta_n \sin\alpha_m(x+a)\cos\beta_n(y+b)|_{y=\pm b}, \quad \text{at } y=\pm b . \end{aligned}\right\} \quad (4.37)$$

Consequently the above boundary value problem can be reduced to the following:

differential equation. $\nabla_1^4\bar{\chi} = 0$ $\cdots\cdots$ (4.38a)

boundary conditions. $\bar{\chi} = 0$ along the boundary $\cdots$ (4.38b)

and the conditions of (4.37).

Now we shall take case I ($m=2m'$, $n=2n'$). Taking account of (4.38) we may put

$$\begin{aligned} \bar{\chi} &= \sum_r A_r(b\cosh\alpha b\sinh\alpha y - y\sinh\alpha b\cosh\alpha y)\sin\alpha x + \\ &+ \sum_s B_s(a\cosh\beta a\sinh\beta x - x\sinh\beta a\cosh\beta x)\sin\beta y , \end{aligned} \quad \cdots \quad (4.39)$$

in which $\alpha = \frac{r\pi}{2a}$, $\beta = \frac{s\pi}{2b}$, $r=2r'$, $s=2s'$. Then Eqs. (4.37) may be rewritten into the forms

$$\left.\begin{aligned} \frac{\partial\bar{\chi}}{\partial x} &= (-1)^{n'+1}\alpha_m \sin\beta_n y , \quad &\text{at} \quad x = \pm a , \\ \frac{\partial\bar{\chi}}{\partial y} &= (-1)^{m'+1}\beta_n \sin\alpha_m x , \quad &\text{at} \quad y = \pm b . \end{aligned}\right\} \quad (4.40)$$

From (4.39) and (4.40) we get the following relations:

$$\sum_r A_r(b\cosh\alpha b\sinh\alpha y-y\sinh\alpha b\cosh\alpha y)\,\alpha(-1)^{r'}+\sum_s B_s\frac{1}{2}(2\beta a-$$
$$-\sinh 2\beta a)\sin\beta y=(-1)^{n'+1}\alpha_m\sin\beta_n y\,,\quad \text{at } x=\pm a \quad\cdots (4.41a)$$

$$\sum_r A_r\frac{1}{2}(2\alpha b-\sinh 2\alpha b)\sin\alpha x+\sum_s B_s(a\cosh\beta a\sinh\beta x-x\sinh\beta a\times$$
$$\times\cosh\beta x)\beta(-1)^{s'}=(-1)^{m'+1}\beta_n\sin\alpha_m x\,,\quad \text{at } y=\pm b \quad\cdots (4.41b)$$

By the use of expansions of the types (4.17) we obtain the relations between the coefficients in equations (4.41).

$$\left.\begin{aligned}\sum_r A_r\frac{(-1)^{r'+s'+1}4a^2b^2r'^2s'\sinh^2\alpha b}{\pi(b^2r'^2+a^2s'^2)^2}+B_s\frac{1}{2}(2\beta a-\sinh 2\beta a)\\ =(-1)^{n'+1}\alpha_m\,,\quad \text{for}\quad n=s\,,\\ =0\,,\quad \text{for}\quad n\neq s\,.\end{aligned}\right\}\quad(4.42a)$$

$$\left.\begin{aligned}A_r\frac{1}{2}(2\alpha b-\sinh 2\alpha b)+\sum_s B_s\frac{(-1)^{r'+s'+1}4a^2b^2r's'^2\sinh^2\beta a}{\pi(b^2r'^2+a^2s'^2)^2}\\ =(-1)^{m'+1}\beta_n\,,\quad \text{for}\quad m=r\,,\\ =0\,,\quad \text{for}\quad m\neq r\,.\end{aligned}\right\}\quad(4.42b)$$

From these relations two sequences $\{A_r\}$, $\{B_s\}$ may be determined.

In case II ($m=2m'+1$, $n=2n'+1$) equations (4.37) become

$$\left.\begin{aligned}\frac{\partial\bar\chi}{\partial x}&=\pm(-1)^{n'}\alpha_m\cos\beta_n y\,, &&\text{at}\quad x=\pm a\,,\\ \frac{\partial\bar\chi}{\partial y}&=\pm(-1)^{m'}\beta_n\cos\alpha_m x\,, &&\text{at}\quad y=\pm b\,.\end{aligned}\right\}\quad(4.43)$$

Whence we may write for $\bar\chi$

$$\bar\chi=\sum_r A_r(b\sinh\alpha b\cosh\alpha y-y\cosh\alpha b\sinh\alpha y)\cos\alpha x+$$
$$+\sum_s B_s(a\sinh\beta a\cosh\beta x-x\cosh\beta a\sinh\beta x)\cos\beta y\,,\qquad(4.44)$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=2r'+1$, $s=2s'+1$.

By (4.43), (4.44) we have

$$\sum_r A_r(-1)^{r'}\alpha(b\sinh\alpha b\cosh\alpha y-y\cosh\alpha b\,\text{snih}\,\alpha y)+\sum_s B_s\frac{1}{2}\times$$
$$\times(2\beta a+\sinh 2\beta a)\cos\beta y=(-1)^{n'+1}\alpha_m\cos\beta_n y\,,\quad\cdots\cdots (4.45a)$$

$$\sum_r A_r\frac{1}{2}(2\alpha b+\sinh 2\alpha b)\cos\alpha x+\sum_s B_s(-1)^{s'}\beta(a\sinh\beta a\cosh\beta x+$$
$$-x\cosh\beta a\sinh\beta x)=(-1)^{m'+1}\beta_n\cos\alpha_m x\,,\quad\cdots\cdots\cdots (4.45b)$$

Utilizing developments of the types

$$\left.\begin{aligned}\cosh \alpha y &= \frac{2}{b}\sum_{s'=0}^{\infty}\cos\frac{(2s'+1)\pi}{2b}y\int_0^b \cosh\frac{r\pi}{2a}y\cos\frac{(2s'+1)\pi}{2b}y\,dy\,,\\ y\sinh\alpha y &= \frac{2}{b}\sum_{s'=0}^{\infty}\cos\beta y\int_0^b y\sinh\alpha y\cos\beta y\,dy\,,\end{aligned}\right\}\quad(4.46)$$

we readily obtain from (4.45)

$$\sum_r A_r\frac{(-1)^{r'+s'}8a^2b^2r^2s\cosh^2 ab}{\pi(b^2r^2+a^2s^2)^2}+B_s\frac{1}{2}(2\beta a+\sinh 2\beta a)$$

$$\left.\begin{aligned}&=(-1)^{n'+1}\alpha_m\,, &&\text{for}\quad n=s\,,\\ &=0\,, &&\text{for}\quad n\neq s\,.\end{aligned}\right\}\quad(4.47\text{a})$$

$$A_r\frac{1}{2}(2ab+\sinh ab)+\sum_s B_s\frac{(-1)^{r'+s'}8a^2b^2rs^2\cosh^2\beta a}{\pi(b^2r^2+a^2s^2)^2}$$

$$\left.\begin{aligned}&=(-1)^{m'+1}\beta_n\,, &&\text{for}\quad m=r\,,\\ &=0\,, &&\text{for}\quad m\neq r\,.\end{aligned}\right\}\quad(4.47\text{b})$$

In case III ($m=2m'$, $n=2n'+1$) the conditions (4.37) are written as

$$\frac{\partial\bar{\chi}}{\partial x}=(-1)^{n'+1}\alpha_m\cos\beta_n y\,,\qquad \text{at}\quad x=\pm a\,,\qquad(4.48\text{a})$$

$$\frac{\partial\bar{\chi}}{\partial y}=\pm(-1)^{m'}\beta_n\sin\alpha_m x\,,\qquad \text{at}\quad y=\pm b\,,\qquad(4.48\text{b})$$

and similarly we may put for $\bar{\chi}$

$$\begin{aligned}\bar{\chi}=&\sum_r A_r(b\sinh ab\cosh\alpha y-y\cosh ab\sinh\alpha y)\sin\alpha x+\\ &+\sum_s B_s(a\cosh\beta a\sinh\beta x-x\sinh\beta a\cosh\beta x)\cos\beta y\,,\end{aligned}\qquad(4.49)$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=2r'$, $s=2s'+1$.

When we substitute formula (4.49) into Eqs. (4.48), the resulting equations are

$$\begin{aligned}&\sum_r A_r(b\sinh ab\cosh\alpha y-y\cosh ab\sinh\alpha y)\alpha(-1)^{r'}+\\ &+\sum_s B_s\frac{1}{2}(2\beta a-\sinh 2\beta a)\cos\beta y=(-1)^{n'+1}\alpha_m\cos\beta_n y\,,\end{aligned}\qquad(4.50\text{a})$$

$$\begin{aligned}&\sum_r A_r\frac{1}{2}(2ab+\sinh 2ab)+\sum_s B_s(a\cosh\beta a\sinh\beta x+\\ &-x\sinh\beta a\cosh\beta x)\beta(-1)^{s'}=(-1)^{m'+1}\beta_n\sin\alpha_m x\,.\end{aligned}\qquad(4.50\text{b})$$

So that in the same way as above we obtain the relations.

$$\sum_r A_r \frac{(-1)^{r'+s'} 8a^2b^2r^2s\cosh^2 ab}{\pi(b^2r^2+a^2s^2)^2} + B_s\frac{1}{2}(2\beta a - \sinh 2\beta a)$$
$$\left.\begin{aligned} &= (-1)^{n'+1}\alpha_m, \quad &&\text{for} \quad s=n, \\ &= 0, &&\text{for} \quad s\neq n. \end{aligned}\right\} \quad (4.51a)$$

$$A_r\frac{1}{2}(2ab+\sinh 2ab) + \sum_s B_s \frac{(-1)^{r'+s'+1} 8a^2b^2rs^2\sinh^2\beta a}{\pi(b^2r^2+a^2s^2)^2}$$
$$\left.\begin{aligned} &= (-1)^{m'+1}\beta_n, \quad &&\text{for} \quad r=m, \\ &= 0, &&\text{for} \quad r\neq m. \end{aligned}\right\} \quad (4.51b)$$

In case IV ($m=2m'+1$, $n=2n'$) we write for reference

$$\left.\frac{\partial\overline{\chi}}{\partial x}\right|_{x=\pm a} = \pm(-1)^{n'}\alpha_m\sin\beta_n y, \quad \left.\frac{\partial\overline{\chi}}{\partial y}\right|_{y=\pm b} = (-1)^{m'+1}\beta_n\cos\alpha_m x, \quad (4.52)$$

$$\overline{\chi} = \sum_r A_r Y_r(y)\cos\alpha x + \sum_s B_s X_s(x)\sin\beta y, \quad \cdots\cdots\cdots\cdots\cdots\cdots \ (4.53)$$

in which

$$X_s(x) = a\sinh\beta a\cosh\beta x - x\cosh\beta a\sinh\beta x,$$
$$Y_r(y) = b\cosh ab\sinh ay - y\sinh ab\cosh ay,$$
$$\alpha = \frac{r\pi}{2a}, \quad \beta = \frac{s\pi}{2b}, \quad r=2r'+1, \quad s=2s',$$

and from these

$$\sum_r A_r Y_r(y)\alpha(-1)^{r'} + \sum_s B_s\frac{1}{2}(2\beta a+\sinh 2\beta a)\sin\beta y = (-1)^{n'+1}\alpha_m\sin\beta_n y, \quad \cdots\cdots \ (4.54a)$$

$$\sum_r A_r\frac{1}{2}(2ab-\sinh 2ab)\cos\alpha x + \sum_s B_s X_s(x)\beta(-1)^{s'} = (-1)^{m'+1}\beta_n\cos\alpha_m x. \quad \cdots\cdots \ (4.54b)$$

Whence similarly from (4.54) we obtain the following relations:

$$\sum_r A_r \frac{(-1)^{r'+s'+1} 8a^2b^2r^2s\sinh^2 ab}{\pi(b^2r^2+a^2s^2)^2} + B_s\frac{1}{2}(2\beta a + \sinh 2\beta a)$$
$$\left.\begin{aligned} &= (-1)^{n'+1}\alpha_m, \quad &&\text{for} \quad n=s. \\ &= 0, &&\text{for} \quad n\neq s. \end{aligned}\right\} \quad (4.55a)$$

$$A_r\frac{1}{2}(2ab-\sinh 2ab) + \sum_s B_s \frac{(-1)^{r'+s'} 8a^2b^2rs^2\cosh^2\beta a}{\pi(b^2r^2+a^2s^2)^2}$$

$$\left.\begin{aligned} &= (-1)^{m'+1}\beta_n\,, \qquad \text{for } m = r\,, \\ &= 0\,, \qquad\qquad\quad \text{for } m \neq r\,. \end{aligned}\right\} \qquad (4.55\text{b})$$

Thus it will be supposed that $\bar{\chi}$, namely $w_{0,2}$, has been determined, although the above process of calculation analogous to that of Hencky may be said to be approximate one. In any way a boundary value problem of this kind could not be dealt with readily, since there seems to be no other better approximate methods. It will be needless to say that the Fourier expansions used in course of calculation are valid in the whole needed ranges.

In order to make use of the formulae (2.7) and (2.8) of (I) we have to find the forms of Θ_1 χ_1' and χ'. It is obvious from (4.32) and (4.36) that

$$w_{0,2} = \frac{1}{E}\left\{(1+\nu)\chi_1'+h^2\Theta_1\right\} = \frac{-\bar{J}}{k^4}\bar{\chi}\,, \qquad \cdots\cdots\cdots\cdots (4.56)$$

and, performing the operation ∇_1^2 on (4.56), we easily obtain the expression for Θ_1. At first we shall take case I ($m=2m'$, $n=2n'$) for simplicity in the sequel. Now we have

$$\Theta_1 = \frac{-2E}{(1-\nu)}\frac{\bar{J}}{k^4}\left(\sum_r A_r\alpha\sinh\alpha b\sinh\alpha y\sin\alpha x+\sum_s B_s\beta\sinh\beta a\sinh\beta x\sin\beta y\right), \qquad \cdots\cdots\cdots\cdots (4.57\text{a})$$

and by (4.56) and (4.57a)

$$\begin{aligned} \chi_1' = \frac{E}{(1-\nu^2)}\frac{\bar{J}}{k^4}\Big[&\sum_r A_r\Big\{\big(2h^2\alpha\sinh\alpha b-(1-\nu)b\cosh\alpha b\big)\sinh\alpha y+ \\ &+(1-\nu)\sinh\alpha b\cdot y\cosh\alpha y\Big\}\sin\alpha x+\sum_s B_s\Big\{\big(2h^2\beta\sinh\beta a-(1-\nu)a\times \\ &\times\cosh\beta a\big)\sinh\beta x+(1-\nu)\sinh\beta a\cdot x\cosh\beta x\Big\}\sin\beta y\Big]\,, \qquad \cdots (4.57\text{b}) \end{aligned}$$

so that

$$\begin{aligned} \chi' = z\chi_1'+\frac{(2-\nu)}{6(1+\nu)}z^3\Theta_1 = \frac{E}{(1-\nu^2)}\frac{\bar{J}}{k^4}\Big[&\sum_r A_r\Big\{\Big(2h^2\alpha\sinh\alpha b-\frac{(2-\nu)}{3}z^2\alpha\times \\ &\times\sinh\alpha b-(1-\nu)b\cosh\alpha b\Big)\sinh\alpha y+(1-\nu)\sinh\alpha b\cdot y\cosh\alpha y\Big\}\sin\alpha x\cdot z+ \\ &+\sum_s B_s\Big\{\Big(2h^2\beta\sinh\beta a-\frac{(2-\nu)}{3}z^2\beta\sinh\beta a-(1-\nu)a\cosh\beta a\Big)\sinh\beta x+ \\ &+(1-\nu)\sinh\beta a\cdot x\cosh\beta x\Big\}z\cdot\sin\beta y\Big]\,, \qquad \cdots\cdots\cdots\cdots (4.57\text{c}) \end{aligned}$$

Once these basic functions are obtained, solutions of the second kind or basic parts of generalized plane stress solutions can easily be found by the help of formulae (2.7) of (I); for stresses

$$\frac{(1-\nu^2)}{E}\cdot\frac{k^4}{\bar{J}}\sigma_{x,2}=\sum_r A_r\alpha z\Big[\Big\{-2\nu\sinh\alpha b+\Big(2h^2-\frac{(2-\nu)}{3}z^2\Big)\alpha^2\sinh\alpha b+$$
$$-(1-\nu)\alpha b\cosh\alpha b\Big\}\sinh\alpha y+(1-\nu)\alpha\sinh\alpha b\cdot y\cosh\alpha y\Big]\sin\alpha x+$$
$$+\sum_s B_s\beta z\Big[\Big\{-2\sinh\beta a-\Big(2h^2-\frac{(2-\nu)}{3}z^2\Big)\beta^2\sinh\beta a+(1-\nu)\beta a\times$$
$$\times\cosh\beta a\Big\}\sinh\beta x-(1-\nu)\beta\sinh\beta a\cdot x\cosh\beta x\Big]\sin\beta y, \quad\cdots (4.58a)$$

$$\frac{(1-\nu^2)}{E}\cdot\frac{k^4}{\bar{J}}\sigma_{y,2}=\sum_r A_r\alpha z\Big[\Big\{-2\sinh\alpha b+-\Big(2h^2-\frac{(2-\nu)}{3}z^2\Big)\alpha^2\sinh\alpha b+$$
$$+(1-\nu)\alpha b\cosh\alpha b\Big\}\sinh\alpha y-(1-\nu)\alpha\sinh\alpha b\cdot y\cosh\alpha y\Big]\sin\alpha x+$$
$$+\sum_s B_s\beta z\Big[\Big\{-2\nu\sinh\beta a+\Big(2h^2-\frac{(2-\nu)}{3}z^2\Big)\beta^2\sinh\beta a-(1-\nu)\beta a\times$$
$$\times\cosh\beta a\Big\}\sinh\beta x+(1-\nu)\beta\sinh\beta a\cdot x\cosh\beta x\Big]\sin\beta y, \quad\cdots (4.58b)$$

$$\frac{(1-\nu^2)}{E}\cdot\frac{k^4}{\bar{J}}\tau_{xy,}=-\sum_r A_r\alpha z\Big[\Big\{\Big(2h^2-\frac{2-\nu}{3}z^2\Big)\alpha^2\sinh\alpha b-(1-\nu)\alpha b\times$$
$$\times\cosh\alpha b+(1-\nu)\sinh\alpha b\Big\}\cosh\alpha y+(1-\nu)\alpha\sinh\alpha b\cdot y\sinh\alpha y\Big]\cos\alpha x+$$
$$-\sum_s B_s\beta z\Big[\Big\{\Big(2h^2-\frac{(2-\nu)}{3}z^2\Big)\beta^2\sinh\beta a-(1-\nu)\beta a\cosh\beta a+(1-\nu)\times$$
$$\times\sinh\beta a\Big\}\cosh\beta x+(1-\nu)\beta\sinh\beta a\cdot x\sinh\beta x\Big]\cos\beta y, \quad\cdots (4.58c)$$

$$\tau_{xz,2}=-\frac{E}{(1-\nu^2)}\frac{\bar{J}}{k^4}\Big(\sum_r A_r\alpha^2\sinh\alpha b\sinh\alpha y\cos\alpha x+\sum_s B_s\beta^2\sinh\beta a\times$$
$$\times\cosh\beta x\sin\beta y\Big)(h^2-z^2), \quad\cdots\cdots (4.58d)$$

$$\tau_{yz,2}=-\frac{E}{(1-\nu^2)}\frac{\bar{J}}{k^4}\Big(\sum_r A_r\alpha^2\sinh\alpha b\cosh\alpha y\sin\alpha x+\sum_s B_s\beta^2\sinh\beta a\times$$
$$\times\sinh\beta x\cos\beta y\Big)(h^2-z^2), \quad\cdots\cdots (4.58e)$$

and for displacements

$$-(1-\nu)\frac{k^4}{\bar{J}}\cdot u_2=\sum_r A_r\Big[\Big\{\Big(2h^2-\frac{2-\nu}{3}z^2\Big)\alpha\sinh\alpha b-(1-\nu)b\cosh\alpha b\Big\}\times$$
$$\times\sinh\alpha y+(1-\nu)\sinh\alpha b\cdot y\cosh\alpha y\Big]\alpha z\cos\alpha x+\sum_s B_s\Big[\Big\{\Big(2h^2-\frac{2-\nu}{3}z^2\Big)\times$$

$$\times \beta^2 \sinh\beta a - (1-\nu)\beta a \cosh\beta a + (1-\nu)\sinh\beta a\Big\} \cosh\beta x +$$

$$+ (1-\nu)\beta\sinh\beta a \cdot x\sinh\beta x\Big] z\sin\beta y\,, \qquad \cdots\cdots\cdots \quad 4.59a$$

$$-(1-\nu)\frac{k^4}{J}\cdot v_2 = \sum_r A_r\Big[\Big\{\Big(2h^2 - \frac{2-\nu}{3}z^2\Big)\ \alpha^2\sinh\alpha b - (1-\nu)\alpha b\cosh\alpha b +$$

$$+ (1-\nu)\sinh\alpha b\Big\}\ \cosh\alpha y + (1-\nu)\alpha\sinh\alpha b\cdot y\sinh\alpha y\Big] z\sin\alpha x +$$

$$+ \sum_s B_s\Big[\Big\{\Big(2h^2 - \frac{2-\nu}{3}z^2\Big)\ \beta\sinh\beta a - (1-\nu)\alpha\cosh\beta a\Big\}\sinh\beta x +$$

$$+ (1-\nu)\sinh\beta a\cdot x\cosh\beta x\Big]\beta z\cos\beta y\,, \qquad \cdots\cdots\cdots \quad 4.59b$$

$$\frac{(1-\nu)k^4}{J}w_2 = \sum_r A_r\Big[\Big\{\nu z^2\alpha\sinh\alpha b - (1-\nu)\,b\cosh\alpha b\Big\}\sinh\alpha y + (1-\nu)\times$$

$$\times \sinh\alpha b\cdot y\cosh\alpha y\Big]\sin\alpha x + \sum_s B_s\Big[\Big\{\nu z^2\beta\sinh\beta a - (1-\nu)\,\alpha\cosh\beta a\Big\}\times$$

$$\times \sinh\beta x + (1-\nu)\sinh\beta a\cdot x\cosh\beta x\Big]\sin\alpha y\,. \qquad \cdots\cdots \quad 4.59c$$

Thus we have obtained the solutions of the second kind to the problem of clamped plate in case I but we shall abridge the similar expressions corresponding to the other three cases and only brief mention of the forms of Θ, χ_1' and χ' in these cases will be made in the following, for reference.

$$\Theta_1 = \frac{-2E}{(1-\nu)}\frac{J}{k^4}\left\{\sum_r A_r\alpha\begin{cases}\cosh\\ \cosh\alpha b\\ \sinh\end{cases}\begin{cases}\cosh\\ \cosh\alpha y\\ \sinh\end{cases}\cdot\begin{cases}\cos\\ \sin\alpha x\\ \cos\end{cases} + \sum_s B_s\beta\begin{cases}\cosh\\ \sinh\beta a\\ \cosh\end{cases}\times\right.$$

$$\left.\times\begin{cases}\cosh\\ \sinh\beta x\\ \cosh\end{cases}\begin{cases}\cos\\ \cos\beta y\\ \sin\end{cases}\right\}, \qquad \cdots\cdots\cdots \quad (4.60a)$$

$$\frac{(1-\nu^2)}{E}\frac{k^4}{J}\chi_1' = \frac{(1-\nu^2)}{E}\frac{k^4}{J}\frac{\chi'}{z}\bigg|_{z=0} \qquad \cdots\cdots\cdots \quad (4.60b)$$

$$\frac{(1-\nu^2)}{E}\frac{k^4}{J}\chi' = \sum_r A_r z\left[\left\{\Big(2h^2 - \frac{2-\nu}{3}z^2\Big)\alpha\begin{cases}\cosh\\ \cosh\alpha b\\ \sinh\end{cases} - (1-\nu)b\begin{cases}\sinh\\ \sinh\alpha b\\ \cosh\end{cases}\right\}\times\right.$$

$$\times\begin{cases}\cosh\\ \cosh\alpha y\\ \sinh\end{cases} + (1-\nu)\begin{cases}\cosh\\ \cosh\alpha b\\ \sinh\end{cases}\cdot y\begin{cases}\sinh\\ \sinh\alpha y\\ \cosh\end{cases}\Bigg]\cdot\begin{cases}\cos\\ \sin\alpha x\\ \cos\end{cases} + \sum_s B_s z\times$$

$$\times\left[\left\{\left(2h^2-\frac{2-\nu}{3}z^2\right)\beta\begin{bmatrix}\cosh\\ \sinh\\ \cosh\end{bmatrix}\beta a-(1-\nu)a\begin{bmatrix}\sinh\\ \cosh\\ \sinh\end{bmatrix}\beta a\right\}\cdot\begin{bmatrix}\cosh\\ \sinh\\ \cosh\end{bmatrix}\beta x+\right.$$

$$\left.+(1-\nu)\begin{bmatrix}\cosh\\ \sinh\\ \cosh\end{bmatrix}\beta a\cdot x\begin{bmatrix}\sinh\\ \cosh\\ \sinh\end{bmatrix}\beta x\right]\cdot\begin{bmatrix}\cos\\ \cos\\ \sin\end{bmatrix}\beta y\,, \qquad \cdots\cdots\cdots\cdots\cdots\cdots \quad (4.60\text{c})$$

As regards the expressions (4.60) the rows of the first, second and third correspond to cases II, III and IV respectively and $\{A_r\}$ and $\{B_s\}$ must be those determined from (4.47), (4.51) and (4.55). As in the case of simply supported rectangular plate, suffixes are to be such that

case II $(m=2m'+1,\ n=2n'+1,\ r=2r'+1,\ s=2s'+1)$,
case III $(m=2m',\ n=2n'+1,\ r=2r',\ s=2s'+1)$,
case IV $(m=2m'+1,\ n=2n',\ r=2r'+1,\ s=2s')$,

$$\alpha=\frac{r\pi}{2a}\,,\quad \beta=\frac{s\pi}{2b}\;.$$

$$\frac{E}{(1-\nu^2)}\frac{\bar{J}}{k^4}=\frac{p}{(1-\nu)k}\frac{1}{(\sinh 2kh-2kh)}\left\{2(1-\nu)\cosh kh+kh\sinh kh\right\}\;.$$

In the preceding calculation we used the same symbols A_r, B_s for the coefficients in the expressions for solutions of the first and second kinds, that is to say, because it appeared that no confusion would arise, we did not distinguish them by using primes but it may be desirable to give primes to A_r and B_s in the solutions of the second kind. Now we can get general solutions by the superposition of three kinds of solutions to the clamped plate problem. We have, for example, by (4.28a), (4.58a) and (4.25) of (I) in case I $(m=2m',\ n=2n')$

$$\sigma_x=\sigma_{x,1}+\sigma_{x,2}+\sigma_{x,3}=\sum_r A_r\frac{\alpha}{(1+\nu)}\Big[-\left\{\nu(-2+\alpha^2z^2)\sinh\alpha b+\right.$$

$$\left.+(1+\nu)\alpha b\cosh\alpha b\right\}\sinh\alpha y+(1+\nu)\alpha\sinh\alpha b\cdot y\cosh\alpha y\Big]\sin\alpha x+$$

$$+\sum_s B_s\frac{\beta}{(1+\nu)}\Big[\left\{(2+\nu\beta^2z^2)\sinh\beta a)+(1+\nu)\beta a\cosh\beta a\right\}\sinh\beta x+$$

$$-(1+\nu)\beta\sinh\beta a\cdot x\cosh\beta x\Big]\sin\beta y+\sum_r A'_r\frac{E}{(1-\nu^2)}\frac{\bar{J}}{k^4}\alpha z\Big[\left\{-2\nu\sinh\alpha b+\right.$$

$$\left.+\left(2h^2-\frac{2-\nu}{3}z^2\right)\alpha^2\sinh\alpha b-(1-\nu)\alpha b\cosh\alpha b\right\}\sinh\alpha y+(1-\nu)\alpha\times$$

$$\times\sinh\alpha b\cdot y\cosh\alpha y\Big]\sin\alpha x+\sum B'_s\frac{E\bar{J}}{(1-\nu^2)k^4}\beta z\Big[\left\{-2\sinh\beta a+\right.$$

$$-\left(2h^2-\frac{2-\nu}{3}z^2\right)\beta^2\sinh\beta a+(1-\nu)\beta a\cosh\beta a\Big\}\sinh\beta x-(1-\nu)\beta\sinh\beta a\cdot$$

$$\times\, x\cosh\beta x\Big]\sin\beta y+\frac{K}{k^2}\Big[\Big\{2\nu\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}\Big)\sinh 2kh\sinh k(z+h)+\Big(4\nu\frac{\partial^2 v}{\partial y^2}+$$

$$+3\frac{\partial^2 v}{\partial x^2}\Big)kh\sinh k(z-h)+\frac{\partial^2 v}{\partial x^2}\Big\{-kh\cosh 2kh\sinh k(z+h)+2k^2h^2\times$$

$$\times\cosh k(z-h)+kz\,(\sinh 2kh\cosh k(z+h)+2kh\cosh k(z-h)\Big\}\Big]\,. \quad (4.61)$$

and by (4.28b), (4.59a) and (4.29b) of (I) in case I

$$u=u_1+u_2+u_3=\sum_r A_r\frac{1}{E}\Big[\Big\{\nu\alpha^2z^2\sinh\alpha b+(1+\nu)\alpha b\cosh\alpha b\Big\}\sinh\alpha y+$$

$$-(1+\nu)\alpha\sinh\alpha b\cdot y\cosh\alpha y\Big]\cos\alpha x+\sum_s B_s\frac{1}{E}\Big[\Big\{(3-\nu+\nu\beta^2z^2)\sinh\beta a+$$

$$+(1+\nu)\beta a\,\mathrm{bosh}\,\beta a\Big\}\cosh\beta x-(1+\nu)\beta\sinh\beta a\cdot x\sinh\beta x\Big]\sin\beta y+$$

$$+\sum_r A_r'\frac{(-1)}{(1-\nu)}\frac{\bar J}{k^4}\Big[\Big\{\Big(2h^2-\frac{2-\nu}{3}z^2\Big)\alpha\sinh\alpha b-(1-\nu)b\cosh\alpha b\Big\}\sinh\alpha y+$$

$$+(1-\nu)\sinh\alpha b\cdot y\cosh\alpha y\Big]\alpha z\cos\alpha x+\sum_s B_s'\frac{(-1)}{1-\nu}\frac{\bar J}{k^4}\Big[\Big\{\Big(2h^2-\frac{2-\nu}{3}z^2\Big)\beta^2\times$$

$$\times\sinh\beta a-(1-\nu)\beta a\cosh\beta a+(1-\nu)\sinh\beta a\Big\}\cosh\beta x+(1-\nu)\beta\times$$

$$\times\sinh\beta a\cdot x\sinh\beta x\Big]z\sin\beta y-\frac{(1+\nu)}{E}\frac{K}{k^3}\frac{\partial v}{\partial x}\Big[\Big\{(2\nu-1)\sinh 2kh+kh\times$$

$$\cosh 2kh\Big\}\sinh k(z+h)+(4\nu-3)kh\sinh k(z-h)-2k^2h^2\cosh k(z-h)-kz\times$$

$$\times\Big\{\sinh 2kh\cosh k(z+h)+2kh\cosh k(z-h)\Big\}\Big]\,. \quad \cdots\cdots\cdots\cdots \quad (4.62)$$

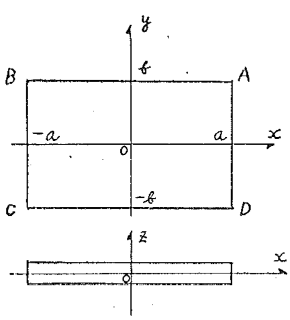
These are the solutions to the problem of clamped rectangular thick plate under sinusoidal pressure $pv(x,y)=p_{mn}\sin\alpha_m(x+a)\sin\beta_n(y+b)$, $(m=2m',\ n=2n')$.

As stated above at the end of Sec. III, when loading function is general and can be expanded in the like of (3.32), solutions may not be hard to obtain by utilizing the solutions in the above four cases. Solutions for the simply supported plate under variable normal load outlined in Sec. III may be said to be the direct extension of Navier's solutions in the theory of thin plate and solutions described in Sec. IV may be called the indirect extension. These solutions satisfy all equations of equilibrium and compatibility and Kirchhoff's four boundary conditions, as frequently mentioned. Anyhow we could solve the correspondingly difficult problems applying the method of Love, though the solutions obtained may not lend themselves readily to numerical computations. Moreover, the difficult-seeming mixed boundary value problems can be easily dealt with, following the mode of attack described above. So a brief mention of mixed problems concerned with rectangular thick plate will be offered in the next section.

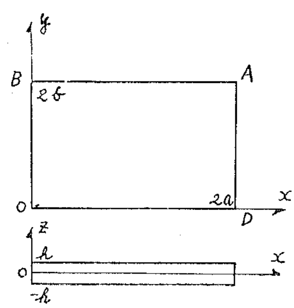
§ V. On the Mixed Bouudary Value Problems Relative to Rectangular Thick Plate.*

(a). On Several Typical Cases.

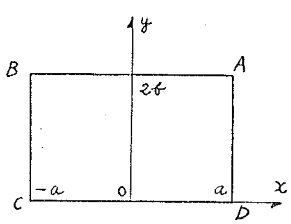
It will be worth while to note that by the aid of the systematic, ingenious, if somewhat lengthy, method of Love we can comparatively easily treat the mixed problems of thick plate, though mixed boundary conditions seem to be under some restriction. That is to say, when mixed conditions are general, it seems to be impossible to tide over the difficulty to solve the problem without making some approximation. Herein we shall discuss chiefly the simply connected plate laden with sinusoidal or variable pressure. Mixed problems of thin plate subjected to a uniform pressure are often treated but that is not the case with thin plate loaded with variable pressure, probably because the latter problem is too troublesome, and solutions for variably loaded thick plate with mixed boundary seem to be rarely seen. Now we shall proceed to investigate several cases. Always in the following



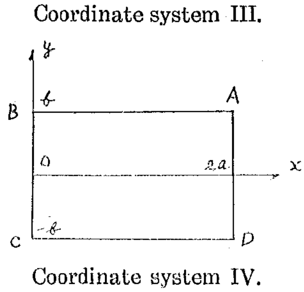
Coordinate system I.



Coordinate system II.



Coordinate system III.



Coordinate system IV.

Fig. 2

we shall chiefly discuss the problem of thick plate under a given normal load, whose loading function is a v-function

$$p_{mn}v_{mn}(x,y) \equiv pv(x,y) = p\sin\alpha_m(x+a)\sin\beta_n(y+b)\,, \quad \cdots\cdots (5.1)$$

$\left(\alpha_m = \frac{m\pi}{2a},\ \beta_n = \frac{n\pi}{2b},\ m = 2m',\ n = 2n'\right)$ which is referred to coordinate system I in Fig. 2.

One of the coordinate systems shown in Fig. 2 will be taken in each case for symmetry reasons.

Case A. When Thick Plate with Two Opposite Edges $\overline{BA}$, $\overline{CD}$ Clamped and the Other Opposite Edges $\overline{BC}$, $\overline{AD}$ Simply Supported is Stressed by the Pressure (5.1).

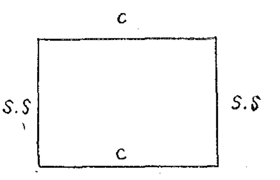


Fig. 3
c : clamped edge.
s. s : simply supported edge.

In this case it is convenient to take the coordinate system I. The boundary conditions are

$$u_0 = u_{0,1} + u_{0,3} = 0\,, \quad v_0 = v_{0,1} + v_{0,3} = 0\,,$$

$$w_0 = w_{0,2} + w_{0,3} = 0 \quad \frac{\partial w_0}{\partial y} = \frac{\partial w_{0,2}}{\partial y} + \frac{\partial w_{0,3}}{\partial y} = 0\,, \quad \text{at } y = \pm b\,,$$

$$\cdots\cdots (5.2a)$$

$$T_1 = T_{1,1} + T_{1,3} = 0\,, \quad S_1 = S_{1,1} + S_{1,3} = 0\,,$$

$$G_1 = G_{1,2} + G_{1,3} = 0\,, \quad w_0 = 0\,, \quad \text{at } x = \pm a\,. \quad \cdots\cdots (5.2b)$$

In the first place we shall determine the two basic quantities Θ_0 and χ_0 observing the boundary conditions regarding u_0, v_0, T_1 and S_1. From the form of $T_{1,3}$ at $x = \pm a$ the following expression for χ'' can be taken in case I,

$$\chi'' = 2h\chi_0 - \frac{1}{3}\frac{\nu}{1+\nu}h^3\Theta_0 = \sum_r (A_r \sinh\alpha y + C_r y\cosh\alpha y)\sin\alpha x +$$

$$+ \sum_s B_s(a\cosh\beta a\sinh\beta x - x\sinh\beta a\cdot\cosh\beta x)\sin\beta y\,, \quad \cdots\cdots (5.3)$$

$$\alpha = \frac{r\pi}{2a}\,, \quad \beta = \frac{s\pi}{2b}\,, \quad r = 2r',\ s = 2s'\,.$$

On inserting formula (5.3) into equation $\Theta_0 = \frac{1}{2h}\nabla_1^2\chi''$, we obtain the expressions for Θ, and then for χ_0.

Hence, using these expressions, from formula

$$\delta_1 = Eu_0 = \xi - (1+\nu)\frac{\partial \chi_0}{\partial x} + J\frac{\partial v}{\partial x} = \bar{\chi}_1 + J\frac{\partial v}{\partial x} \ ,$$

and condition

$$u_0 = 0 \ , \qquad \text{at} \quad y = \pm b \ ,$$

$\bar{\chi}_1$ can be written as

$$\bar{\chi}_1 = \sum_r D_r \alpha(1+\nu)(b\cosh\alpha b\sinh\alpha y - y\sinh\alpha b\cosh\alpha y)\cos\alpha x +$$
$$+ \sum_s B_s \frac{1}{2h}\Big[\Big\{\Big(\nu - 1 + \frac{\nu}{3}h^2\beta^2\Big)\sinh\beta a - (1+\nu)\,\beta a\cosh\beta a\Big\}\cosh\beta x +$$
$$+ (1+\nu)\,\beta\sinh\beta a\cdot x\sinh\beta x\Big]\sin\beta y \ , \qquad (5.4)$$

in which D_r is a coefficient to be determined from such relations that

$$C_r = 2h\sinh\alpha b\cdot D_r \ ,$$
$$A_r = -2h\frac{1}{3(1+\nu)\alpha}\{(6+\nu h^2\alpha^2)\sinh\alpha b + 3(1+\nu)\,\alpha b\cosh\alpha b\}\,D_r . \qquad (5.5)$$

Accordingly it results that

$$\chi'' = -\sum_r A_r 2h\Big[\frac{1}{3(1+\nu)\alpha}\{(6+\nu h^2\alpha^2)\sinh\alpha b + 3(1+\nu)\alpha b\cosh\alpha b\} \times$$
$$\times \sinh\alpha y - \sinh\alpha b\cdot y\cosh\alpha y\Big]\sin\alpha x + \sum_s B_s 2h(a\cosh\beta a\sinh\beta x +$$
$$- x\sinh\beta a\cosh\beta x)\sin\beta y \ , \qquad (5.6a)$$

$$\Theta_0 = \sum_r A_r 2\alpha\sinh\alpha b\sinh\alpha y\sin\alpha x - \sum_s B_s 2\beta\sinh\beta a\sinh\beta x\sin\beta y \ , \qquad (5.6b)$$

$$\chi_0 = \sum_r A_r\Big[\Big\{-b\cosh\alpha b - \frac{2}{(1+\nu)\alpha}\sinh\alpha b\Big\}\sinh\alpha y + \sinh\alpha b\cdot y\cosh\alpha y\Big] \times$$
$$\times \sin\alpha x + \sum_s B_s\Big[\Big\{a\cosh\beta a - \frac{1}{3}\frac{\nu}{(1+\nu)}h^2\beta\sinh\beta a\Big\}\sinh\beta x +$$
$$- \sinh\beta a\cdot x\cosh\beta x\Big]\sin\beta y \ , \qquad (5.6c)$$

$$\bar{\chi}_1 = \sum_r A_r (1+\nu)(\alpha b\cosh\alpha b\sinh\alpha y - \alpha y\sinh\alpha b\cdot\cosh\alpha y)\cos\alpha x +$$
$$+ \sum_s B_s\Big[\Big\{(\nu - 1 + \frac{\nu}{3}h^2\beta^2)\sinh\beta a - (1+\nu)\,\beta a\cosh\beta a\Big\}\cosh\beta x +$$
$$+ (1+\nu)\beta\sinh\beta a\cdot x\sinh\beta x\Big]\sin\beta y \ , \qquad (5.6d)$$

$$\bar{\chi}_2 = \sum_r A_r \Big[\Big\{ (3-\nu)\sinh\alpha b + (1+\nu)\,\alpha b\cosh\alpha b \Big\} \cosh\alpha y - (1+\nu)\alpha\sinh\alpha b \times$$

$$\times\ y\sinh\alpha y \Big] \sin\alpha x + \sum_s B_s \Big[\Big\{ \Big(2+\frac{\nu}{3}h^2\beta^2\Big)\sinh\beta a - (1+\nu)\,\beta a\cosh\beta a \Big\} \times$$

$$\times\ \sinh\beta x + (1+\nu)\,\beta\sinh\beta a\cdot x\cosh\beta x \Big] \cos\beta y\,, \qquad \cdots\cdots\cdots\cdots\cdots (5.6e)$$

in which we replace D_r and B_s in the preceding formulae by A_r and $2hB_s$ respectively. Now we are left with the following two conditions unsatisfied.

$$S_1 = 0\,, \qquad \text{at} \quad x = \pm a\,, \qquad \cdots\cdots\cdots\cdots\cdots (5.7a)$$

$$v_0 = 0\,, \qquad \text{at} \quad y = \pm b\,. \qquad \cdots\cdots\cdots\cdots\cdots (5.7b)$$

By (4.7a), (3.4) and (2.1b) of (I) we must have

$$S_1 = S_{1,1} + S_{1,3} = \frac{-\partial^2\chi''}{\partial x\,\partial y} + Q\frac{\partial^2 v}{\partial x\,\partial y} = \sum_r A_r 2h\alpha \Big[\frac{1}{3(1+\nu)} \Big\{ (6+\nu h^2\alpha^2) \times$$

$$\times\ \sinh\alpha b + 3(1+\nu)\,\alpha b\cosh\alpha b \Big\} \cosh\alpha y - \sinh\alpha b\big(\cosh\alpha y + \alpha y\sinh\alpha y\big) \Big] \times$$

$$\times\ \cos\alpha x - \sum_s B_s 2h\beta \Big\{ (\beta a\cosh\beta a - \sinh\beta a)\cosh\beta x - \beta\,\text{snih}\,\beta a \times$$

$$\times\ x\sinh\beta x \Big\} \cos\beta y + Q\frac{\partial^2 v}{\partial x\,\partial y} = 0\,, \qquad \text{at } x = \pm a\,. \quad \cdots\cdots\cdots\cdots (5.8)$$

In the same way as before from (5.8) we get the relation:

$$\begin{aligned} &\sum_r A_r \frac{2h(-1)^{r'+s'}\varepsilon_{s'} b r'^2 \sinh^2\alpha b}{(b^2r'^2 + a^2s'^2)} \Big\{ \frac{\nu(h^2\alpha^2-6)}{3(1+\nu)} + \frac{2b^2r'^2}{(b^2r'^2+a^2s'^2)} \Big\} + \\ &-B_s h\beta(2\beta a - \sinh 2\beta a) = (-1)^{n'+1}\alpha_m\beta_n Q\,, \quad \text{for } n = s\,, \\ &\qquad\qquad = 0\,, \qquad\qquad \text{for } n \neq s\,, \end{aligned} \qquad (5.9)$$

in which $\varepsilon_0 = 1$, $\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \cdots\cdots = 2$.

And by (5.7b), (5.6e), (4.5) and (4.3) there exists the formula

$$\delta_2 = \bar{\chi}_2 + J\frac{\partial v}{\partial y} = 0\,, \qquad \text{at} \quad y = \pm b\,,$$

that is,

$$\sum_r A_r \Big\{ (3-\nu)\sinh\alpha b\cosh\alpha b + (1+\nu)\,\alpha b \Big\} \sin\alpha x + \sum_s B_s \Big[\Big\{ \Big(2+\frac{\nu}{3}h^2\beta^2\Big) \times$$

$$\times\ \sinh\beta a - (1+\nu)\,\beta a\cosh\beta a \Big\} \sinh\beta x + (1+\nu)\,\beta\sinh\beta a\cdot x\cosh\beta x \Big] \times$$

$$\times\ (-1)^{s'} + J\beta_n(-1)^{m'}\sin\alpha_m x = 0\,. \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5.10a)$$

From (5.10a) another relation is obtained, which is given by

$$A_r \left\{(3-\nu)\sinh \alpha b \cosh \alpha b + (1+\nu)\alpha b\right\} + \sum_s B_s (-1)^{r'+s'} \frac{2}{3} \frac{b^2 r' \sinh^2 \beta a}{\pi (b^2 r'^2 + a^2 s'^2)^2} \times$$
$$\times \left\{\nu(6-h^2\beta^2)a^2 s'^2 - (6+\nu h^2 \beta^2) b^2 r'^2\right\}$$
$$\left.\begin{aligned} &= (-1)^{m'+1} J \frac{n'\pi}{b}, \quad &&\text{for} \quad m = r, \\ &= 0, &&\text{for} \quad m \neq r. \end{aligned}\right\} \qquad (5.10\text{b})$$

In the next place we shall determine the forms of basic quantities Θ_1, χ'_1 and χ'. By the condition $w_0 = 0$ along the boundary we may take $\bar{\chi}$ in the formula (4.36) to be of the form

$$\bar{\chi} = \sum_r A_r (b \cosh \alpha b \sinh \alpha y - y \sinh \alpha b \cosh \alpha y) \sin \alpha x +$$
$$+ \sum_s B_s (a \cosh \beta a \sinh \beta x - x \sinh \beta a \cosh \beta x) \sin \beta y, \qquad \cdots\cdots (5.11)$$

in case I with $r = 2r'$, and $s = 2s'$. However, taking account of the fact that by (4.36) of (I)

$$G_{1,3} = 0, \text{ at } x = \pm a \text{ and hence } G_{1,2} = 0, \text{ at } x = \pm a.$$

and, further, $G_{1,2}$ is expressible in the form (2.10) of (I) by the use of formula $w_{0,2} = \dfrac{-\bar{J}}{k^4}\bar{\chi}$, it would be apparent that

$$B_s \equiv 0. \qquad \cdots\cdots (5.12)$$

Therefore we see that

$$w_0 = \frac{-\bar{J}}{k^4} \left\{\sum_r A_r (b \cosh \alpha b \sinh \alpha y - y \sinh \alpha b \cosh \alpha y) \sin \alpha x + (vx, y)\right\} \qquad \cdots\cdots (5.13)$$

satisfies the conditions

$$w_0 = 0 \text{ and } G_1 = 0, \quad \text{at} \quad x = \pm a,$$

and when the condition $\dfrac{\partial w_0}{\partial y} = 0$, at $y = \pm b$ is considered, sequence $\{A_r\}$ reduces to

$$A_m = (-1)^{m'} \frac{\pi n}{b} \Big/ (2\alpha b - \sinh 2\alpha b). \qquad \cdots\cdots (5.14)$$

Consequently we can readily determine the forms of Θ_1 and χ'_1 by means of the formula $w_{0,2}=\frac{1}{E}\left\{h^2\Theta_1+(1+\nu)\chi_1'\right\}$.

$$\Theta_1=\frac{-E}{(1-\nu)}\frac{\bar{J}}{k^4}A_m 2\alpha\sinh\alpha b\sinh\alpha y\sin\alpha x\,, \quad\cdots\cdots\cdots\cdots \text{(5. 15a)}$$

$$\chi_1'=\frac{E}{(1+\nu)}\frac{\bar{J}}{k^4}A_m\left\{\left(-b\cosh\alpha b+\frac{2}{(1-\nu)}h^2\alpha\sinh\alpha b\right)\sinh\alpha y+\right.$$
$$\left.+\sinh\alpha b\cdot y\cosh\alpha y\right\}\sin\alpha x\,. \quad\cdots\cdots\cdots\cdots \text{(5. 15b)}$$

From (5.9), (5.10b) and (5.14) all coefficients in the expressions for four basic quantities Θ_0, χ_0, Θ_1 and χ_1' can be obtained, so the problem in question may be said to have been solved virtually. By (5.6) we have

$$\chi=\chi_0-\frac{1}{2}\frac{\nu}{1+\nu}z^2\Theta_0=\sum_r A_r\left[\left\{-b\cosh\alpha b-\frac{1}{(1+\nu)\alpha}(2+\nu z^2\alpha^2)\sinh\alpha b\right\}\times\right.$$
$$\left.\times\sinh\alpha y+\sinh\alpha b\cdot y\cosh\alpha y\right]\sin\alpha x+\sum_s B_s\left[\left\{a\cosh\beta a+\frac{\nu}{1+\nu}\times\right.\right.$$
$$\left.\left.\times\left(z^2-\frac{1}{3}h^2\right)\beta\sinh\beta a\right\}\sinh\beta x-\sinh\beta a\cdot x\cosh\beta x\right]\sin\beta y. \quad \text{(5. 16)}$$

By (5.15) we have for χ'

$$\chi'=z\chi_1'+\frac{2-\nu}{6(1+\nu)}z^3\Theta_1=\frac{E}{(1-\nu^2)}\frac{\bar{J}}{k^4}A_m z\left[\left\{-(1-\nu)b\cosh\alpha b+2h^2\alpha\sinh\alpha b+\right.\right.$$
$$\left.\left.-\frac{2-\nu}{3}\alpha\sinh\alpha b\cdot z^2\right\}\sinh\alpha y+(1+\nu)\sinh\alpha b\cdot y\cosh\alpha y\right]\sin\alpha x\,.$$
$$\cdots\cdots\cdots\cdots \text{(5. 17)}$$

For example, normal stress σ_x will be given by

$$\sigma_x=\sigma_{x,1}+\sigma_{x,2}+\sigma_{x,3}\,, \quad\cdots\cdots\cdots\cdots \text{(5. 18)}$$

with

$$\sigma_{x,1}=\frac{\partial^2\chi}{\partial y^2}=\sum_r A_r\frac{\alpha}{(1+\nu)}\left[-\left\{(1+\nu)\alpha b\cosh\alpha b+(z^2\alpha^2-2)\nu\sinh\alpha b\right\}\sinh\alpha y+\right.$$
$$\left.+(1+\nu)\alpha\sinh\alpha b\cdot y\cosh\alpha y\right]\sin\alpha x-\sum B_s\frac{\beta}{(1+\nu)}\left[\left\{(1+\nu)\beta a\cosh\beta a+\right.\right.$$
$$\left.\left.+\nu\left(z^2-\frac{1}{3}h^2\right)\beta^2\sinh\beta a\right\}\sinh\beta x-(1+\nu)\beta\sinh\beta a\cdot x\cosh\beta x\right]\sin\beta y,$$

$$\sigma_{x,2}=\frac{E}{(1-\nu^2)}\frac{\bar{J}}{k^4}A_m\alpha z\left[\left\{-(1-\nu)\alpha b\cosh\alpha b+\left(2\nu+2h^2\alpha^2-\frac{(2-\nu)}{3}\alpha^2z^2\right)\times\right.\right.$$
$$\left.\left.\times\sinh\alpha b\right\}\sinh\alpha y+(1+\nu)\alpha\sinh\alpha b\cdot y\cosh\alpha y\right]\sin\alpha x\,,$$

taking $\sigma_{x,3}$ from (4.25) of (I). We omit the solutions in the other three cases than the above case, since we can proceed along quite a similar way and needed descriptions are so lengthy.

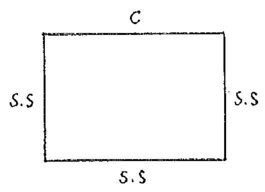


Fig. 4

Case B. When Thick Plate with One Edge Clamped and Other Three Edges Simply Supported is Stressed by the Pressure (5.1).

We shall take the coordinate system I in Fig. 2 for convenience' sake, though it would be proper that the coordinate system III should be referred to in this case, if the edge $\overline{AB}$ is clamped. The boundary conditions are as follows:

$$\left.\begin{array}{lll} u_0=0,\ v_0=0,\ w_0=0, & \dfrac{\partial w_0}{\partial y}=0, & \text{at}\quad y=b, \\ T_1=0,\ S_1=0,\ G_1=0, & w_0=0, & \text{at}\quad x=\pm a, \\ T_2=0,\ S_2=0,\ G_2=0, & w_0=0, & \text{at}\quad y=-b. \end{array}\right\} \tag{5.19}$$

At first we shall want to obtain the solutions of the first kind in the same fashion as before. In view of the boundary conditions for T we may put χ'' in the form

$$\chi''=2h\chi_0-\frac{1}{3}\frac{\nu}{1+\nu}h^3\Theta_0=\sum_r\Big\{A_r\sinh\alpha(y+b)+C_r(y+b)\cosh\alpha y+$$
$$+E_r(y+b)\sinh\alpha y\Big\}\sin\alpha x+\sum_s B_s(a\cosh\beta a\sinh\beta x-x\sinh\beta a\cosh\beta x)\times$$
$$\times\sin\beta(y+b)\,. \tag{5.20}$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=2r'$ and $s=s'$. It is obvious that resultant forces T's derived from (5.20) satisfy the required edge conditions at the three edges. Then by the condition $u_0=\dfrac{1}{E}\Big(\bar{\chi}_1+J\dfrac{\partial v}{\partial x}\Big)=0$ at $y=b$ the expression for $\bar{\chi}_1$ obtained from (5.20) yields the following relations:

$$A_r=\frac{-C'_r}{(1+\nu)}\Big\{\sinh\alpha b+(1+\nu)\,\alpha b\cosh\alpha b+\frac{1}{6}\nu h^2\alpha^2\sinh\alpha b\Big\}+$$
$$-\frac{E'_r}{(1+\nu)}\Big\{\cosh\alpha b+(1+\nu)\,\alpha b\sinh\alpha b+\frac{1}{6}\nu h^2\alpha^2\cosh\alpha b\Big\}\,, \tag{5.21a}$$

$$C_r=\alpha\sinh\alpha b\cosh\alpha b\cdot C'_r\,,\quad E_r=\alpha\sinh\alpha b\cosh\alpha b\cdot E'_r\,. \tag{5.21b}$$

Hereafter we shall drop primes over C_r and E_r on the right-hand sides of equations (5.21) for simplification.

Now we may write for basic quantities Θ_0, χ_0 and χ.

$$\Theta_0 = \frac{1}{h}\Big\{\sum_r (C_r \sinh \alpha y + E_r \cosh \alpha y)\alpha^2 \sinh \alpha b \cosh \alpha b \sin \alpha x +$$
$$-\sum_s B_s \beta \sinh \beta a \sinh \beta x \sin \beta (y+b)\Big\}, \quad \cdots\cdots (5.22a)$$

$$\chi_0 = \chi|_{z=0}, \quad \cdots\cdots (5.22b)$$

$$\chi = \sum_r C_r \frac{1}{2h}\cdot\frac{1}{(1+\nu)}\Big[-\Big\{\sinh \alpha b + (1+\nu)\alpha b \cosh \alpha b + \frac{1}{6}\nu h^2\alpha^2 \sinh \alpha b\Big\}\times$$
$$\times \sinh \alpha(y+b) + \alpha \sinh \alpha b \cosh \alpha b\Big\{(1+\nu)(y+b)\cosh \alpha y - \nu\left(z^2 - \frac{1}{3}h^2\right)\times$$
$$\times \alpha \sinh \alpha y\Big\}\Big]\sin \alpha x + \sum_r E_r \frac{1}{2h(1+\nu)}\Big[-\Big\{\cosh \alpha b + (1+\nu)\alpha b \sinh \alpha b +$$
$$+\frac{1}{6}\nu h^2\alpha^2 \cosh \alpha b\Big\}\sinh \alpha(y+b) + \alpha \sinh \alpha b \cosh \alpha b\Big\{(1+\nu)(y+b)\sinh \alpha y +$$
$$-\nu\left(z^2 - \frac{1}{3}h^2\right)\alpha \cosh \alpha y\Big\}\Big]\sin \alpha x + \sum_s B_s \frac{1}{2h(1+\nu)}\Big[\Big\{(1+\nu)a \cosh \beta a +$$
$$+\nu\left(z^2 - \frac{1}{3}h^2\right)\beta \sinh \beta a\Big\}\sinh \beta x - (1+\nu)x \sinh \beta a \cosh \beta x\Big]\sin \beta(y+b).$$
$$\cdots\cdots (5.22c)$$

And χ'' (5.20) is by (5.21)

$$\chi'' = \sum_r C_r\Big[\frac{-1}{(1+\nu)}\Big\{\sinh \alpha b + (1+\nu)\alpha b \cosh \alpha b + \frac{1}{6}\nu h^2\alpha^2 \sinh \alpha b\Big\}\times$$
$$\times \sinh \alpha(y+b) + \frac{1}{2}\alpha \sinh 2\alpha b\cdot(y+b)\cosh \alpha y\Big]\sin \alpha x +$$
$$+\sum_r E_r\Big[\frac{-1}{(1+\nu)}\Big\{\cosh \alpha b + (1+\nu)\alpha b \sinh \alpha b + \frac{1}{6}\nu h^2\alpha^2 \cosh \alpha b\Big\}\times$$
$$\times \sinh \alpha(y+b) + \frac{1}{2}\alpha \sinh 2\alpha b\cdot(y+b)\sinh \alpha y\Big]\sin \alpha x + \sum_s B_s(a \cosh \beta a \times$$
$$\times \sinh \beta x - x \sinh \beta a \cosh \beta x)\sin \beta(y+b). \quad \cdots\cdots (5.23)$$

Now we must have three equations to determine three sequences of coefficients $\{C_r\}$, $\{E_r\}$ and $\{B_s\}$. The condition $\delta_2 = Ev_0 = \bar{\chi}_2 + J\dfrac{\partial v}{\partial y} = 0$, at $y=b$ requires that

$$\Big[\sum_r C_r \frac{\alpha}{4h}\Big\{\left(2+\frac{\nu}{3}h^2\alpha^2\right)\sinh \alpha b \cosh 2\alpha b + \left(1-\nu-\frac{\nu}{3}h^2\alpha^2\right)\cosh \alpha b \times$$

$$\times \sinh 2\alpha b + 2(1+\nu)\alpha b \cosh \alpha b \cosh 2\alpha b - 2(1+\nu)\alpha b \sinh \alpha b \sinh 2\alpha b \Big\} +$$

$$+ \sum_r E_r \frac{\alpha}{4h} \Big\{ \Big(2 + \frac{\nu}{3} h^2\alpha^2\Big) \cosh \alpha b \cosh 2\alpha b + \Big(1 - \nu - \frac{\nu}{3} h^2\alpha^2\Big) \sinh \alpha b \times$$

$$\times \sinh 2\alpha b + 2(1+\nu)\alpha b \sinh \alpha b \cosh 2\alpha b - 2(1+\nu)\alpha b \cosh \alpha b \sinh 2\alpha b \Big\} \Big] \sin \alpha x +$$

$$+ \sum_s B_s \frac{1}{2h} \Big[\Big\{ 2\sinh \beta a - (1+\nu)\beta a \cosh \beta a + \frac{1}{3}\nu h^2\beta^2 \sinh \beta a \Big\} \sinh \beta x +$$

$$+ (1+\nu)\sinh \beta a \cdot \beta x \cosh \beta x \Big] \cos 2b\beta = (-1)^{m'+1} J \beta_n \sin \frac{\pi m' x}{a} , \qquad (5.24)$$

and equation (5.24) furnishes the relation

$$C_r \frac{\alpha}{2} \Big\{ \Big(2 + \frac{\nu}{3} h^2\alpha^2\Big) \sinh \alpha b \cosh 2\alpha b + \Big(1 - \nu - \frac{\nu}{3} h^2\alpha^2\Big) \cosh \alpha b \sinh 2\alpha b +$$

$$+ 2(1+\nu)\alpha b \cosh \alpha b \cosh 2\alpha b - 2(1+\nu)\alpha b \sinh \alpha b \sinh 2\alpha b \Big\} +$$

$$E_r \frac{\alpha}{2} \Big\{ \Big(2 + \frac{\nu}{3} h^2\alpha^2\Big) \cosh \alpha b \cosh 2\alpha b + \Big(1 - \nu - \frac{\nu}{3} h^2\alpha^2\Big) \sinh \alpha b \sinh 2\alpha b +$$

$$+ 2(1+\nu)\alpha b \sinh \alpha b \cosh 2\alpha b - 2(1+\nu)\alpha b \cosh \alpha b \sinh 2\alpha b \Big\} +$$

$$+ \sum_s B_s \frac{(-1)^{r'+s} 4b^2 r'}{h\pi(b^2r^2 + a^2s^2)} \sinh^2 \beta a \Big\{ -\Big(2 + \frac{1}{3}\nu h^2\beta^2\Big) + \frac{(1+\nu)a^2s^2}{(b^2r^2 + a^2s^2)} \Big\}$$

$$\left. \begin{aligned} &= (-1)^{m'+1} 2hJ\beta_n , && \text{for} \quad m = r , \\ &= 0 , && \text{for} \quad m \neq r . \end{aligned} \right\} \qquad (5.25)$$

Next from the condition

$$S_2 = \frac{\partial^2 \chi''}{\partial x \partial y} - Q \frac{\partial^2 v}{\partial x \partial y} = 0 , \qquad \text{at} \quad y = -b ,$$

where the expression (5.23) is to be substituted in $\frac{\partial^2 \chi''}{\partial x \partial y}$ and the second term comes from (3.4), we obtain the relation among coefficients:

$$C_r \frac{\alpha^2}{(1+\nu)} \Big\{ + \frac{(1+\nu)}{2} (2\alpha b - \sinh 2\alpha b) \cosh \alpha b + \Big(1 + \frac{\nu}{6} h^2\alpha^2\Big) \sinh \alpha b +$$

$$+ E_r \frac{\alpha^2}{(1+\nu)} \Big\{ \frac{(1+\nu)}{2} \Big(2\alpha b + \sinh 2\alpha b\Big) \sinh \alpha b + \Big(1 + \frac{\nu}{6} h^2\alpha^2\Big) \cosh \alpha b \Big\} +$$

$$+ \sum_s B_s \frac{(-1)^{r'} \varepsilon_{r'} 2ab^2r^2s^2 \sinh^2 \beta a}{(b^2r^2 + a^2s^2)^2}$$

$$\left. \begin{aligned} &= (-1)^{m'+1} Q \alpha_m \beta_n , && \text{for} \quad m = r , \\ &= 0 , && \text{for} \quad m \neq r , \end{aligned} \right\} \qquad (5.26)$$

in which

$$\varepsilon_0=1, \quad \varepsilon_1=\varepsilon_2=\varepsilon_3=\cdots=2 .$$

Then we have to apply the last condition,

$$S_1=\frac{-\partial^2\chi''}{\partial x\partial y}+Q\frac{\partial^2 v}{\partial x\partial y}=0, \qquad \text{at } x=\pm a .$$

In order to consider this condition it would be appropriate to take the coordinate system III as shown in Fig. 2. This condition can be re-written into the following form, referred to coordinate system III.

$$\begin{aligned}
&\sum_{r} C_r\Big[\frac{-1}{(1+\nu)}\Big\{\sinh\alpha b+(1+\nu)\,\alpha b\cosh\alpha b+\frac{1}{6}\nu h^2\alpha^2\sinh\alpha b\Big\}\cosh\alpha y+\\
&\quad+\frac{1}{2}\sinh 2\alpha b\Big\{\cosh\alpha(y-b)+\alpha y\sinh\alpha(y-b)\Big\}\Big]\alpha^2(-1)^{r'}+\\
&\quad+\sum_{r} E_r\Big[\frac{-1}{(1+\nu)}\Big\{\cosh\alpha b+(1+\nu)\alpha b\sinh\alpha b+\frac{1}{6}\nu h^2\alpha^2\cosh\alpha b\Big\}\cosh\alpha y+\\
&\quad+\frac{1}{2}\sinh 2\alpha b\Big\{\sinh\alpha(y-b)+\alpha y\cosh\alpha(y-b)\Big\}\Big]\alpha^2(-1)^{r'}+\\
&\quad+\sum_{s} B_s\frac{1}{2}\beta(2\beta a-\sinh 2\beta a)\cos\beta y=Q\alpha_m\beta_n\cos\beta_n y . \qquad\cdots\cdots (5.27)
\end{aligned}$$

When we represent the left-hand side of equation (5.27) by Fourier cosine series for range (0, 2b) and equate coefficients of similar terms on both sides of the resulting equation, we obtain the required relation.

$$\begin{aligned}
&\sum_{r} C_r(-1)^{r'+s}\varepsilon_s\frac{2b\pi}{a}\frac{r'^3\sinh\alpha b\sinh 2\alpha b}{(b^2r^2+a^2s^2)}\Big[\frac{-1}{(1+\nu)}\Big\{\Big(1+\frac{\nu}{6}h^2\alpha^2\Big)+\\
&\quad-\frac{(1+\nu)}{2}(1+(-1)^s)\Big\}-\frac{(1+(-1)^s)}{2}\frac{(b^2r^2-a^2s^2)}{(b^2r^2+a^2s^2)}\Big]+\\
&\quad+\sum_{r} E_r(-1)^{r'+s}\varepsilon_s\frac{2b\pi}{a}\frac{r'^3\cosh\alpha b\sinh 2\alpha b}{(b^2r^2+a^2s^2)}\Big[\frac{-1}{(1+\nu)}\Big\{\Big(1+\frac{\nu}{6}h^2\alpha^2\Big)+\\
&\quad-\frac{(1+\nu)}{2}(1-(-1)^s)\Big\}-\frac{(1-(-1)^s)}{2}\frac{(b^2r^2-a^2s^2)}{(b^2r^2+a^2s^2)}\Big]+B_s\frac{\beta}{2}(2\beta a-\sinh 2\beta a)\\
&\qquad\qquad\left.\begin{aligned}&=\alpha_m\beta_n Q, &&\text{for}\quad n=s,\\ &=0, &&\text{for}\quad n\neq s.\end{aligned}\right\}\qquad (5.28)
\end{aligned}$$

Consequently we can determine the coefficients $\{C_r\}$, $\{E_r\}$ and $\{B_s\}$ from (5.25), (5.26) and (5.28) by successive approximation.

In the next place we shall obtain solutions of the second kind. The boundary conditions to be considered herein are relative to the quantities w_0, $\frac{\partial w_0}{\partial y}$, G_1 and G_2. Noting the condition $w_0=0$ along the boundary, $\bar{\chi}$ in the formula $w_0=\frac{-\bar{J}}{k^4}\left\{v(x,y)+\bar{\chi}(x,y)\right\}$ can be written as

$$\bar{\chi}=\left\{\sum_r A_r(b\cosh\alpha b\sinh\alpha y-y\sinh\alpha b\cosh\alpha y)+\sum_r B_r(b\sinh\alpha b\cosh\alpha y+\right.$$
$$\left.-y\cosh\alpha b\sinh\alpha y)\right\}\sin\alpha x+\sum_s C_s(a\cosh\beta a\sinh\beta x-x\sinh\beta a\cosh\beta x)\times$$
$$\times\sin\beta(y+b)\,, \qquad \text{5.29)}$$

with $\alpha=\frac{r\pi}{2a}$, $\beta=\frac{s\pi}{2b}$, $r=2r'$ and $s=s'$, which refers, of course, to the coordinate system I in Fig. 2.

By the condition $G_1=0$, at $x=\pm a$ we have

$$C_s=0 \qquad (5.30)$$

and, further, according to the condition $G_2=0$, at $y=-b$ the relation between A_r and B_r is given by $B_r\cdot\cosh^2\alpha b=A_r\cdot\sinh^2\alpha b$ so that we put

$$A_r=\cosh^2\alpha b A_r', \quad B_r=\sinh^2\alpha b\cdot B_r' \qquad (5.31)$$

and drop primes always in the sequel as before.

Finally, if we apply the condition $\frac{\partial w_0}{\partial y}=0$, at $y=b$, it is easily seen that

$$\left.\begin{aligned} A_r&=(-1)^{m'}2n\pi\cdot\frac{1}{b(\sinh 4\alpha b-4\alpha b)}\,, && \text{for } r=m\,,\\ &=0\,, && \text{for } r\neq m\,. \end{aligned}\right\} \qquad (5.32)$$

As a consequence we may write $\bar{\chi}$ in the form

$$\bar{\chi}=A_m\left\{\cosh^2\alpha b\cdot Y_1+\sinh^2\alpha b\cdot Y_2\right\}\sin\alpha_m x\,, \qquad (5.33)$$

in which

$$Y_1=b\cosh\alpha b\cdot\sinh\alpha y-y\sinh\alpha b\cosh\alpha y\,,$$
$$Y_2=b\sinh\alpha b\cosh\alpha y-y\cosh\alpha b\sinh\alpha y\,. \qquad (\alpha=\alpha_m)$$

With the aid of formula $w_{0,2}=\frac{-\bar{J}}{k^4}\bar{\chi}=\frac{1}{E}\left\{(1+\nu)\chi_1'+h^2\Theta_1\right\}$ from (5.33) we obtain

$$\Theta_1=\frac{E}{(1-\nu)}\frac{\bar{J}}{k^4}\nabla_1^2\bar{\chi}=\frac{-E}{(1-\nu)}\frac{\bar{J}}{k^4}A_m\alpha\sinh 2\alpha b\sinh\alpha(y+b)\sin\frac{m'\pi x}{a}\,, \quad (5.34a)$$

$$\chi_1' = \chi'/z\big|_{z=0}\ , \qquad \cdots\cdots (5.34b)$$

$$\chi' = z\chi_1' + \frac{(2-\nu)}{6(1+\nu)} z^3\Theta_1 = \frac{-E}{(1+\nu)}\frac{\bar{J}}{k^4} z A_m \Big[Y_1\cosh^2\alpha b + Y_2\sinh^2\alpha b + + \frac{1}{(1-\nu)}\alpha\sinh 2\alpha b\cdot\sinh\alpha(y+b)\Big\{\frac{(2-\nu)}{6}z^2 - h^2\Big\}\Big]\sin\alpha x\ . \cdots (5.34c)$$

From these basic quantities we can easily obtain the solutions to the problem concerned. For instance, normal stress σ_x is given by

$$\sigma_x = \sigma_{x,1} + \sigma_{x,2} + \sigma_{x,3}\ , \qquad \cdots\cdots (5.35)$$

where by (5.22c)

$$\sigma_{x,1} = \frac{\partial^2\chi}{\partial y^2} = \sum_r C_r \frac{\alpha^2}{2h(1+\nu)}\Big[-\Big\{\sinh\alpha b + (1+\nu)\alpha b\cosh\alpha b + + \frac{1}{6}\nu h^2\alpha^2\sinh\alpha b\Big\}\sinh\alpha(y+b) + \frac{1}{2}\sinh 2\alpha b\Big\{(1+\nu)\alpha(y+b)\times \times\cosh\alpha y + \Big(2(1+\nu) + \nu\alpha^2\big(\frac{1}{3}h^2 - z^2\big)\Big)\sinh\alpha y\Big\}\Big]\sin\alpha x + - \sum_r E_r \frac{\alpha^2}{2h(1+\nu)}\Big[-\Big\{\cosh\alpha b + (1+\nu)\alpha b\sinh\alpha b + \frac{1}{6}\nu h^2\alpha^2\cosh\alpha b\Big\}\times \times\sinh\alpha(y+b) + \frac{1}{2}\sinh 2\alpha b\Big\{2(1+\nu) + \nu\alpha^2\big(\frac{1}{3}h^2 - z^2\big)\Big\}\cosh\alpha y\Big]\times \times\sin\alpha x - \sum_s B_s \frac{\beta^2}{2h(1+\nu)}\Big[\Big\{(1+\nu)a\cosh\beta a + \nu\beta\sinh\beta a\cdot\big(z^2 - \frac{1}{3}h^2\big)\Big\}\times \times\sinh\beta x - x\sinh\beta a\cosh\beta x\Big]\sin\beta(y+b)\ ,$$

and by (5.34)

$$\sigma_{x,2} = \frac{\partial^2\chi'}{\partial y^2} + \frac{z\Theta_1}{(1+\nu)} = \frac{-E}{(1-\nu^2)}\frac{\bar{J}}{k^4}A_m\alpha\Big[\text{snih}\,2\alpha b\cdot\sinh\alpha(y+b) + (1-\nu)\times \times\Big\{\cosh^2\alpha b\Big((\alpha b\cosh\alpha b - 2\sinh\alpha b)\sinh\alpha y - \alpha y\sinh\alpha b\cdot\cosh\alpha y\Big) + + \sinh^2\alpha b\Big((\alpha b\sinh\alpha b - 2\cosh\alpha b)\cosh\alpha y - \alpha y\cosh\alpha b\sinh\alpha y\Big)\Big\} + + \alpha^2\sinh 2\alpha b\sinh\alpha(y+b)\cdot\Big(\frac{2-\nu}{6}z^2 - h^2\Big)\Big]z\sin\alpha x\ , \quad (\alpha = \alpha_m)\ .$$

For $\sigma_{x,3}$ the form (4.25) of (I) can be used .

Case C. When Thick Plate with Two Adjacent Edges Clamped and the Other Two Adjacent Edges Simply Supported is Loaded by the Pressure (5.1).

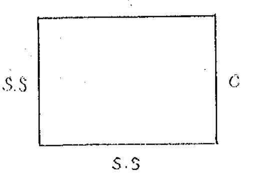


Fig. 5

At first we shall take the coordinate system

I as shown in Fig. 2 for convenience' sake. The boundary conditions in this case are

$$\left.\begin{array}{ll} u_0=v_0=w_0=\dfrac{\partial w_0}{\partial x}=0\,, & \text{at} \quad x=a\,, \\ u_0=v_0=w_0=\dfrac{\partial w_0}{\partial y}=0\,, & \text{at} \quad y=b\,, \\ T_1=S_1=G_1=w_0=0\,, & \text{at} \quad x=-a\,, \\ T_2=S_2=G_2=w_0=0\,, & \text{at} \quad y=-b\,. \end{array}\right\} \quad \cdots\cdots (5.36)$$

Owing to the boundary condition for T at $x=-a$ and $y=-b$ the following form for χ'' can be taken

$$\chi''=\sum_r \{A_r \sinh\alpha(y+b)+C_r(y+b)\cosh\alpha y+E_r(y+b)\sinh\alpha y\}\sin\alpha(x+a)+$$
$$+\sum_s \{B_s\sinh\beta(x+a)+D_s(x+a)\cosh\beta x+F_s(x+a)\sinh\beta x\}\times$$
$$\times\sin\beta(y+b)\,, \quad \cdots\cdots (5.37)$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=r'$ and $s=s'$. r' and s' are positive integers. Of course, we can adopt the expression (5.37), whether m and n are even or odd integers. On substituting the expressions for Θ_0 and χ_0 derived from (5.37) into the formula $\delta_1=Eu_0=\bar{\chi}_1+J\dfrac{\partial v}{\partial x}$ and letting the boundary conditions $u_0=0$, at $y=b$ be satisfied, we obtain the relation among coefficients.

$$A_r=\frac{-2}{(1+\nu)}\Big[C'_r\Big\{\sinh\alpha b+(1+\nu)\alpha b\cosh\alpha b+\frac{\nu}{6}h^2\alpha^2\sinh\alpha b\Big\}+$$
$$+E'_r\Big\{\cosh\alpha b+(1+\nu)\alpha b\sinh\alpha b+\frac{\nu}{6}h^2\alpha^2\cosh\alpha b\Big\}\Big]\,, \quad \cdots\cdots (5.38)$$
$$C_r=\alpha\sinh 2\alpha b\cdot C'_r\,, \qquad E_r=\alpha\sinh 2\alpha b\cdot E'_r$$

and from now on we shall delete the primes over C_r and E_r on the right-hand sides of these equations. Similarly by the condition $v_0=0$, at $x=a$ we have

$$B_s=\frac{-2}{(1+\nu)}\Big[D'_s\Big\{\sinh\beta a+(1+\nu)\beta a\cosh\beta a+\frac{\nu}{6}h^2\beta^2\sinh\beta a\Big\}+$$
$$+F'_s\Big\{\cosh\beta a+(1+\nu)\beta a\sinh\beta a+\frac{\nu}{6}h^2\beta^2\cosh\beta a\Big\}\Big]\,, \quad \cdots\cdots (5.39)$$

$D_s=\beta\sinh 2\beta a\cdot D'_s$, $F_s=\beta\sinh 2\beta a\cdot F'_s$ and drop the primes as before. Accordingly the representation of Θ_0, χ_0 and χ can be taken in the forms

$$\Theta_0 = \sum_r \frac{\alpha^2 \sinh 2\alpha b}{h} (C_r \sinh \alpha y + E_r \cosh \alpha y) \sin \alpha (x+a) +$$

$$+ \sum_s \frac{\beta^2 \sinh 2\beta a}{h} (D_s \sinh \beta x + F_s \cosh \beta x) \sin \beta (y+b) , \quad \cdots (5.40a)$$

$$\chi_0 = \chi|_{z=0} , \quad \cdots\cdots (5.40b)$$

$$\chi = \sum_r C_r \frac{1}{2h} \Big[\frac{-2}{(1+\nu)} \Big\{ \sinh \alpha b + (1+\nu) \alpha b \cosh \alpha b + \frac{\nu}{6} h^2 \alpha^2 \sinh \alpha b \Big\} \times$$

$$\times \sinh \alpha (y+b) + \alpha (y+b) \sinh 2\alpha b \cdot \cosh \alpha y + \frac{\nu}{1+\nu} \Big(\frac{1}{3} h^2 - z^2 \Big) \alpha^2 \times$$

$$\times \sinh 2\alpha b \cdot \sinh \alpha y \Big] \sin \alpha (x+a) + \sum_r E_r \frac{1}{2h} \Big[\frac{-2}{(1+\nu)} \Big\{ \cosh \alpha b + (1+\nu) \alpha b \times$$

$$\times \sinh \alpha b + \frac{\nu}{6} h^2 \alpha^2 \cosh \alpha b \Big\} \sinh \alpha (y+b) + \alpha (y+b) \sinh 2\alpha b \sinh \alpha y +$$

$$+ \frac{\nu}{1+\nu} \Big(\frac{1}{3} h^2 - z^2 \Big) \alpha^2 \sinh 2\alpha b \cosh \alpha y \Big] \sin \alpha (x+a) +$$

$$+ \sum_s D_s \frac{1}{2h} \Big[\frac{-2}{(1+\nu)} \Big\{ \sinh \beta a + (1+\nu) \beta a \cosh \beta a + \frac{\nu}{6} h^2 \beta^2 \sinh \beta a \Big\} \times$$

$$\times \sinh \beta (x+a) + \beta (x+a) \sinh 2\beta a \cdot \cosh \beta x + \frac{\nu}{1+\nu} \Big(\frac{1}{3} h^2 - z^2 \Big) \beta^2 \times$$

$$\times \sinh 2\beta a \sinh \beta x \Big] \sin \beta (y+b) + \sum_s F_s \frac{1}{2h} \Big[\frac{-2}{(1+\nu)} \Big\{ \cosh \beta a + (1+\nu) \beta a \times$$

$$\times \sinh \beta a + \frac{\nu}{6} h^2 \beta^2 \cosh \beta a \Big\} \sinh \beta (x+a) + \beta (x+a) \sinh 2\beta a \cdot \sinh \beta x +$$

$$+ \frac{\nu}{1+\nu} \Big(\frac{1}{3} h^2 - z^2 \Big) \beta^2 \sinh 2\beta a \cosh \beta x \Big] \sin \beta (y+b) . \quad \cdots\cdots (5.40c)$$

Then, using the expressions obtained above, quantities $\bar{\chi}_1$, $\bar{\chi}_2$, $S_{1,1}$ and $S_{2,1}$, which are necessary to the following calculation, are found to be

$$(-h)\bar{\chi}_1 = \Big[\sum_r C_r \alpha \Big\{ - \Big(\sinh \alpha b + (1+\nu) \alpha b \cosh \alpha b + \frac{\nu}{6} h^2 \alpha^2 \sinh \alpha b \Big) \sinh \alpha (y+b) +$$

$$+ \frac{1}{2} \sinh 2\alpha b \Big(\Big(2 + \frac{\nu}{3} h^2 \alpha^2 \Big) \sinh \alpha y + (1+\nu) \alpha (y+b) \cosh \alpha y \Big) \Big\} +$$

$$+ \sum_r E_r \alpha \Big\{ - \Big(\cosh \alpha b + (1+\nu) \alpha b \sinh \alpha b + \frac{\nu}{6} h^2 \alpha^2 \cosh \alpha b \Big) \sinh \alpha (y+b) +$$

$$+ \frac{1}{2} \sinh 2\alpha b \Big(\Big(2 + \frac{\nu}{3} h^2 \alpha^2 \Big) \cosh \alpha y + (1+\nu) \alpha (y+b) \sinh \alpha y \Big) \Big\} \Big] \times$$

$$\times \cos\alpha(x+a)+\Big[\sum_s D_s\beta\Big\{-\Big(\sinh\beta a+(1+\nu)\beta a\cosh\beta a+\frac{\nu}{6}h^2\beta^2\sinh\beta a\Big)\times$$

$$\times \cosh\beta(x+a)+\frac{1}{2}\sinh 2\beta a\Big(\Big(\nu-1+\frac{\nu}{3}h^2\beta^2\Big)\cosh\beta x+(1+\nu)\beta(x+a)\times$$

$$\times \sinh\beta x\Big)\Big\}+\sum_s F_s\beta\Big\{-\Big(\cosh\beta a+(1+\nu)\beta a\sinh\beta a+\frac{\nu}{6}h^2\beta^2\cosh\beta a\Big)\times$$

$$\times \cosh\beta(x+a)+\frac{1}{2}\sinh 2\beta a\Big(\Big(\nu-1+\frac{\nu}{3}h^2\beta^2\Big)\sinh\beta x+(1+\nu)\beta\times$$

$$\times (x+a)\cosh\beta x\Big)\Big\}\Big]\sin\beta(y+b)\,, \qquad \cdots\cdots\cdots (5.41\text{a})$$

$$(-h)\bar{\chi}_2=\Big[\sum_r C_r\alpha\Big\{-\Big(\sinh\alpha b+(1+\nu)\alpha b\cosh\alpha b+\frac{\nu}{6}h^2\alpha^2\sinh\alpha b\Big)\times$$

$$\times \cosh\alpha(y+b)+\frac{1}{2}\sinh 2\alpha b\Big(\Big(\nu-1+\frac{\nu}{3}h^2\alpha^2\Big)\cosh\alpha y+(1+\nu)\alpha(y+b)\times$$

$$\times \sinh\alpha y\Big)\Big\}+\sum_r E_r\alpha\Big\{-\Big(\cosh\alpha b+(1+\nu)\alpha b\sinh\alpha b+\frac{\nu}{6}h^2\alpha^2\cosh\alpha b\Big)\times$$

$$\times \cosh\alpha(y+b)+\frac{1}{2}\sinh 2\alpha b\Big(\Big(\nu-1+\frac{\nu}{3}h^2\alpha^2\Big)\sinh\alpha y+(1+\nu)\alpha\times$$

$$\times (y+b)\cosh\alpha y\Big)\Big\}\Big]\sin\alpha(x+a)+\sum_s\Big[D_s\beta\Big\{-\Big(\sinh\beta a+(1+\nu)\beta a\cosh\beta a+$$

$$+\frac{\nu}{6}h^2\beta^2\sinh\beta a\Big)\sinh\beta(x+a)+\frac{1}{2}\sinh 2\beta a\Big(\Big(2+\frac{\nu}{3}h^2\beta^2\Big)\sinh\beta x+$$

$$+(1+\nu)\beta(x+a)\cosh\beta x\Big)\Big\}+F_s\beta\Big\{-\Big(\cosh\beta a+(1+\nu)\beta a\sinh\beta a+$$

$$+\frac{\nu}{6}h^2\beta^2\cosh\beta a\Big)\sinh\beta(x+a)+\frac{1}{2}\sinh 2\beta a\Big(\Big(2+\frac{\nu}{3}h^2\beta^2\Big)\cosh\beta x+$$

$$+(1+\nu)\beta(x+a)\sinh\beta x\Big)\Big\}\Big]\cos\beta(y+b)\,, \qquad \cdots\cdots\cdots (5.41\text{b})$$

$$S_{1,1}=-S_{2,1}=-\frac{\partial^2\chi''}{\partial x\,\partial y}=\frac{-2}{(1+\nu)}\sum_r\Big[C_r\alpha^2\Big\{-\Big(\sinh\alpha b+(1+\nu)\alpha b\cosh\alpha b+$$

$$+\frac{\nu}{6}h^2\alpha^2\sinh\alpha b\Big)\cosh\alpha(y+b)+\frac{(1+\nu)}{2}\sinh 2\alpha b\Big(\cosh\alpha y+\alpha(y+b)\times$$

$$\times \sinh\alpha y\Big)\Big\}+E_r\alpha^2\Big\{-\Big(\cosh\alpha b+(1+\nu)\alpha b\sinh\alpha b+\frac{\nu}{6}h^2\alpha^2\cosh\alpha b\Big)\times$$

$$\times \cosh\alpha(y+b)+\frac{(1+\nu)}{2}\sinh 2\alpha b\Big(\sinh\alpha y+\alpha(y+b)\cosh\alpha y\Big)\Big\}\Big]\cos\alpha(x+a)+$$

$$+\frac{-2}{(1+\nu)}\sum_s\Big[D_s\beta^2\Big\{-\Big(\sinh\beta a+(1+\nu)\beta a\cosh\beta a+\frac{\nu}{6}h^2\beta^2\sinh\beta a\Big)\times$$

$$\times \cosh\beta(x+a)+\frac{(1+\nu)}{2}\sinh 2\beta a\Big(\cosh\beta x+\beta(x+a)\sinh\beta x\Big)\Big\}+$$
$$+F_s\beta^2\Big\{-\Big(\cosh\beta a+(1+\nu)\beta a\sinh\beta a+\frac{\nu}{6}h^2\beta^2\cosh\beta a\Big)\cosh\beta(x+a)+$$
$$+\frac{(1+\nu)}{2}\sinh 2\beta a\Big(\sinh\beta x+\beta(x+a)\cosh\beta x\Big)\Big\}\Big]\cosh\beta(y+b). \quad (5.41c)$$

Thus we can deal with the boundary conditions in regard to quantities u_0, v_0 and S_1, S_2, which are left unsatisfied, by letting the expressions refer to the coordinate system II in Fig. 2 and be represented by Fourier series for range (0, 2a) or (0, 2b).

From the condition $Eu_0=\bar{\chi}_1+J\frac{\partial v}{\partial x}=0$, at $x=a$, in which $\bar{\chi}_1$ is to be represented by (5.41a), it is found that

$$\sum_r C_r\frac{(-1)^r ars}{(b^2r^2+a^2s^2)}\sinh ab\sinh 2ab\Big\{-\Big(1+\frac{\nu}{6}h^2a^2\Big)+\frac{(1+\nu)b^2r^2}{(b^2r^2+a^2s^2)}\Big(1+(-1)^s\Big)\Big\}+$$
$$+\sum_r E_r\frac{(-1)^r ars}{(s^2r^2+a^2s^2)}\cosh ab\sinh 2ab\Big\{\Big(1+\frac{\nu}{6}\Big)h^2a^2+\frac{(1+\nu)b^2r^2}{(b^2r^2+a^2s^2)}\times$$
$$\times\Big(-1+(-1)^s\Big)\Big\}+D_s\beta\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\sinh\beta a-(1+\nu)\beta a\cosh\beta a+$$
$$+\frac{(\nu-3)}{2}\cosh\beta a\sinh 2\beta a\Big\}+F_s\beta\Big\{-\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\cosh\beta a+(1+\nu)\beta a\times$$
$$\times\sinh\beta a+\frac{(\nu-3)}{2}\sinh\beta a\sinh 2\beta a\Big\}$$
$$\begin{aligned}&=hJ\alpha_m, && \text{for } s=n,\\ &=0, && \text{for } s\neq n.\end{aligned}\quad\begin{pmatrix}m=2m',\\ n=2n'.\end{pmatrix}\quad (5.42)$$

Similarly, using formula (5.41b), from the condition $Ev_0=\bar{\chi}_2+J\frac{\partial v}{\partial y}=0$, at $y=b$ we have the formula

$$C_r\alpha\Big\{\Big(1+\frac{\nu}{6}h^2\alpha^2\Big)\sinh ab-(1+\nu)ab\cosh ab+\frac{(\nu-3)}{2}\cosh ab\sinh 2ab\Big\}+$$
$$+E_r\alpha\Big\{-\Big(1+\frac{\nu}{6}h^2\alpha^2\Big)\cosh ab+(1+\nu)ab\sinh ab+\frac{(\nu-3)}{2}\sinh ab\times$$
$$\times\sinh 2ab\Big\}+\sum_s D_s\frac{(-1)^s brs}{(b^2r^2+a^2s^2)}\sinh\beta a\sinh 2\beta a\Big\{-\Big(1+\frac{\nu}{6}h^2\beta^2\Big)+$$
$$+\frac{(1+\nu)a^2s^2}{(b^2r^2+a^2s^2)}\Big(1+(-1)^r\Big)\Big\}+\sum_s F_s\frac{(-1)^s brs}{(b^2r^2+a^2s^2)}\cosh\beta a\sinh 2\beta a\times$$
$$\times\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)+\frac{(1+\nu)a^2s^2}{(b^2r^2+a^2s^2)}\Big(-1+(-1)^r\Big)\Big\}$$

$$\left.\begin{aligned}&= hJ\beta_n\,, \quad \text{for} \quad m = r\,,\\ &= 0\,, \qquad \text{for} \quad m \neq r\,.\end{aligned}\right\} \tag{5.43}$$

Then by means of the condition $S_1 = \dfrac{-\partial^2\chi''}{\partial x\partial y} + Q\dfrac{\partial^2 v}{\partial x\partial y} = 0$, at $x = -a$ which refers, needless to notice, to the coordinate system I, using (5.41c), (3.4), we obtain the following relation:

$$\begin{aligned}&\sum_r C_r(-1)^s\varepsilon_s\frac{b\pi}{4a}\frac{r^3\sinh\alpha b\sinh 2\alpha b}{(b^2r^2+a^2s^2)}\left\{-\left(1+\frac{\nu}{6}h^2\alpha^2\right)+\frac{(1+\nu)a^2s^2}{(b^2r^2+a^2s^2)}\times\right.\\ &\quad\times\left.\left(1+(-1)^s\right)\right\}+\sum_r E_r(-1)^s\varepsilon_s\frac{b\pi}{4a}\frac{r^3\cosh\alpha b\sinh 2\alpha b}{(b^2r^2+a^2s^2)}\left\{-\left(1+\frac{\nu}{6}h^2\alpha^2\right)+\right.\\ &\quad+\left.\frac{(1+\nu)a^2s^2}{(b^2r^2+a^2s^2)}\left(1-(-1)^s\right)\right\}+D_s\beta^2\left\{-\left(1+\frac{\nu}{6}h^2\alpha^2\right)\sinh\alpha b+\right.\\ &\quad+\left.\frac{(1+\nu)}{2}\cosh\beta a\left(\sinh 2\beta a-2\beta a\right)\right\}+F_s\beta^2\left\{-\left(1+\frac{\nu}{6}h^2\alpha^2\right)\cosh\alpha b+\right.\\ &\quad-\left.\frac{(1+\nu)}{2}\sinh\beta a\left(\sinh 2\beta a+2\beta a\right)\right\}\end{aligned}$$

$$\left.\begin{aligned}&=\frac{(1+\nu)}{2}Q\alpha_m\beta_n\,, \quad \text{for} \quad n = s\,,\\ &= 0\,, \qquad\qquad\quad \text{for} \quad n \neq s\,.\end{aligned}\right\} \tag{5.44}$$

Lastly from the condition $S_2 = \dfrac{\partial^2\chi''}{\partial x\partial y} - Q\dfrac{\partial^2 v}{\partial x\partial y} = 0$, at $y = -b$, referred to the coordinate system I in Fig. 2, we have a similar relation

$$\begin{aligned}&C_r\alpha^2\left\{-\left(1+\frac{\nu}{6}h^2\alpha^2\right)\sinh\alpha b+\frac{(1+\nu)}{2}\cosh\alpha b(\sinh 2\alpha b-2\alpha b)\right\}+\\ &\quad+E_r\alpha^2\left\{-\left(1+\frac{\nu}{6}h^2\alpha^2\right)\cosh\alpha b-\frac{(1+\nu)}{2}\sinh\alpha b\,(\sinh 2\alpha b+2\alpha b)\right\}+\\ &\quad+\sum_s D_s(-1)^r\varepsilon_r\frac{a\pi}{4b}\frac{s^3\sinh\beta a\sinh 2\beta a}{(b^2r^2+a^2s^2)}\left\{-\left(1+\frac{\nu}{6}h^2\beta^2\right)+\frac{(1+\nu)b^2r^2}{(b^2r^2+a^2s^2)}\times\right.\\ &\quad\times\left.\left(1+(-1)^r\right)\right\}+\sum_s F_s(-1)^r\varepsilon_r\frac{a\pi}{4b}\frac{s^3\cosh\beta a\sinh 2\beta a}{(b^2r^2+a^2s^2)}\times\\ &\quad\times\left\{-\left(1+\frac{\nu}{6}h^2\beta^2\right)+\frac{(1+\nu)b^2r^2}{(b^2r^2+a^2s^2)}\left(1-(-1)^r\right)\right\}\end{aligned}$$

$$\left.\begin{aligned}&=\frac{(1+\nu)}{2}Q\alpha_m\beta_n\,, \quad \text{for} \quad m = r\,,\\ &= 0\,, \qquad\qquad\quad \text{for} \quad m \neq r\,.\end{aligned}\right\} \tag{5.45}$$

Once the relations as (5.42) to (5.45) are found, the expressions for sequences of coefficients can be determined by the method of successive approximation and accordingly solutions of the first kind to the whole boundary value problem may be regarded as obtained.

In the next place we shall determine the forms of basic functions Θ_1, χ'_1 and χ' as preparatory work to obtain solutions of the second kind or basic parts of generalized plane stress solutions. First we take the coordinate system I in Fig. 2 for convenience. From the condition $w_0=0$, at $x=\pm a$ and at $y=\pm b$, $\bar{\chi}$ in $w_0=\frac{-\bar{J}}{k^4}\bar{\chi}+v(x,y)$, (4.36) can be written in the form

$$\bar{\chi} = \left(\sum_r A_r Y_1 + \sum_r B_r Y_2\right)\sin\alpha(x+a) + \left(\sum_s C_s X_1 + \sum_s D_s X_2\right)\sin\beta(y+b) ,$$

in which

$$\alpha = \frac{r\pi}{2a}, \quad \beta = \frac{s\pi}{2b}, \quad r=r',\ s=s', \qquad \cdots\cdots\cdots\cdots\cdots\cdots \quad (5.46)$$

and
$$Y_1 = b\cosh\alpha b\sinh\alpha y - y\sinh\alpha b\cosh\alpha y ,$$
$$Y_2 = b\sinh\alpha b\cosh\alpha y - y\cosh\alpha b\sinh\alpha y ,$$
$$X_1 = a\cosh\beta a\sinh\beta x - x\sinh\beta a\cosh\beta x ,$$
$$X_2 = a\sinh\beta a\cosh\beta x - x\cosh\beta a\sinh\beta x ,$$

in spite of what positive integers the suffixes m and n involved in the loading function may take. Then the substitution for $G_{1,2}$ and $G_{2,2}$ in (2.10) of (I), for example,

$$G_{1,2} = -D\left(\frac{\partial^2 w_{0,2}}{\partial x^2} + \nu\frac{\partial^2 w_{0,2}}{\partial y^2}\right) + \frac{8+\nu}{10}Dh^2\frac{\partial^2}{\partial y^2}\nabla_1^2 w_{0,2}, \quad D = \frac{2Eh^3}{3(1-\nu^2)} ,$$

from (5.46) by means of the formula $w_{0,2}=\frac{-\bar{J}}{k^4}\bar{\chi}$ and the application of the boundary condition such that the resulting formulae $G_{1,2}$ and $G_{2,2}$ vanish at the edges $x=-a$ and $y=-b$ respectively, yield the following relations:

$$\left.\begin{aligned} A_r &= \cosh^2\alpha b\cdot A'_r , & B_r &= \sinh^2\alpha b\cdot A'_r , \\ C_s &= \cosh^2\beta a\cdot B'_s , & D_s &= \sinh^2\beta a\cdot B'_s . \end{aligned}\right\} \qquad \cdots\cdots\cdots \quad (5.47)$$

and we shall hereafter drop primes of A'_r and B'_s as before. Consequently $\bar{\chi}$ (5.46) can be furnished by

$$\bar{\chi} = \sum_r A_r\left(\cosh^2\alpha b\cdot Y_1 + \sinh^2\alpha b\cdot Y_2\right)\sin\alpha(x+a) + \sum_s B_s\left(\cosh^2\beta a \times\right.$$

$$\times X_1+\sinh^2\beta a\cdot X_2\Big)\sin\beta(y+b)\,. \qquad \cdots\cdots\cdots\cdots\cdots\cdots (5.48)$$

Now we are left with the boundary conditions $\frac{\partial w_0}{\partial y}=0$, at $y=b$; $\frac{\partial w_0}{\partial x}=0$, at $x=a$, that is to say, by the formula $w_0=\frac{-\bar{J}}{k^4}(\bar{\chi}+v)$ in case I ($m=2m'$, $n=2n'$), which exclusively we are discussing in this section,

$$\frac{\partial\bar{\chi}}{\partial y}=-\beta_m\sin\alpha_m(x+a),\ \text{at}\ y=b\,;\ \frac{\partial\bar{\chi}}{\partial x}=-\alpha_m\sin\beta_n(y+b),\ \text{at}\ x=a\,. \qquad \cdots\cdots\cdots\cdots (5.49)$$

And, if we let equations (5.49), into which the expression for $\bar{\chi}$ (5.48) is to be substituted, be referred to the coordinate system II and be expanded in Fourier series for range (0, 2a) and (0, 2b) in terms $\sin\frac{r\pi}{2a}x$ and $\sin\frac{s\pi}{2b}y$ and the coefficients of like terms be equated in the resulting equations, we find the following relations:

$$A_r\frac{1}{4}(4ab-\sinh 4ab)+\sum_s B_s\frac{(-1)^{r+s+1}2a^2b^2rs^2\sinh^2 2\beta a}{\pi(b^2r^2+a^2s^2)^2}\begin{cases}=-\beta_n\,, & \text{for}\quad r=m\,,\\ =0\,, & \text{for}\quad r\neq m\,.\end{cases} \qquad (5.50\text{a})$$

$$\sum_r A_r\frac{(-1)^{r+s+1}2a^2b^2r^2s\sinh^2 2\alpha b}{\pi(b^2r^2+a^2s^2)^2}+B_s\frac{1}{4}(4\beta a-\sinh 4\beta a)\begin{cases}=-\alpha_m\,, & \text{for}\quad n=s\,,\\ =0\,, & \text{for}\quad n\neq s\,.\end{cases} \qquad (5.50\text{b})$$

From these relations $\{A_r\}$ and $\{B_s\}$ can be determined and hence it may be said we have attained virtually the present purpose.

By virtue of the formula $w_{0,2}=\frac{-\bar{J}}{k^4}\bar{\chi}=\frac{1}{E}\{h^2\Theta_1+(1+\nu)\chi_1'\}$ we get the following basic functions:

$$\Theta_1=\frac{-E}{(1-\nu)}\frac{\bar{J}}{k^4}\Big\{\sum_r A_r\alpha\sinh 2\alpha b\sinh\alpha(y+b)\sin\alpha(x+a)+$$

$$+\sum_s B_s\beta\sinh 2\beta a\sinh\beta(x+a)\sin\beta(y+b)\Big\}\,, \qquad \cdots\cdots\cdots\cdots (5.51\text{a})$$

$$\chi_1'=-\Big\{\frac{\bar{J}}{k^4}\bar{\chi}+\frac{h^2}{E}\Theta_1\Big\}\frac{E}{(1+\nu)}=\frac{\chi'}{z}\Big|_{z=0}\,, \qquad \cdots\cdots\cdots\cdots (5.51\text{b})$$

$$\chi' = z\chi_1' + \frac{2-\nu}{6(1+\nu)} z^3 \Theta_1 = \frac{-E}{(1+\nu)} \frac{J}{k'} z \Big[\sum_r A_r \Big\{ \cosh^2 \alpha b \cdot Y_1 + \sinh^2 \alpha b \cdot Y_2 +$$

$$+ \frac{1}{(1-\nu)} \Big(\frac{(2-\nu)}{6} z^2 - h^2 \Big) \alpha \sinh 2\alpha b \sinh \alpha (y+b) \Big\} \sin \alpha (x+a) +$$

$$+ \sum_s B_s \Big\{ \cosh^2 \beta a \cdot X_1 + \sinh^2 \beta a \cdot X_2 + \frac{1}{(1-\nu)} \Big(\frac{(2-\nu)}{6} z^2 - h^2 \Big) \times$$

$$\times \beta \sinh 2\beta a \cdot \sinh \beta (x+a) \Big\} \sin \beta (y+b) \Big]. \qquad \text{(5. 51c)}$$

By means of the formulae (5.51) the solutions of the second kind can be obtained with ease. Therefore, we get the solutions to the problem in this section by superposing the three kinds of solutions. As an instance, normal stress σ_x may be written by (5.41c), (5.51) and (4.25) of (I)

$$\sigma_x = \sigma_{x,1} + \sigma_{x,2} + \sigma_{x,3} , \qquad \text{(5. 52)}$$

in which

$$\sigma_{x,1} = \frac{\partial^2 \chi}{\partial y^2} = \sum_r C_r \frac{\alpha^2}{2h} \Big[\frac{-2}{(1+\nu)} \Big\{ \sinh \alpha b + (1+\nu) \alpha b \cosh \alpha b +$$

$$+ \frac{\nu}{6} h^2 \alpha^2 \sinh \alpha b \Big\} \sinh \alpha (y+b) + \Big\{ 2 + \frac{\nu}{1+\nu} \Big(\frac{1}{3} h^2 - z^2 \Big) \alpha^2 \Big\} \times$$

$$\times \sinh 2\alpha b \cdot \sinh \alpha y + \alpha (y+b) \sinh 2\alpha b \cdot \cosh \alpha y \Big] \sin \alpha (x+a) +$$

$$+ \sum_r E_r \frac{\alpha^2}{2h} \Big[\frac{-2}{(1+\nu)} \Big\{ \cosh \alpha b + (1+\nu) \alpha b \sinh \alpha b + \frac{\nu}{6} h^2 \alpha^2 \cosh \alpha b \Big\} \times$$

$$\times \sinh \alpha (y+b) + \Big\{ 2 + \frac{\nu}{1+\nu} \Big(\frac{1}{3} h^2 - z^2 \Big) \alpha^2 \sinh 2\alpha b \cosh \alpha y \Big\} + \alpha (y+b) \times$$

$$\times \sinh 2\alpha b \cdot \sinh \alpha y \Big] \sin \alpha (x+a) - \sum_s D_s \frac{\beta^2}{2h} \Big[\frac{-2}{(1+\nu)} \Big\{ \sinh \beta a +$$

$$+ (1+\nu) \beta a \cosh \beta a + \frac{\nu}{6} h^2 \beta^2 \sinh \beta a \Big\} \sinh \beta (x+a) + \frac{\nu}{(1+\nu)} \Big(\frac{1}{3} h^2 - z^2 \Big) \times$$

$$\times \beta^2 \sinh 2\beta a \sinh \beta x + \beta (x+a) \sinh 2\beta a \cosh \beta x \Big] \sin \beta (y+b) +$$

$$- \sum_s F_s \frac{\beta^2}{2h} \Big[\frac{-2}{(1+\nu)} \Big\{ \cosh \beta a + (1+\nu) \beta a \sinh \beta a + \frac{\nu}{6} h^2 \beta^2 \cosh \beta a \Big\} \times$$

$$\times \sinh \beta (x+a) + \frac{\nu}{(1+\nu)} \Big(\frac{1}{3} h^2 - z^2 \Big) \beta^2 \sinh 2\beta a \cosh \beta x + \beta (x+a) \times$$

$$\times \sinh 2\beta a \sinh \beta x \Big] \sin \beta (y+b) ,$$

$$\sigma_{x,2} = -\frac{E}{(1+\nu)}\frac{\bar{J}}{k^4} z \sum_r A_r \alpha \Big[\alpha b \left(\cosh^3 \alpha b \sinh \alpha y + \sinh^3 \alpha b \cdot \cosh \alpha y \right) +$$

$$-\frac{1}{2}\alpha \sinh 2\alpha b \cdot y \cosh \alpha (y+b) + \frac{1}{(1-\nu)}\left\{ \nu + \alpha^2 \left(\frac{2-\nu}{6} z^2 - h^2 \right) \right\} \times$$

$$\times \sinh 2\alpha b \cdot \sinh \alpha (y+b) \Big] \sin \alpha (x+a) + \frac{E}{(1+\nu)}\frac{\bar{J}}{k^4} z \times$$

$$\times \sum_s B_s \beta \Big[\beta \cosh^2 \beta a \cdot X_1 + \beta \sinh^2 \beta a \cdot X_2 + \frac{1}{(1-\nu)}\left\{ -1 + \beta^2 \left(\frac{2-\nu}{6} z^2 - h^2 \right) \right\} \times$$

$$\times \sinh 2\beta a \cdot \sinh \beta (x+a) \Big] \sin \beta (y+b) ,$$

$$\sigma_{x,3} = \frac{-K}{k^2}\Big[(2\nu\beta_n^2 + \alpha_m^2) \sinh 2kh \cdot \sinh k(z+h) + (4\nu\beta_n^2 + 3\alpha_m^2) kh \sinh k(z-h) +$$

$$+ \alpha_m^2 \Big\{ -kh \cosh 2kh \sinh k(z+h) + 2k^2h^2 \cosh k(z-h) + kz \times$$

$$\times \left(\sinh 2kh \cdot \cosh k(z+h) + 2kh \cosh k(z-h) \right) \Big\} \Big] \sin \alpha_m (x+a) \sin \beta_n (y+b) ,$$

in which $$K \equiv K_{mn} = p_{mn} \Big/ (\sinh^2 2kh - 4k^2h^2) ,$$

$$k \equiv k_{mn} = \sqrt{\alpha_m^2 + \beta_n^2} .$$

It will be needless to say that these expressions are referred to the coordinate system I in Fig. 2 and owing to the property of the problem under consideration the expressions and relations among coefficients can be utilized in other cases than case I by making slightest alterations.

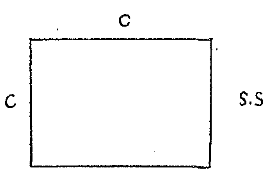


Fig. 6

Case D. When Thick Plate with One Edge Simply Supported and the Others Clamped is Bent by the Pressure (5.1).

Also in this case we can proceed quite similarly, so we shall give only a brief description of the process of calculations. At first we choose the coordinate system I and let the edge $x=a$ be simply supported. We shall determine Θ_0 and χ_0 by putting χ'' in the expression of the following form in view of the boundary condition

$$T_1 = T_{1,1} + T_{1,3} = 0 , \quad \text{at} \quad x = a ;$$

$$\chi'' = \sum_r \left(A_r \sinh \alpha y + C_r y \cosh \alpha y \right) \sin \alpha (x+a) + \sum_s \Big\{ B_s \sinh \beta (x-a) +$$

$$+D_s(x-a)\cosh\beta x+F_s(x-a)\sinh\beta x\Big\}\sin\beta y\,, \qquad\cdots\cdots\cdots\cdots (5.53)$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=r'$ and $s=2s'$.

If we derive Θ_0 and χ_0 from χ'' (5.53) and substitute them in $Eu_0=Eu_{0,1}+Eu_{0,3}=\bar{\chi}_1+J\dfrac{\partial v}{\partial x}$ and apply the condition $u_0=0$, at $y=\pm b$, A_r and C_r are found readidly to be expressions of the forms

$$\left.\begin{aligned}A_r&=\frac{2}{(1+\nu)}\frac{1}{\alpha}\left\{\left(1+\frac{\nu}{6}h^2\alpha^2\right)\sinh\alpha b+\frac{(1+\nu)}{2}\alpha b\cosh\alpha b\right\}A'_r\,,\\ C_r&=-\sinh\alpha b\cdot A'_r\,,\end{aligned}\right\}\qquad(5.54)$$

and further, applying the condition

$$Ev_0=Ev_{0,1}+Ev_{0,3}=\bar{\chi}_2+J\frac{\partial v}{\partial y}=0\,,\quad\text{at}\quad x=-a\,.$$

we see that

$$\begin{aligned}B_s&=\frac{2}{(1+\nu)\beta}\Big[-B'_s\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\sinh\beta a+(1+\nu)\,\beta a\cosh\beta a\Big\}+\\&\quad+C'_s\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\cosh\beta a+(1+\nu)\,\beta a\sinh\beta a\Big\}\Big], \qquad\cdots\cdots\cdots (5.55)\\ D_s&=\sinh 2\beta a\cdot B'_s\,,\quad F_s=\sinh 2\beta a\cdot C'_s\,.\end{aligned}$$

We shall delete dashes of A'_r, B'_s and C'_s below.
Then we may write for χ'', Θ_0, χ_0, and χ

$$\begin{aligned}\chi''&=\sum_r A_r\alpha\Big[\Big\{\Big(1+\frac{\nu}{6}h^2\alpha^2\Big)\frac{2}{(1+\nu)}\sinh\alpha b+\alpha b\cosh\alpha b\Big\}\sinh\alpha y+\\&\quad-\alpha y\sinh\alpha b\cdot\cosh\alpha y\Big]\sin\alpha(x+a)+\sum_s B_s\frac{1}{\beta}\Big[\beta(x-a)\sinh 2\beta a\cosh\beta x+\\&\quad-\frac{2}{(1+\nu)}\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\sinh\beta a+(1+\nu)\beta a\cosh\beta a\Big\}\sinh\beta(x-a)\Big]\times\\&\quad\times\sin\beta y+\sum_s C_s\frac{1}{\beta}\Big[\beta(x-a)\sinh 2\beta a\sinh\beta x+\frac{2}{(1+\nu)}\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\times\\&\quad\times\cosh\beta a+(1+\nu)\beta a\sinh\beta a\Big\}\sinh\beta(x-a)\Big]\sin\beta y\,, \qquad\cdots\cdots (5.56a)\end{aligned}$$

$$\Theta_0=\frac{1}{h}\Big[-\sum_r A_r\alpha\sinh\alpha b\cdot\sinh\alpha y\sin\alpha(x+a)+\sum_s\Big\{B_s\beta\sinh 2\beta a\sinh\beta x+$$

$$+ C_s\beta\sinh 2\beta a\cdot\cosh\beta x\Big\}\sin\beta y\Big], \quad\cdots\cdots\cdots (5.56b)$$

$$\chi_0 = \chi|_{z=0}, \quad\cdots\cdots\cdots (5.56c)$$

$$\chi = \sum_r A_r\frac{1}{2h(1+\nu)\alpha}\Big[\Big\{(2+\nu z^2\alpha^2)\sinh\alpha b+(1+\nu)\alpha b\cosh\alpha b\Big\}\sinh\alpha y +$$
$$-(1+\nu)\alpha y\sinh\alpha b\cosh\alpha y\Big]\sin\alpha(x+a)+\sum_s B_s\frac{1}{2h(1+\nu)\beta}\Big[(1+\nu)\beta\times$$
$$\times(x-a)\sinh 2\beta a\cosh\beta x-2\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\sinh\beta a+(1+\nu)\beta a\cosh\beta a\Big\}\times$$
$$\times\sinh\beta(x-a)+\nu\Big(\frac{1}{3}h^2-z^2\Big)\beta^2\sinh 2\beta a\sinh\beta x\Big]\sin\beta y+$$
$$+\sum_s C_s\frac{1}{2h(1+\nu)\beta}\Big[(1+\nu)\beta(x-a)\sinh 2\beta a\sinh\beta x+2\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\times$$
$$\times\cosh\beta a+(1+\nu)\beta a\sinh\beta a\Big\}\sinh\beta(x-a)+\nu\Big(\frac{1}{3}h^2-z^2\Big)\times$$
$$\times\beta^2\sinh 2\beta a\cosh\beta x\Big]\sin\beta y. \quad\cdots\cdots\cdots (5.56d)$$

And the forms of $\bar{\chi}_1$ and $\bar{\chi}_2$ can be written as

$$(-h)\bar{\chi}_1=-hEu_{0,1}=\sum_r A_r\frac{(1+\nu)}{2}\alpha\Big(b\cosh\alpha b\sinh\alpha y-y\sinh\alpha b\cosh\alpha y\Big)\times$$
$$\times\cos\alpha(x+a)+\sum_s B_s\Big[-\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\sinh\beta a+(1+\nu)\beta a\cosh\beta a\Big\}\times$$
$$\times\cosh\beta(x-a)+\frac{1}{2}\sinh 2\beta a\Big\{\Big(\nu-1+\frac{\nu}{3}h^2\beta^2\Big)\cosh\beta x+(1+\nu)\beta\times$$
$$\times(x-a)\sinh\beta x\Big\}\Big]\sin\beta y+\sum_s C_s\Big[\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\cosh\beta a+$$
$$+(1+\nu)\beta a\sinh\beta a\Big\}\cosh\beta(x-a)+\frac{1}{2}\sinh 2\beta a\Big\{\Big(\nu-1+\frac{\nu}{3}h^2\beta^2\Big)\times$$
$$\times\sinh\beta x+(1+\nu)\beta(x-a)\cosh\beta x\Big\}\Big]\sin\beta y, \quad\cdots\cdots\cdots (5.57a)$$

$$(-h)\bar{\chi}_2=-hEv_{0,1}=\sum_r A_r\Big[\frac{1}{2}\Big\{(3-\nu)\sin\alpha b+(1+\nu)\alpha b\cosh\alpha b\Big\}\cosh\alpha y+$$
$$-\frac{(1+\nu)}{2}\alpha\sinh\alpha b\cdot y\sinh\alpha y\Big]\sin\alpha(x+a)+\sum_s B_s\Big[-\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\times$$
$$\times\sinh\beta a+(1+\nu)\beta a\cosh\beta a\Big\}\sinh\beta(x-a)+\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\times$$
$$\times\sinh 2\beta a\cdot\sinh\beta x+\frac{(1+\nu)}{2}\beta\sinh 2\beta a\cdot(x-a)\cosh\beta x\Big]\cos\beta y+$$

$$+\sum_s C_s\Big[\Big\{\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\cosh\beta a+(1+\nu)\,\beta a\sinh\beta a\Big\}\sinh\beta(x-a)+$$
$$+\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\sinh 2\beta a\cosh\beta x+\frac{(1+\nu)}{2}\beta\sinh 2\beta a\cdot(x-a)\sinh\beta x\Big]\cos\beta y$$
$$\cdots\cdots\cdots\cdots\cdots\cdots \quad (5.57\text{b})$$

Now we shall apply the three conditions left unsatisfied concerning solutions of the first kind. Let the expressions for shearing force and displacement on the middle plane be referred to the coordinate system IV in Fig. 2 and the resulting expressions be expanded in Fourier series. First, according to the boundary condition $S_1=S_{1,1}+S_{1,3}=\dfrac{-\partial^5\chi''}{\partial x\partial y}+Q\dfrac{\partial^2 v}{\partial x\partial y}=0$, at $x=a$, which is referred to coordinate system I, we obtain the relation among coefficients

$$\sum_r A_r\frac{(-1)^{r+s'}\varepsilon_s b r^2\sinh^2 ab}{(b^2r^2+a^2s^2)}\Big\{\frac{\nu}{1+\nu}\Big(-2+\frac{1}{3}h^2a^2\Big)+\frac{2b^2r^2}{(b^2r^2+a^2s^2)}\Big\}+$$
$$+B_s\beta\Big\{\frac{-1}{(1+\nu)}\Big(2+\frac{\nu}{3}h^2\beta^2\Big)\sinh\beta a+(\sinh 2\beta a-2\beta a)\cosh\beta a\Big\}+$$
$$+C_s\beta\Big\{\frac{1}{(1+\nu)}\Big(2+\frac{\nu}{3}h^2\beta^2\Big)\cosh\beta a+(\sinh 2\beta a+2\beta a)\sinh\beta a\Big\}=$$
$$\left.\begin{aligned}&=(-1)^{n'}Q\alpha_m\beta_n\,, &&\text{for}\quad n=s\,,\\ &=0\,, &&\text{for}\quad n\neq s\,.\end{aligned}\right\}\quad(5.58)$$

By the condition $Eu_0=\bar{\chi}_1+J\dfrac{\partial v}{\partial x}=Eu(x,y,0)=0$, at $x=-a$, referred to the coordinate system I, we have the relation

$$\sum_r A_r(-1)^{s'+1}\frac{(1+\nu)}{\pi}\frac{4a^2b^2r^2s\sinh^2 ab}{(b^2r^2+a^2s^2)^2}+B_s\Big[\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\sinh\beta a+$$
$$+\Big\{-(1+\nu)\beta a+\frac{\nu-3}{2}\sinh 2\beta a\Big\}\cosh\beta a\Big]+C_s\Big[\Big(1+\frac{\nu}{6}h^2\beta^2\Big)\cosh\beta a+$$
$$-\Big\{(1+\nu)\beta a+\frac{\nu-3}{2}\sinh 2\beta a\Big\}\sinh\beta a\Big]=$$
$$\left.\begin{aligned}&=(-1)^{n'}h\alpha_m J\,, &&\text{for}\quad n=s\,,\\ &=0\,, &&\text{for}\quad n\neq s\,.\end{aligned}\right\}\quad(5.59)$$

Finally from the conditions $Ev_0=\bar{\chi}_2+J\dfrac{\partial v}{\partial y}=Ev(x,y,0)=0$, at $y=\pm b$, which are referred to the coordinate system I, we obtain similarly

$$A_r\left\{\frac{3-\nu}{4}\sinh 2\alpha b+\frac{(1+\nu)}{2}\alpha b\right\}+\sum_s B_s\frac{(-1)^{r+s'}2b^2r\sinh\beta a\sinh 2\beta a}{\pi(b^2r^2+a^2s^2)}\times$$

$$\times\left\{-\left(1+\frac{\nu}{6}h^2\beta^2\right)+\frac{(1+\nu)a^2s^2(1+(-1)^r)}{(b^2r^2+a^2s^2)}\right\}+\sum_s C_s\times$$

$$\times\frac{(-1)^{r+s'}2b^2r\cosh\beta a\sinh 2\beta a}{\pi(b^2r^2+a^2s^2)}\left\{-\left(1+\frac{\nu}{6}h^2\beta^2\right)-\frac{(1+\nu)a^2s^2(1+(-1)^r)}{(b^2r^2+a^2s^2)}\right\}$$

$$\left.\begin{aligned}&=h\beta_n J, &&\text{for}\quad m=r,\\ &=0, &&\text{for}\quad m\neq r.\end{aligned}\right\}\qquad(5.60)$$

From the relation (5.58) to (5.60) coefficients $\{A_r\}$, $\{B_s\}$ and $\{C_s\}$ can be determined and so we have virtually obtained solutions of the first kind. In the next place we shall get the forms of basic functions Θ_1, χ_1' and χ'. Proceeding according to the same way as in the foregoing cases, the representation of $\bar{\chi}$ in the formula $w_0=\frac{-J}{k^4}\{v(x,y)+\bar{\chi}\}$ can easily be obtained. By noting the condition in regard to w_0 we can take $\bar{\chi}$ in the form

$$\bar{\chi}=\sum_r A_r(b\cosh\alpha b\sinh\alpha y-y\sinh\alpha b\cosh\alpha y)\sin\alpha(x+a)+$$

$$+\sum_s\{B_s(a\cosh\beta a\sinh\beta x-x\sinh\beta a\cosh\beta x)+C_s(a\sinh\beta a\cosh\beta x+$$

$$-x\cosh\beta a\sinh\beta x)\}\sin\beta y\,,\qquad\cdots\cdots\cdots\cdots\qquad(5.61)$$

in which $\alpha=\frac{r\pi}{2a}$, $\beta=\frac{s\pi}{2b}$, $r=r'$, $s=2s'$.

From the condition that bending moment G_1 vanishes at the edge $x=a$ we get the relation

$$B_s=\cosh^2\beta a\cdot B_s'\,,\quad C_s=-\sinh^2\beta a\cdot B_s'\,,\qquad\cdots\cdots\cdots\qquad(5.62)$$

since $G_{1,3}=0$ at $x=a$ and erase prime of B_s'. On applying the boundary conditions that

$$\frac{\partial w_0}{\partial y}=0\,,\ \text{at}\ y=\pm b\ \text{and}\ \frac{\partial w_0}{\partial x}=0\,,\ \text{at}\ x=-a,\ \text{that is,}$$

$\frac{\partial\bar{\chi}}{\partial y}=-\beta_n\sin\alpha_m(x+a)$, $\frac{\partial\bar{\chi}}{\partial x}=(-1)^{n'+1}\alpha_m\sin\beta_n y$, respectively, which are referred to the coordinate system I in Fig. 2, by letting all the expressions be relative to the coordinate system IV in Fig. 2, it is readily found that

$$A_r \frac{1}{2}(2\alpha b - \sinh 2\alpha b) + \sum_s B_s \frac{(-1)^{s'+1} 2\alpha^2 b^2 r s^2 \sinh^2 2\beta a}{\pi (b^2 r^2 + a^2 s^2)^2}$$
$$\left.\begin{aligned} &= -\beta_n, \quad \text{for} \quad m = r, \\ &= 0, \quad \text{for} \quad m \neq r. \end{aligned}\right\} \qquad (5.63a)$$

and

$$\sum_r A_r \frac{(-1)^{s'+1} 8\alpha^2 b^2 r^2 s \sinh^2 \alpha b}{\pi (b^2 r^2 + a^2 s^2)^2} + B_s \frac{1}{4}(4\alpha b - \sinh 4\alpha b)$$
$$\left.\begin{aligned} &= (-1)^{n'+1} \alpha_m, \quad \text{for} \quad s = n, \\ &= 0, \quad \text{for} \quad s \neq n. \end{aligned}\right\} \qquad (5.63b)$$

Hence coefficients $\{A_r\}$ and $\{B_s\}$ can be determined from (5.63) and now we can consider the solutions of the second kind as having been obtained. Though we shall not indicate the solutions to the whole problem, we write basic quantities for reference.

$$\Theta_1 = \frac{E}{(1-\nu)} \frac{\bar{J}}{k^4} \nabla_1^2 \bar{\chi} = \frac{-E}{(1-\nu)} \frac{\bar{J}}{k^4} \Big\{ \sum_r A_r 2\alpha \sinh \alpha b \sinh \alpha y \sin \alpha (x+a) +$$
$$+ \sum_s B_s \beta \sinh 2\beta a \sinh \beta (x-a) \sin \beta y \Big\}, \qquad \cdots\cdots\cdots\cdots (5.64a)$$

$$\chi_1' = -\frac{E}{(1+\nu)} \Big\{ \frac{\bar{J}}{k^4} \bar{\chi} + \frac{h^2}{E} \Theta_1 \Big\}, \quad = \frac{\chi'}{z} \Big|_{z=0}, \qquad \cdots\cdots\cdots\cdots (5.64b)$$

$$\chi' = z\chi_1' + \frac{2-\nu}{6(1+\nu)} z^3 \Theta_1 = \frac{-E}{(1+\nu)} \frac{\bar{J}}{k^4} z \sum_r A_r \Big[\Big\{ b \cosh \alpha b + \frac{1}{(1-\nu)} \times$$
$$\times \Big(\frac{2-\nu}{3} z^2 - 2h^2 \Big) \alpha \sinh \alpha b \Big\} \sinh \alpha y - y \sinh \alpha b \cosh \alpha y \Big] \sin \alpha (x+a) +$$
$$+ \frac{-E}{(1+\nu)} \frac{\bar{J}}{k^4} z \sum_s B_s \Big[\cosh^2 \beta a \Big\{ a \cosh \beta a + \frac{1}{(1-\nu)} \Big(\frac{2-\nu}{3} z^2 - 2h^2 \Big) \beta \sinh \beta a \Big\} \times$$
$$\times \sinh \beta x - \sinh^2 \beta a \Big\{ a \sinh \beta a + \frac{1}{(1-\nu)} \Big(\frac{2-\nu}{3} z^2 - 2h^2 \Big) \beta \cosh \beta a \Big\} \times$$
$$\times \cosh \beta x - \frac{1}{2} \sinh 2\beta a \cdot x \cosh \beta (x-a) \Big] \sin \beta y. \qquad \cdots\cdots\cdots (5.64c)$$

Though we used similar labels for coefficients in the solutions of the first and second kinds, they are, needless to notice, essentially different.

Case E. When Thick Plate with One Edge Free from Tractions and the Other Edges Simply Supported is Bent by the Pressure (5.1).

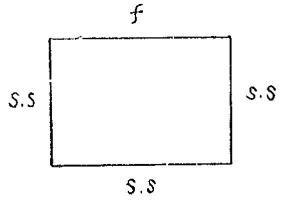


Fig. 7
f: edge free from tractions.

First we choose coordinate system I and take the edge $y=b$ to be free from tractions. So the boundary conditions at this edge are to be represented by the following four conditions:

$$T_2=0, \quad S_2=0, \quad G_2=0 \quad \text{and} \quad N_2-\frac{\partial H_2}{\partial x}=0. \qquad \cdots\cdots (5.65)$$

As we stated in the introduction of the first report, to the order of accuracy of the procedure of Michell and Love we are obliged to do with resultant forces and couples in order to satisfy the boundary conditions and, further, to use Kirchhoff's theorem regarding the boundary conditions, if necessary, to the effect that five conditions concerning T, S, G, N and H can be reduced, as is well known, to four by letting torsional couple H be merged into shearing force N in such way that we let N be replaced by $N-\frac{\partial H}{\partial s}$, s being a length of the bounding curve. Incidentally, it seems to the author that it is well-nigh impossible to satisfy the boundary conditions by employing the method of attack described in this paper without applying this theorem.

Since the boundary conditions needed in this case to obtain the solutions of the first kind are that T and S vanish at the boundary, the solutions or the forms of χ'' are the same as in the case of rectangular thick plate simply supported or as in the case of Sec. III, and hence we omit this process of calculation here.

Next we shall obtain the solutions of the second kind or basic functions Θ_1, χ_1' and χ'. First we take the coordinate system I. We can write $\bar{\chi}$ in the formula $w_0=w_{0,3}+w_{0,2}=\frac{-J}{k^4}(v+\bar{\chi})$ in the following form, discarding the terms of the type

$$\sum_s D_s(a\cosh\beta a\sinh\beta x-x\sinh\beta a\cosh\beta x)\sin\beta(y+b),$$

$$\bar{\chi}=\sum_r\left\{A_r\sinh\alpha(y+b)+B_r(y+b)\cosh\alpha y+C_r(y+b)\sinh\alpha y\right\}\sin\alpha x. \qquad \cdots\cdots (5.66)$$

It is apparent that this expression yields w_0 which vanishes at the boundary except at the edge $y=b$. Then we treat the condition

$$N_2-\frac{\partial H_2}{\partial x}=(N_{2,2}+N_{2,3})-\frac{\partial}{\partial x}(H_{2,2}+H_{2,3})=0\,,\quad \text{at } y=b\,, \qquad \cdots (5.67)$$

in which by (2.10) of (I)

$$\left.\begin{aligned} &N_{2,2}=-D\frac{\partial}{\partial y}\nabla_1^2 w_{0,2}\,,\quad w_{0,2}=\frac{-\bar{J}}{k^4}\bar{\chi}\,,\\ &-H_{1,2}=H_{2,2}=-D(1-\nu)\frac{\partial^2 w_{0,2}}{\partial x\partial y}-\frac{8+\nu}{10}Dh^2\frac{\partial^2}{\partial x\partial y}\nabla_1^2 w_{0,2}\,, \end{aligned}\right\} \qquad (5.68a)$$

and by (4.34b), (4.35) of (I)

$$N_{2,3}=\frac{-p}{k^2}\frac{\partial v}{\partial y}\,,\quad -H_{1,3}=H_{2,3}=\frac{-K}{k^4}\{-(2\nu+1)\sinh^2 2kh+$$

$$+2\nu kh\sinh 2kh(\cosh 2kh-1)+4k^2h^2(1+\nu+\nu\cosh 2kh)\}\times$$

$$\times\frac{\partial^2 v}{\partial x\partial y}=-\bar{Q}\frac{\partial^2 v}{\partial x\partial y}\,. \qquad \cdots\cdots (5.68b)$$

The application of the condition of (5.67) leads to the result

$$A_r(1-\nu)\alpha^3\cosh 2\alpha b+B_r\alpha^2\left\{(1-\nu)2\alpha b\sinh\alpha b+\left(3-\nu+\frac{8+\nu}{5}h^2\alpha^2\right)\times\right.$$

$$\left.\times\cosh\alpha b\right\}+C_r\alpha^2\left\{(1-\nu)2\alpha b\cosh\alpha b+\left(3-\nu+\frac{8+\nu}{5}h^2\alpha^2\right)\sinh\alpha b\right\}$$

$$\left.\begin{aligned} &=(-1)^{m'}\frac{k^4}{D\bar{J}}\beta_n\left(\frac{p}{k^2}+\alpha_m^2\bar{Q}\right)\,,\quad \text{for}\quad m=r\,,\\ &=0\,,\qquad\qquad\qquad\qquad\qquad\quad \text{for}\quad m\neq r\,. \end{aligned}\right\} \qquad (5.69)$$

It is readily seen that bending moment $G_1=G_{1,2}+G_{1,3}$ vanishes at the edges $x=\pm a$ according to the forms of $\bar{\chi}$ (5.66), of $G_{1,2}$ (2.10) of (I) and of $G_{1,3}$ (4.36) of (I). From the condition $G_2=G_{2,3}+G_{2,2}=0$, at $y=-b$ it is found that

$$B_r=\cosh\alpha b\cdot B'_r\,,\qquad C_r=\sinh\alpha b\cdot B'_r \qquad \cdots\cdots (5.70)$$

with B'_r introduced anew. By the condition $G_2=G_{2,2}+G_{2,3}=0$, at $y=b$ we get the relation

$$A_r(1-\nu)\alpha\sinh 2\alpha b+B'_r\left\{\left(2+\frac{8+\nu}{5}h^2\alpha^2\right)\sinh 2\alpha b+2(1-\nu)\alpha b\cosh 2\alpha b\right\}=0\,.$$

$$\cdots\cdots (5.71)$$

Accordingly, from (5.69), (5.70) and (5.71) we obtain the expressions for coefficients:

$$A_m = -\left\{2(1-\nu)\alpha b\cosh 2\alpha b + \left(2+\frac{8+\nu}{5}h^2\alpha^2\right)\sinh 2\alpha b\right\} B''_m ,$$

$$B_m = (1-\nu)\,\alpha\cosh\alpha b\cdot\sinh 2\alpha b\cdot B''_m ,$$

$$C_m = (1-\nu)\,\alpha\sinh\alpha b\cdot\sinh 2\alpha b\cdot B''_m , \quad \text{in which}$$

$$B''_m = \frac{2}{(1-\nu)^2\alpha^3(\sinh 4\alpha b - 4\alpha b)}\cdot(-1)^{m'}\frac{k^4}{D\bar{J}}\beta_n\left(\frac{p}{k^2}+\alpha_m^2\bar{Q}\right) ,$$

$$\left.\begin{array}{ll} & \text{for} \quad r=m , \\ A_r=B_r=C_r=0 , & \text{for} \quad r\neq m . \end{array}\right\} \tag{5.72}$$

Thus solutions of the second kind are very simple. We indicate merely the forms of Θ_1, χ'_1 and χ' below.

$$\Theta_1 = \frac{E}{(1-\nu)}\frac{\bar{J}}{k^4}\nabla_1^2\bar{\chi} = E\frac{\bar{J}}{k^4}B''_m 2\alpha^2\sinh 2\alpha b\sinh\alpha(y+b)\sin\alpha x , \tag{5.73a}$$

$$\chi'_1 = \frac{-E}{(1+\nu)}\frac{\bar{J}}{k^4}\bar{\chi} - \frac{h^2}{(1+\nu)}\Theta_1 = \left.\frac{\chi'}{z}\right|_{z=0} , \quad \cdots\cdots \tag{5.73b}$$

$$\chi' = \frac{E}{(1-\nu)}\frac{\bar{J}}{k^4}B''_m z\left[\left\{(1-\nu)2\alpha b\cosh 2\alpha b + \left(2+(2-\nu)\left(\frac{z^2}{3}-\frac{h^2}{5}\right)\alpha^2\right)\times\right.\right.$$

$$\left.\left.\times\sinh 2\alpha b\right\}\sinh\alpha(y+b) - (1-\nu)\alpha\sinh 2\alpha b\cdot(y+b)\cosh\alpha(y+b)\right]\sin\alpha x .$$

$$\cdots\cdots \tag{5.73c}$$

Case F. When Thich Plate with One Edge Free from Tractions and the Other Edges Clamped is Bent by the Pressure (5.1).

First we let the representations be referred to coordinate system I in Fig. 2 for convenience' sake and the edge $y=b$ be free from tractions, though we should take coordinate system III in Fig. 2 from the beginning in accordance with the feature of the problem in question.

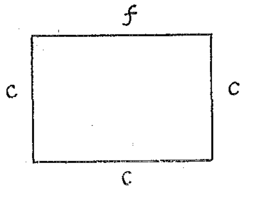


Fig. 8

We shall obtain solutions of the first kind. Since boundary conditions to be considered here are similar to those in case D, that is, case where thick plate with one edge simply supported and the others clamped is bent by the pressure (5.1), we can easily attain the object to obtain basic functions Θ_0, χ_0, χ only by interchanges of x and y, a and b, α and β, r and s.

Hence, now we turn to the problem of obtaining the basic functions

Θ_1, χ_1' and χ' associated with the solutions of the second kind. Boundary conditions are as follows:

$$w_0 = \frac{\partial w_0}{\partial x} = 0, \text{ at } x = \pm a; \; w_0 = \frac{\partial w_0}{\partial y} = 0, \text{ at } y = -b;$$
$$G_2 = N_2 - \frac{\partial H_2}{\partial x} = 0, \text{ at } y = b. \qquad \cdots\cdots (5.74)$$

In view of the conditions concerning w_0 the representation of $\bar{\chi}$ in the formula $w_0 = \frac{-\bar{J}}{k^4}(v+\bar{\chi})$ can be of the form

$$\bar{\chi} = \sum_r \left\{A_r \sinh\alpha(y+b) + B_r (y+b)\cosh\alpha y + C_r (y+b)\sinh\alpha y\right\} \sin\alpha x +$$
$$+ \sum_s D_s (a\cosh\beta a \sinh\beta x - x\sinh\beta a\cosh\beta x)\sin\beta(y+b),$$

in which

$$\alpha = \frac{r\pi}{2a}, \quad \beta = \frac{s\pi}{2b}, \quad r = 2r', \quad s = s'. \qquad \cdots\cdots (5.75)$$

From the condition $G_2 = 0$, at $y = b$ by means of the formula just written and of G_2 (2.10) of (I) we readily get the relations

$$A_r = -\left\{2(1-\nu)\alpha b\cosh\alpha b + \left(2 + \frac{8+\nu}{5}h^2\alpha^2\right)\sinh\alpha b\right\}\alpha B_r' +$$
$$-\left\{2(1-\nu)\alpha b\sinh\alpha b + \left(2 + \frac{8+\nu}{5}h^2\alpha^2\right)\cosh\alpha b\right\}\alpha C_r',$$
$$B_r = (1-\nu)\alpha^2\sinh 2\alpha b B_r', \quad C_r = (1-\nu)\alpha^2 \sinh 2\alpha b C_r', \qquad \cdots\cdots (5.76)$$

since $G_{2,3} = 0$, at $y = b$, as seen from (4.36) of (I), in which B_r' and C_r' are introduced anew for convenience, and we shall delete primes over B_r and C_r hereafter. Now we are left with three conditions

$$\frac{\partial w_0}{\partial x} = 0, \text{ at } x = \pm a; \; \frac{\partial w_0}{\partial y} = 0, \text{ at } y = -b;$$
$$N_2 - \frac{\partial H_2}{\partial x} = 0, \text{ at } y = b. \qquad \cdots\cdots (5.77)$$

These are rewritten in the forms.

$$\sum_r B_r (-1)^{r'}\alpha^2\Big[-\left\{2(1-\nu)\alpha b\cosh\alpha b + \left(2 + \frac{8+\nu}{5}h^2\alpha^2\right)\sinh\alpha b\right\} \times$$
$$\times \sinh\alpha(y+b) + (1-\nu)\alpha\sinh 2\alpha b \cdot (y+b)\cosh\alpha y\Big] +$$

$$+\sum_r C_r(-1)^{r'}\alpha^2\Big[-\Big\{2(1-\nu)\alpha b\sinh\alpha b+\Big(2+\frac{8+\nu}{5}h^2\alpha^2\Big)\cosh\alpha b\Big\}\times$$

$$\times\sinh\alpha(y+b)+(1-\nu)\alpha\sinh 2\alpha b\cdot(y+b)\sinh\alpha y\Big]+$$

$$+\sum_s D_s\frac{1}{2}(2\beta a-\sinh 2\beta a)\sin\beta(y+b)=-\alpha_m\sin\beta_n(y+b)\,,\qquad(5.78\text{a})$$

$$\sum_r B_r\alpha^2\Big[-\Big\{2(1-\nu)\alpha b\cosh\alpha b+\Big(2+\frac{8+\nu}{5}h^2\alpha^2\Big)\sinh\alpha b\Big\}+$$

$$+(1-\nu)\sinh 2\alpha b\cdot\cosh\alpha b\Big]\sin\alpha x+\sum_r C_r\alpha^2\Big[-\Big\{2(1-\nu)\alpha b\sinh\alpha b+$$

$$+\Big(2+\frac{8+\nu}{5}h^2\alpha^2\Big)\cosh\alpha b\Big\}-(1-\nu)\sinh 2\alpha b\sinh\alpha b\Big]\sin\alpha x+$$

$$+\sum_s D_s(a\cosh\beta a\sinh\beta x-x\sinh\beta a\cosh\beta x)\beta=(-1)^{m'+1}\beta_n\sin\alpha_m x\,,$$

$$\cdots\cdots\cdots\cdots\qquad(5.78\text{b})$$

$$\sum_r(1-\nu)\alpha^4\Big[B_r\Big\{\Big(2+\frac{8+\nu}{5}h^2\alpha^2\Big)\sinh\alpha b+(1-\nu)(\sinh 2\alpha b-2\alpha b)\cosh\alpha b\Big\}+$$

$$+C_r\Big\{-\Big(2+\frac{8+\nu}{5}h^2\alpha^2\Big)\cosh\alpha b+(1-\nu)(\sinh 2\alpha b+2\alpha b)\sinh\alpha b\Big\}\Big]\times$$

$$\times\sin\alpha x+\sum_s D_s(-1)^s\beta^2\Big[\Big\{-(1-\nu)\beta a\cosh\beta a+\Big(-2\nu+\frac{8+\nu}{5}h^2\beta^2\Big)\times$$

$$\times\sinh\beta a\Big\}\sinh\beta x+(1-\nu)\beta\sinh\beta a\cdot x\cosh\beta x\Big]$$

$$=\frac{(-1)^{m'}k^4}{D\bar{J}}\beta_n\Big(\frac{p}{k^2}+\alpha_m^2\overline{Q}\Big)\sin\alpha_m x\,.\qquad\cdots\cdots\cdots\qquad(5.78\text{c})$$

By letting equations (5.78) be expressed in terms of coordinates of the reference system III in Fig. 2 and by the aid of Fourier expansions the desired relations among coefficients are obtained as follows:

$$\sum_r(-1)^{r'+s}\frac{\pi r^2s^2\sinh 2\alpha b}{2(b^2r^2+a^2s^2)}\Big[B_r\Big\{\Big(2+\frac{8+\nu}{5}h^2\alpha^2\Big)+\frac{(1-\nu)2b^2r^2}{(b^2r^2+a^2s^2)}\big(1+(-1)^s\big)\Big\}\times$$

$$\times\sinh\alpha b+C_r\Big\{\Big(2+\frac{8+\nu}{5}h^2\alpha^2\Big)+\frac{(1-\nu)2b^2r^2(1-(-1)^s)}{(b^2r^2+a^2s^2)}\Big\}\cosh\alpha b\Big]+$$

$$\left.\begin{aligned}+D_s\frac{1}{2}(2\beta a-\sinh 2\beta a)&=-\alpha_m\,,\quad\text{for}\quad n=s\,,\\&=0\,,\quad\text{for}\quad n\neq s\,.\end{aligned}\right\}\qquad(5.79\text{a})$$

$$B_r\alpha^2\Big\{(1-\nu)(\sinh 2\alpha b-2\alpha b)\cosh\alpha b+\Big(2+\frac{8+\nu}{5}h^2\alpha^2\Big)\sinh\alpha b\Big\}+$$

$$+ C_r \alpha^2 \left\{ -(1-\nu)(\sinh 2\alpha b + 2\alpha b)\sinh \alpha b + \left(2+\frac{8+\nu}{5}h^2\alpha^2\right)\cosh \alpha b \right\} +$$

$$+ \sum_s D_s \frac{(-1)^{r'+1} 8a^2b^2rs^2}{\pi(b^2r^2+a^2s^2)^2} \sinh^2 \beta a$$

$$\left.\begin{aligned} &= (-1)^{m'+1}\beta_n, \quad &&\text{for} \quad m=r, \\ &= 0, &&\text{for} \quad m\neq r, \end{aligned}\right\} \quad (5.79\text{b})$$

$$(1-\nu)\alpha^4 \Big[B_r \left\{ \left(2+\frac{8+\nu}{5}h^2\alpha^2\right)\sinh \alpha b + (1-\nu)(\sinh 2\alpha b - 2\alpha b)\cosh \alpha b \right\} +$$

$$+ C_r \left\{ -\left(2+\frac{8+\nu}{5}h^2\alpha^2\right)\cosh \alpha b + (1-\nu)(\sinh 2\alpha b + 2\alpha b)\sinh \alpha b \right\} \Big] +$$

$$+ \sum_s D_s \frac{(-1)^{r'+s}\pi rs^2 \sinh^2 \beta a}{(b^2r^2+a^2s^2)} \left\{ -\left(-2\nu+\frac{8+\nu}{5}h^2\beta^2\right) + (1-\nu)\frac{2a^2s^2}{(b^2r^2+a^2s^2)} \right\}$$

$$\left.\begin{aligned} &= \frac{(-1)^{m'}k^4}{D\bar{J}}\beta_n\left(\frac{p}{k^2}+\alpha_m^2 Q\right), \quad &&\text{for} \quad m=r, \\ &= 0, &&\text{for} \quad m\neq r. \end{aligned}\right\} \quad (5.79\text{c})$$

From these relations coefficients $\{B_r\}$, $\{C_r\}$ and $\{D_s\}$ can be determined. We do not write the representations of basic functions.

(b). Some Remarks on Other Cases.

As it is bothersome to continue to discuss other conceivable cases in the same detail as in the foregoing, though the descriptions up to now may lack clarity, we shall explain only the gist of the process of approach to problems. Of course, the process of calculation followed in this paper itself is very simple. Therefore, the above examples will be adequate enough to make this process comprehensible. We shall be concerned with the remaining cases only for reference. In the following ten cases, viz., from case G to case P, the bonndary conditions are indicated in figures for brevity and expressions refer to the coordinate system I in Fig. 2 and, needless to say, the pressure of the type (5.1) is applied to the upper surface of the plate.

Case G. It will be easily seen that the solutions of the first kind are of the same forms as in case C, since the boundary conditions to be imposed upon the problem of determining basic functions Θ, χ_0 and χ are similar to those in case C. Hence, we have no need to indicate the process of

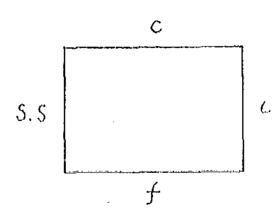


Fig. 9

calculations renewedly here. Then, with a view to obtaining the solutions of the second kind the representation of $\bar{\chi}$ can be taken in the form

$$\bar{\chi} = \sum_r \left\{ A_r \sinh \alpha(y-b) + B_r (y-b)\cosh \alpha y + C_r (y-b)\sinh \alpha y \right\} \sin \alpha(x+a) +$$
$$+ \sum_s \left\{ D_s (a\cosh \beta a \sinh \beta x - x\sinh \beta a \cosh \beta x) + E_s (a\sinh \beta a \cosh \beta x + \right.$$
$$\left. - x\cosh \beta a \sinh \beta x) \sin \beta (y+b) \right. \quad \cdots\cdots\cdots \quad (5.80)$$

in which $\quad \alpha = \dfrac{r\pi}{2a}, \quad \beta = \dfrac{s\pi}{2b}, \quad r = r', \quad s = s',$

r' and s' are positive integers. And by the application of the conditions that $G_{1,2}$ and $G_{2,2}$ vanish at the edges $x = -a$ and $y = -b$ respectively, five sequences of coefficients in formula (5.80) can be readily reduced to three sequences as in the foregoing cases, since $G_{1,3}$ and $G_{2,3}$ tend to zero at the respective edges. Next these three sequences of coefficients can be determined from the remaining conditions by the use of Fourier expansions of equations derived from these conditions in coordinates of the reference system II in Fig. 2 in a similar manner as before.

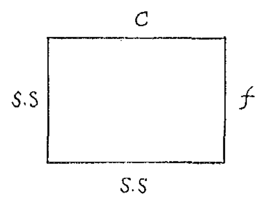


Fig. 10

Case H. On account of the similarity between the boundary conditions in this case and case B which are to be satisfied in order to obtain the solutions of the first kind we can utilize the solutions in the latter case as they are. Then, for the form of $\bar{\chi}$ we can put

$$\bar{\chi} = \sum_r \left\{ A_r (b\cosh \alpha b \sinh \alpha y - y\sinh \alpha b \cosh \alpha y) + B_r (b\sinh \alpha b \cosh \alpha y + \right.$$
$$\left. - y\cosh \alpha b \sinh \alpha y) \right\} \sin \alpha (x+a) + \sum_s \left\{ C_s \sinh \beta (x+a) + D_s (x+a) \times \right.$$
$$\left. \times \cosh \beta x + E_s (x+a) \sinh \beta x \right\} \sin \beta (y+b), \quad \cdots\cdots\cdots \quad (5.81)$$

in which $\quad \alpha = \dfrac{r\pi}{2a}, \quad \beta = \dfrac{s\pi}{2b}, \quad r = r'$ and $s = s'$.

By substituting this expression into the conditions $G_{1,2} = 0$, at $x = \pm a$ and $G_{2,2} = 0$, at $y = -b$ with the aid of formula $w_{0,2} = \dfrac{-\bar{J}}{k^4}\bar{\chi}$, five sequences of coefficients in (5.81) are reduced to two sequences and these are to be determined from the conditional equations

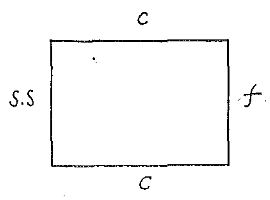


Fig. 11

$$\frac{\partial w_0}{\partial y}=0, \text{ at } y=b \text{ and } N_1-\frac{\partial H_1}{\partial y}=0, \text{ at } x=a,$$

in the manner as explained above.

Case I. Under similar conditions in this case and case A to obtain the solutions of the first kind, solutions to the latter case are easily seen to be applicable to the former case. Next $\bar{\chi}$ can be taken to be of the form

$$\bar{\chi} = \sum_r A_r (b\cosh \alpha b \sinh \alpha y - y \sinh \alpha b \cosh \alpha y) \sin \alpha (x+a) +$$
$$+ \sum_s \left\{ B_s \sinh \beta(x+a) + C_s (x+a) \cosh \beta x + D_s (x+a) \sinh \beta x \right\} \sin \beta y , \qquad (.5.82)$$

in which $\alpha = \frac{r\pi}{2a}$, $\beta = \frac{s\pi}{2b}$, $r=2r'$ and $s=s'$.

By applying the conditions $G_{1,2}=0$, at $x=\pm a$, C_s and D_s can be merged in $\{B_s\}$ and two sequences $\{A_r\}$ and $\{B_s\}$ can be determined from two coniditional equations

$$\frac{\partial w_0}{\partial y}=0, \text{ at } y=+b \quad \text{and} \quad N_1-\frac{\partial H_1}{\partial y}=0, \text{ at } x=a,$$

and, needless to notice, the condition $\frac{\partial w_0}{\partial y}=0$, at $y=-b$ yields no additional result for symmetry reasons.

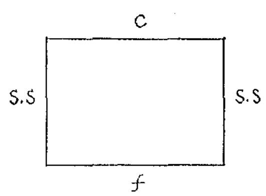


Fig. 12

Case J. Under similar load and conditions applied to the plate to get solutions of the first kind in this case and case B the solutions obtained in the latter are usable. In order to obtain solutions of the second kind the following form of $\bar{\chi}$ can be taken:

$$\bar{\chi} = \sum_r \left\{ A_r \sinh \alpha (y-b) + B_r (y-b) \cosh \alpha y + C_r (y-b) \sinh \alpha y \right\} \sin \alpha x , \qquad (5.83)$$

in which $\alpha = \frac{r\pi}{2a}$, $r=2r'$,

By (5.83) the condition $G_{1,2}=0$, at $x=\pm a$ is satisfied obviously and the condition $G_{2,2}=0$, at $y=-b$ leads to a simple relation among A_r, B_r and C_r. Then the remaining two conditions

$$\frac{\partial w_0}{\partial y}=0, \text{ at } y=b \quad \text{and} \quad N_2-\frac{\partial H_2}{\partial x}=0, \text{ at } y=-b$$

serve to determine unknown coefficients.

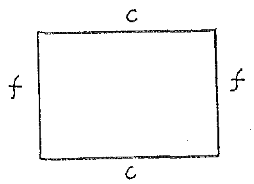


Fig. 13

Case K. For basic functions Θ_0 and χ_0 we can manifestly make use of the forms of the solutions in case A. Thus for $\bar{\chi}$ we can set

$$\bar{\chi}=\sum_r A_r(b\cosh\alpha b\sinh\alpha y-y\sinh\alpha b\cdot\cosh\alpha y)\times$$
$$\times\sin\alpha x+$$
$$+\sum_s(B_s\sinh\beta x+C_s x\cosh\beta x)\sin\beta y\,, \qquad \cdots\cdots (5.84)$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=2r'$ and $s=2s'$. From three condition

$$\frac{\partial w_0}{\partial y}=0, \text{ at } y=\pm b; \quad N_1-\frac{\partial H_1}{\partial y}=0, \quad \text{at } x=\pm a$$

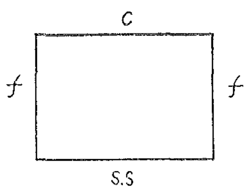


Fig. 14

and $G_{1,2}=0$, at $x=\pm a$,

we can determine three sequences of coefficients.

Case L. For Θ_0 and χ_0 we can take the forms of these basic functions in case B in view of the similar conditions and load. Then $\bar{\chi}$ can be taken to be of the form

$$\bar{\chi}=\sum_r\left\{A_r(b\cosh\alpha b\sinh\alpha y-y\sinh\alpha b\cosh\alpha y)+B_r(b\sinh\alpha b\cosh\alpha y+\right.$$
$$\left.-y\cosh\alpha b\cdot\sinh\alpha y)\right\}\sin\alpha x+\sum_s(C_s\sinh\beta x+D_s x\cosh\beta x)\sin\beta(y+b)\,, \qquad \cdots\cdots (5.85)$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=2r'$ and $s=s'$.

Next the application of two conditions $G_{1,2}=0$, at $x=a$ and $G_{2,2}=0$, at

$y=-b$ yields two simple relations between C_s and D_s and between A_r and B_r respectively. Hence, the remaining unknown sequences of coefficients may be determined from the two conditional equations

$$\frac{\partial w_0}{\partial y}=0\,, \quad \text{at } y=b \text{ and } N_1-\frac{\partial H_1}{\partial y}=0\,, \quad \text{at } x=a\,.$$

From symmetry two conditions $G_{1,2}=0$, at $x=-a$ and $N_1-\frac{\partial H_1}{\partial y}=0$, at $x=-a$ lead to no additional result.

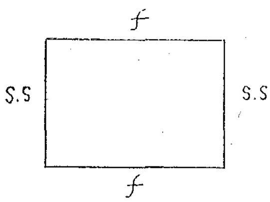


Fig. 15

Case M. Solutions of the first kind are of the same forms as in the case where plates are simply supported at the edges. $\bar{\chi}$ can be put in the following form:

$$\bar{\chi}=\sum_r (A_r \sinh \alpha y+B_r y \cosh \alpha y) \sin \alpha x\,, \quad (5.86)$$

in which $\alpha=\frac{r\pi}{2a}\,, \quad r=2r'\,.$

Condition $G_{1,2}=0$, at $x=\pm a$ is evidently fulfilled by (5.86) and from the condition $G_{2,2}=0$, at $y=\pm b$ a simple relation between A_r and B_r is obtained, therefore, they are determined easily from the condition $N_2-\frac{\partial H_2}{\partial x}=0$, at $y=\pm b$. In this case series $\bar{\chi}$ reduces to a single term.

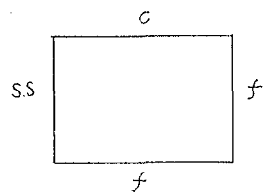


Fig. 16

Case N. For the forms of Θ_0 and χ_0 we can utilize the solutions in case B. The representation of $\bar{\chi}$ can be taken in the form.

$$\bar{\chi}=\sum_r \left\{A_r \sinh \alpha (y-b)+B_r (y-b) \cosh \alpha y+C_r (y-b) \sinh \alpha y\right\} \times$$
$$\times \sin \alpha (x+a)+\sum_s \left\{D_s \sinh \beta (x+a)+E_s (x+a) \cosh \beta x+\right.$$
$$\left.+F_s (x+a) \sinh \beta x\right\} \sin \beta (y+b)\,, \quad \cdots\cdots\cdots\cdots \quad (5.87)$$

in which $\alpha=\frac{r\pi}{2a}\,, \quad \beta=\frac{s\pi}{2b}\,, \quad r=r' \text{ and } s=s'\,.$

By applying the condition $G_{1,2}=0$, at $x=\pm a$, E_s and F_s can be expressed in terms of D_s and by the condition $G_{2,2}=0$, at $y=-b$ A_r is merged in B_r and C_r. Thus three unknown sequences of coefficients are to be determined from the three conditions

$$\frac{\partial w_0}{\partial y}=0\,,\ \text{at}\ y=b;\quad N_1-\frac{\partial H_1}{\partial y}=0,\ \text{at}\ x=a;\quad N_2-\frac{\partial H_2}{\partial x}=0\,,$$

at $y=-b$.

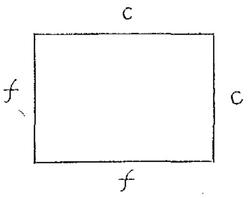


Fig. 17

Case O. The representations of Θ_0 and χ_0 are of the same forms as those in case C. $\bar{\chi}$ can be given by the following form:

$$\bar{\chi}=\sum_r\left\{A_r\sinh\alpha(y-b)+B_r(y-b)\cosh\alpha y+C_r(y-b)\sinh\alpha y\right\}\sin\alpha(x+a)+$$
$$+\sum_s\left\{D_s\sinh\beta(x-a)+E_s(x-a)\cosh\beta x+F_s(x-a)\sinh\beta x\right\}\times$$
$$\times\sin\beta(y+b)\,, \qquad\qquad (5.88)$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=r'$, $s=s'$.

By virtue of the conditions $G_{1,2}=0$, at $x=-a$ and $G_{2,2}=0$, at $y=-b$ D_s is expressible in terms of E_s, F_s and A_r in terms of B_r, C_r respectively. Thus the remaining four sequences of coefficients can be determined from the following four conditions:

$$\frac{\partial w_0}{\partial x}=0\,,\ \text{at}\ x=a;\quad \frac{\partial w_0}{\partial y}=0\,,\ \text{at}\ y=b;$$

$$N_1-\frac{\partial H_1}{\partial y}=0\,,\ \text{at}\ x=-a\,;$$

$$N_2-\frac{\partial H_2}{\partial y}=0\,,\ \text{at}\ y=-b\,.$$

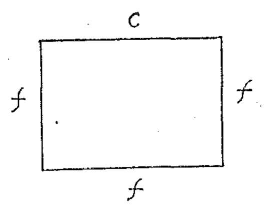


Fig. 18

Case P. Plane stress solutions to the problem are of the same forms as those in case B. The representation of $\bar{\chi}$ can be the expression of the form

$$\bar{\chi}=\sum_r\left\{A_r\sinh\alpha(y-b)+B_r(y-b)\cosh\alpha y+C_r(y-b)\sinh\alpha y\right\}\sin\alpha x+$$
$$+\sum_s(D_s\sinh\beta x+F_s x\cosh\beta x)\sin\beta(y+b)\,, \qquad\qquad (5.89)$$

in which $\alpha=\dfrac{r\pi}{2a}$, $\beta=\dfrac{s\pi}{2b}$, $r=2r'$ and $s=s'$.

Five sequences of coefficients in $\bar{\chi}$ (5.89) are to be determined from five conditions,

$$G_{1,2}=0,\ \text{at}\ x=a;\quad G_{2,2}=0,\ \text{at}\ y=-b;$$
$$\frac{\partial w_0}{\partial y}=0,\ \text{at}\ y=b;\quad N_1-\frac{\partial H_1}{\partial y}=0,\ \text{at}\ x=a;\quad N_2-\frac{\partial H_2}{\partial x}=0,$$
$$\text{at}\ \ y=-b.$$

The conditions $G_{1,2}=0$, at $x=-a$ and $N_1-\frac{\partial H_1}{\partial y}=0$, at $x=-a$ are, of course, unnecessary from symmetry reasons. As stated before, in the above examples we took only case I ($m=2m'$, $n=2n'$) for the sake of simplification, where the sinusoidal pressure applied to the upper surface of the plate is expressible in the form (5.1). Accopdingly, when we take one of the other three cases, slight changes must be made in the preceding expressions of this section. The needed modifications may be readily found, applying such procedures as explained in Secs. III and IV. It is evident that modifications are very simple, if the solutions are obtained under circumstances that boundary conditions to be imposed on two opposite edges are not similar, that is, if the expressions for χ'' or $\bar{\chi}$, referred to coordinate system I, are neither even nor odd functions of x or y. At any rate, to the degree of accuracy furnished by the procedure of Love, solutions of the first and second kinds can be obtained to some extent separately; this fact is of interest and serves to save laborious calculations. Thus problems of thick plate under variable normal load, though this section is concerned with a sinusoidal normal load only, and various boundary conditions may be treated readily, employing the method of Love, to say nothing of the regret that we have to apply Kirchhoff's four boundary conditions. And yet other types of mixed boundary conditions than those stated above are conceivable, confining ourselves to the problem of rectangular thick plate.

(c). A Few Remarks upon Other Particular Various Edge Conditions.

Herein it is not our purpose to discuss composite rectangular thick plate problem, because a composite plate is composed of plates of various thicknesses and elastic constants and we are under such restraints that we have to apply Kirchhoff's eight conditions of continuity at the joining lines of the composite plate, though this problem may be treated by resorting to the method to be briefly explained below regarding the following complicated problems. Furthermore, we cannot solve a problem of thick plate, whose thickness varies continuously, by employing the procedure of Love. Hence, the situation in the vicinity of the joining lines may be said to be severe.

In the next place problems of continuous rectangular thick plate[4] under any variable load can be solved in a similar manner easily, if continuous rectangular thick plates are defined as those which are simply supported at all edges and intersecting lines which are perpendicular to their edges. It appears that we cannot but treat each span of continuous plate separately and, hence, we need one coordinate system for each span. Now we take a case where a continuous plate has three spans as shown in Fig. 19. The conditions at the intermediate lines to represent the continuity of plate will be as follows:

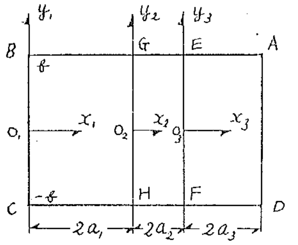


Fig. 19

on the line $\overline{GH}$

$$\left.\begin{aligned} {}_1u_0 &= {}_2u_0 , \quad {}_1v_0 = {}_2v_0 , \\ {}_1T_1 &= {}_2T_1 , \quad {}_1S_1 = -{}_2S_1 , \end{aligned}\right\} \quad \cdots\cdots (5.90a)$$

for the solutions of the first kind;

$${}_1w_0 = {}_2w_0 = 0, \quad \frac{\partial\, {}_1w_0}{\partial x} = \frac{\partial\, {}_2w_0}{\partial x} , \quad \cdots\cdots (5.90b)$$

${}_2G_1 = {}_1G_1$ for the solutions of the second kind.

on the line $\overline{EF}$

$${}_2u_0 = {}_3u_0 , \quad {}_2v_0 = {}_3v_0 , \quad {}_2T_1 = {}_3T_1 \quad \cdots\cdots (5.91a)$$

${}_2S_1 = -{}_3S_1$, for the solutions of the first kind;

$${}_2w_0 = {}_3w_0 = 0, \quad {}_2G_1 = {}_3G_1 , \quad \frac{\partial\, {}_2w_0}{\partial x} = \frac{\partial\, {}_3w_0}{\partial x} , \quad \cdots\cdots (5.91b)$$

for the solutions of the second kind,

in which subscripts 1, 2, 3 are placed to the left of labels to indicate that quantites represented by labels with these subscripts refer to the left, middle and right side span respectively. First, we expand the intensity of load applied to each span in Fourier series in each coordinate system and then obtain particular solutions for single sinusoidal terms in double Fourier series by applying the calculating process explained in the first report. If we proceed to calculate with a view to obtaining the solutions of the first kind in the same manner as above concerning each portion of the plate, we are left with eight sequences of coefficients undetermined after we apply the edge conditions and four of those sequences belong to the middle span of plate and these coefficients are to be determined from eight conditions at the intermediate lines (5.90a), (5.91a). When we employ the foregoing process of calculation to get the solutions of the second kind, all coefficients can be determined from the conditions at the edges and intermediate lines. So finally we obtain complete solutions to the problem by combining three kinds of solutions as before.

We turn to cases where boundary conditions vary abruptly across the intermediate points on the edges and there exists no portion which is supported in the interior of the bounding curve. If we investigate a problem as indicated in Fig. 20, as an instance, we first divide the whole domain into two rectangles by an intersecting line through point F, across which the boundary condition varies, and draw two coordinate systems $(x_1, y_1), (x_2, y_2)$. Then we treat the two rectangles separately in the same manner as in the preceding examples for a while and finally carry out the continuation-process between the solutions relative to two rectangles by means of the conditions that

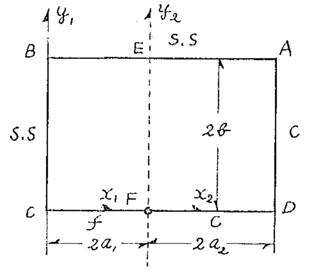


Fig. 20

for the solutions of the first kind

$$
{}_1u_0 = {}_2u_0\,, \quad {}_1v_0 = {}_2v_0\,, \quad {}_1T_1 = {}_2T_1
$$

$$
{}_1S_1 = -{}_2S_1\,, \qquad \text{at } x_2 = 0 \text{ or at } x_1 = a_1\,, \qquad \cdots\cdots\cdots \ (5.92\text{a})
$$

for the solutions of the second kind

$$
{}_1w_0 = {}_2w_0\,, \ \frac{\partial_1 w_0}{\partial x_1} = \frac{\partial_2 w_0}{\partial x_2}\,, \ \ {}_1G_1 = {}_2G_1
$$

$$
{}_1N_1 - \frac{\partial_1 H_1}{\partial y} = -\left({}_2N_1 - \frac{\partial_2 H_1}{\partial y_2}\right), \ \text{at } x_2 = 0 \text{ or at } x_1 = a_1, \quad (5.92\text{b})
$$

in which subscripts 1, 2 placed to the left of labels denote that the labels belong to the left side compartment and right side one respectively. Thus, when we undertake to get the solutions of the first kind, four sequences of coefficeints, left unknown after we apply the edge conditions, can be determined from (5.92a). Also the conditions (5.92b) together with edge conditions serve to determine coefficients in the solutions of the second kind. Of course, the degree of accuracy of the solutions in the vicinity of the intersecting line is inevitably the same as near the boundary. And so in order to solve such a problem as shown in Fig. 21, following the procedure of Love, we will have to draw intersecting lines perpendicular to the edges through intermediate points on the boundary F, G, I, across which the boundary conditions vary suddenly. Therefore, we have six rectangular compartments and we can proceed in the same manner as above, applying conditions similar to (5.92), but the computation will be very laborious. Now we shall explain, for reference, the derivation of the solutions to the problem shown in Fig. 20 in more detail. Let a given intensity of load applied to the upper surface of compartments of plate be expanded in double Fourier series in the coordinate systems (x_1, y_1), (x_2, y_2) respectively. In the following we consider only a pair of single terms

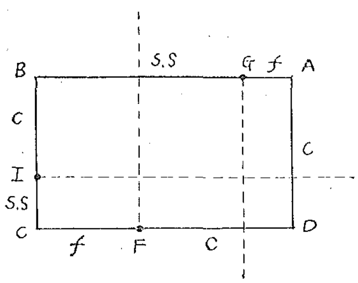


Fig. 21

$$ {}_1p_{mn}\sin{}_1\alpha_m x_1 \sin \beta_n y_1 \,, \qquad {}_2p_{mn}\sin{}_2\alpha_m x_2 \sin{}_2\beta_n y_2 \,, $$

in which $$ {}_1\alpha_m=\frac{m\pi}{2a_1}\,,\quad {}_1\beta_n=\frac{n\pi}{2b}\,,\quad {}_2\alpha_m=\frac{m\pi}{2a_2}\,,\quad {}_2\beta_n=\frac{n\pi}{2b}\,, \tag{5.93} $$

$(m=2m',\ n=2n')$ For the purpose of obtaining the solutions of the first kind we may put ${}_1\chi''=2h_1\chi_0-\frac{1}{3}\frac{\nu}{1+\nu}h^3{}_1\Theta_0$ in the following form:

$$ {}_1\chi''=\sum_r A_r\Big\{b\cosh{}_1\alpha b\sinh{}_1\alpha(y_1-b)-(y_1-b)\sinh{}_1\alpha b\cosh{}_1\alpha(y_1-b)\Big\}\times $$
$$ \times\sin{}_1\alpha x_1+\sum_s\left(B_s\sinh{}_1\beta x_1+C_s x_1\cosh{}_1\beta x_1+D_s x_1\sinh{}_1\beta x_1\right)\sin{}_1\beta(y_1-b)\,, \tag{5.94} $$

in which $${}_1\alpha=\frac{r\pi}{2a_1}\,,\quad {}_1\beta=\frac{s\pi}{2b}\,,\quad r=2r' \text{ and } s=s'\,.$$

For ${}_2\chi''$ we can put in view of the condition for T

$$ {}_2\chi''=\sum_r\Big\{E_r\sinh{}_2\alpha(y_2-2b)+F'_r(y_2-2b)\cosh{}_2\alpha y_2+G_r(y_2-2b)\sinh{}_2\alpha y_2\Big\}\times $$
$$ \times\sin{}_2\alpha x_2+\sum_s H_s\Big(\sinh{}_2\beta x_2+I_s\cosh{}_2\beta x_2+J_s x_2\cosh{}_2\beta x_2+ $$

$$+ K_s x_2 \sinh {}_2\beta x_2 \big) \sin {}_2\beta y \ , \qquad \cdots\cdots (5.95)$$

in which $\quad {}_2\alpha = \dfrac{r\pi}{2a_2}, \quad {}_2\beta = \dfrac{s\pi}{2b}, \quad r = r', \quad s = s'$.

We obtain two simple relations among E_r, F_r, G_r and among H_s, I_s, J_s, K_s from the conditional equations

${}_2u_0 = 0$, at $y_2 = 0$ and ${}_2v_0 = 0$, at $x_2 = 2a_2$ respectively. Thus we can determine four sequences of coefficients in (5.94) and five sequences in (5.95) from five edge conditions

$${}_1S_2 = 0, \text{ at } y_1 = 2b; \quad {}_1S_1 = 0, \text{ at } x_1 = 0; \quad {}_2u_0 = 0, \text{ at } x_2 = 2a_2;$$

$${}_2v_0 = 0, \text{ at } y_2 = 0; \quad {}_2S_2 = 0, \text{ at } y_2 = 2b,$$

and the conditions on the intermedite line (5.92a). Next in order to get the solutions of the second kind we may set ${}_1\bar{\chi}$ and ${}_2\bar{\chi}$ in the following forms according to the conditions regarding ${}_1w_0$ and ${}_2w_0$.

$$\bar{\chi}_1 = \sum_r \Big\{ A_r \sinh {}_1\alpha (y_1 - 2b) + B_r (y_1 - 2b) \cosh {}_1\alpha y_1 + C_r (y_1 - 2b) \sinh {}_1\alpha y \Big\} \times$$

$$\times \sin {}_1\alpha x_1 + \sum_s \Big\{ D_s \sinh {}_1\beta x_1 + E_s x_1 \cosh {}_1\beta x_1 + F_s x_1 \sinh {}_1\beta x_1 \Big\} \sin {}_1\beta y_1 \ ,$$

in which

$${}_1\alpha = \frac{r\pi}{2a_1}, \quad {}_1\beta = \frac{s\pi}{2b}, \quad r = r', \quad s = s', \qquad \cdots\cdots (5.96)$$

and

$${}_2\bar{\chi} = \sum_r \Big[G_r \Big\{ b \cosh {}_2\alpha b \sinh {}_2\alpha (y_2 - b) - (y_2 - b) \sinh {}_2\alpha b \cosh {}_2\alpha (y_2 - b) \Big\} +$$

$$+ H_r \Big\{ b \sinh {}_2\alpha b \cosh {}_2\alpha (y_2 - b) - (y_2 - b) \cosh {}_2\alpha b \sinh {}_2\alpha (y_2 - b) \Big\} \Big] \sin {}_2\alpha x_2 +$$

$$+ \sum_s \Big\{ I_s \sinh {}_2\beta (x_2 - 2a_2) + J_s (x_2 - 2a_2) \cosh {}_2\beta x_2 + K_s (x_2 - 2a_2) \sinh {}_2\beta x_2 \Big\} \times$$

$$\times \sin {}_2\beta y_2 \ , \qquad \cdots\cdots (5.97)$$

in which $\quad {}_2\alpha = \dfrac{r\pi}{2a_2}, \quad {}_2\beta = \dfrac{s\pi}{2b}, \quad r = r', \quad s = s'$.

Two simple relations among A_r, B_r and C_r are found from the conditions ${}_1G_2 = 0$, at $y_1 = 0$ and ${}_1G_2 = 0$, at $y_1 = 2b$ and one simple relation among D_s, E_s and F_s can be obtained from ${}_1G_1 = 0$, at $x_2 = 0$. Further, from the condition ${}_2G_2 = 0$, at $y_2 = 2b$ we easily get a simple relation between G_r and H_r. Accordingly, we can determine three sequences of coefficients in (5.95) and four sequences in (5.97) from three edge conditions

$$ {}_1N_2-\frac{\partial_1 H_2}{\partial x}=0\,,\ \text{at}\ y_1=0\,;\ \frac{\partial_2 w_0}{\partial x_2}=0\,,\ \text{at}\ x_2=2a_2\,; $$

$\frac{\partial_2 w_0}{\partial y_2}=0$, at $y_2=0$, and the conditions on the intermediate line (5.92b).

As is readily understood, solutions obtained concerning one compartment of the plate do not hold in other compartments and, hence, of course, we cannot write down solutions which are valid in the whole domain. That is a matter of some regret. Yet it is manifest that, if we apply the procedure described above, we may easily treat problems of thick plates whose edge lines are parallel with or perpendicular to

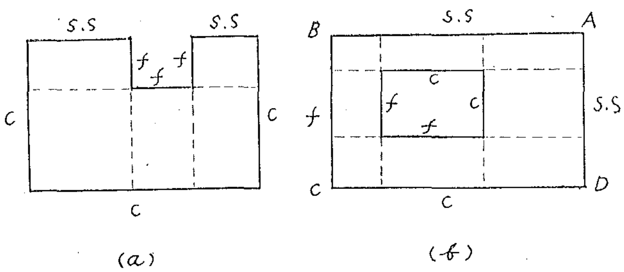


Fig. 22

one coordinate axis and, further, whose domains are multiply-connected or which have holes, for example, problems of plates which have such forms as shown in Fig. 22. These situations are, needless to notice, similar to those in the theory of thin plate. But their numerical calculation will be very troublesome, since for the determination of solutions to the problem as indicated in Fig. 22b we must deal virtually with eight thick plates. Here we only point out that even such intricate problems are not beyond the scope of the presented method of obtaining solutions to the problem of moderately thick plate under any variable load and various edge conditions. Additionally, though this paper is not concerned with the problem of thick plate under variable tangential load, we can obviously solve it in the same way as in the case of variable normal load. Further, the discussion of plate, whose bounding curve is such that it is convenient to refer expressions to other cylinder coordinates than cartesian, shall be postponed to later reports.

Finally the author wishes to express his heart-felt thanks to the members of the Department of Mechanical Engineering, and especially to prof. R. Kuno for his having read the manuscript.

Notes and References.

1) K. Hata: Memoirs of the Faculty of Engineering, Hokkaido Univ., Vol. 9, No. 3, p. 428.

2) A. E. H. Love: The Mathematical Theory of Elasticity, 4th Ed., Cambridge University Press, London, 1934, pp. 467–473,

3) R. V. Southwell: Philosophical Magazine, Vol. 21, 7th series, (Feb, 1936) p. 201.

4) S. Timoshenko: Theory of Plates and Shells, McGraw-hill Book Company, 1940, p. 232.

*) Gist of the contents in the section V "on the mixed boundary value problems relative to the rectangular thick plate" was read at the Third Japan National Congress for Applied Mechanics, Sept. 9, 1953.

Errata: In Eqs. (21.4) of the first report the following alteration should be made:

$$F-\alpha\varphi_3=F'+\frac{4}{4-\alpha}\Phi'_1+\frac{\alpha-2}{4-\alpha}x\frac{\partial\Phi'_1}{\partial x},\quad F'=\Phi'_0+x\frac{\partial\Phi'_1}{\partial x};\quad \varphi_2=0,$$

and Eq. (22.2) and $A'\alpha fz$ on page 436 of the first report should be altered respectively as follows:

$$F_1=-\frac{h^2}{2(1+\nu)}\Theta+A'2(1-\nu)f,\quad A'(\alpha-1)fz.$$