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Note	Appendix: General conception of the circle diagrams for electric power transmission circuit with a sending and a receiving end / Koji Ogushi
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**Tripping Characteristics of Protective Relays on  
Transmission Network,  
Expressed by Circle Diagram Method.**

(Part II)

By

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§ 2. Apparent impedance circle diagram and  
Impedance relays.

The impedance relay operates in proportion to the apparent impedance or absolute value of impedance at its setting point. The locus of apparent impedance or its inverse value, apparent admittance, is a circle on the co-ordinate plane to show watt and wattless component of power at the relay setting point or any other distance point on the network. As the co-ordinate Units, Power, admittance, impedance or *p.u.* of complex values may be taken. The admittance co-ordinate plane,  $(g+jb)$  is preferable, when its own voltage is dependent on some other terminal voltage of the network. The impedance co-ordinate plane,  $(r+jx)$  has inverse figures on the admittance co-ordinate plane.

The impedance co-ordinate is often used for expressing the impedance relay characteristics. However, admittance or complex power,  $W=(P+jQ)$  co-ordinate will be convenient, when it is desired to express the impedance relay characteristics together with the power circle diagram.

I. Impedance relays on a transmission line with a single power sending end.

(1) Trip area on  $W_1$ -plane.

As shown in fig. 7, impedance relays are arranged in a straight transmission line, relay 1 at the middle point of the line, relay 2 at the receiving end and relay 3 at a feeder line of the receiving end. First of all will be considered the apparent impedance  $|Z_2|$  at the receiving end 2. The apparent impedance circles on  $W_1$ -plane, keeping the sending voltage constant and taking him as the reference phase,

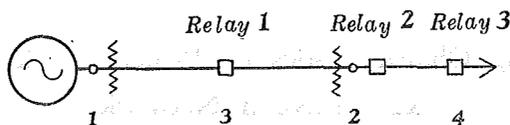


Fig. 7. Impedance Relay.

are given by the following equation.

$$|Z_2| \epsilon^{j\theta_2} = \left| \frac{-W_1 + \frac{D}{B} E_1^2}{W_1 - \frac{C}{A} E_1^2} \times \frac{B}{A} \right| \epsilon^{j\theta_2} \dots \dots \dots (14)$$

and shown in fig. 8.

The ratio of distances from any point  $W_1$  on the circle locus to the short circuit point  $C_{e1}$  and the open circuit point  $C_{i1}$ , which are

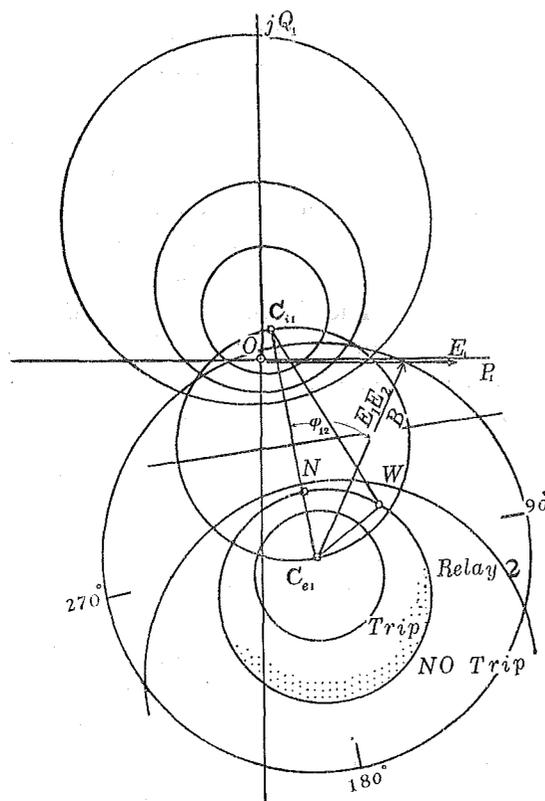


Fig. 8. Trip Area Relay 2 on  $W_1$ -plane.

the well known centers of the power circle diagram and the apparent current diagram, are always constant. Therefore, taking the length  $\overline{C_{e1} C_{i1}}$ , as the scale of apparent impedance ohms, the circle with its center on  $\overline{C_{e1} C_{i1}}$  line, intersecting rectangle with the  $\overline{C_{e1} C_{i1}}$  diameter circle, through the point,  $N$  of the value,  $Z_2 = \frac{C_{e1} N B}{C_{i1} N A}$  ohms, is the required apparent impedance circle with the above values. It is clear that this circle will give at  $C_{e1}$  point, Zero ohm and at  $C_{i1}$  point,  $\infty$  ohms.

The inside of the circle is the tripping area of relay 2 in fig. 7, adjusted so as to operate at  $Z_2$  ohm.

Similarily, one can get

the impedance circles to show the trip areas on  $W_1$ -plane of relay 1 and relay 2. These areas may be approximately as shown in fig. 9; the relays operate around the short circuit points, which are obtained by assuming the relay setting points as receiving power ends. It is convenient to show the power circle diagram on the same co-ordinate plane, by which the normal operating conditions can be directly read off. From fig. 9, it can be understood that the impedance relays are more effective for selective action of fault than the over current relay explained in the preceding chapter.

But, impedance relays have not been used in Japan. This relay is usually connected to line to earth.

On a transmission system, connecting two synchronous machines at both its ends, the power angle  $\phi_{12}$  shown in fig. 9 may be taken as a large

angle, if their frequencies slightly differ to with one another or if frequency swing occurs. Actions of the relays are easily known from fig. 9, showing that relay 1, which is settled at the middle point of the line in fig. 7 is in the tripping area, while the other relays are not.

(2) Trip area on  $Y_2$ -plane, keeping the sending, voltage,  $E_1$  constant.

Because the terminal voltage depends on another terminal voltage, admittance plane  $Y_2 = g + jb$  may be taken, assuming lagging and receiving power as positive.

It is self evident that the operating area of relay 2 is outside of the circle with the radius  $Y = \frac{1}{Z}$ , the inverse value of the defined

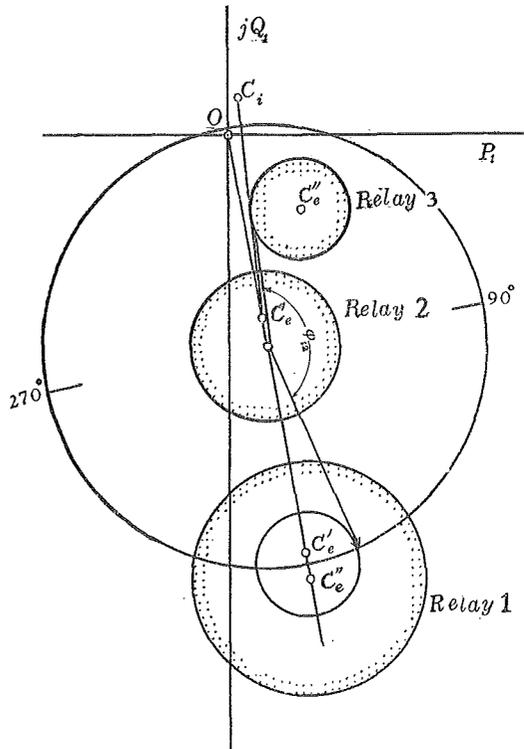


Fig. 9. Trip Area Impedance Relay on  $W_1$ -plane.

impedance of the relay.

This  $Y_2$ -plane, having variable voltage does not directly express its own terminal powers. Therefore, one may consider the power circle diagram, watt and wattless power circle diagrams as shown in fig. 10,

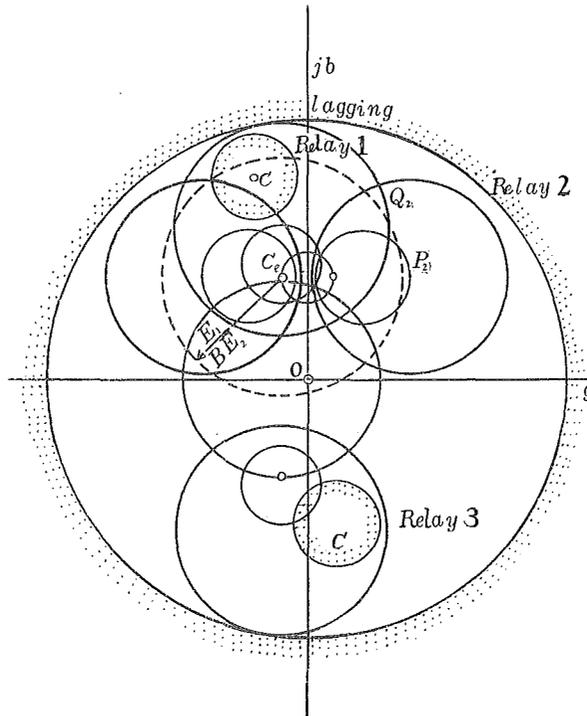


Fig. 10. Trip Area on  $y_2$ -plane Impedance Relay.

which are the circles having their centers at  $-\frac{A}{B}$ , horizontal and vertical lines through  $-\frac{A}{B}$  point as well known.

The tripping area of relay 1 connected at the middle point of the transmission line is inside the impedance locus

$$|Z_1|_{\epsilon^{j\theta_1}} = \left| \frac{y_2 + \frac{A'}{B'}}{y_2 + \frac{C'}{D'}} \frac{B'}{D'} \right|_{\epsilon^{j\theta_1}} \dots\dots\dots (15)$$

which is the circle having its center on the line through the short

circuit point  $C'_e = \frac{A'}{B'}$  and the open circuit point  $C'_i = \frac{C'}{D'}$ , considering the relay setting point, 3, as a sending power end.

The operating area of relay 1 may be obtained in the same way as indicated in fig. 8, and equation (14) as well as for relay 3 at the feeder line. They are in fig. 9 which expresses evidently that the impedance relay has a property to protect around the faulted points.

(3) Tripping area on  $Z$ -plane.

The impedance co-ordinate is the inverse of the admittance co-ordinate above explained. It is easy to get the tripping area on  $Z$ -plane by the theory of inversion.

For example, the tripping area of the three relays to show on the  $Z_2$ -plane of the receiving end are obtained from the inversion of fig. 10 as showing in fig. 11.  $Z = (r + jx)$ -plane is more often used, however, it has the same meaning as the  $Y$ -plane.

II. Impedance relays on a transmission network with many power stations and substations.

A transmission network has in most cases many power stations and substations.

In the practical case, this problem is important, however, it is difficult to formulate a general analytical method. This may be the problem to treat in each cases.

Let the terminals 1 and 2 in fig 12 be the sending and receiving power end having a direct connector  $\overline{12}$  and some branch connectors from them. Remembering that impedance at any point is defined as the ratio of its voltage and current it is evident that

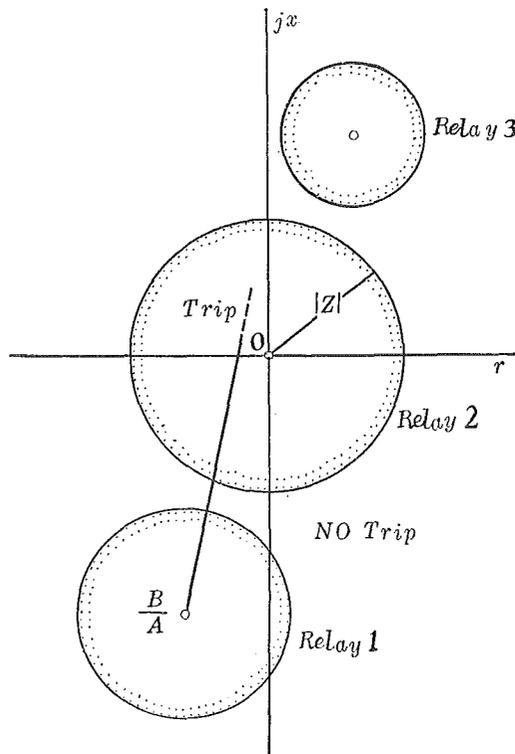


Fig. 11. Impedance Relay on  $Z$ -plane.

impedance can be taken in different ways, because in this network with branch connectors, current may be taken as total current or a branch current and voltage depends on many other power stations connected. Let a most simple case be considered. The impedance relay 1 is connected on the feeder of the receiving power end 2 which is connected with power stations 1 and 3 as shown in fig. 12. The tripping area of this relay will be shown on  $Y_1$ -plane of sending power end 1. Then, the impedance of the relay setting point 2 is

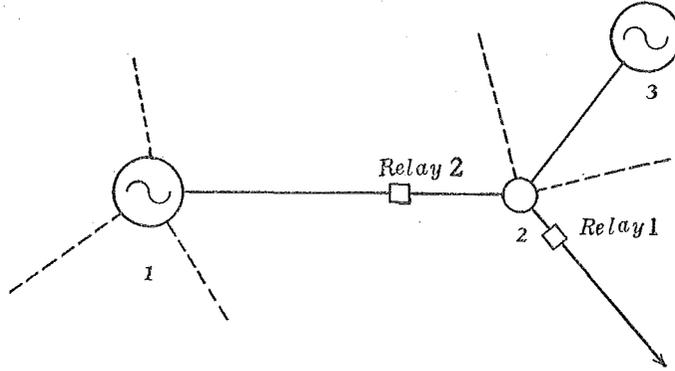


Fig. 12.  $n$ -Terminal Network.

$$Z_2 = \frac{E_2}{I_1} = \frac{E_2}{I_2 + I_3} = \frac{DE_1 - BI_1}{-CE_1 + AI_1 - C'I_3 + A'I_3}$$

$$|Z_2| \varepsilon^{j\theta_2} = \left| \frac{\frac{D}{B} - y_1}{-\frac{C}{A} - \frac{C'}{A} \frac{E_3}{E_1} + \frac{A'}{A} \frac{I_3}{E_1} + y_1} \times \frac{B}{A} \right| \varepsilon^{j\theta_2} \dots (16)$$

As compared with fig. 8, the open circuit point,  $C_i$  is changed into  $\left(-\frac{C}{A} - \frac{C'}{A} \frac{E_3}{E_1} + \frac{A'}{A} \frac{I_3}{E_1}\right)$  instead of  $-\frac{C}{A}$ . It depends on the other interconnected power station, 3, of which conditions to be satisfied must be given.

If the point,  $C_i$  is determined by the given conditions, the impedance circles to show the tripping area of relay 1 can be obtained by the same method as explained before. As shown in fig. 13, it is clear that the relay gives protection around the short circuit point  $C_s$ , the operation is nearly independent of the other connected power station. The above admittance unit is easily changed to power unit,

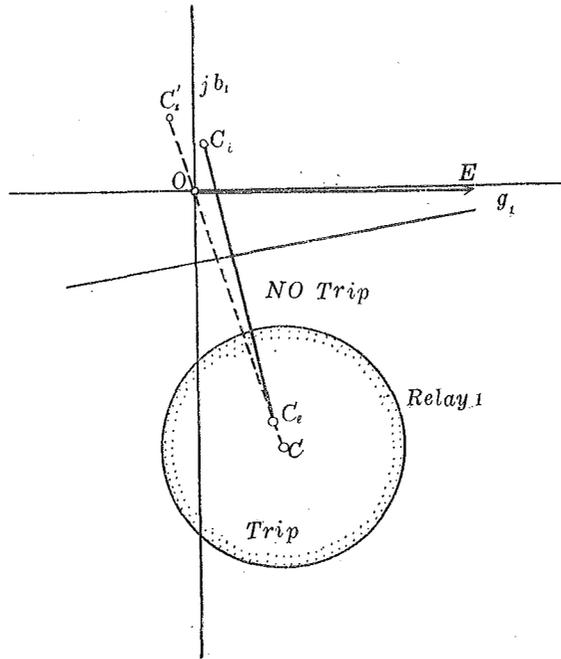


Fig. 13. Trip Area Impedance Relay  $n$ -Terminal.

by multiplying the square of the given terminal 1 voltage,  $E_1$ .

The tripping area on the above  $Y_1$  co-ordinate plane of relay 2, connected at an intermediate point of the connector  $\overline{12}$  is self evident independently shown on the other connected power station.

#### Example 2.

The transmission line, 160 Km, 60 KV, 15 MVA of example 1 has three impedance relays at the receiving end, relay 2, the middle point of the transmission line, relay 1, and a point on feeder line, relay 3, as shown in fig. 8. Express their tripping area on  $W_1$ -plane and  $Y_2$ -plane, keeping the sending power end voltage  $\overline{E}_1=1$  constant. The general transmission circuit constants without generator reactance between the terminals 1 and 2 are

$$A = D = 0.926 + j0.00185$$

$$B = 0.1014 + j0.407$$

$$C = -0.00233 + j0.351 \quad \text{P. U.}$$

These impedance relays are assumed to trip when their impedance

becomes  $\frac{1}{3}$  of the short circuit impedance  $|C_e|$ . The feeder relay is connected at the point  $Z=j0.5$  P.U. from the receiving power end 2, or from the sending power end 1, line constants are

$$A' = 0.926 + j0.0185, \quad C' = -0.00233 + j0.351$$

$$B' = 0.0921 + j0.870, \quad D' = 0.751 + j0.0173.$$

Solution:—

Tripping area on  $W_1$ -plane.

The short circuit and open circuit point, relating to the relay 2 setting point on  $W_1$  plane-with  $E_1$  as reference voltage are

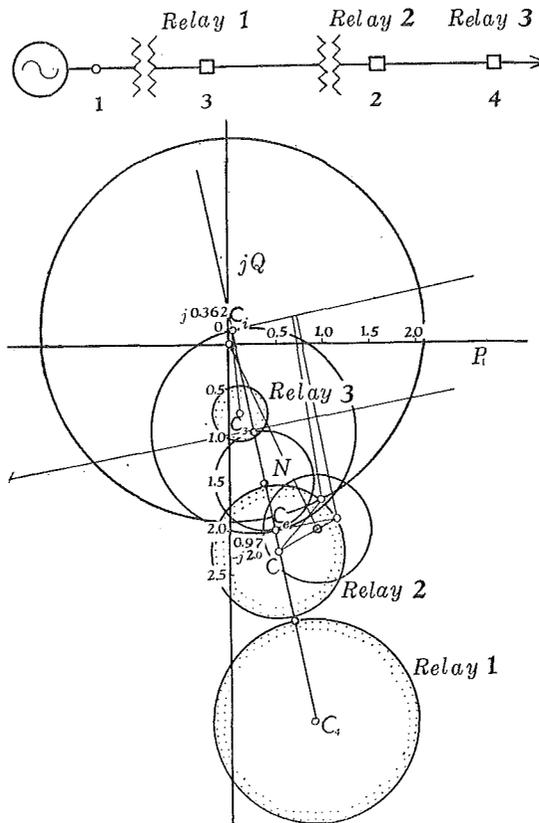


Fig. 14. Impedance Relay.

$$C_e = \frac{D}{B} E_1^2 = 0.975 - j 209 ,$$

$$C_i = \frac{C}{A} E_1^2 = 0.068 + j 0.362$$

Hence, the tripping area of the relay is inside the impedance circle through  $N$ , the point at a quarter distance of  $\overline{C_e C_i}$  from  $C_e$  intersecting rectangle with the circle, having diameter  $\overline{C_e C_i}$  as shown in fig. 14.

For relay 1, it is assumed that its setting point is at exactly at the mid-point of the transmission line. Hence, taking the short circuit and open circuit points as  $C_e' = C_2$ ,  $C_i' = \frac{1}{2} C_i$ , the tripping area of relay 1 may be obtained by the same method as above.

For relay 3 at the receiving feeder, one gets

$$C_e'' = \frac{D'}{B'} E_1^2 = 0.1 - j 0.735, \quad C_i'' = \frac{C'}{A'} E_1^2 = 0.068 + j 0.362 .$$

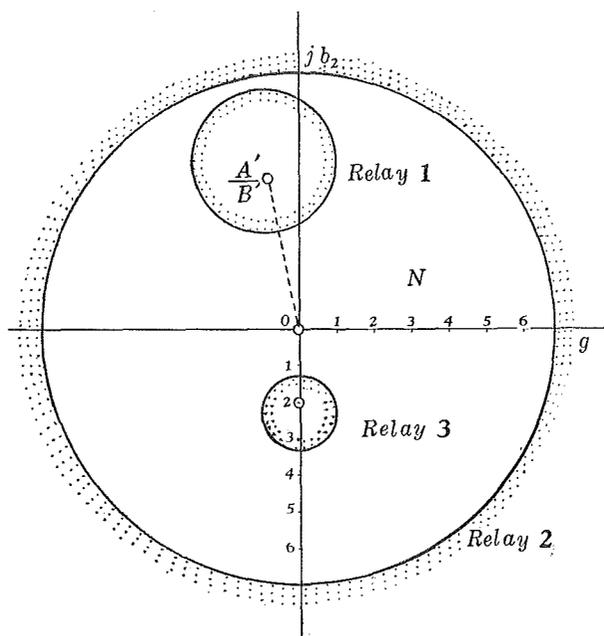


Fig. 15. Impedance Relay on  $y_2$ -plane sending  $E_1$  constant.

Their impedance circles are as shown in fig. 14.

Relay 2 operates at a trisection of the short circuit impedance. Hence,  $\frac{1}{3} \times \left| \frac{\mathbf{B}}{\mathbf{D}} \right| \left| \frac{1}{3(0.975 - j2.09)} \right| = \frac{1}{3 \cdot 2.3}$  ohm impedance circle is the circle with radius 6.9 mho, center at origin on the admittance plane. The tripping area is outside of this circle.

The trip area for relay 1 is the same as the former case in fig. 14, except the unit. The lagging power is taken as positive in this case. With reference to relay 3, the short circuit point is  $\frac{1}{j0.5} = -j2$  on  $\mathbf{Y}_2$ -plane and the open circuit point is at the origin. They are shown as in fig. 15.

(to be continued)