APPENDIX

General Conception of the Circle Diagrams for Electric Power Transmission Circuit with a Sending and a Receiving End.

By

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§ 1. The fundamental circles for circle diagram method.

In a general transmission circuit, various electric characteristics can be graphically expressed on the co-ordinate plane of watt and wattless power at its sending or receiving end by using the fundamental circles, intersecting each other at rectangeles. Such characteristics can as well be derived from the general circuit constants $\boldsymbol{A}\ \boldsymbol{B}\ \boldsymbol{C}\ \boldsymbol{D}$.

The co-ordinates to express a. c. complex quantities at the receiving and the sending end are 3 kinds as,

(1) Watt and wattless power, $\boldsymbol{W}=P+jQ$, M V A, with its own terminal voltage keeping constant.

(2) Admittance, $\boldsymbol{Y}=g+jb$, mho with its own terminal voltage depending on the other one side or both sides constant terminal voltages.

(3) Impedaece, $\boldsymbol{Z}=r+jx=\dfrac{1}{\boldsymbol{Y}}$, ohm,

For the 1st kind of co-ordinate, the various units may be easily exchanged for each other as shown in equations,

$$\boldsymbol{Y}=(g+jb)\text{ mho}=(P+jQ)\div E^2(\text{KV}),$$

$$(p+jq)\text{ p. u.}=(P+jQ)\div\text{Base M V A,} \quad \text{and}$$

$$(\boldsymbol{I}=I_r+{}_jI_i)\text{KV.}=(P+jQ)\div E(\text{K V.})$$

For the 2nd kind of co-ordinate, the circle diagrams including voltage terms take completely different form from the 1st kind, for examples as power and loss although those not including them are the same as the 1st kind as transmission efficiency.

The 3rd kind of co-ordinate is the inversion of the 1st and 2nd kind of co-ordinates.

The values to be shown in circle diagrams are the magnitude of the terminal voltages, the phase angle difference of both the terminal voltages, so called power angle, apparent currents, power factors, apparent admittances, apparent impedances, resistances, conductances, reactances, susceptances, watt power, wattless power of each terminal and the values expressing the ratio or difference of the above values as the transmission efficiency, transmission loss etc. Moreover it is known that a quantity to satisfy the condition to be on the two circle loci at a same time, will be expressed by a stright line, called loss line, output line or efficiency scale.

It is remarkable that the above mentioned many circles can be derived from the following fundamental circles. Let $W_1=P_2+jQ_1$ and $W_2=P_2+jQ_2$ be the co-ordinate planes at the sending and the receivig end, connected through a general transmission circuit with its circuit constants, $\boldsymbol{A}\,\boldsymbol{B}\,\boldsymbol{C}\,\boldsymbol{D}$ as shown in fig. 1. It is assumed that in W_1-plane the sending power and the leading wattless power are positive in the

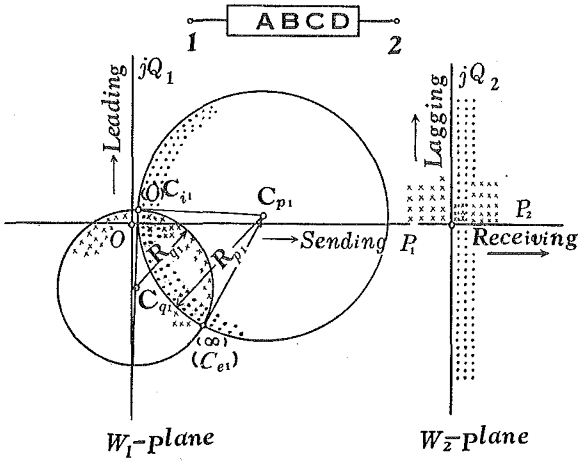

Fig. 1. Transformation of W_2-plane on W_1-plane.

W_2-plane the receiving power and lagging wattless power are positive, taking their own voltages as reference phase. If the terminal voltage E_1 be kept constant, every point on W_2-plane are transfigured on W_1-plane, changing the rectangular co-ordinate axis of W_2-plane into two at rectangle intersecting circles as follows,

$$\left.\begin{aligned} C_{p1} &= \frac{CB_k + DA_k}{AB_k + A_kB} E_1^2 \\ R_{p1} &= \frac{1}{AB_k + A_kB} E_1^2 \end{aligned}\right\} \quad \cdots\cdots\cdots (1)$$

$$\left.\begin{aligned} C_{q1} &= \frac{CB_k - DA_k}{AB_k - A_kB} E_1^2 \\ R_{q1} &= \frac{1}{AB_k - A_kB} E_1^2 \end{aligned}\right\} \quad \cdots\cdots\cdots (2)$$

As shown in fig. 1, the right half side of the vertical axis of W_2-plane, i. e. every value of the receiving power comes in the inside of the C_{p1} circle, and the upper half of the horizontal axis i. e. every lagging reactive power comes in the inside of the C_{q1} circle. The origin and the infinite point of W_2-axis are changed into the following intersecting points of the two circles.

$$C_{i1} = \frac{C}{A} E_1^2 \quad \cdots\cdots\cdots (3)$$

$$C_{e1} = \frac{D}{B} E_1^2 \quad \cdots\cdots\cdots (4)$$

Now, every electrical characteristic of the receiving end, 2 can be read off directly on W_1-plane as desired without using W_2-plane. The above rectangle intersecting circles are available instead of the rectangular co-ordinate of W_2-plane. They may be called the fundamental circles of circle diagrams. They can be treated as a circle co-ordinate axis. These fundamental circles are very important for practical purposes, because every transmission characteristic is derived from them. Further another explanation will be attempted.

The C_{p1} circle is the locus on W_1-plane with E_1 volt, when the receiving load is pure reactance or has zero power factor. The C_{q1} circle is that when the load is pure resistance of 1 power factor. It is called Heyland circle diagram for the induction motor.

In the same way as above, W_1-plane can be transfigurel on W_2-plane, keeping its voltage E_2 contstant, into the two rectangle intersecting circles as shown in fig. 2 and equations (5) (6) (7) (8)

$$\left.\begin{aligned} C_{p2} &= -\frac{CB_k + AD_k}{DB_k + D_kB} E_2^2 \\ R_{p2} &= \frac{1}{DB_k + D_kB} E_2^2 \end{aligned}\right\} \quad \cdots\cdots\cdots (5)$$

$$\left.\begin{aligned} C_{q2} &= \frac{CB_k - AD_k}{DB_k + D_kB} E_2^2 \\ R_{q2} &= \frac{1}{DB_k + D_kB} E_2^2 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (6)$$

$$C_{e2} = -\frac{A}{B} E_2^2 \quad \cdots\cdots\cdots\cdots (7)$$

$$C_{i2} = -\frac{C}{D} E_2^2 \quad \cdots\cdots\cdots\cdots (8)$$

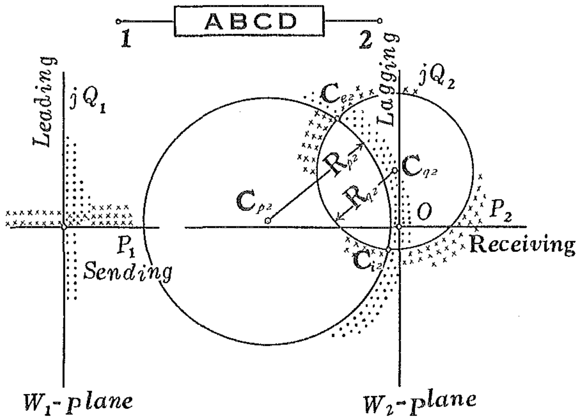

Fig. 2. Transformation of W_1-plane on W_2-plane.

In the W_2-plane, sending powers, P_1 and leading wattless powers, Q_1 are shown in the outsides of C_{p2} and C_{q2} circles respectively.

From the proceeding descriptions, it is concluded that the co-ordinate axis for expressing the complex quantities at a terminal in a general transmission line can be transfigured on another co-ordinate plane, changing its axis into two fundamental circles of the circle diagram method.

§ 2. Derivation of electrical quantities from the fundamental circles.

I. A co-ordinate plane with its own voltage taken as reference phase and kept constant.

(1) Watt and wattless power.

These values are easily gotten from the mean distance to the fundamental circles defined by equation (9) and fig. 3., taking (+) sign for the outside of circles,

$$\left.\begin{aligned} P &= \frac{p(p \pm 2R_p)}{2R_p} \\ Q &= \frac{q\,(q \pm 2R_q)}{2R_q} \end{aligned}\right\} \cdots (9)$$

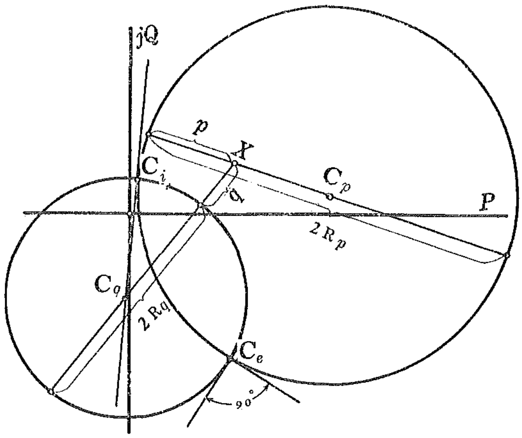

Fig. 3. Circle co-ordinate C_p and C_q circles intersecting at rectangle.

These equations are also applicable for the rectanger co-ordinate, taking $R=\infty$. At the centers, p and q are equal to R_p and R_q respectively. One gets $P=\frac{R_p}{2}$ or $Q=\frac{R_q}{2}$. These values are the maximum possible transmission watt power or wattless power, when the transmission voltage is defined. It can be said therefore that the centers of the fundamental circles are the max. watt power point and the max. wattless power point respectively.

(2) Absolute values and phase angle difference of the sending and receiving voltages. Power circle diagram.

This well-known power circle diagram of transmission circuit has its center at the intersecting point of the fundamental circles, $\boldsymbol{C}_{e2} = \frac{\boldsymbol{D}}{\boldsymbol{B}} E_1^2$ or $\boldsymbol{C}_{e2} = -\frac{\boldsymbol{A}}{\boldsymbol{B}} E_2^2$, and has its radius as $R = \left|\frac{\boldsymbol{E}_1 \boldsymbol{E}_2}{\boldsymbol{B}}\right|$.

Therefore, the absolute value of the other terminal voltage can be easily read off from the radius. For zero or other terminal voltage, the radius becomes zero and the center shows the short circuit point. The sending power circle diagram which has its center at $\boldsymbol{C}_{e1}$ is shown in fig. 4.

The phase angle difference between the two terminal voltages, so called power angle ϕ_{12} is obtained from the angle between the radius through the given working point on the power circle and the zero phase angle radius. This zero phase angle radius makes angle $(-180° -\phi_B)$ in $\boldsymbol{W}_1$-plane and angle $-\phi_B$ in $\boldsymbol{W}_2$-plane to horizontal axis, where ϕ_B is the angle of the transfer impedance $\boldsymbol{B}$. The power angle ϕ_{12} must be read clockwise on $\boldsymbol{W}_1$-plane and countercrockwise in $\boldsymbol{W}_2$-plane in most cases.

Dividing by E_1^2 or E_2^2, one obtains the admittance unit co-ordinate, in which the radius of the circle diagrams change into $\frac{1}{\boldsymbol{B}}\frac{E_2}{E_2}$ or $\frac{1}{\boldsymbol{B}}\frac{E_1}{E_2}$.

(2)a Relation between equivalent π line and power circle diagram.

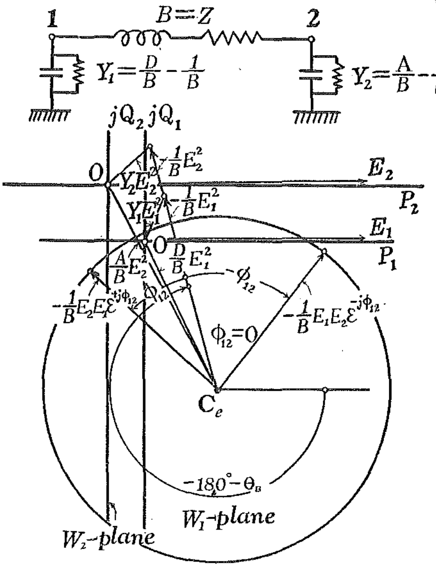

Fig. 4. Relation between Equivalent π line and power circle diagram.

Here is considered a modified power circle diagram taking the receiving power at negative side and the leading power at positive side in W_2-plane in the same way as in W_1-plane so as to be able to use a single circle as shown in fig. 4. Then, this circle diagram may more clearly express not only the relation between the driving impedances $\frac{\boldsymbol{A}}{\boldsymbol{B}}$ or $\frac{\boldsymbol{D}}{\boldsymbol{B}}$, transfer impedance $\frac{1}{\boldsymbol{B}}$ and power angles ϕ_{12}, but also the impedance of the equivalent π-line $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$, $\boldsymbol{Z}$ of a general transmission line, as $\boldsymbol{Y}_1=\frac{\boldsymbol{D}}{\boldsymbol{B}}-\frac{1}{\boldsymbol{B}}$, $\boldsymbol{Z}=\boldsymbol{B}$, $\boldsymbol{Y}_2=\frac{\boldsymbol{A}}{\boldsymbol{B}}-\frac{1}{\boldsymbol{B}}$.

In the cases, of short transmission line, $\boldsymbol{Y}_1\simeq\boldsymbol{Y}_2\simeq 0$ and $\boldsymbol{W}_1$-plane agress with $\boldsymbol{W}_2$-plane using admittance unit.

(3) Manitude and phase angle of another terminal current. Apparent current circle and power factor circle.

On a circle with its center at an intersecting point of the fundamental circles, $\boldsymbol{C}_{i1}=\frac{\boldsymbol{C}}{\boldsymbol{A}}\boldsymbol{E}_1^2$ or $\boldsymbol{C}_{i2}=-\frac{\boldsymbol{C}}{\boldsymbol{D}}\boldsymbol{E}_2^2$ and its radius $R=\left|\frac{I_2E_1}{\boldsymbol{A}}\right|$ or $\left|\frac{\boldsymbol{I}_1\boldsymbol{E}_2}{\boldsymbol{D}}\right|$, the magnitude of the other terminal current is constant. Therefore the circle with its center at the open circuit point of the other terminal is used for apparent current circle diagram.

A circle through the intersecting points $\boldsymbol{C}_e$, $\boldsymbol{C}_i$ of the fundamental circles has a definite power factor, of which the angle is directly measured at the $\boldsymbol{C}_e$ or $\boldsymbol{C}_i$ point.

The fundamental circles belong to the above circles, having zero power factor ($\boldsymbol{C}_p$) and 1 power factor ($\boldsymbol{C}_q$).

(4) The conductance g, the susceptance jb, the resistance r, and the reactance jx of the other terminal. The circles as their loci.

On every circle through the short circuit point C_e and having their centers on connecting line of the short circuit point and the max. watt point $\overline{C_e C_p}$, the terminal conductances g are constant, while on the circles intersecting at a right angle to the above circles, i. e. the circles having their centers on the connecting line of the short circuit point and the max. wattless point, $\overline{C_e C_q}$, the values of terminal susceptance jb, are constant.

In the same manner, the constant terminal resistance, r circles and the constant terminal reactance, jx circles are obtained by taking the open circuit point, C_i instead of the short circuit point, C_e in the above case.

As an example to illustrate the above circles, let the conductance circle $(C_{g1} R_{g1})$ be taken, having a constant conductance g_2 at the receiving end on W_1-plane in fig. 5. The value of g_2 can be graphically obtained as follows. By drawing a circle with its radius as $\left|\frac{E_1}{B}\right|$ and its center at C_{e1}, get the intersecting points with R_{p1} and R_{g1} circle, M and N. Draw two normal lines $\overline{MND}$ and $\overline{NF}$ to the $\overline{C_{e1} C_{p1}}$ line. Then, the length $\overline{C_{e1} D}$ shows the conductance or the real part of short circuit admittance $\frac{D}{B}$ and the length $\overline{DF}$ shows the required conductance, g_2, on the constant conductance circle in the inside as well as the outside of the fundamental circle R_{p1}.

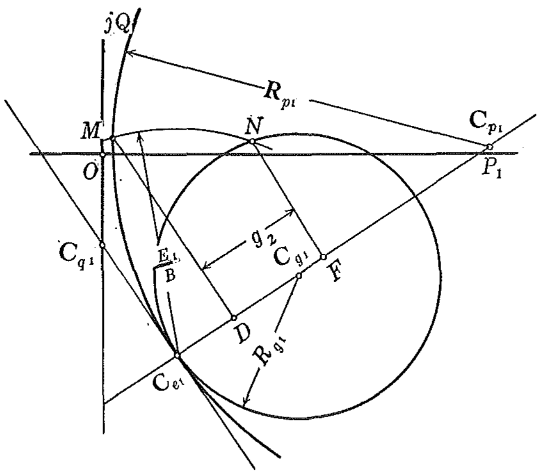

Fig. 5.
Conductance circle and graphical method to obtain g_2.

(5) Apparent admittance and apparent impedance.

The apparent admittance of the terminal, $|Y| = \sqrt{g^2+b^2}$ and its inverse values, the values of apparent impedance $|Z| = \frac{1}{Y} = \sqrt{r^2+x^2}$ are constant on the circles, which intersect at right angles with the circle, having its diameter $\overline{C_e C_i}$ and have their centers on the $\overline{C_e C_i}$ line. Apparent admittance circles and apparent impedance circles are same things. As an example of this method, the admittance circle on W_1-plane is shown in fig. 6. The required apparent admittance of the receiving end is $|Y_2| = \frac{C_{y1}T}{C_{y1}C_{i1}}\left|\frac{A}{B}\right|$, where C_{y1} is the center of the admittance circle.

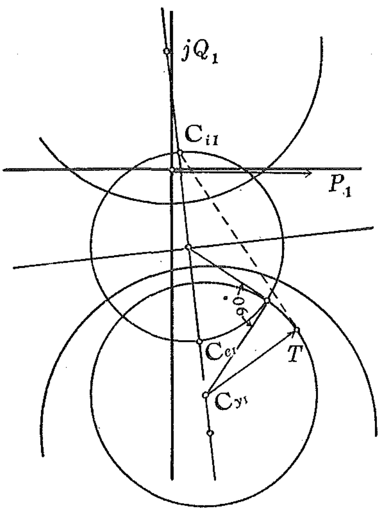

Fig. 6. Apparent admittance or impedance circles.

(6) Min. transmission loss point, max. transmission efficiency point and max. transmission efficiency line. Efficiency circle and loss circle.

The min. transmission loss point, C_v is the nearest point of the fundamental circle, C_p to the vertical axis. The max. transmission efficiency points, B_η, which are gotten from the efficiency circle diagram, have co-ordinates $B_\eta = \pm\sqrt{\alpha^2-R_p^2}+j\beta$, where $\alpha+j\beta = C_p$. These relations in W_1-plane are shown as fig. 7. These two points are in most cases near to the origin of the coördinate plane. For the practical purpose, the max. efficiency line i. e. the horizontal line through the center of the fundamental circle C_p is taken as a best condition for a desired transmission power.

The loss circles are the circles with their center at the above min. loss point, C_e on which the transmission losses are constant.

The transmission efficiency circles to give a constant transmission efficiency have their centers at the above max. transmission efficiency line and intersect at right angles with the circle with diameter $2B_\eta = 2\sqrt{\alpha^2-R_p^2}$, passing through the above max. efficiency points. The transmission efficiency is given by the ratio of the radius R_p of the fundamental circle and the abscissa of the center of the efficiency circle, $\overline{C'_v C_{p1}}$ in fig. 7 can be taken as the efficiency scale.

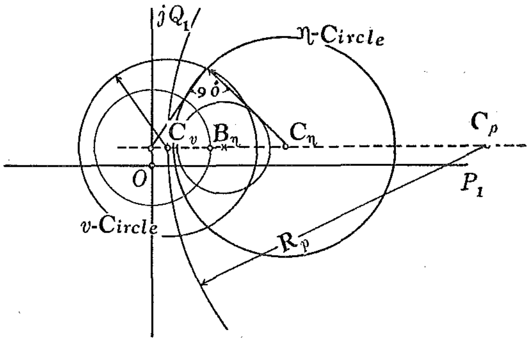

Fig. 7. Efficiency circle and loss circle.

(7) Radical axis of two circles.

If the sending or the receiving power is designated to operate on a locus of a circle, various electrical quantities are expressed graphically by the so-called radical axis of two circles or the straight line through the intersecting points of the above operating circle and the circle which expresses an electrical quantity. For example, in a constant voltage power transmission system, a sending power operates on a

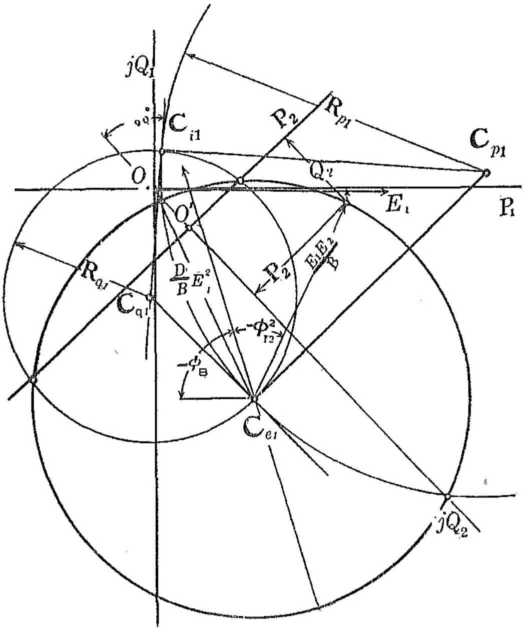

Fig. 8. Output lines P_2 & Q_2 of power circle diagram.

sending power circle diagram. The radical axis of this power circle diagram with the fundamental circles R_p & R_q are OP_2 and OQ_2 as shown in fig. 8. This is just the inverted figure of the well known receiving end power circle diagram in $\boldsymbol{W}_2$-plane. In the same way, one obtains out put line, efficiency line, loss line or resistance line which is called slip line in Heyland circle diagram.

They are well known in the cases of electric machiney. However, this method may also be applied to a transmission line.

(8) Power circle diagrams for frequency swing.

Under the frequency swing condition of two Synchronous machines at the two ends of a transmission line, a large power angle between $\boldsymbol{E}_1$ and $\boldsymbol{E}_2$, is required, resulting in the occurrence of abnormal small voltage points at the middle points of the transmission line. This phenomenon may be easily explained by using power circle diagram as shown in fig. 9. The working point to show a sending power must be always on the circle of which the center and the radius are at

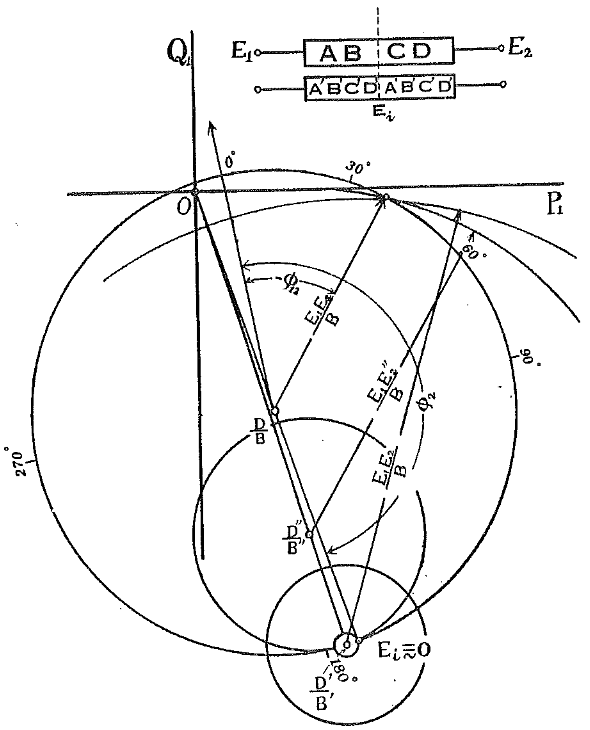

Fig. 9. Frequency swing circles.

$\frac{D}{B}E_1^2$ and $\frac{E_1E_2}{B}$, taking E_2 voltage of the receiving-end, while at an intermediate point, i, on the transmission line, taking its voltage E_i as the standard, the same sending power must be shown on a power circle of which the center is at $\frac{D^1}{B_1}E_1^2$ and the radius $\frac{E_1E_i}{B_1}$. As the value $\frac{D_1}{B_1}$ may be about twice that of $\frac{D}{B}$, the intermediate voltage E_i must be very small as shown in fig. 9, when the power angle ϕ_{12} becomes large or the frequency swings. The various electrical quantities can be easily known by the general rules of circle diagram.

(9) Difference value circle diagrams for various difference quantities between the sending and the receiving-ends.

The difference between various quantities, which are expressed by circle loci at the sending or the receiving-end, and their corresponding values at the opposite and are in general expressed also by circles, for example as line loss, wattless power difference, power factor difference, conductance difference, etc. But, complex quantities are not included. The above relations are easily proveded from the fact that the 2nd order equations of circles do not change the coefficiencts of 2nd order variables by introducing the above difference value in them. These circle diagrams are available to study the line loss or the pilot wire system relay protection.

II. Admittance co-ordinate plane, Y_2 with its own voltage is dependent and the other terminal voltage is kept constant.

In the most cases, the sending power-end has a constant voltage generator and the receiving power-end has no device to adjust its voltage.

In this case, the admittance co-ordinate may be avilable. This expression is convenient for the study of relays. Let us consider that receiving voltage is relaible to the sending voltage. The first group of circles, which do not include the receiving voltage is quite the same as explained in the preceeding section, as power factor, transmission efficiency conductance and susceptance etc. It is to be remembered that the conductance circle to show the conductance at sending-end is same as that of watt power cirle, because by multiplying the constant sending voltage the same figure is gotten.

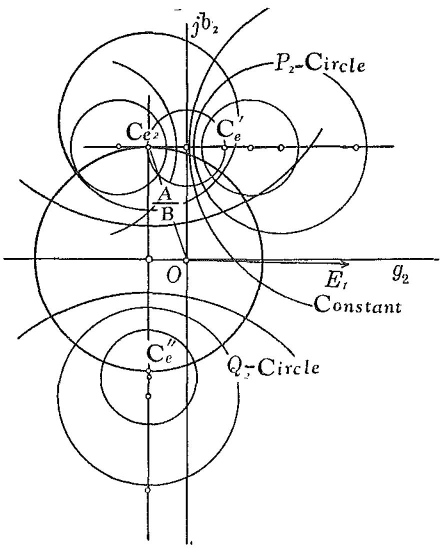

Fig. 10. Watt power and wattless power on receiving admittance co-ordinate plane, when only the sending and voltage is kept constant.

(1) Watt and wattless power.

The second group of circles has the term of dependent voltage at receiving end as the watt power and wattless power at receiving end. Fig. 10 shows the circle diagrams of watt power, P_2, wattless power, Q_2 and the receiving voltage on the receiving end admittance play, $\boldsymbol{Y}_2$. $\boldsymbol{C}_{e2}$ is the driving admittance point, $-\frac{\boldsymbol{A}}{\boldsymbol{B}}$. $\boldsymbol{C}_{e2}'$ and $\boldsymbol{C}_{e2}''$ are the image points with respect to the co-ordinate axis.

Then, the circles, having their centers on $\overline{\boldsymbol{C}_{e2}\boldsymbol{C}_{e2}'}$ line and intersecting at a right angle with the diameter $\overline{\boldsymbol{C}_{e2}\boldsymbol{C}_{e2}'}$ circle are the watt power circles, from which the receiving power, P_2 is obtained by $P_2 = \frac{E_1^2}{2|\boldsymbol{B}|^2 \times \overline{\boldsymbol{C}_{e2}\boldsymbol{C}_1}}$, Similarly, the circles, having their centers on $\overline{\boldsymbol{C}_{e2}\boldsymbol{C}_{e2}''}$ line and intersecting at right angles with the diameter $\overline{\boldsymbol{C}_{e2}\boldsymbol{C}_{e2}''}$ circle are the wattless power circles, from which the receiving wattless power, Q_2 is obtained by $Q_2 = \frac{E_1^2}{2|\boldsymbol{B}|^2 \times \overline{\boldsymbol{C}_{e2}\boldsymbol{C}_2}}$.

(2) Sendig or receiving voltae and their phase angles.

These are the power circle diagrams §2 I (2) expressed in admittance unit as shown in fig. 11.

$$\left.\begin{aligned} g_2 + jb_2 &= -\frac{\boldsymbol{A}}{\boldsymbol{B}} + \frac{1}{\boldsymbol{B}}\left|\frac{\boldsymbol{E}_1}{\boldsymbol{E}_2}\right| \varepsilon^{j\phi_{12}} \\ g_1 + jb_1 &= \frac{\boldsymbol{D}}{\boldsymbol{B}} + \frac{1}{\boldsymbol{B}}\left|\frac{\boldsymbol{E}_2}{\boldsymbol{E}_1}\right| \varepsilon^{-j\phi_{12}} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (10)$$

are the receiving and sending circle equations.

(3) Frequency swing circles on admittance plane at a certain point of a transmission line, connecting two Synchronous machines.

On the admittance plane, $y_p = g_p + jb_p$, at a certain intermediate point, p, of a transmission line connecting two synchronous machines

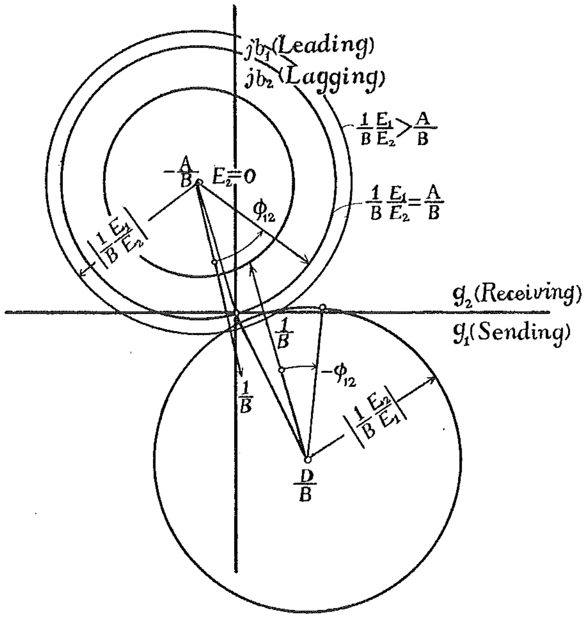

Fig. 11. Power circle diagrams on admittance co-ordinate plane.

at its ends, the power circle between the terminal 1 and p in fig. 12 is

$$y_p = -\frac{A_1}{B_1} + \frac{1}{B_1}\frac{E_1}{E_p} .$$

From the voltages between the intermediate point and the power-receiving end, $E_2 = D_2 E_p - B_2 y_p E_2$, one gets $E_p = \frac{E_2}{D_2 + B y_p}$. By introducing this value in the above equation,

$$y_p = \frac{-A_1 + D_2 \frac{E_1}{E_2}}{B_1 + B_2 \frac{E_1}{E_2}} = \frac{-A_1 + D_2 n \varepsilon^{j\phi_{12}}}{B_1 + B_2 n \varepsilon^{j\phi_{12}}} \qquad \cdots\cdots\cdots\cdots\cdots\cdots (11)$$

whene n is voltage ratio $\left|\frac{E_1}{E_2}\right|$ and ϕ_{12} is the power angle. The locus, expressed by the above equation is that of the frequency swing circles as shown in fig. 12, if the frequency swings of ϕ_{12} changes. They intersect as rights angles to each other with the locus, when the terminal

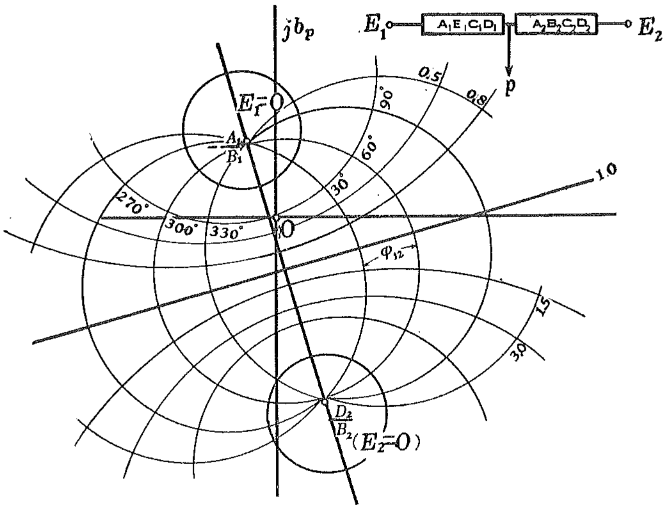

Fig. 12. Frequency swing circles.

voltage ratio, n changes. They have short circuit pornts $-\frac{\boldsymbol{A}_1}{\boldsymbol{B}_1}$ and $\frac{\boldsymbol{D}_2}{\boldsymbol{B}_2}$ as their poles. These curves are useful for the out of step relay.

III. Impedance co-ordinate plane.

The impedance co-ordinate plane is more often used for the study of relays. Inversions of the above described circles on admittance planes are the required circles on the impedance plane.

For example, inversions of the concentric receiving power circles in fig. 11, of which the axes are g_2, jb_2, the center is at $-\frac{\boldsymbol{A}}{\boldsymbol{B}}$ and the radius is $\left|\frac{1}{\boldsymbol{B}}\frac{\boldsymbol{E}_1}{\boldsymbol{E}_2}\right|$, are excentric circles as shown in fig. 13 and by the following equation,

$$R + jx = \frac{\boldsymbol{B}}{-\boldsymbol{A}+\left|\frac{\boldsymbol{E}_1}{\boldsymbol{E}_2}\right|\varepsilon^{j\phi_{12}}} \qquad \cdots\cdots \quad (12)$$

The circle $\frac{\boldsymbol{A}}{\boldsymbol{B}}\frac{E_1}{E_2}=1$ in fig. 11 changes into a straight line. One gets a group of circles intersecting at right angles with their poles,

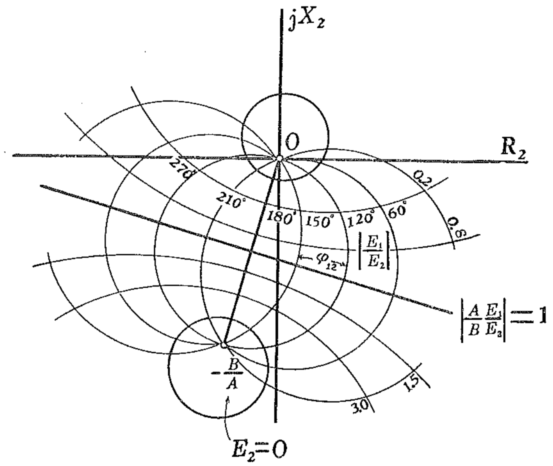

Fig. 13. Receiving power circle diagram expressed on receiving load impedance plane.

the short circuit impedance point $\frac{B}{A}$ and the co-ordinate origin, if $\left|\frac{E_1}{E_2}\right|$ or ϕ_{12} varies respectively.

The circle diagrams on the impedance plane are, in general, complicated for constant voltage system.