Density Current in River Mouth with Small Tidal Range

By

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Abstract

The present paper is an attempt of investigating the penetration of salt water into river mouth on the basis of field observations performed in the mouth of the Ishikari.

Introduction

Penetration of salt water into a river mouth is an interesting problem of density current. The authors performed several field observations in the mouth

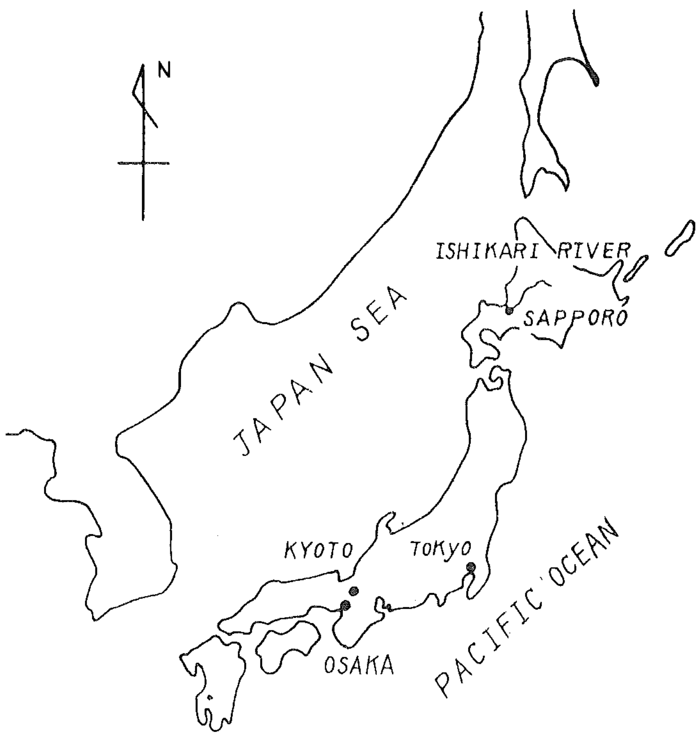

Fig. 1 a. Location of the Ishikari.

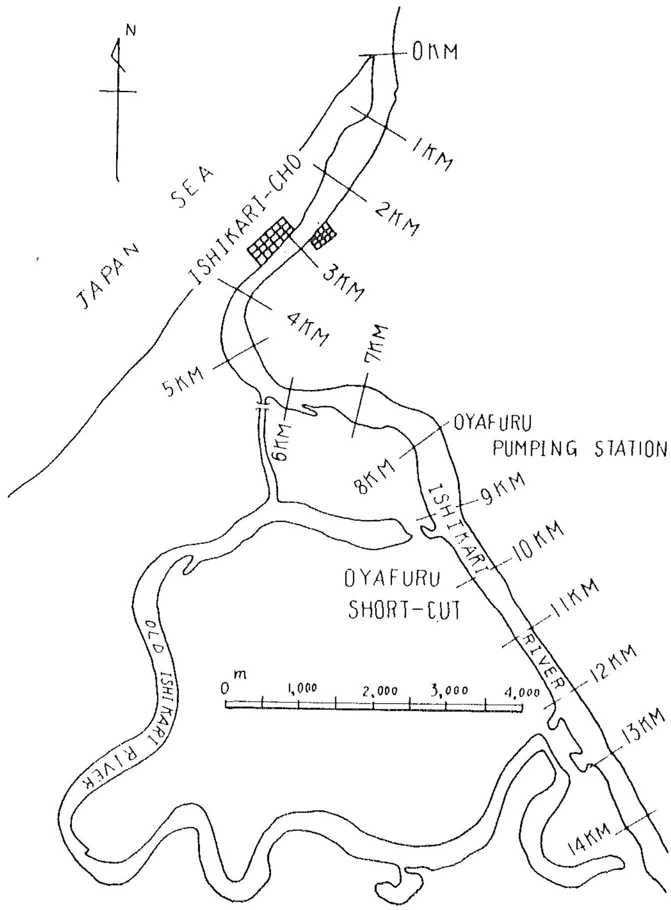

Fig. 1b. Lower reaches of the Ishikari.

of the Ishikari to study the mechanism of the penetration of salt water. The original purpose of the observations was in obtaining a favorable measure of reducing the salinity hazard in its irrigated areas.

The Ishikari drains the Ishikari Plain, Hokkaido Pref., in northern district of Japan and flows into Japan Sea. Its basin area and length are about 14,600 km^2 and 300 km, respectively.

The period from April to May is a high water season for this river because of snow melt. However, the period from June to August is a low water season because of a relatively small precipitation, in spite of a high demand for irrigation water of good quality. According to the records in 1954, for example, the monthly mean discharges are as shown in Table 1.

TABLE 1 Monthly mean discharges at Ebetsu City in 1954

Month	April	May	June	July	August
Mean Discharge (m^3/s)	1,413	935	490	359	596

In low water season, penetrated salt water has often been observed even at Ebetsu City located as far as about 30 km upstream from the mouth, and a considerable salinity hazard has been detected in the fields downstream. According to the statistics from 1954 to 1956, the annual discharge distribution of this river was as follows:

95-days discharge … 599 m^3/s 185-days discharge … 388 m^3/s

275-days discharge … 293 m^3/s 355-days discharge … 258 m^3/s

While, as will be described later, the critical discharge at which the salt wedge is completely washed away from the mouth was estimated to be nearly 500 m^3/s. Therefore, a penetration of salt water and a stratified flow regime must be expected for more than 200 days every year.

In this meaning, the present purpose of our observations was pointed to the two technical problems: (1) the relation between salinity contents in upper fresh-water layer and the distance from the mouth, (2) the length and the shape of salt wedge. In order to give an answer to the first problem, it is necessary to know the mixing mechanism of fresh water and salt water through the surface of salt wedge. In other words, entrainment of salt water in the upper fresh-water layer must first be investigated. For the purpose of clarifying the second problem, it seems to be essential to evaluate the magnitude of shearing stress acting on the surface of salt wedge.

Since Japan Sea into which the Ishikari flows has a relatively small tidal range of only 30 cm in its great diurnal range, the results obtained by employing a basic assumption of steady motion in the following analysis were almost satisfactory for the present purposes.

Entrainment of salt water by upper fresh-water flow

Salt water begins to penetrate into the river mouth when the river discharge decreases under a certain critical value. If an intense turbulence exists in the mouth, river water and penetrated salt water would be mixed and almost uniform density-distribution will be realized. This condition of almost uniform density-distribution has often been observed in a shallow river mouth with great tidal range.

In the case of the Ishikari, however, the tidal range and further the surface slope of river are so small that the mixing of fresh water and salt water is negligible. Therefore, the so-called salt wedge is clearly formed. Under such conditions, the increase of salinity contents of river water flowing on the salt wedge is very small. However, the restriction for salinity contents of irrigation water is very severe. Rice, for example, is said to be very sensitive to salinity as it would be damaged even by 1‰ of salinity concentration. Consequently, the evaluation of high accuracy of such a delicate increase in salinity contents of river water is required.

G. H. Keulegan[1] studied the interfacial instability and mixing in the stratified flow. According to his results the criterion of interfacial instability is given by

$$\theta = \left(\nu_2 g \frac{\Delta\rho}{\rho_1}\right)^{1/3} / u_c = 0.178 \tag{1}$$

where

ν_2 = kinetic viscosity of lower fluid
ρ_1 = density of upper fluid
$\Delta\rho = \rho_2 - \rho_1$, ρ_2 = density of lower fluid
u_c = critical velocity
g = acceleration of gravity

Substitution of $\nu_2 = 0.01$ cm²/s, $g = 980$ cm/s², and $\Delta\rho/\rho_1 = 0.02$ into eq. (1) gives

$$u_c = 3.3 \text{ cm/s}$$

Further, Keulegan gave the following expression for the rate of mixing i. e., the upward velocity v at the boundary surface:

$$v = K(u - 1.15\, u_c) \tag{2}$$

where

K = a coefficient
u = relative velocity of fluids in two layers

and the value of K determined by his experiments was a constant:

$$K = 3.5 \times 10^{-4} \tag{3}$$

Then the increase of salinity content of upper layer can be calculated by the

1) G. H. Keulegan: Interfacial instability and mixing in stratified flows, Journ. of Res., Nat. Bur. Stand., Vol. 43, 1949.

use of eq. (2). However, as H. Stommel[2] pointed out, the numerical value of K given by eq. (3) can not be extrapolated to the large scale of a river mouth.

The authors evaluated the numerical value of K by the following procedure. Equations of mass conservation and volume conservation in stratified flow can be obtained by assuming zero velocity of lower layer:

$$\left.\begin{aligned} d\,(A_1\rho_1 u_1)/dx &= B\rho_2 v \\ d\,(A_1 u_1)/dx &= Bv \end{aligned}\right\} \tag{4}$$

where

A_1 = sectional area of upper layer
B = width of upper layer
ρ_1 = density of upper layer
ρ_2 = density of lower layer
u_1 = velocity of upper layer
v = mixing velocity of both layers

Calculation of $d\rho_1/dx$ in eq. (4) under the assumption of $d\rho_2/dx \fallingdotseq 0$ leads to

$$\frac{d\varepsilon}{dx} + \frac{Bv}{A_1 u_1}\varepsilon = 0 \tag{5}$$

where

$$\varepsilon = (\rho_2 - \rho_1)/\rho_2$$

Since u_c is generally very small compared with u_1, substitution of eq. (2) into eq. (5) gives

$$\varepsilon = \varepsilon_0 \exp\left[-\int_{x_0}^{x} (K/h)\,dx\right] \tag{6}$$

where

$\varepsilon_0 = \varepsilon$ at point $x = x_0$
h = depth of upper layer

When the salinity concentrations of both layers and depth of upper layer are determined by field observations, the value of K can be calculated by eq. (6).

Fig. 2 shows a result of measurements of velocity and Cl concentrations in water of the Ishikari.

2) H. Stommel: The role of density currents in estuaries, Proc. Minesota Int. Hyd. Conv., 1953.

As will be seen in Fig. 2 three distinct layers of Cl concentrations can be found. The first is the surface layer in which Cl concentration and its vertical gradient are very small. The second is the transition layer in which Cl concentration increases rapidly. The third is the bottom layer of uniformly high Cl concentration. The depth of the surface layer gradually decreases in the direction downstream and the one of the transition layer increases in the same direction. The position of zero velocity is at the middle of the transition layer.

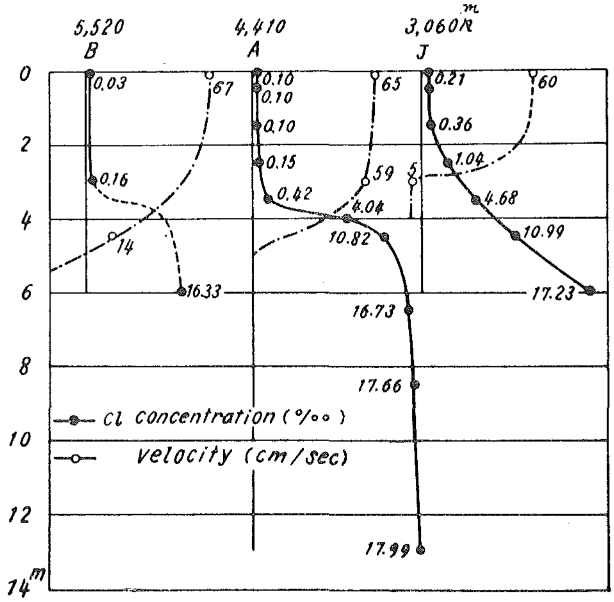

Fig. 2. Velocity- and salinity-distributions in the Ishikari.

The values of K calculated by eq. (6) differs according to the definition of upper layer whether it includes the transition layer. The choice of its definition may depend on the purpose of calculation. From the view point of practical use, the transition layer is excluded and the depth of surface layer is used for h in eq. (6) in the present case, because the intakes of irrigation water are in the surface layer.

Seven field observations were performed during the period of 1942 and 1957 in the Ishikari. Relations between mean salinity concentration in the surface layer and the distance from the mouth are shown in Fig. 3.

In all cases, the salinity concentration varies exponentially with the distance from the mouth. Moreover, the rate of variation of salinity concentration seem to be relatively unchanged. These facts are quite important for practical purpose as well as they are of interests in treating the mixing of fresh water and salt water.

Transforming eq. (6) into a equation of salinity, one has

$$\frac{s_{2x}-s_{1x}}{s_{20}-s_{10}} = \exp\left[-\int_{x_0}^{x} (K/h)\, dx\right] \qquad (7)$$

where

s_{1x} = salinity concentration in the surface layer at point x

s_{2x} = salinity concentration in the bottom layer at point x

$s_{\,0}$ = salinity concentration in the surface layer at point x_0

s_{20} = salinity concentration in the bottom layer at point x_0

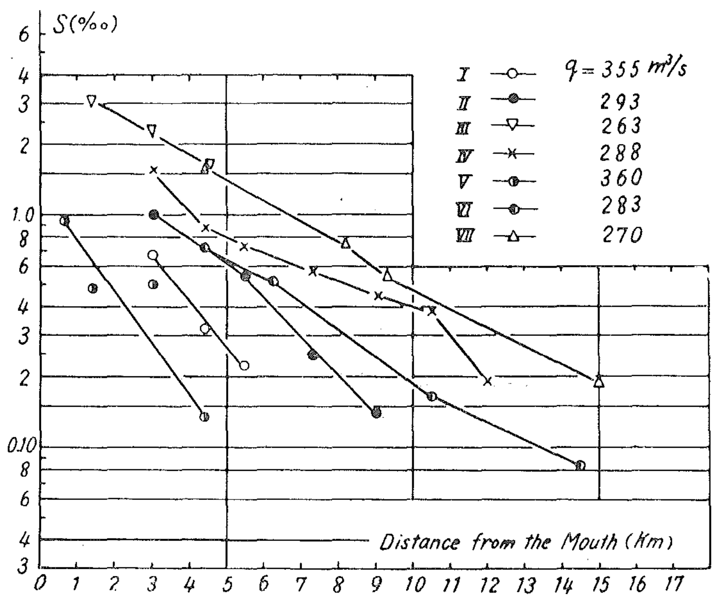

Fig. 3. Salinity concentration in the surface layer in the Ishikari.

When $s_{1x} \ll s_{2x}$, $s_{10} \ll s_{20}$ and $s_{2x}=s_{20}$ are assumed, eq. (7) is approximately reduced to

$$1-\frac{s_{1x}-s_{10}}{s_{20}} \doteqdot \exp\left[-\int_{x_0}^{x} (K/h)\, dx\right] \qquad (8)$$

As are shown in Fig. 3, s_{1x} increases exponentially so that the value of (K/h) has to increase as the absolute value of salinity concentration increases. When the shape of salt wedge for a certain discharge is considered, the depth of surface layer would remain comparatively unchanged in a long distance excepting the part near the mouth. As an exponential variation of salinity exists in the layer of constant depth upstream of this part, the value of K necessarily increases as the salinity concentration of surface layer increases. In other words, K must have a tendency to decrease with the distance from the mouth.

The values of K calculated by eq. (6) are shown in Fig. 4.

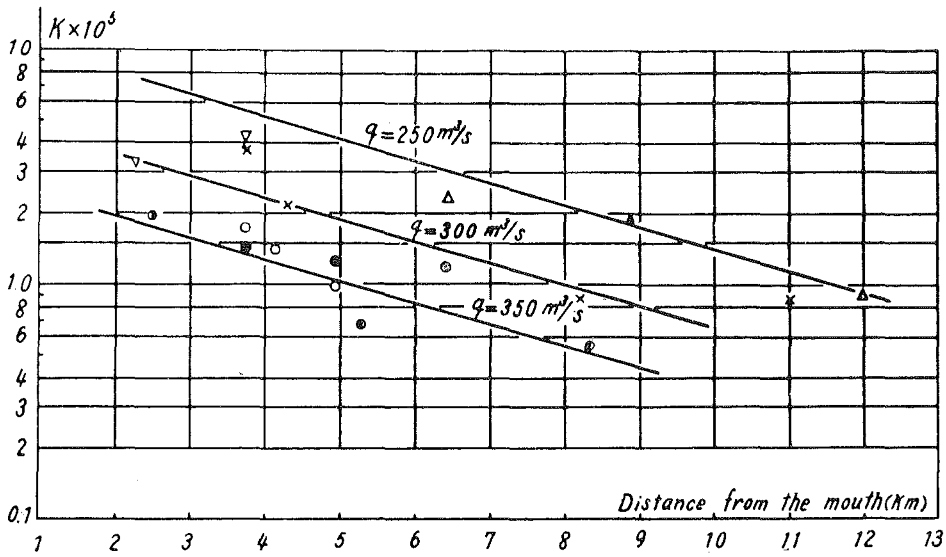

Fig. 4. Values of K in the Ishikari.

As was expected, K shows an exponential decrease with distance. When the river discharge decreases, the depth of surface layer also decreases. Namely, the relation of K and discharge is left unknown. A tendency that K depends upon the river discharge can be seen in Fig. 4. In Fig. 4, full lines show the representative values of K for three different discharges. The fact K is a function of discharge means that the variation of depth of surface layer with discharge is not so large as to result in the increase of salinity shown in Fig. 3. The rough figure of the mean value of K is 1.7×10^{-5} and this is about one twentieth of Keulegan's value given in eq. (3).

In Fig. 3, the river discharge of each observation is also indicated. As will be naturally expected, salinity concentration at any point in the river changes with the discharge and increases as the discharge becomes small. The relation between the salinity concentration and the discharge prepared from the observed values in Fig. 3 are shown in Fig. 5.

The plotted points shown in Fig. 5 are the results obtained at $x=3$ km and

4.4 km. The other lines were estimated by inter- or extra-polation of Fig. 3.

According to Fig. 5, the critical discharge at which the salt wedge is completely washed away from the mouth can be estimated to be about 480 m³/s. H. Stommel & G. Farmer[3], and J. Schijf & J. Schönfeld[4] have shown inde-

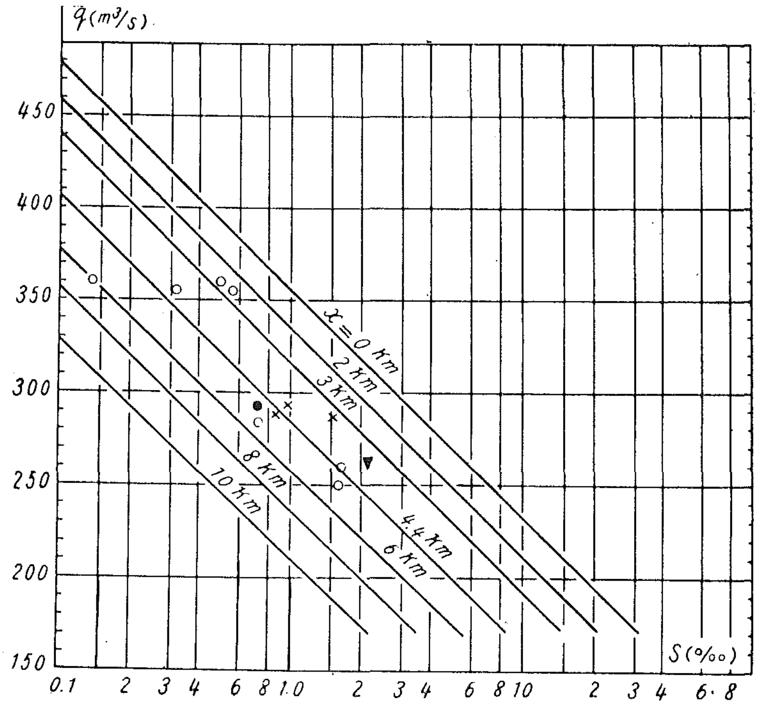

Fig. 5. Relation between the surface salinity and the river discharge.

pendently that the internal Froude number becomes unity at the river mouth.

$$F_i = u_i^2/\varepsilon g h_1 = 1 \tag{9}$$

Then the critical discharge q_0 becomes

$$q_0 = \sqrt{\varepsilon g} \cdot A \cdot \sqrt{D} \tag{10}$$

where

A = total sectional area of the river at the mouth

D = total depth at the mouth

3) H. Stommel & G. Farmer: Abrupt change in width in two layer open channel flow, Journ. Mar. Res., Vol. XI, No. 2, 1952.

4) J. B. Schijf and J. C. Schönfeld: Theoretical considerations on the motion of salt and fresh water, Proc. Minesota Int. Hyd. Conv., 1953.

$$\varepsilon = (\rho_2 - \rho_1)/\rho_2$$

In the Ishikari, $A=600\ \mathrm{m}^2$, $D=3\ \mathrm{m}$. When $\varepsilon=0.02$, $q_z=450\ \mathrm{m/s}$ resulted. This value agrees fairly well with the estimated value of $480\ \mathrm{m}^3/\mathrm{s}$.

Austausch coefficient (turbulent diffusivity) in the stratified flow

Salt water transfered from the bottom layer diffuses into the river-water layer by the turbulent diffusion. As are shown in Fig. 2. diffusion of salinity is concentrated in a relatively thin transition layer which contacts with the bottom layer, and only a small quantity of salinity diffuses into the upper surface layer. The distribution of salinity shown in Fig. 2 would resemble to that of the fine suspended sand in ordinary rivers. Such a distribution of salinity suggests that the turbulence in the water flowing on the salt wedge is presumably decayed to a considerable extent. T. Hamada[5] evaluated the value of eddy viscosity η from his results of velocity measurements in the mouth of the Shinano-river in Niigata Pref., Japan, in 1947 and reported that turbulence in river water decreases in the direction downstream. According to his results, η of the river water on salt wedge was about 100~20 c.g.s.

The authors evaluated the diffusivity η in the Ishikari[6] from the salinity distribution. The data used were obtained in the field observations shown in Fig. 3. The steady state salinity distribution will be considered. Let x-axis be horizontal and z-axis be vertical upward. The condition of continuity of salinity is expressed by

$$\frac{\partial}{\partial z}\left(\eta\frac{\partial s}{\partial z}\right) = \frac{\partial(us)}{\partial x} \tag{11}$$

where

s = salinity concentration

u = velocity of flow

η = diffusivity

When the surface condition of $(\partial s/\partial z)_{z=a}=0$ is considered, η at any depth z becomes

$$\eta = \int_a^z \frac{\partial(us)}{\partial x}\,dz \Big/ \left(\frac{\partial s}{\partial z}\right)_{z=z} \tag{12}$$

The results of calculation of this equation are shown in Table 2.

5) T. Hamada: Density current problems in an estuary, Proc. Minesota Int. Hyd. Conv., 1953.

6) H. Fukushima: On the eddy diffusion in the water layers in estuary, Bull. Faculty of Eng. Hokkaido University, No. 12, 1955.

TABLE 2 Values of turbulent diffusivity in the Ishikari

z (m)	u (cm/s)	s (Cl‰)	us	Δus	$\left(\frac{\partial s}{\partial z}\right)_z$	$\left(\frac{\partial s}{\partial z}\right)_{\mathrm{mean}}$	η (c.g.s.)
(I) St. B							
0	67	0.03	2.01	4.5	0.04×10^{-2}	0.02×10^{-2}	11.5
0.5	59	0.05	2.95	3.4	0.05	0.03	19.0
1.5	49	0.10	4.90	1.3	0.04	0.05	15.8
2.5	38	0.14	5.32	2.2	0.04	0.16	20.0
3.0	32	0.16	5.12	14.9	0.04	0.15	36.7
3.5	26	0.18	4.68	19.7	0.08	3.59	29.8
4.0	20	0.22	4.40	22.6			
St. A							
0	65	0.10	6.5	5.7	$0.0\ \times10^{-2}$	$0.0\ \times10^{-2}$	
0.5	64	0.10	6.4	5.2	0.0	0.08	8.5
1.5	62	0.10	6.2	14.7	0.05	0.37	45.6
2.5	60	0.15	7.5	19.5	0.28	0.53	59.2
3.0	59	0.29	23.0	13.9	0.26		
3.5	58	0.42	24.4	33.5	7.24		
4.0	57	4.04	230				
St. J							
0	58	0.21	12.2		-0.02×10^{-2}		
0.5	58	0.20	11.6		0.16		
1.5	58	0.36	20.9		0.68		
2.5	26	1.04	27.0		0.78		
3.0	-5	1.82	-9.1				
(II) St. A							
0	67	0.38	25.5		-0.02×10^{-2}		
0.5	64	0.37	20.7		-0.01		
1.5	59	0.36	21.2		0.11		
2.5	55	0.47	25.9				
3.0	52						
St. B							
0	37	0.14	5.2	20.3	0.02×10^{-2}		46.0
0.5	36	0.15	5.4	15.3	0.03		76.6
1.5	33.5	0.18	4.1	15.1	0.29		12.6
2.5	31	0.47	16.6	11.3	0.64		7.3
3.0	30	0.79					
(II) St. A							
0	49	0.37	18.1	4.9	0.02×10^{-2}		9.2
0.5	47	0.38	17.9	1.0	0.00	0.04×10^{-2}	6.5
1.5	44	0.38	16.7	1.0	0.05	0.25	1.3
2.5	41	0.43	17.6	4.5			
3.0	39						
St. J							
0	48	0.48	23.0		-0.08×10^{-2}		
0.5	43	0.44	18.9		0.08		
1.5	34	0.52	17.7		0.04		
2.5	23	0.96	22.1				
3.0	20	12.51	25.0				
(III) St. A							
0	59	0.81	48	17	0.00×10^{-2}	0.03×10^{-2}	21.0
0.5	59	0.81	48	16	0.01	0.03	60.5
1.5	59	0.82	48	22	0.39	0.53	6.8
2.5	59	1.21	71	66			

z (m)	u (cm/s)	s (Cl‰)	us	Δus	$\left(\frac{\partial s}{\partial z}\right)_z$	$\left(\frac{\partial s}{\partial z}\right)_{mean}$	η (c.g.s.)
St. J							
0	64	1.02	65		0.06×10^{-2}	0.03×10^{-2}	7.4
0.5	64	1.05	67		0.04	0.03	26.2
1.5	64	1.09	70		0.67	0.53	0.5
2.5	64	2.14	137				
St. K							
0	45	1.72	77.4	12.4	0.04×10^{-2}		
0.5	45	1.74	78.3	11.3	0.04		
1.5	45	1.78	80.0	10.0	4.04		
2.5	45	5.82	262	125			
(V) St. A							
0	59	0.06	3.54	3.14	0.00×10^{-2}	0.02×10^{-2}	11.6
1.0	56	0.06	3.36	3.31	0.00	0.02	23.9
2.0	53	0.06	3.18	2.37	0.00	0.21	3.1
3.0	50	0.05	2.75	4.18	−0.005	0.47	2.1
4.0	47	0.05	2.35	5.55	−0.005	0.78	1.4
5.0	11						
St. J							
0	47	0.14	6.58	4.4	0.04×10^{-2}	0.02×10^{-2}	13.1
1.0	37	0.18	6.67	3.8	0.04	0.02	24.5
2.0	25	0.22	5.55	4.3	0.41	3.26	0.0
3.0	11	0.63	6.93	235	0.95		
4.0	5	1.58	7.90		1.56		
5.0	−2	3.14	−6.28				
St. K							
0	45	0.25	11.0	6.8	-0.005×10^{-2}	0.08×10^{-2}	11.5
1.0	43	0.245	10.5	48.5	−0.005	0.39	19.3
2.0	41	0.24	9.9		6.11		
3.0	38	6.35	242				
St. L							
0	81	0.22	17.8		0.16×10^{-2}		
0.5	70	0.38	26.6		0.62		
1.0	59	1.00	59.0		0.56		
1.5	48	1.56	74.9				

As will be seen in Table 2, the maximum value of η which was obtained in observation II is 76.6 c.g.s., and more frequent values fall between 30 and 10 c.g.s. The velocity distributions were measured at only several stations near the mouth, and the decay of η could not be discussed. However, the order of magnitude of η obtained in the Ishikari is rather smaller compared with those in ordinary rivers. And this facts simply shows that the turbulence decays in the river water flowing on the salt wedge. According to the results shown in Table 2, the values of $(\partial s/\partial z)$ in the upper layer $10^{-1}\sim10^{-2}$ [‰/m].

Shape of wedge and shearing stress on its surface

Analytical attempts of determining the shape of salt wedge have been made by H. Farmer & G. Morgan[7] and J. Schijf & J. Schönfeld[8]. In their practical application, however, one has to know the magnitude of shearing stress acting on the surface of salt wedge. So far as the authors are aware of the sufficient data have not yet been prepared to evaluate the shearing stress on the surface of salt wedge. The results calculated by the use of the data obtained in the Ishikari will be presented. When the notations shown in Fig. 6 are used, the equations of steady motion in the stratified flow can be given respectively for the upper and lower layers:

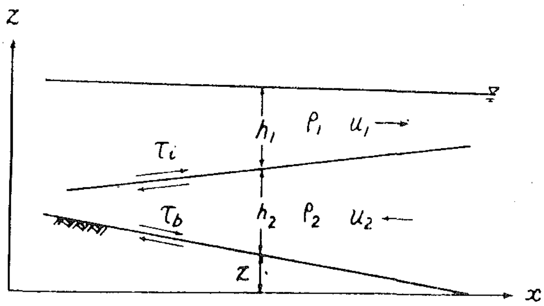

Fig. 6. Definition sketch of stratified flow system.

$$u_1\frac{du_1}{dx}+g\frac{dh_1}{dx}+g\frac{dh_2}{dx}+\frac{\tau_i}{\rho_1 h_1}-gi_b+\frac{1}{2}\cdot\frac{gh_1}{\rho_1}\cdot\frac{d\rho_1}{dx}=0 \tag{13}$$

$$u_2\frac{du_2}{dx}+\frac{\rho_1}{\rho_2}\cdot g\cdot\frac{dh_1}{dx}+g\frac{dh_2}{dx}-\left(\frac{\tau_i-\tau_b}{\rho_2 h_2}\right)$$
$$-gi_b+\frac{1}{2}\cdot\frac{gh_2}{\rho_2}\cdot\frac{d\rho_2}{dx}+\frac{gh_1}{\rho_1}\cdot\frac{d\rho_1}{dx}=0 \tag{14}$$

where

τ_i = shearing stress on tde surface of salt wedge

τ_b = shearing stress on the bed

i_b = $-dz/dx$=slope of the bed

J. Schijf & J. Schönfeld obtained the following equation by eliminating dh/dx from eqs. (13) and (14), under the assumption of $u_2=0$ and $d\rho/dx=d\rho_2/dx=0$:

$$\varepsilon g\frac{dh_1}{dx}+u_1\frac{du_1}{dx}+\frac{\tau_i D}{\rho_m h_1(Dh_1)}=0 \tag{51}$$

7) H. Farmer and G. Morgan: The salt wedge, Proc. Third Conf. Coastal Eng., 1952.
8) see 4).

where

$$D = h_1 + h_2 = \text{total depth}$$

$$\rho_m = \frac{1}{2}(\rho_1 + \rho_2)$$

When a frictional coefficient of Chézy's type C_i is introduced, τ_i is expressed as

$$\tau_i = \frac{\rho_m g u_1^2}{4C_i^2} \qquad (16)$$

Substitution of eq. (16) into eq. (15) gives

$$\varepsilon g \frac{dh_1}{dx} + u_1 \frac{du_1}{dx} + \frac{g u_1^2 D}{4C_i^2 h_1(D-h_1)} = 0 \qquad (17)$$

While, the equation of continuity is given by

$$\frac{dq}{dx} = \frac{d(Bh_1u_1)}{dx} = 0 \qquad (18)$$

where

B = width of the river-water layer

q = discharge of the river

when $u_1\ du_1/dx$ is eliminated between eq. (17) and eq. (18), the result can be reduced to

$$\frac{dh_1}{dx} = \frac{1}{\left(\varepsilon g - \dfrac{q^2}{B^2h_1^3}\right)} \left[\frac{q^2}{B^3h_1^2}\frac{dB}{dx} - \frac{gq^2D}{4C_i^2B^2h_1^3(D-h_1)}\right] \qquad (19)$$

By comparing the results of numerical integration of eq. (19) with those of field observations, one can determined the value of C_i^2. An example of the results obtained in the Ishikari is shown in Table 3. The value of h_1 used in the present case is the depth for zero velocity of flow (See Fig. 2).

It can be seen in Table 3 that the coefficient C_i^2 is not a constant but has a tendency to decrease slightly towards the mouth. And the shearing stress obtained by eq. (16) seems to increase towards the mouth. The magnitudes of τ_i are about 0.2~1.0 dyne/cm^2.

As is described above, the values of C_i^2 vary from place to place on one salt wedge. Furthermore, it was found that the values of C_i^2 were affected by the river discharge. Consequently, an accurate determination of the shape of salt wedge by the use of eq. (19) must be said to be difficult at the present stage of knowledge until a sufficient knowledge of shearing stress can be obatined.

TABLE 3 Shearing stress on the surface of the salt wedge observed in the mouth of the Ishikari

Distance from the mouth (m)	11,000	10,500	10,000	9,500	9,000	8,500	8,000	7,500	7,500	6,500	6,000
q (m^3/s)					283						
$\varepsilon = \frac{\rho_2 - \rho_1}{\rho_2}$					0.02						
h_1 (m)	3.26	3.25	3.20	3.15	3.10	3.05	2.99	2.93	2.88	2.83	2.73
D (m)	5.51	4.95	4.15	5.57	3.96	3.55	4.00	4.46	4.97	8.15	7.57
B (m)	219	250	288	238	378	459	368	324	282	214	186
C_i^2 (m/s^2)	30,000	20,000	30,000	20,000	10,000	30,000	30,000	6,800	9,000	5,000	10,000
τ_i (dyne/cm^2)	0.15	0.23	0.16	0.25	0.51	0.18	0.18	0.84	0.66	1.22	0.66

As have been shown by H. Farmer & G. Morgan and J. Schijf & J. Schönfeld, the shape of salt wedge is rather easy to be determined when the width and the depth of river are assumed to be constant. Although these assumptions are not generally applicable, their analytical results would be useful for the practical purpose when a correction factor of the apparent shearing stress for the above assumptions is found. However, as are shown in eq. (15), for example, their results involve another unknown quantity of total depth D. It is attempted to remove this ambiguous point in the present paper.

In place of eqs. (15) and (16), the following approximate relations are employed:

$$\varepsilon g \frac{dh_1}{dx} + u_1 \frac{du_1}{dx} + \frac{2\tau_i}{\rho_1 h_1} = 0 \tag{20}$$

$$\tau_i = \rho_1 g u_1^2 / 2 C_f^2 \tag{21}$$

When the width of river is assumed to be constant, integration of eq. (20) gives

$$-x = \frac{C_f^2}{g} \left\{ \frac{1}{4} h_1 \left(\frac{h_1}{h_c} \right)^3 - h_1 + \frac{3}{4} h_c \right\} = \frac{C_f^2}{g} F\left(\frac{h_1}{h_c} \right) \tag{22}$$

where

$$h_c = \sqrt[3]{\frac{q^2}{\varepsilon g B^2}}$$

The values of C_f^2 will differ from those given in Table 3, because eq. (20) is an approximate expression and eq. (22) is derived under the assumption of

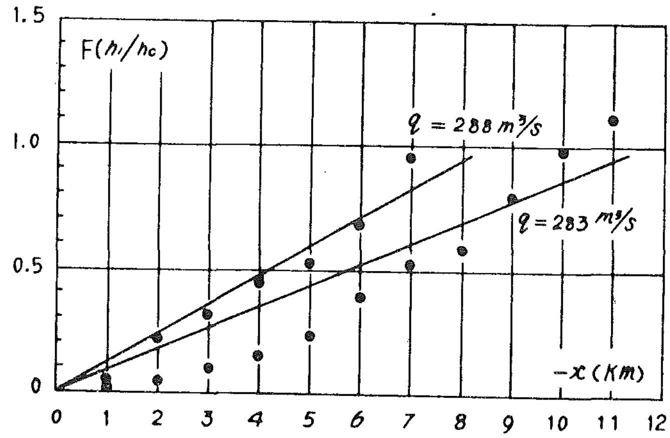

Fig. 7. Comparison of the relation (22) with the observed results

TABLE 4 Compraison of calculated depth of river-water layer with the observed depth

Distance from the mouth (km)		1	2	3	4	5	6	7	8	9	10	11
$q=288$ (m^3/s)	Measured depth h_1 (m)	2.35	2.60	2.80	2.90	2.95	3.05	3.20				
	Calculated depth h_1 (m)	2.59	2.73	2.83	2.92	3.00	3.07	3.13				
$q=283$ (m^3/s)	Measured depth h_1 (m)	2.30	2.40	2.50	2.60	2.70	2.80	2.90	2.99	3.10	3.20	3.26
	Calculated depth h_1 (m)	2.51	2.64	2.73	2.82	2.89	2.94	2.99	3.05	3.08	3,14	3.18
$q=270$ (m^3/s)	Measured depth h_1 (m)								2.65			
	Calculated depth h_1 (m)								2.98			
$q=178$ (m^3/s)	Measured depth h_1 (m)								2.30			
	Calculated depth h_1 (m)								2.34			

constant width.

The relation in eq. (22) is examined by the use of the field data in shown Fig. 7.

Though the value of C_f^2/g varies more or less with discharge, it may be regarded as a constant for one salt wedge. The depth of upper layer h_1 calculated by eq. (22) and Fig. 7 are compared with the observed depths in Table 4. Although several assumptions were employed in the above processes, the agreement between analysis and observation was unexpectedly good.

In order to examine the effects of river discharge on the value of C_f^2/g, the cases $q=270\ \mathrm{m^3/s}$ and $170\ \mathrm{m^3/s}$ are calculated in Table 4. These calculations are based on the value of C_f^2/g for $q=283\ \mathrm{m^3/s}$. The calculated depths agree fairly well with the observed results. Therefore, when the discharge is relatively small, it would be concluded that the variation of the value of C_f^2/g is comparatively small so that its value may be regarded as a constant independent of the river discharge.

Summary

This paper consists of three parts. In the first part, entrainment of salt water by upper fresh-water flow is discussed. Two distinct layers of salinity concentrations are found in the river water flowing on the salt wedge. The one is the surface layer of very low salinity concentration and the other is the transition layer of considerbly high salinity concentration. When the rate of mixing proposed by G. Keulegan $v=K\ (u-1.15\ u_c)$ is applied in the surface layer, coefficient K was not a constant and was found to vary with the river discharge and the distance from the mouth. Next, a critical discharge at which the salt wedge is completely washed away is studied. The calculated value of $q_0=450\ \mathrm{m^3/s}$ agreed well with the observed value of $q_0 \doteqdot 480\ \mathrm{m^3/s}$.

In the second part, turbulent diffusivity calculated from the salinity distribution in the river water flowing on the salt wedge is discussed. Salinity gradient is large in the transition layer and is very small in the upper surface layer. Therefore, the diffusivity in these layers was expected to be small and was 10$\sim$30 c.g.s. The small value of diffusivity would show the decay of turbulence in the water flowing on the salt wedge.

In the last part, the shape of salt wedge and the magnitude of shearing stress on the surface of wedge are discussed. The rough figure of the magnitudes of shearing stress were 1 dyne/cm^2 or less. And it was found that the shearing stress varies with discharge and place. As an accurate calculation of the shape of salt wedge is difficult for the present knowledge of shearing stress, an approximate method is proposed.