Charts for Estimating the Bending Moments Caused by Thermal Deformation of Uniform Rectangular Rigid Frames

By

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Synopsis

This paper presents the results of studies on the bending moments in the beams, footing beams and columns of uniform rectangular rigid frames caused by the temperature difference between the overground part of frame and its footing. Some theoretical analysis is attempted and 240 examples are computed by changing the number of spans, length of beam and column, relative stiffness of beam and column, etc. The results are summarized in several charts for the convenience of practical estimation of these bending moments.

1. Introduction

Certain bending moments will be caused in the beams and columns of rigid frames, when some vertical loads or horizontal loads are applied. The method of calculation of these bending moments has already been derived in the decade between 1920 and 1930.

On the other hand, another type of bending moment will be caused in the members of rigid frame, when the temperature of each part of frame is different. It is named the thermal bending moment in this paper. Some investigations on this kind of bending moment were established for most simple cases, for instance, for one storied frames whose footings were perfectly fixed. However, there is very few investigations for muliple-storied rigid frames.

This paper presents the summaries of the studies on the thermal bending moments in uniform rectangular rigid frames of multiple stories. At first, some theoretical analysea are attempted on the general features of thermal bending moments. Then 240 examples are computed by a modified "moment distribution method" and the results are summarized in several charts for practical estimation of thermal bending moment in uniform rectangular rigid frames. The values of the bending moments can be easily derived from the charts with a sufficient accuracy, so for as the number of spans, relative stiffness ratio of beams and footing beams against the column, dimension of frames and the temperature difference between the overground frames and their footing beams are known.

2. Assumptions

Following assumptions and conditions are set up:

(1) The object is limited in a uniform rectangular rigid frame.

(2) The whole of overground frame is exposed in the same temperature and there is a certain temperature difference between the overground part of frame and its footings.

(3) The length change of columns is neglected, because the axial forces in the columns caused by thermal deformation are usually very small.

3. Symbols

h: length of column

l: ,, of beams and footing beams

A: sectional area of beam including floor slabs

J_c: moment of inertia of the section of column

J_b: ,, ,, of beam including floor slabs

J_f: ,, ,, of footing beam

K_c: relative stiffness of column $=J_c/h$; this is taken as a standard K_0

K_b: ,, of beam $=J_b/l$

K_f: ,, of footing beam $=J_f/l$

k_b: relative stiffness ratio of beam $=K_b/K_0$

k_f: ,, of footing beam $=K_f/K_0$

E: elastic modulus of the material of frame

α: temperature coefficient of length change of material

t: effective temperature difference between the overground frame and its footing or footing beam

$B=6K_0l/(Ah^2)$

$C=6EK_0\alpha tl/h$

M: bending moment at the fixed end of the member

4. General feature of thermal bending moment

When there is some temperature difference between the overground frame and its footing beam, the length of the beam and the footing beam would be different. In this case, the beam and the footing beam will be constrained by each other, because they are rigidly connected to the columns which have some stiffnesses. Consequently, each of beam and footing beam will be subjected to certain axial forces. These axial forces would increase with the increase of number of spans and with the increase of relative stiffness of the column, and causes the secondary length change of beams and footing beams. It is easily realized that the axial forces of beams and footing beams in the vicinity of the central part of frame should be larger than those of the external part of the frame, and that the "slopes" and the "deflections" of members would be different at each panel point.

5. Effect of relative stiffness ratio of beam on thermal bending moments

The thermal bending moments of one-storied uniform frames of 1 to 8 spans, whose columns are perfectly fixed on the ground, are theoretically derived after the "slope-deflection method".

For example, the thermal bending moment at the top of the most external column of a four-spanned frame can be expressed by

$$\frac{M}{C} = -\frac{k_b}{D}\left\{\left(14k_b+10\right)+B\left(14k_b+2\right)\right\}$$

where,

k_b = relative stiffness ratio of beam

$C = 6EK_0\alpha tl/h$

$D = 2^2(2k_b+1)(k_b+1) - k_b^2 + B(42k_b^2+48k_b+6) + B^2(28k_b^2+12k_b+1)$

$B = 6K_0l/(Ah^2)$

In the case of a frame without roof floor slab, B can be written in the following form :

$$B = \frac{6K_0l}{Ah^2} = \frac{6K_0l}{bd\cdot\dfrac{d^2}{12l}\cdot\dfrac{12lh^2}{d^2}} = \frac{6K_0\cdot d^2}{12K_b\cdot h^2} = \frac{1}{2k_b}\left(\frac{d}{h}\right)^2$$

where b and d show the width and depth of the beam respectively.

Thus, the thermal bending moment can be expressed as a function of k_b, d/h and C.

Several examples of the relation between M/C and k_b are shown in Fig. 1, by taking d/h as a parameter. The condition of $d/h=0$ means the special case in which the length change of beam due to axial force is not taken into

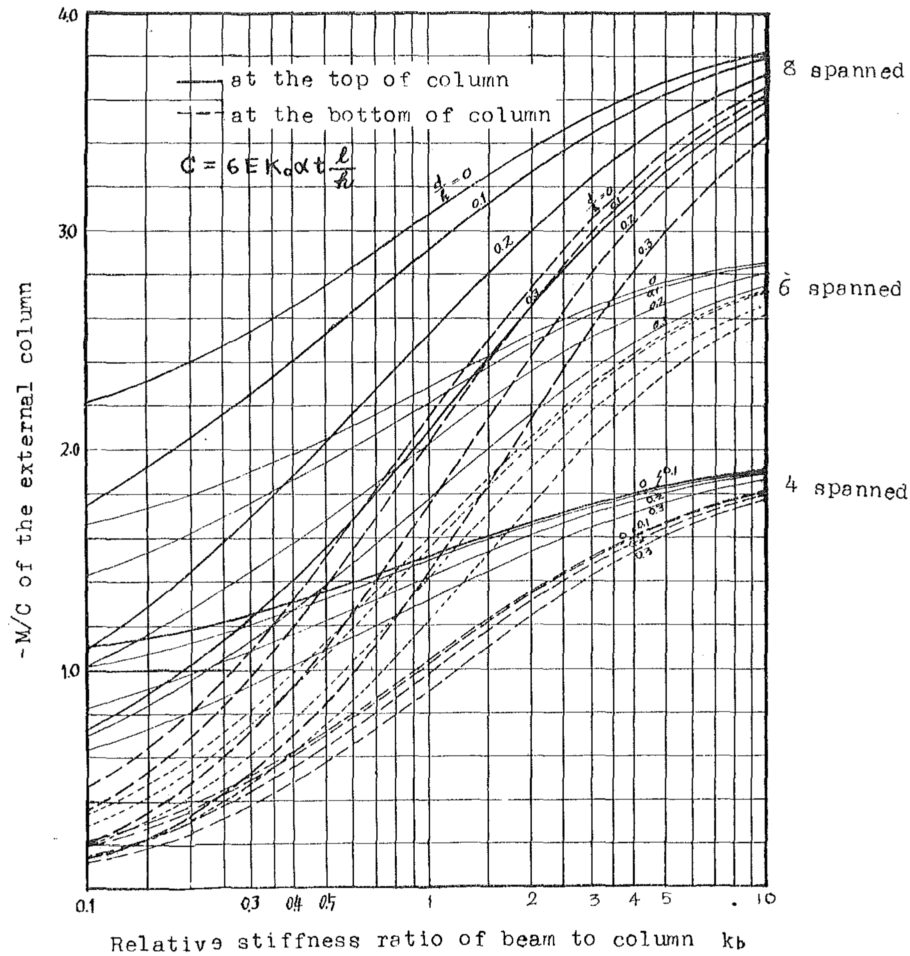


Fig. 1. M/C of the most external column in one-storied frames of 4~8 spanned.

consideration. In this case the value of M/C becomes the largest for all values of d/h, when k_b is kept equal. The figure also shows that the value of M/C increases with the value of k_b and that it has a tendency to converge to a certain value for each frame, when k_b becomes comperatively larger.

Fig. 2 shows an example of the feature of thermal bending moments of each column of these frames. When the other conditions are equal, the larger the number of spans, the larger the thermal bending moment of the columns connected to the most external span.

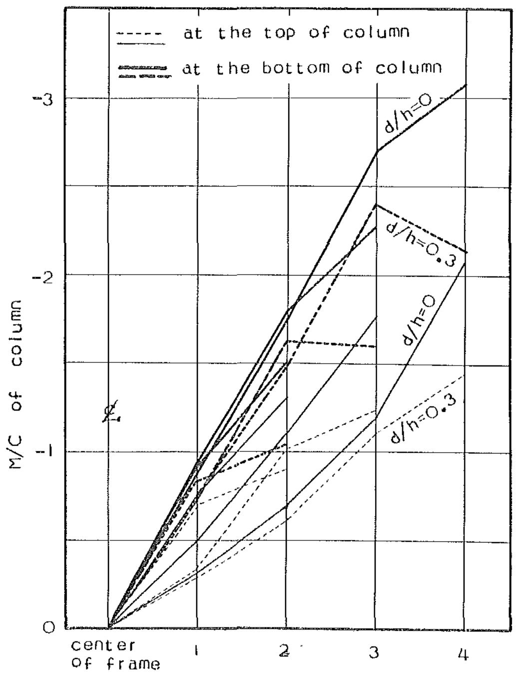


Fig. 2. M/C of each column of one-storied frames of 4~8 spans.

6. Effect of the numbers of stories

Another theoretical analysis is attempted for two-spanned frames of 2~4 stories, after the "slope-deflection method". The thermal bending moments are also expressed as functions of k_b, d/h and C, by neglecting the presence of floor slab, when all beams and columns had the equal dimensions, respectively. Some examples of the relation between M/C and k_b are shown in Fig. 3. The results of analysis show that;

(1) thermal bending moments of beams and columns of the first floor are the largest and decrease rapidly for upper stories, and they can be practically neglected in the stories upper than the third floor,

(2) in the most external column, the sign of shearing force of the second floor is opposite to that of the first floor,

(3) when k_b is larger than about 1.0, the magnitude of thermal bending moments of beams and columns at the first and second floors do not vary so much for frames whose number of stories is larger than three.

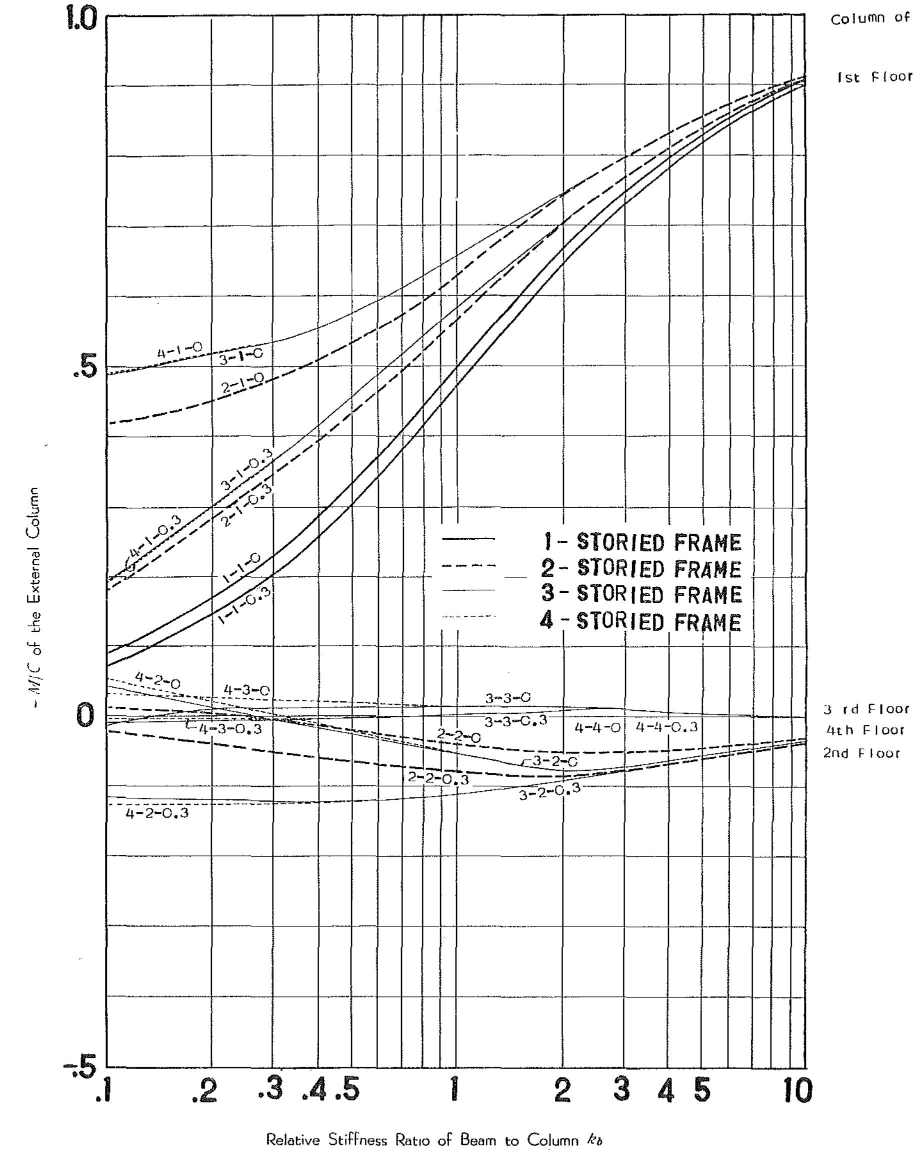


Fig. 3 (a). Thermal bending moments at the top of the most external column of the frame.
The figures mean ; (number of stories of frame)-(the position of column)—(the value of d/h.)

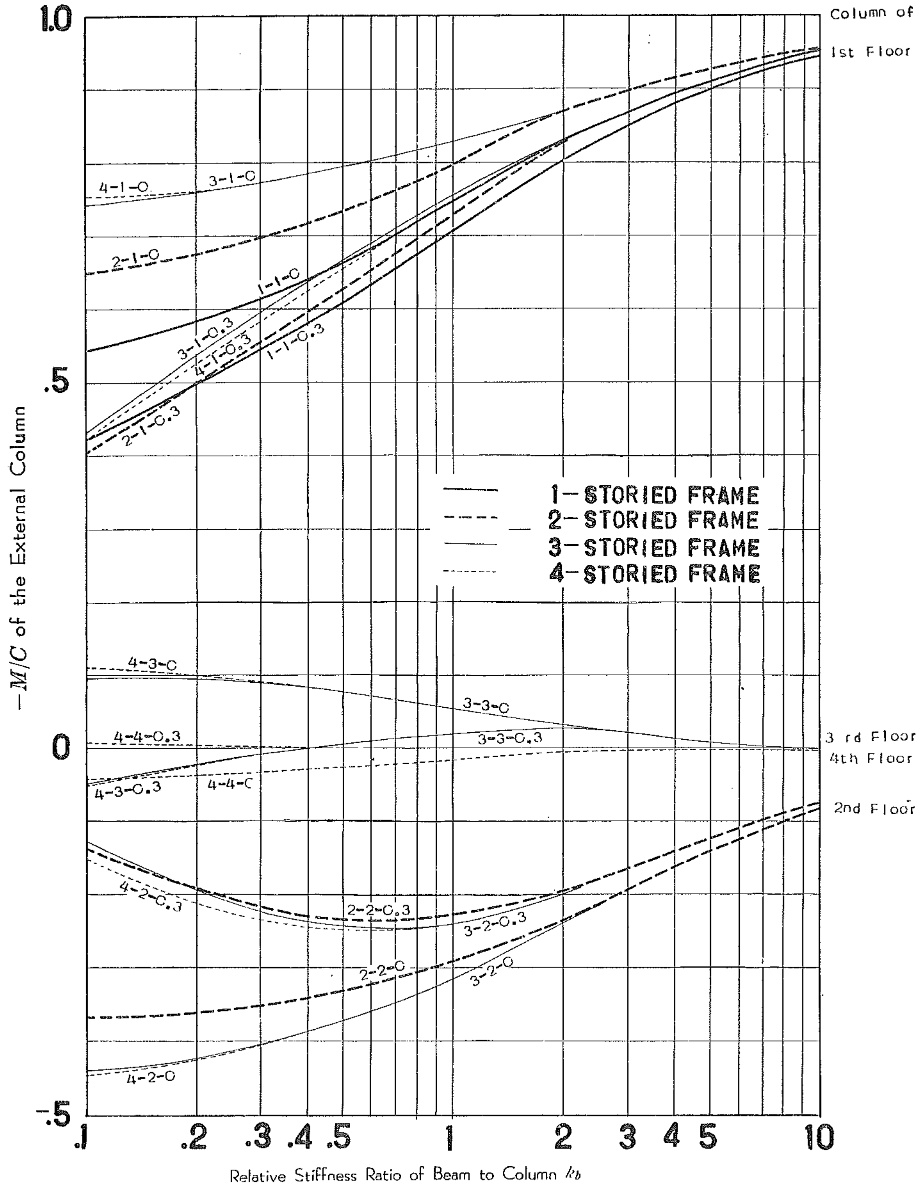
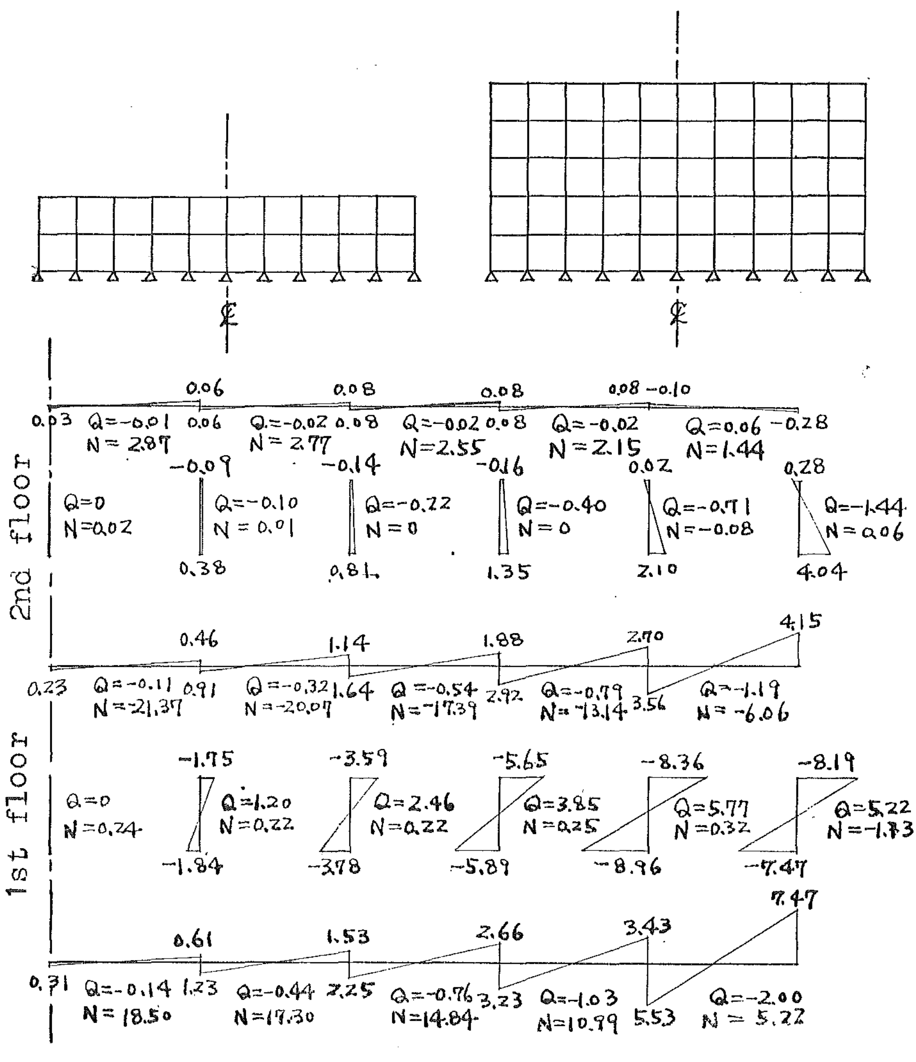


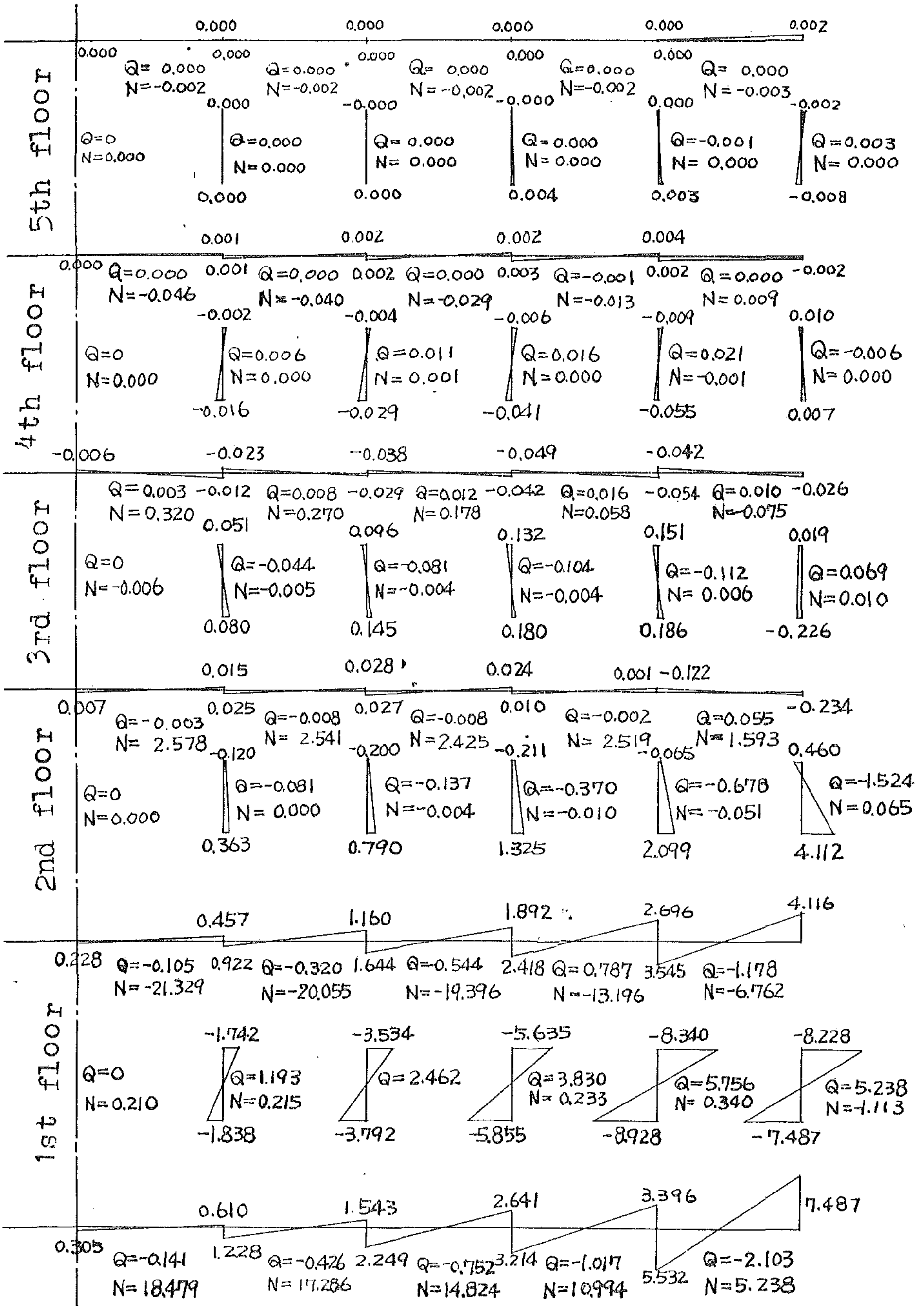
Fig. 3 (b). Thermal bending moments at the bottom of the most external column of the frame.
The figures mean ; (number of stories of frame)-(the position of column)-(the value of d/h.)

The other examples are also numerically computed by the author's method described in paragraph 8, for ten-spanned frames with 1∼5 stories. Some results are shown in Fig. 4. It is confirmed that the features of thermal bending moments of multiple-storied frames are approximately represented by those of the two-storied frames. The error should be very small.



center line of frame

Fig. 4 (a). M(tm), Q(ton), N(ton) of two storied frame.



center line of frame

Fig. 4 (b). *M* (ton), Q (tm), *N* (ton) of five storied frame.

7. Effect of fixing conditions of footings

It is comparatively easy to find thermal bending moments of frames whose footings are perfectly fixed on the ground. However, the actual condition of footings seldom satisfies this ideal condition.

The authors' survey on the movement of an actual reinforced concrete building under temperature variations showed a considerable displacement of footings. Therefore, a theoretical analysis is attempted for four kinds of fixing conditions of footings, i. e.,

(a) footing is perfectly fixed,
(b) footings cannot move, but can rotate according to the stiffness of footing beam,
(c) footings can move in the horizontal direction and can rotate as like as (b),
(d) footings can move in any directions and also can rotate as like as (b).

Table 1 shows a result of numerical substitution into the theoretical formulae for those four conditions. It can be seen that the thermal bending moments would remarkably decrease with the degree of release of the footing from a rigid fixing.

The authors have concluded to take the condition (c) for practical eastimation of thermal bending moment by considering the results of the above-mentioned field observation.

TABLE 1. Shearing forces of columns of one-storied frame

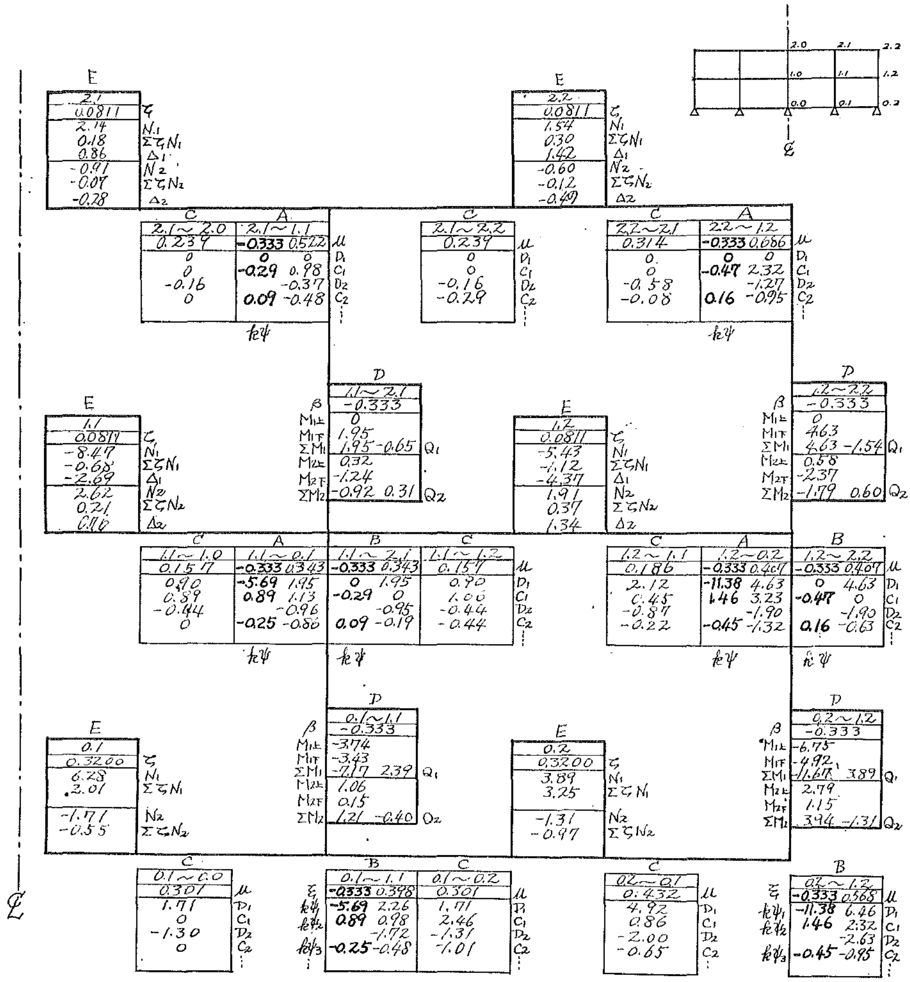
condition of footing	column				
	center	No. 2	No. 3	No. 4	external
perfectly fixed	1.16 (1.52)	2.39 (1.45)	3.75 (1.44)	5.52 (1.37)	5.45 (1.74)
semi-fixed	1.085 (1.41)	2.228 (1.35)	3.443 (1.32)	5.513 (1.28)	4.033 (1.29)
movable in horizontal direction	0.770 (1.00)	1.047 (1.00)	2.609 (1.00)	4.040 (1.00)	3.128 (1.00)
movable in any direction	0.0001 (0.0001)	0.0002 (0.0001)	0.0013 (0.0005)	0.0304 (0.0075)	0.2909 (0.093)

dimension of frame: $l = 6.5$ m $h = 3.0$ m
column 50×50 cm^2 $E = 210\, t/\text{cm}^2$
beam 30×50 cm^2 $\alpha = 12 \times 10^{-6}$
footing beam 30×70 cm^2 $t = 10°$C

The figures in the blackets show the ratios of shearing forces by taking (c) case as a standard.

8. Method for the numerical computation of thermal bending moment

Generally speaking, it is very complicated to express the thermal bending moments of multiple-storied frames of multiple spans by the theoretical formulae. For the convenience of practical calculation, two kinds of following



$$\beta = -\frac{1}{h},\quad \zeta = \frac{6K_0 l}{A},\quad \xi = -\frac{k}{h},\quad \Delta = -\Psi h = 6K_0 \Sigma \frac{Nl}{A}$$

Fig. 5. An Example of numerical computation after the modified "moment distribution method."

methods for numerical computation can be deviced under the condition that each of the beam and thes footing beam has no "deflection", i. e., each panel point will move only in the horizontal direction.

(1) After the "slope-deflection method"

The form of a numerical computation by the "slope-deflection method" for vertical loads or horizontal loads has already been deviced by Mr. Ikebe[1] and the others. Some additional columns should be added to this form to adjust the variations of "slopes" and "deflections" due to length changes of beams and footing beams caused by axial forces.

(2) After the "moment distribution method"

The form of a numerical computation for vertical loads or horizontal loads was already proposed by Mr. Cross[2], Professor Muto[3], etc. Some additional columns should be added to these forms for the calculation of thermal addieffects. Fig. 5 shows an example of this method of computation.

9. Numerical computations

The authors have already suggested that

(a) the features of main thermal bending moments of multiple-storied frame are approximately represented by those of the two-storied frame which has the same dimensions of members and the same spans,

(b) the condition of footing should be assumed to be elastically fixed by footing beams and can move only in the horizontal direction.

After these suggestions, the authors have intended to get further materials for two-storied frames for the convenience of practical estimation of thermal bending moments. 240 kinds of rectangular frames of two-storied uniform type which have 8 to 18 spans are numerically computed by the above-mentioned method for various lengths of spans, the heights of columns and the relative stiffness ratios of beams. The results of calculations are as follows, if the other conditions be equal:

(1) The thermal bending moments of the beams and footing beams connected to the most external span show the largest values in the same floor. However, the value of the thermal bending moment in the most external column is not always the largest in accordance with the ratio of k_b/k_c. (See Fig. 2). In many cases, the second column has the largest bending moment.

(2) The general feature of thermal bending moments of members in the second floor is similar to that of the first floor, but each of the corresponding value is considerably smaller than that of the first floor.

(3) The thermal bending moments of the central beam, footing beam and column show the smallest values and those of the other beams, footing beams

and columns, excepting those of the most external span, increase almost linearly with the distance between the center of frame and the column concerned.

10. Charts for estimating thermal bending moments caused by temperature difference in the frames

Next charts show the relation between k_b, k_f/k_b, B and M/C for the beams, footing beams and columns of the external span of the frames with two stories of multiple spans of 8, 10, 14 and 18.

11. Application of the charts

If the frame is given, k_b, k_f/k_b, B and C can be calculated. Select the chart of corresponding number of spans and take the value of k_b on the abscissa, and find the coordinate on the group of lines of k_f/k_b. Then the corresponding point will be decided on the group of lines on the right-hand side. The abscisa of this point shows the value of M/C. And thus the thermal bending moment M of columns, beams and footing beams indicated in the figures above the charts will be estimated. M at the external end of each beam and footing beam can be obtained from the balance of bending moments at each panel point.

For the frames whose number of spans are not indicated in these charts, M can be estimated by an interpolation between the values of two adjacent examples.

The thermal bending moment of the center column in the frames of even number of spans must be zero. In the case of the frames of odd number of spans, an imaginary column could be assumed at the middle of the frame, and this imaginary column should also have no stress.

The thermal bending moments of an arbitarary column and those at the internal ends of beams and footing beams, which are placed between the center of frame and the third panel point from the end of frame, can be estimated by assuming that they decrease linearly with the distance between above-mentioned center column and the panel point concerned. The thermal bending moments at the external end of each beam and footing beam can also be obtained from the balance of bending moment at each panel point.

For one-storied frame, the thermal bending moments can be derived by unloading the bending moments at the bottoms of columns in the second story of the two-storied frame which has the same number of spans. The effect of these unloading must be distributed proportionally to the relative stiffness ratios of the beams and the columns at each panel point. The maximum error of these approxima tion is about 10% for 10-spanned frame.

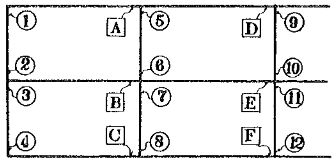
Chart 1. 8 Spanned Frame

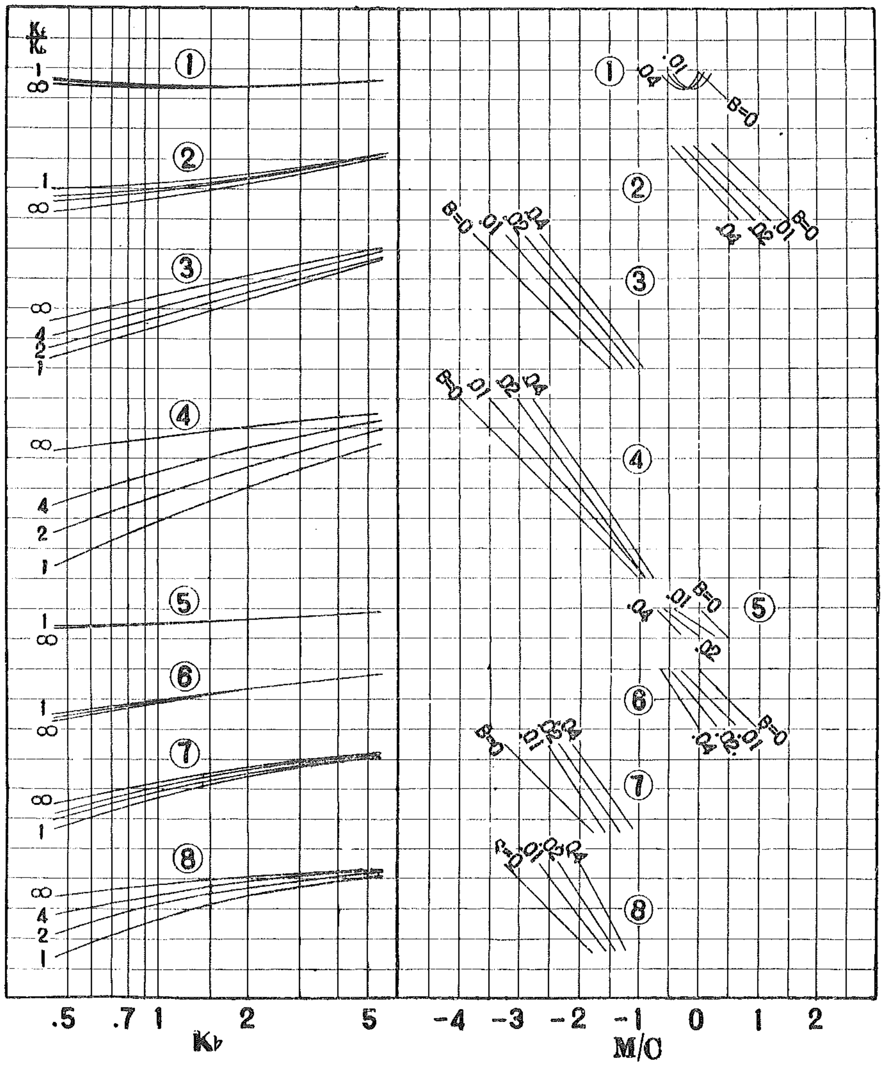
k_b = relative stiffness ratio of beam

k_f = " of footing beam.

$B = 6K_0 l/(Ah^2)$

$C = 6EK_0 \alpha t l/h$





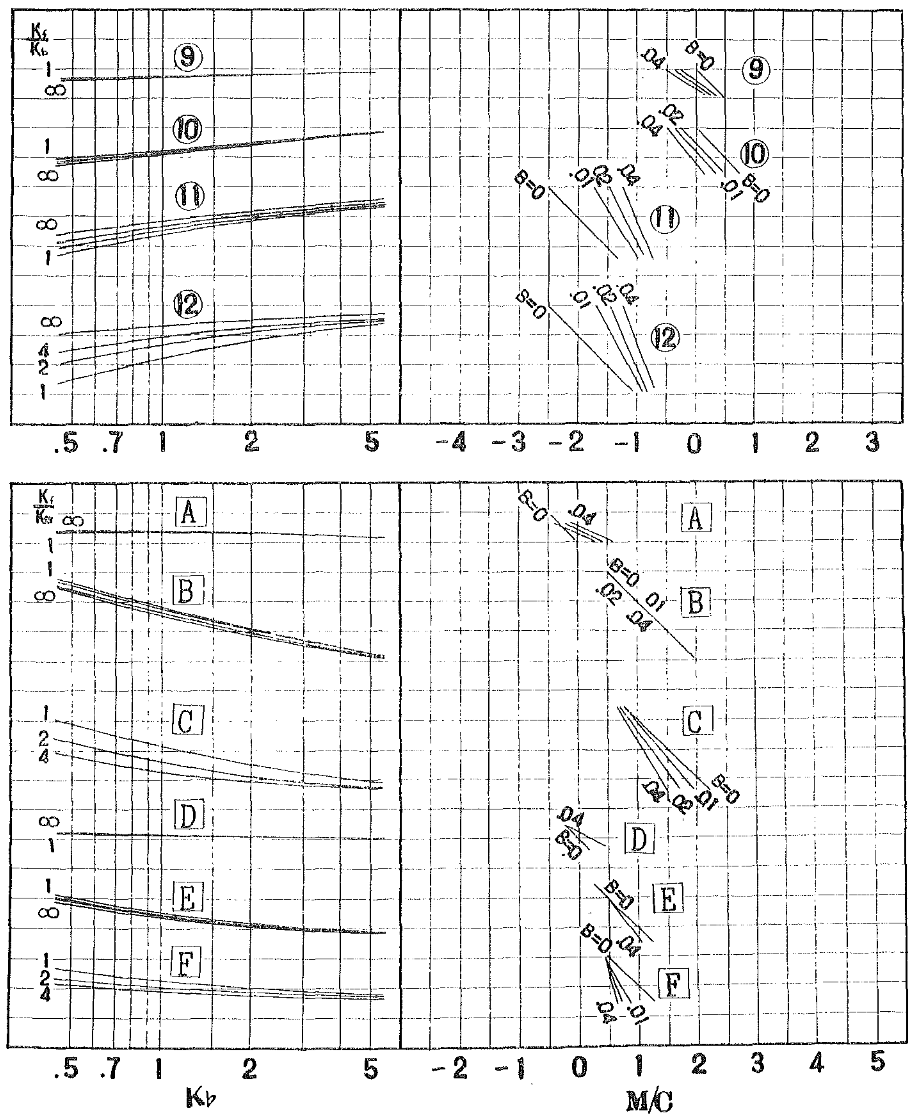

Kf/Kb
⑨
⑩
⑪
⑫
.5 .7 1 2 5
-4 -3 -2 -1 0 1 2 3
A
B
C
D
E
F
-2 -1 0 1 2 3 4 5
Kb
M/C

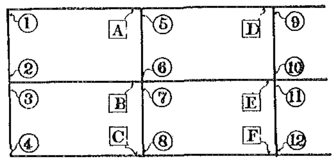
Chart 2. 10 Spanned Frame

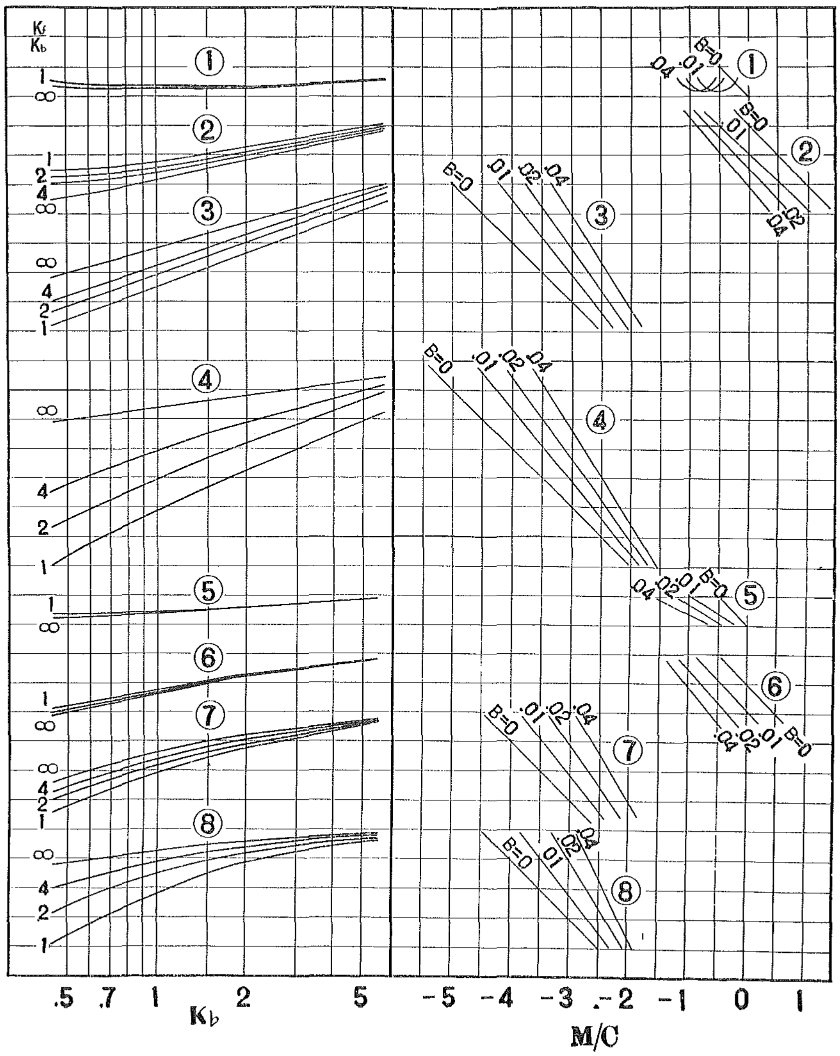
k_b = relative stiffness ratio of beam

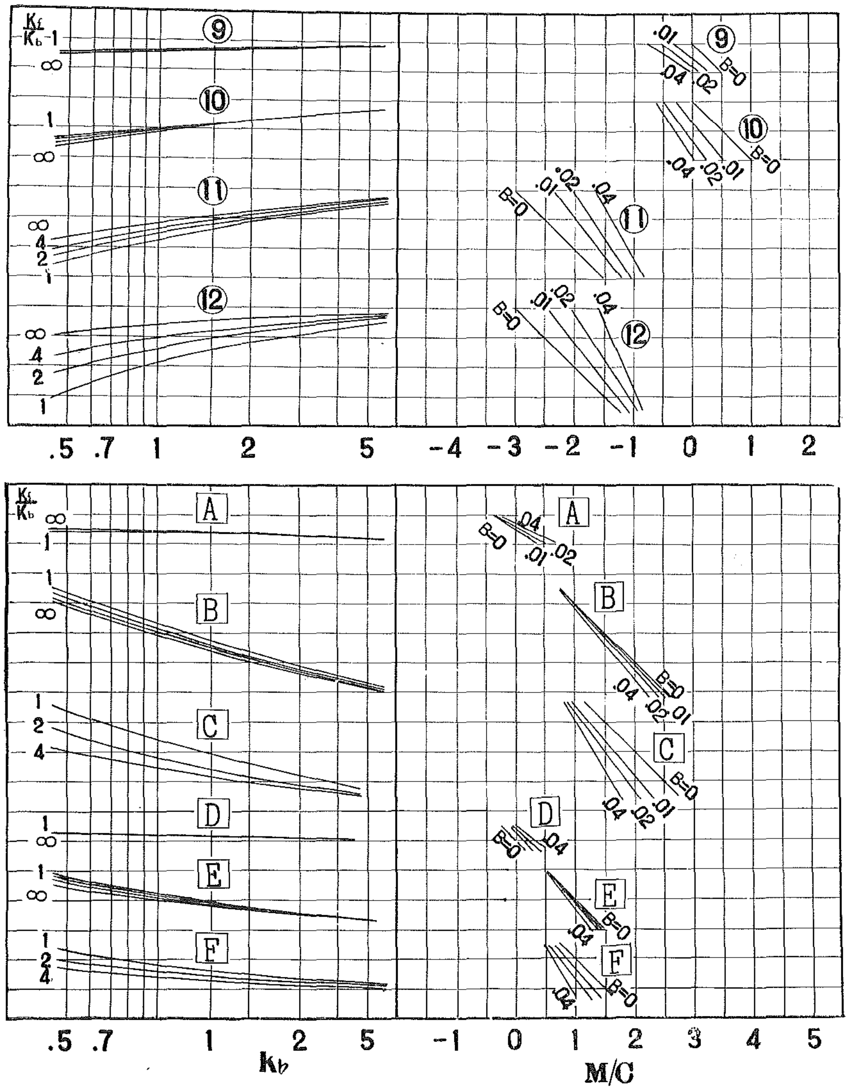
k_f = " of footing beam

$B=6K_0l/(Ah^2)$

$C=6EK_0\alpha tl/h$







Kf/Kb
9
10
11
12
A
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C
D
E
F
B=0
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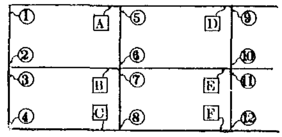
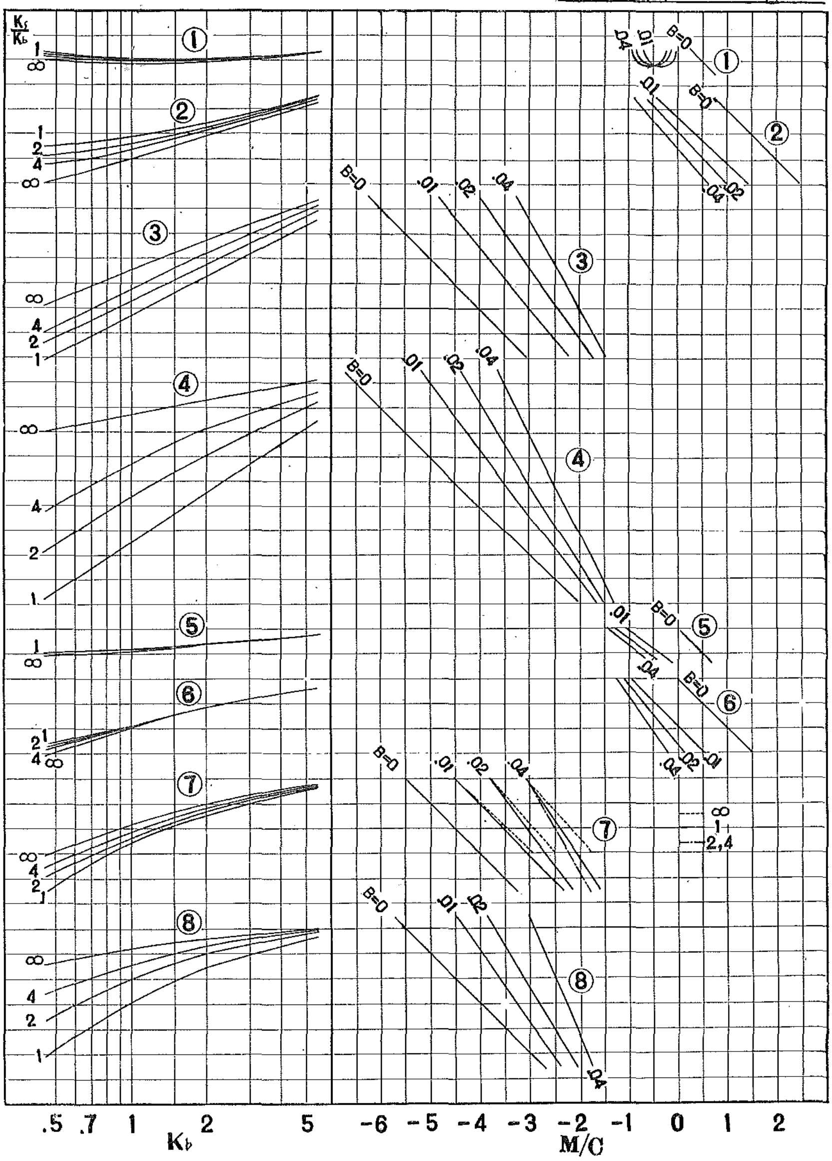
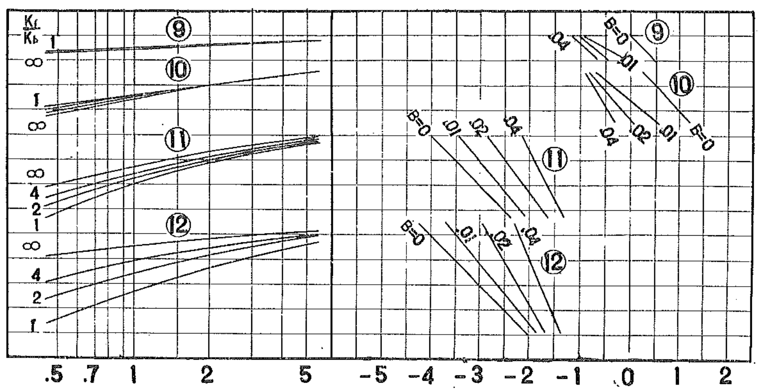
.02
.04
.5 .7 1 2 5
-4 -3 -2 -1 0 1 2
-1 0 1 2 3 4 5
Kb
M/C

Chart 3. 14 Spanned Frame
k_b = relative stiffness ratio of beam
k_f = " of footing beam
$B = 6K_0l/(Ah^2)$
$C = 6EK_0\alpha tl/h$






Kf/Kb
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B=0
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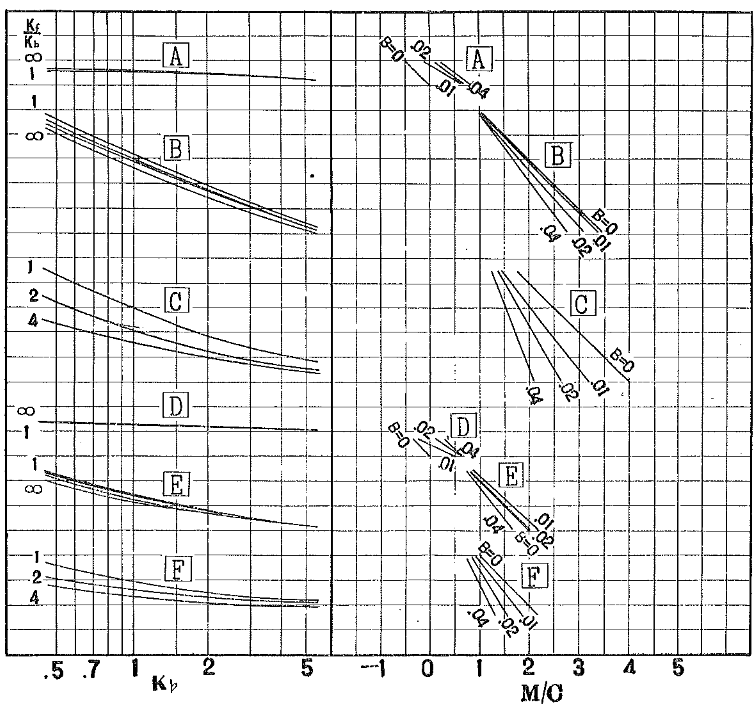
Kf/Kb
A
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B=0
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.02
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.5 .7 1 2 5 -1 0 1 2 3 4 5
Kb
M/C

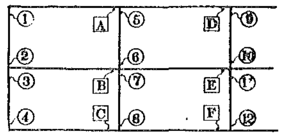
Chart 4. 18 Spanned Frame

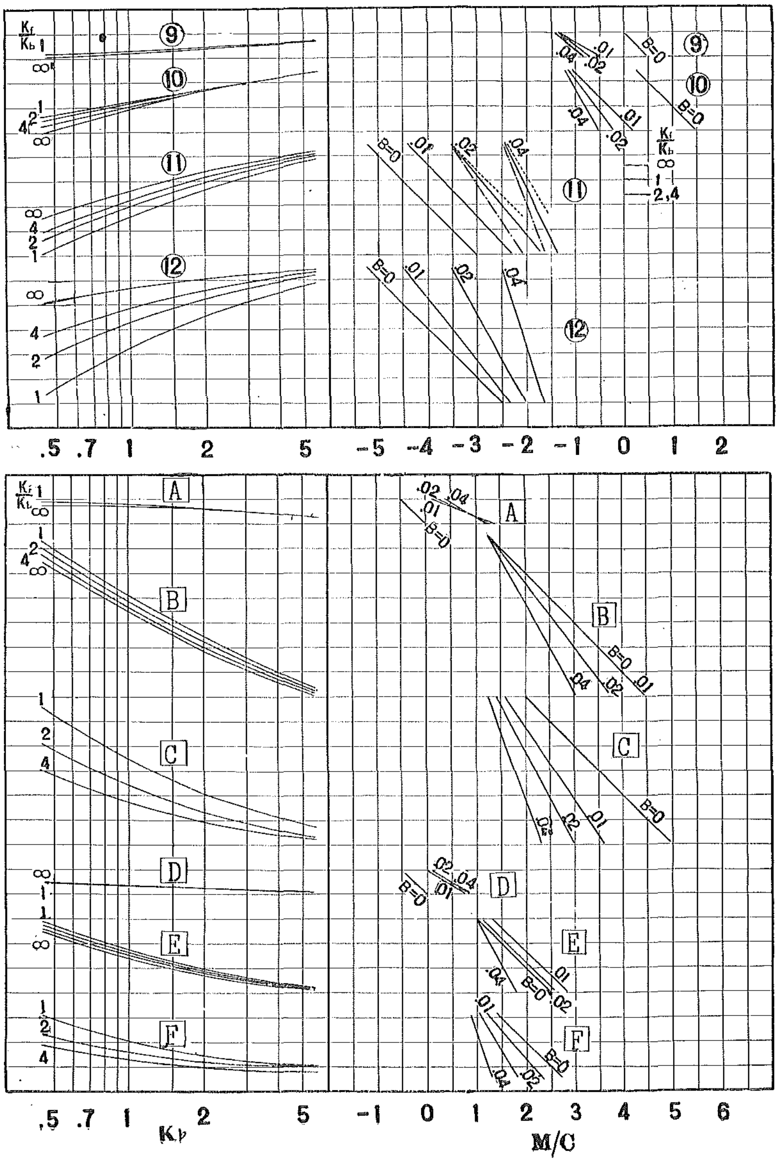
k_b = relative stiffness ratio of beam

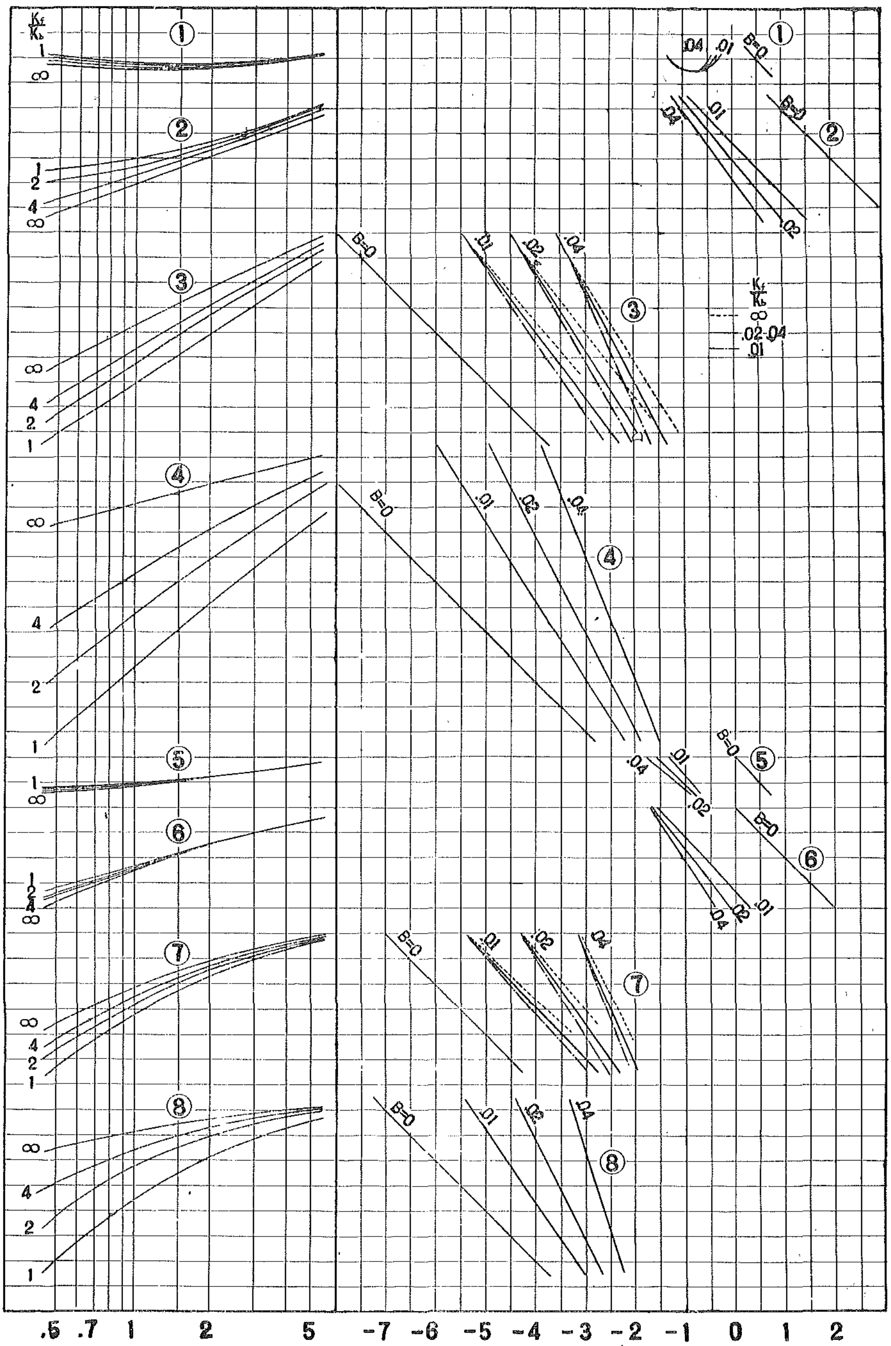
k_f = " of footing beam

$B = 6K_0 l/(Ah^2)$

$C = 6EK_0 \alpha t l/h$






Kf/Kb
.5 .7 1 2 5
Kb
-7 -6 -5 -4 -3 -2 -1 0 1 2
M/C
B=0
.01
.02
.04

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