Phase of the Diurnal Variation of Water Temperature of a River

By

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Abstract

It has been observed that the feature of the diurnal variation of water temperature at the upper stream of a river is distinctly different from that at the mid-stream. At the upper stream, the mean diurnal temperature of water becomes higher and the amplitude of the diurnal variation increases as the water flows down. But the phase lag of air and water temperature is not so large, even at a station located far from the source.

At the mid-stream, on the other hand, the mean temperature and the amplitude are almost constant, and the phase lag becomes larger with flow.

The author made an attempt to solve an equation of the water temperature variation under appropriate boundary conditions, and could explain why the different variations occur at the upper and the mid-stream. The reasonability of the solution for the mid-stream was assured by temperature records at the Tokachi River with a good agreement.

1. Introduction

The variation of water temperature of a river is controlled by thermal conditions of the environment under which the river water flows down from the source to the river mouth, and also by mechanical conditions of the river itself. As the variation of the temperature is very complicated, it is quite interesting from the geophysical view point to investigate the process of the variation. And as the water at the down-stream is used for agriculture, it is also very important from the view point of practical use.

The heat transfer between the external environment and the water through the river surface is chiefly performed in the following manner:

(1) heat gain by solar radiation.

(2) heat transfer between water and air near the river surface by contact.

(3) heat loss by evaporation from the river surface.

(4) heat loss by long-wave radiation from the surface.

Owing to the heat transfer in these ways, the variation of water temperature occurs in accordance with the condition of flow-current velocity, depth of water and width of the river.

From this thermodynamical view point, Magono et al. and Takatuki et al. studied on the relation between heat transfer and water temperature rise in the Bisei River[1], a branch of the Tokachi River, and the Ōi River[2], respectively. Many studies[3] also on the temperature rise of water-warming ponds have been published from the same point of view.

However, report on the phase lag of air and water temperature or on the causes of different variations of water temperature at the upper and the mid-stream has not yet been found. In this paper, the author will deal chiefly with the phase of the diurnal variation of water temperature on the basis of the new solutions of the equation of temperature variation and observations made on water temperature along the Tokachi River.

2. Observational Results

In 1952 the author and others observed air and water temperature and

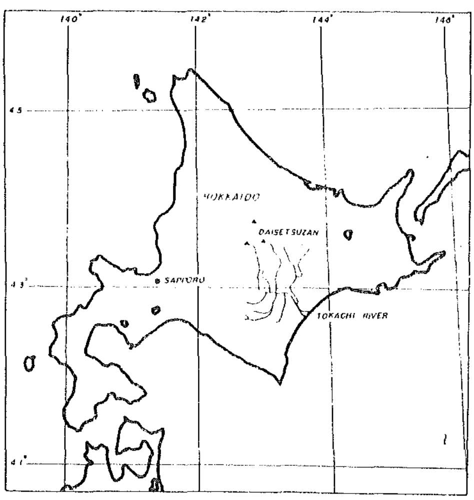


Fig. 1-a.

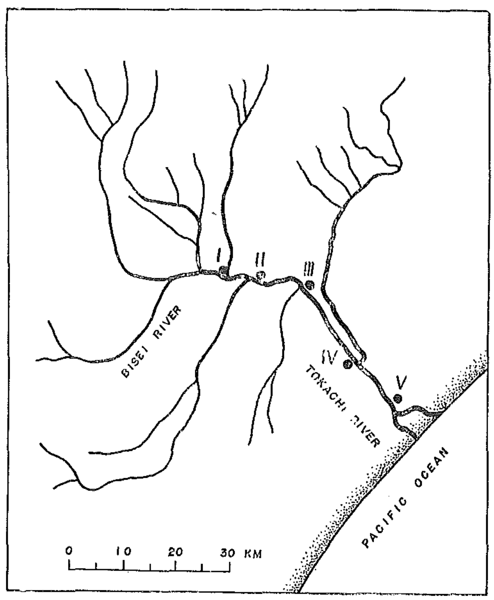


Fig. 1-b.

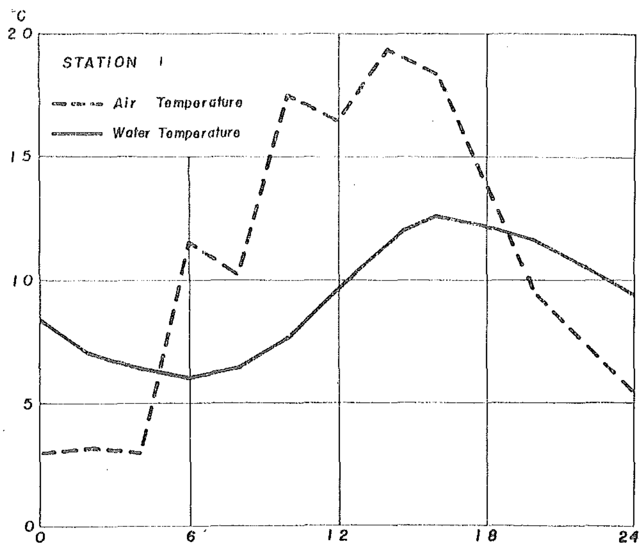
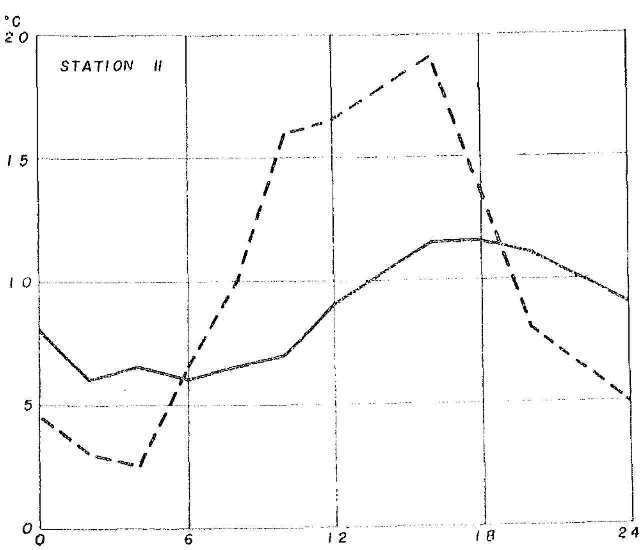

°C
20
15
10
5
0
STATION I
Air Temperature
Water Temperature
0
6
12
18
24

Fig. 2-a.


°C
20
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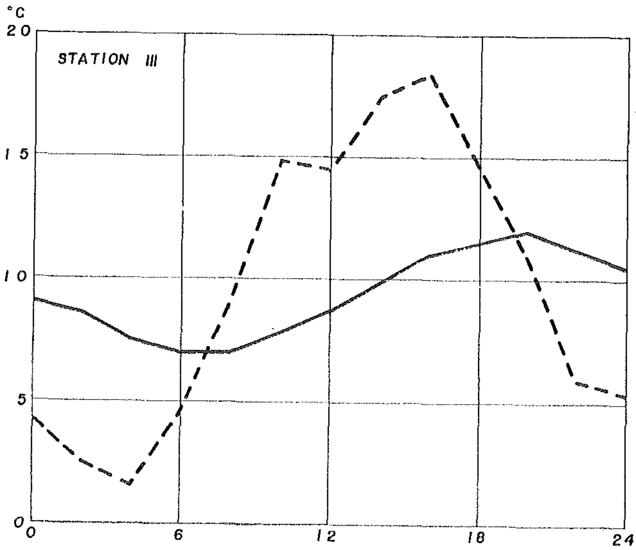
STATION II
0
6
12
18
24
MAY 9, 1952

Fig. 2-b.


°C
20
STATION III
15
10
5
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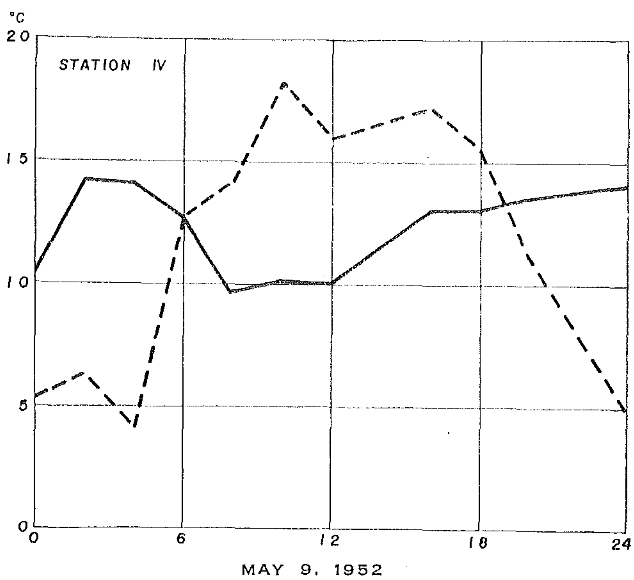
0
6
12
18
24

Fig. 2-c.


°C
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STATION IV
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18
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MAY 9, 1952

Fig. 2-d.

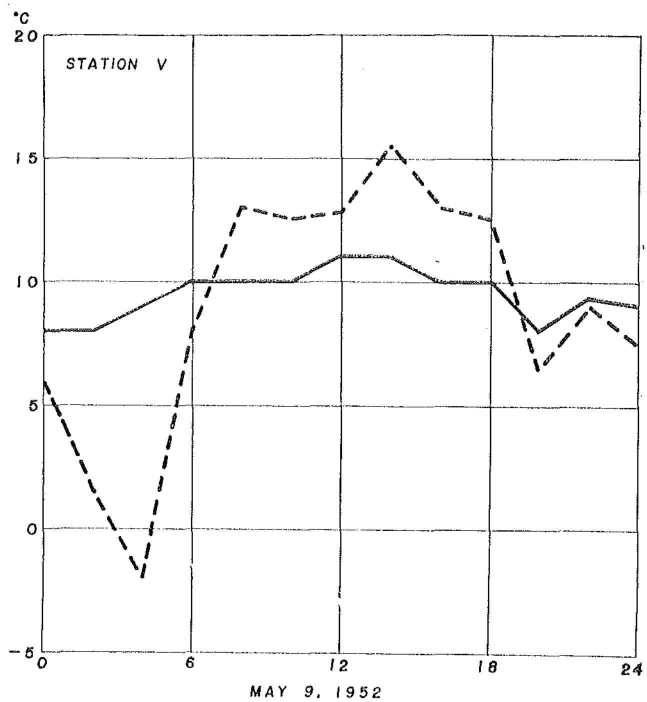

Fig. 2-e.

the current velocity of flow at five stations along the Tokachi River shown in Fig. 1[4]. The observations were carried out at intervals of one or two hours, for four consecutive days from 7th to 11th May, 1952. Some of the observed values of air and water temperature are shown in Fig. 2. From Fig. 2, it is seen that the phase lag of the variations of air and water temperature is about two hours at the station I, and it becomes larger as the water flows down. At the station II, the phase lag is four hours, at III six hours and at IV it is about ten hours. And so the highest water temperature has been observed about at 12 o'clock p.m. at station IV. The reasoning of the large phase lag at these stations will be discussed in the following section.

3. The Equation and its Solution

By introducing a symbol D/Dt to denote a differentiation following the motion of a water mass of a river, the differential equation of the variation of water temperature can be given as

$$C\rho \frac{D\theta}{Dt} = \frac{1}{H} \left\{ k(T-\theta) + R \right\} \qquad (1)$$

where C and ρ are the specific heat and the density of water, θ and T are the temperature of water and air near the river surface, k the heat transfer coefficient and H the mean depth of the river. The terms $k(T-\theta)$ and R respectively mean the quantity of heat which the water received from air and the other thermal sources mentioned at section 1, through unit area per unit time. The term $k(T-\theta)+R$ can also be given in another form $k^*(T^*-\theta)$, where k^* is the apparent heat transfer coefficient and T^* containing R the apparent air temperature.

And so Eq. (1) becomes

$$C\rho \frac{D\theta}{Dt} = \frac{1}{H} k^*(T^*-\theta)$$

or

$$\frac{\partial\theta}{\partial t} + V\frac{\partial\theta}{\partial x} = \frac{k^*}{C\rho H}(T^*-\theta) \qquad (2)$$

where V is the mean velocity of flow.

Moreover, if it is assumed that the diurnal variations of θ and T^* can be represented by sine functions, Eq. (2) gives the following relation by denoting mean values of θ and T^* as $\bar{\theta}$ and $\bar{T}^*$:

$$V\frac{d\bar{\theta}}{dx} = \frac{k^*}{C\rho H}(\bar{T}^*-\bar{\theta}) \qquad (3)$$

This is the relation between $\bar{\theta}$ and $\bar{T}^*$.

When $\bar{T}^*=$constant, the solution of Eq. (3) can easily be obtaind and it becomes as follows:

$$\bar{T}^*-\bar{\theta} = e^{-\frac{k^*}{c\rho VH}x}(\bar{T}^*-\theta_0) \qquad (4)$$

where θ_0 is the mean diurnal value of θ at the source located at $x=0$. Therefore when x approaches to ∞, the value of $\bar{\theta}$ approaches exponentially to the value of $\bar{T}^*$. When the contact with the air is the only process through which the water receives heat, a relation between the mean values of air and water temperature can be given by Eq. (3) or Eq. (4).

When T^* and θ are represented as

$$T^* = \bar{T}^*(x) + T\, e^{i\sigma t}$$
$$\theta = \bar{\theta}(x) + X(x)\cdot e^{i\sigma t}$$

the process of solving the differential equation (1) can be simplified by assuming that the amplitude of the variation of water temperature at the source is negligiblly small, namely with the boundary condition $X(x)=0$ at $x=0$.

Then, the real part of the solution of Eq. (2) gives the result one is seeking for:

$$\theta=\bar{\theta}-T\,e^{-\frac{k^*}{c\rho VH}x}\,\frac{k^{*2}\cos\sigma\left(t-\dfrac{x}{V}\right)+k^*\,C\rho\sigma H\sin\sigma\left(t-\dfrac{x}{V}\right)}{k^{*2}+(C\rho\sigma H)^2}$$

$$+\,T\,\frac{k^{*2}\cos\sigma t+k^*\,C\rho\sigma H\sin\sigma t}{k^{*2}+(C\rho\sigma H)^2}$$

where V and H are considered to be independent on x and t.

By putting

$$\cos\varphi=\frac{k^*}{\sqrt{k^{*2}+(C\rho\sigma H)^2}}$$

$$\sin\varphi=\frac{C\rho\sigma H}{\sqrt{k^{*2}+(C\rho\sigma H)^2}}$$

θ can be written as

$$\theta=\bar{\theta}-T\,e^{-\frac{k^*}{c\rho VH}x}\cdot\cos\varphi\cdot\cos\left\{\sigma\left(t-\frac{x}{V}\right)-\varphi\right\}+T\cos\varphi\cdot\cos(\sigma t-\varphi)$$

Therefore, the water temperature at $x\rightarrow\infty$ can be given by

$$\theta=\bar{\theta}+T\cos\varphi\cdot\cos(\sigma t-\varphi) \tag{5}$$

4. Discussion

By the use of the above result, for example, the phase lag φ of θ and T^* at a station located far from the source can be estimated with actual values of C, ρ, σ, H and k^*. Namely, if C, ρ, σ, H and k^* are given as

$$C=1 \quad (\text{cal/g}\,^\circ\text{C})$$

$$\rho=1 \quad (\text{g/cm}^3)$$

$$\sigma=\frac{2\pi}{24\times 60\times 60} \quad (1/\text{sec})$$

$$k^*=1\times 10^{-3} \quad (\text{cal/sec}\cdot\text{cm}^2\cdot{}^\circ\text{C})$$

the phase lag becomes

$$\frac{\varphi}{\sigma}=3.7 \text{ hours for } H=20\text{ cm},$$

$$\frac{\varphi}{\sigma}=5.0 \text{ hours for } H=50\text{ cm}.$$

As the most powerful solar radiation is measured about at 12 o'clock a.m., it is assumed that T^* will take the highest value at about the same time. Therefore, from the estimated values of phase lag, one can predict the time of the highest water temperature at about 5 o'clock p.m. at station I of the Tokachi River and 4 o'clock p.m. at Shinseibashi of the Bisei River. But a too large phase lag as at stations II, III, IV and V cannot be predicted by Eq. (5).

Therefore, it can be seen that at the mid-stream, as in the domain of the stations I, II, III, IV along the Tokachi River, the temperature variation occurs owing to a different mechanism from that at the upper stream. Next, an equation that gives the variation of water temperature at the mid-stream will be introduced.

If it is assumed that θ and T^* in Eq. (3) are represented by

$$T^* = \bar{T}^* + T\, e^{i\sigma}$$

$$\theta = \bar{\theta} + \Theta \cdot X(x) \cdot e^{i\sigma\left(t-\frac{\varphi}{\sigma}\right)}$$

the boundary condition becomes

$$X(x) = 1 \quad \text{at} \quad x = 0.$$

If it is further assumed that the heat transfer between water and the external environment is negligible $\left(\frac{k^*}{C\rho H} \fallingdotseq 0\right)$, a solution of Eq. (3) can be given by

$$\theta = \bar{\theta} + \Theta \cos \sigma \left(t - \frac{\varphi}{\sigma} - \frac{x}{V}\right)$$

Also when the mean velocity V is a function of distance from the station $x=0$, Eq. (3) can be solved similarly in the same way.

In this case, the solution becomes

$$V = V(x)$$

$$\theta = \bar{\theta} + \Theta \cos \sigma \left(t - \frac{\varphi}{\sigma} - \int_0^x \frac{dx}{V}\right) \qquad (6)$$

It is seen in Eq. (6), that the phase lag of the variations of water temperature at two stations x_1 and x_2 is equal to the value of $\int_{x1}^{x2} \frac{dx}{V}$, which is the time water mass takes to flow down from x_1 to x_2.

In the present case, the mean velocity may be given by $V = V(x)$, since the flow was almost steady when the observations along the Tokachi River were carried out.

TABLE I.

Station	Distance (Km)	Width of river (m)	Mean depth (m)	Mean velocity (m/sec)	Phase lag of water Temperature (hr)	
					obs.	cal.
I	0	57	1.8	1.07	0	0
II	8.8	100	1.4	1.46	2.0	2.0
III	19.0	103	1.6	1.35	4.0	4.0
IV	36.0	80	3.5	0.74	8.0	9.0
V	48.5	185	2.1	0.77	20.0	13.6

Therefore, the variation of water temperature must be represented by Eq. (6). A comparison of the observational values of phase lag of water temperature computed by $\int_0^x \frac{dx}{V}$ at all the stations is shown in Table 1 by taking the reference phase at station 1 as a standard. A good coincidence between observed and calculated values can be seen except at station V. This discrepancy at station V must be explained by a different mechanism of flow.

The width, the mean depth of water and the mean velocity of flow at every station are also presented in Table 1.

5. Conclusion

In a natural river, an exchange of heat exists between water and the external environment. Therefore, water temperature at the upper stream may be easily controlled by the thermal condition of the external environment, because the discharge is comparatively small there.

The relation of the mean diurnal values of air temperature (or T^*) and water temperature is given by Eq. (3). The amplitude of the variation of water temperature becomes larger with flow, and approaches to $T \cos \varphi$ at $x \rightarrow \infty$ as will be seen in Eq. (5). The phase lag of the variations of θ and T^* also increases and will be φ at $x \rightarrow \infty$.

On the other hand, at the mid- stream where the discharg increases rapidly as at the stations along the Tokachi River by gathering some branch rivers, the flow has a large heat capacity and may be hardly influenced by the heat transfer. Therefore, the heat is transported only with the flow of water mass, and the phase lag of the variations of water temperature at two stations is equal to the time required to flow down from one station to the other. As a result, it can be said that the phase lag of air and water temperature increases with flow at the mid-stream of a river.

Acknowledgement

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References

1) N. Magono, M. Kumai and A. Higasi: On water Temperature of River, Bull. Inst. Agr. Phys, 2 (1952) 41–51.
2) T. Takatuki, I. Takahasi and S. Tezima: Study on the Effect of the Water Power Works on the Temperature of Irrigation Water (IV), J. Agr. Engin. Soc. Japan, 24 (1956) 277–284.
3) Y. Mihara, Z. Uchijima, K. Ōnuma, S. Yamamoto and M. Hagiwara: Thermal Efficiency of Warming Ponds for Cold Irrigation water, Bll.Nat. Inst. Agr. Sci. A, No. 4 (1955) 45–92. etc.
4) Dept. App. Physics, Faculty of Engineering, Hokkaido Univ.: The Survey Report on the Flow of the Tokachi River, 1953.