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Stresses in Thin Plates of Cantilever Type Stretched out around an External Angle

By

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Introduction

Although most problems of free edged rectangular plates have already been solved, stresses of the plates in the title remain to be solved, which are important in the practical application for the cases of pent-roofs or canopies of reinforced concrete.

As there is no information on stresses in the cases above mentioned we have no reliable methods of reinforcement of the concrete, hence sometimes the risk of cracking or collapsing of cantilever plates cannot be avoided.

The present paper discusses a solution for such plates subject to uniform load using the method of finite differrences which was previously applied to the solution of the cases of rectangular plates with a thin or open quadrangle in the center[*)].

1. Nomenclature and the Subdivision of the Surface of the Plates

The following definitions are given here:

$$N = \frac{Eh^3}{12(1-\nu^2)}$$

$$\Psi = \frac{N}{\lambda^2} w$$

$$SA = \lambda A, \quad SA = \lambda' A. \quad (\gamma = \lambda/\lambda')$$

with N: Flexural rigidity of plate
Ψ: Deflection
h: Thickness of plate
A: Reaction
ν: Poisson's ratio
SA: Resultant reaction
λ, λ': Length of subdivision of plate

Each surface of the plates is subdivided into equal small squares with every point of intersection numbered as shown in Fig. 1.

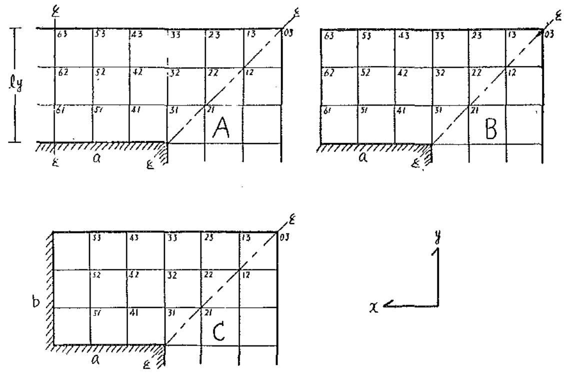


Fig. 1.

2. Setting Up the Equations

The finite difference equations at all such points derived from the governing equations for rectangular plates with a thin quadrangle in the center are shown in Table 1.

At the same time the equation for Point 31 which may come into question and the equation for determining the resultant reaction at the reentering corner point are schematized in Figs. 2–1, 2–2 and 3.

And further, these two equations are so simplified by substituting $\nu=0$, $\lambda=\lambda'$ and $n=\infty$ in them as shown in Fig. 4 and 5, respectively.

TABLE 1–A

	ψ_{03}	ψ_{13}	ψ_{12}	ψ_{23}	ψ_{22}	ψ_{21}	ψ_{33}	ψ_{32}	ψ_{31}	ψ_{43}	ψ_{42}	ψ_{41}	ψ_{53}	ψ_{52}	ψ_{51}	ψ_{63}	ψ_{62}	ψ_{61}	
03	3	-6	2	1															0.02778
13	-3	9.5	-6	-4	3		0.5												0.05556
12	2	-12	18	4	-16	2		2											0.11111
23	0.5	-4	2	8	-6	1	-4	2		0.5									0.05556
22		3	-8	-6	21	-8	2	-8	3		1								0.11111
21			2	2	-16	20		4	-16			2							0.11111
33		0.5		-4	2		8	-6	1	-4	2		0.5						0.05556
32			1	2	-8	2	-6	19	-8	2	-8	2		1					0.11111
31					3	-8	1	-8	22.5		2	-8			1				0.11111
43				0.5			-4	2		8	-6	1	-4	2		0.5			0.05556
42					1		2	-8	2	-6	19	-8	2	-8	2		1		0.11111
41						1		2	-8	1	-8	21		2	-8			1	0.11111
53							0.5			-4	2		8.5	-6	1	-4	2		0.05556
52								1		2	-8	2	-6	20	-8	2	-8	2	0.11111
51									1		2	-8	1	-8	22		2	-8	0.11111
63										1			-8	4		8	-6	1	0.05556
62											2		4	-16	4	-6	19	-8	0.11111
61												2		4	-16	1	-8	21	0.11111

TABLE 1–B

	ψ_{03}	ψ_{13}	ψ_{12}	ψ_{23}	ψ_{22}	ψ_{21}	ψ_{33}	ψ_{32}	ψ_{31}	ψ_{43}	ψ_{42}	ψ_{41}	ψ_{53}	ψ_{52}	ψ_{51}	ψ_{63}	ψ_{62}	ψ_{61}	
03	3	-6	2	1															0.02778
13	-3	9.5	-6	-4	3		0.5												0.05556
12	2	-12	18	4	-16	2		2											0.11111
23	0.5	-4	2	8	-6	1	-4	2		0.5									0.05556
22		3	-8	-6	21	-8	2	-8	3		1								0.11111
21			2	2	-16	20		4	-16			2							0.11111
33		0.5		-4	2		8	-6	1	-4	2		0.5						0.05556
32			1	2	-8	2	-6	19	-8	2	-8	2		1					0.11111
31					3	-8	1	-8	22.5		2	-8			1				0.11111
43				0.5			-4	2		8	-6	1	-4	2		0.5			0.05556
42					1		2	-8	2	-6	19	-8	2	-8	2		1		0.11111
41						1		2	-8	1	-8	21		2	-8			1	0.11111
53							0.5			-4	2		7.5	-6	1	-3	2		0.05556
52								1		2	-8	2	-6	18	-8	2	-6	2	0.11111
51									1		2	-8	1	-8	20		2	-6	0.11111
63										0.5			-3	2		3	-3	0.5	0.02778
62											1		2	-6	2	-3	7.5	-4	0.05556
61												1		2	-6	0.5	-4	8.5	0.05556

TABLE 1–C

	ψ_{03}	ψ_{13}	ψ_{12}	ψ_{23}	ψ_{22}	ψ_{21}	ψ_{33}	ψ_{32}	ψ_{31}	ψ_{43}	ψ_{42}	ψ_{41}	ψ_{53}	ψ_{52}	ψ_{51}	
03	3	-6	2	1												0.02778
13	-3	9.5	-6	-4	3		0.5									0.05556
12	2	-12	18	4	-16	2		2								0.11111
23	0.5	-4	2	8	-6	1	-4	2		0.5						0.05556
22		3	-8	-6	21	-8	2	-8	3		1					0.11111
21			2	2	-16	20		4	-16			2				0.11111
33		0.5		-4	2		8	-6	1	-4	2		0.5			0.05556
32			1	2	-8	2	-6	19	-8	2	-8	2		1		0.11111
31					3	-8	1	-8	22.5		2	-8			1	0.11111
43				0.5			-4	2		8	-6	1	-4	2		0.05556
42					1		2	-8	2	-6	19	-8	2	-8	2	0.11111
41						1		2	-8	1	-8	21		2	-8	0.11111
53							0.5			-4	2		8.5	-6	1	0.05556
52								1		2	-8	2	-6	20	-8	0.11111
51									1		2	-8	1	-8	22	0.11111

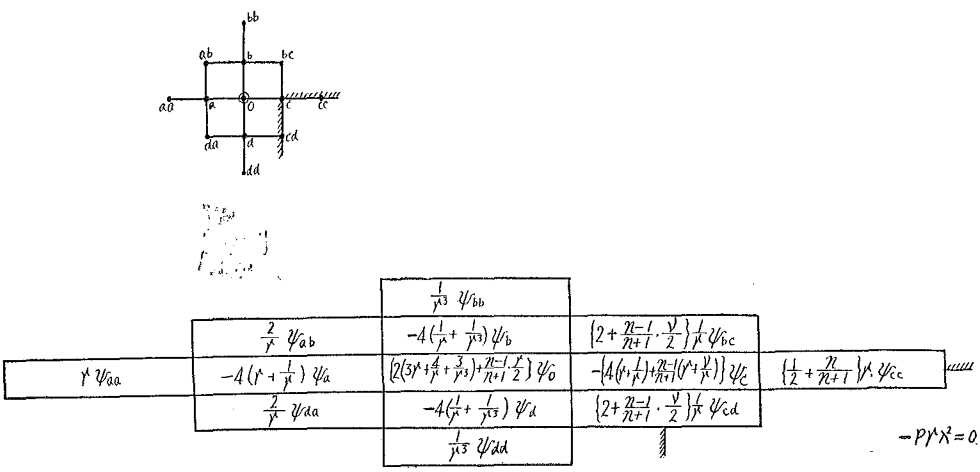


Fig. 2–1.

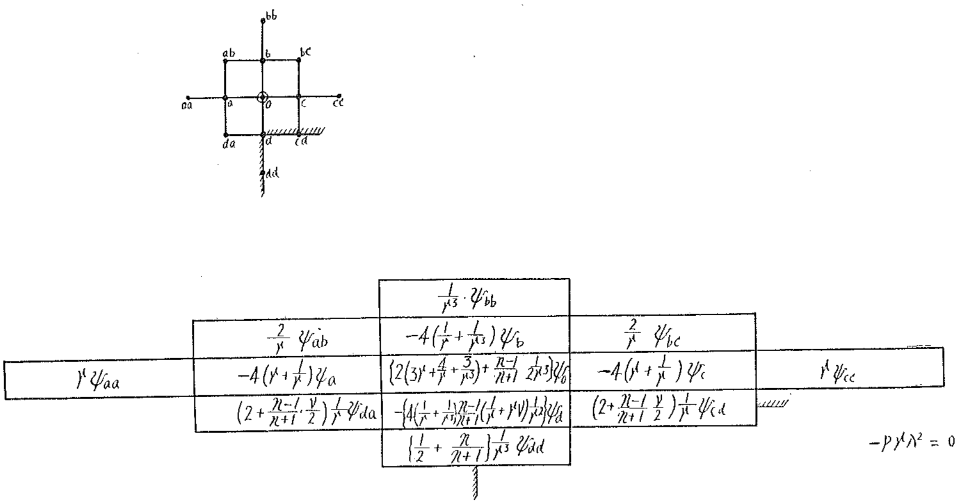


Fig. 2-2.

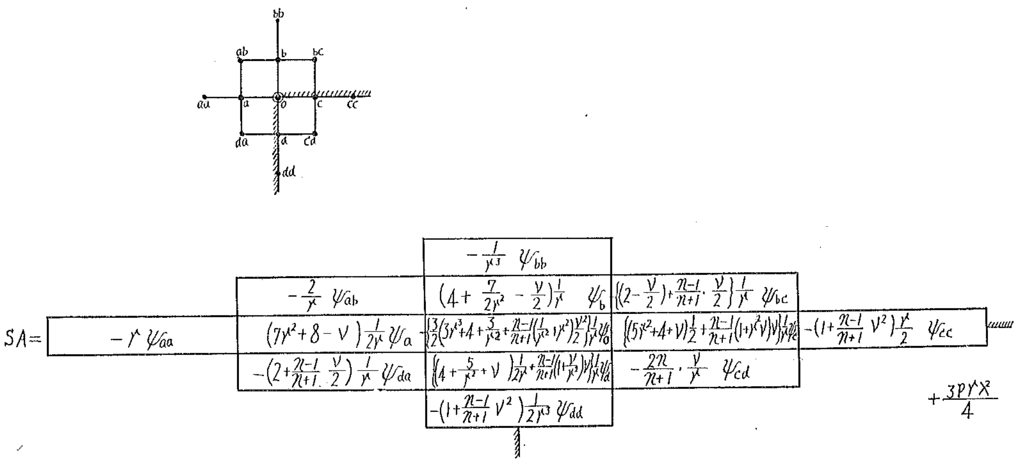


Fig. 3.

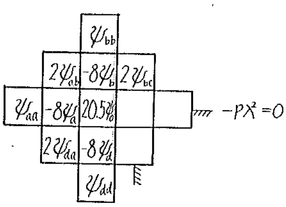


Fig. 4.

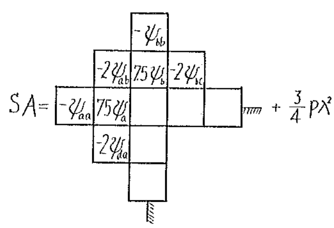
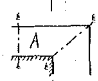


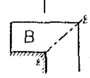
Fig. 5.

TABLE 2-A, B, C Col. I Deflection Ψ (Pl_x^2)
Col. II Bending Moment M_y (")
Col. III " M_x (")
Col. IV Torsional Moment M_{xy} (")
Col. V Reaction A (Pl_x)

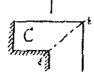
1.45410 0 0.10430 0	1.50625 0 0.12303 0.02981	1.68143 0 0.17074 0.04613	2.02735 0 0.20673 0.02603	2.58000 0 0.13551 0.00531	3.26816 0 0.04761 0.01194	4.00393 0 0 0
0.80614 0.10116 0.07212 0	0.84220 0.09593 0.09559 0.03712	0.97385 0.07194 0.16555 0.06551	1.27105 0.01839 0.25211 0.03668	1.82036 0.02936 0.15281 0.01159	2.52248 0.04356 0.04356 0.01650	
0.25933 0.28746 0.02948 0	0.27407 0.29404 0.04939 0.04193	0.33820 0.29743 0.13080 0.10721	0.53313 0.20477 0.36201 0.07834	1.09008 0.17334 0.17334 0.00491		
0 0.51866 0 0 0.68334	0 0.548l4 0 0 0.63258	0 0.67640 0 0 0.51882	0 1.06626 1.06626 0 6.01556			



1.36715 0 0 0	1.49412 0 0.07994 0.04380	1.70103 0 0.15432 0.05344	2.06226 0 0.20304 0.02919	2.62653 0 0.13531 0.00466	3.32611 0 0.04765 0.01195	4.07334 0 0 0
0.74199 0.11303 0 0.04275	0.83461 0.09624 0.06105 0.05471	0.98828 0.06939 0.15384 0.07361	1.29579 0.01564 0.25210 0.03944	1.85540 0.02838 0.15394 0.06336	2.56895 0.04361 0.04361 0.01677	
0.22986 0.28227 0 0.04631	0.27134 0.29193 0.03210 0.06157	0.34492 0.29844 0.12646 0.11530	0.54496 0.20587 0.36765 0.08054	1.11265 0.17506 0.17506 0.00566		
0 0.45972 0 0 0.20124	0 0.54268 0 0 0.72633	0 0.68984 0 0 0.58212	0 1.08992 1.08992 0 6.18351			



0 0 0.50478 0 2.45484	0.25239 0 0.16675 0.12716	0.67153 0 0.08460 0.14270	1.17527 0 0.07081 0.10904	1.74982 0 0.00956 0.06420	2.33393 0 0.03350 0.03911	2.88454 0 0 0
0 0 0.29244 0 0.60015	0.14622 0.01159 0.12477 0.12724	0.41721 0.00032 0.09550 0.15399	0.78370 0.03358 0.12723 0.11398	1.27742 0.02006 0.04282 0.05995	1.81396 0.01657 0.01657 0.04247	
0 0 0.10328 0 0.04101	0.05164 0.04294 0.05929 0.10430	0.16257 0.09207 0.08505 0.15937	0.35855 0.06660 0.23043 0.12542	0.78496 0.06605 0.06605 0.06164		
0 0 0 0 0.22650	0 0.10328 0 0 0.00804	0 0.32514 0 0 0.35559	0 0.71710 0.71710 0 5.02194			



3. Solution of the Equations

The equations are solved simultaneously by the method of elimination and stresses are found as respectively shown in Tables 2-A, B, C.

4. Stress Diagrams

The distributions of bending moments M_x and M_y, torsional moment M_{xy}, reaction A and principal stresses are diagramed below. (Figs. 6-1, 7-1, 8-1)

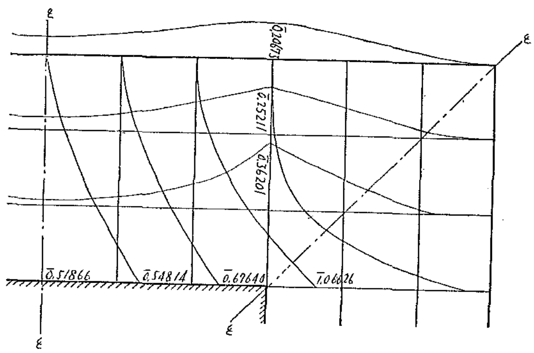


Fig. 6-1.

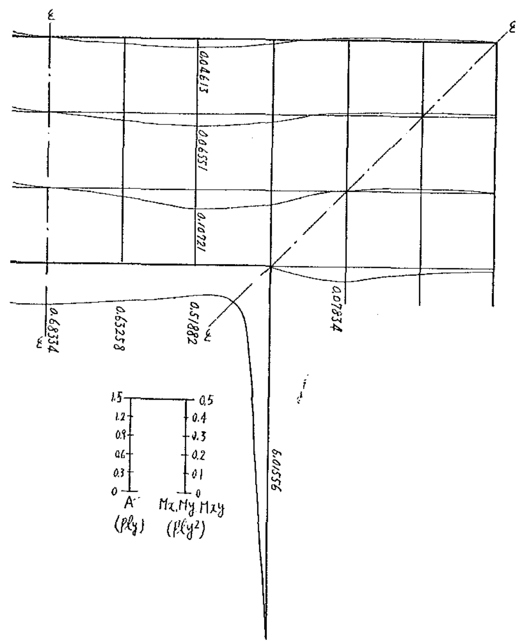


Fig. 6-2.

show M_x and M_y, Figs. 6-2, 7-2, 8-2 M_{xy} and A, Figs. 9-A, 9-B, 9-C the principal stress diagram of each plate, respectively.)

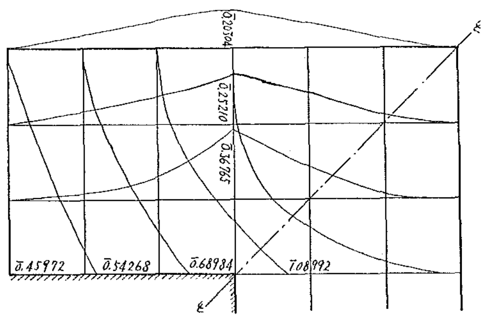


Fig. 7-1.

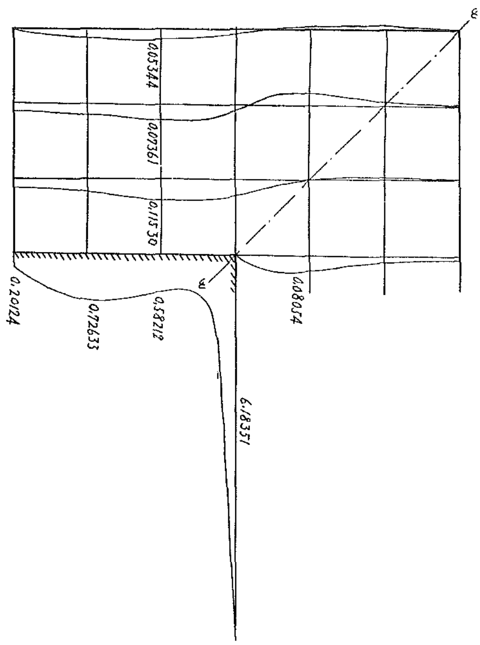


Fig. 7-2.

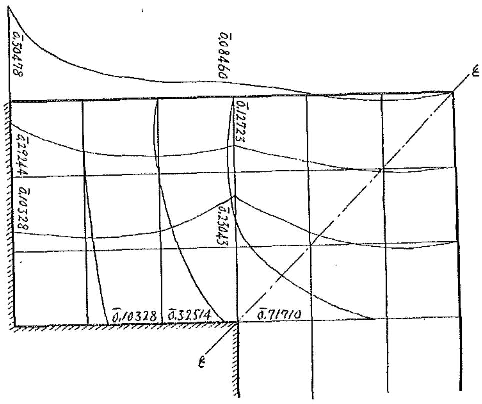


Fig. 8-1.

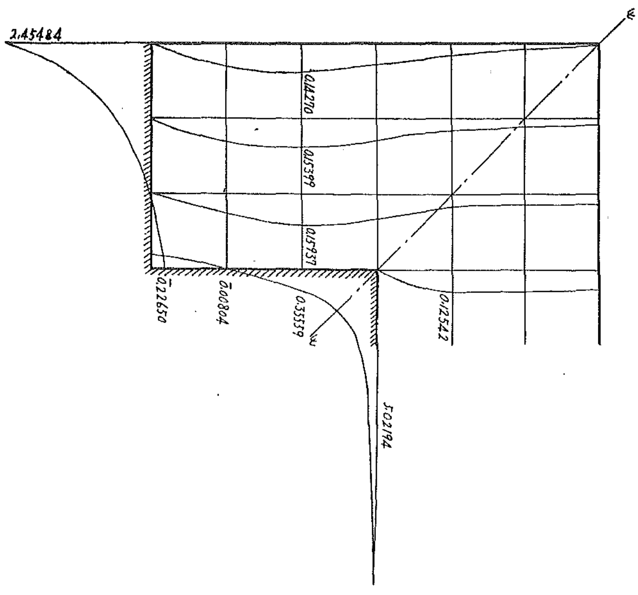
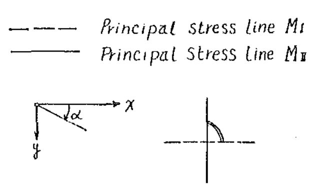

2.45484

Fig. 8-2.


Principal stress line M_I
Principal Stress Line M_{II}
x
y
α

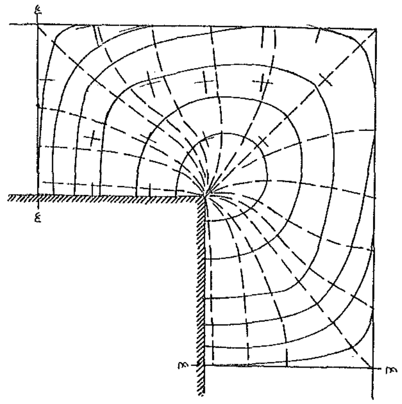


Fig. 9–A.

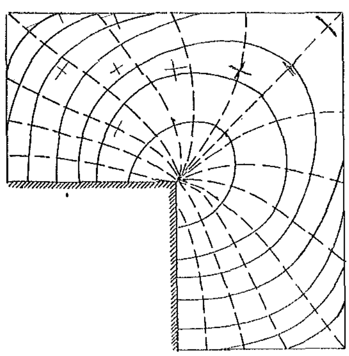


Fig. 9–B.

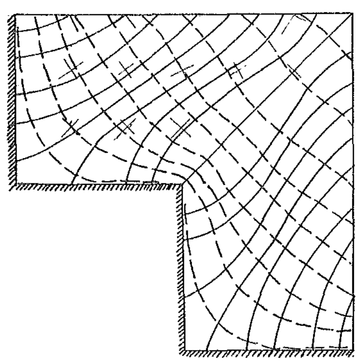


Fig. 9–C.

Conclusions

The results for each plates are examined and respectively summarized as follows :

Plate A : Bending moments M_x, M_y and reaction A are highly concentrated at the reentering corner of the plates. Especially M_y is larger than twice the value for the corresponding cantilever and almost two thirds of the total load is concentrated at that corner.

Plate B : The effect of the transition when the line of symmetry of Plate A is made free is appreciable by the fact that stresses are concentrated at the reentering corner slightly higher than in case of Plate A, so that M_y along such free edge is made rather smaller than that of the corresponding cantilever.

Plate C: Stress concentration at the corner is smaller than in case of Plate A or B since stresses are distributed along the side edge, induced by the transition from free (Plate B) to built-in edges. The characteristics of plate C are discriminated from those of Plate A or B by the fact that the distributions of the stresses in Plate C are considerably similar to those in a rectangular plate with two adjacent edges built in and the others free. Figs 10–1 and 10–2 explain the concentration of M_x, M_y along the extension line of the reentering built-in edges down the corner.

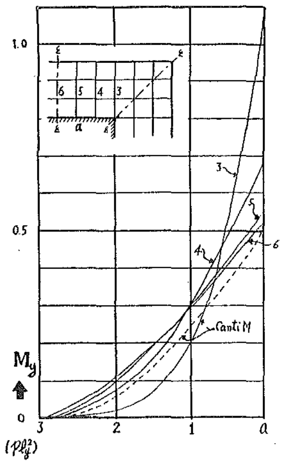


Fig. 10–1.

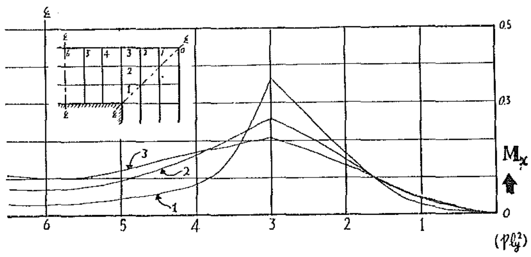


Fig. 10–2.

In Figs. 11–A, B and C is compared the bending moment Mn (principal bending moment) of each plate with that of the corresponding cantilever which diagonally spans from its reentering corner to external corner.

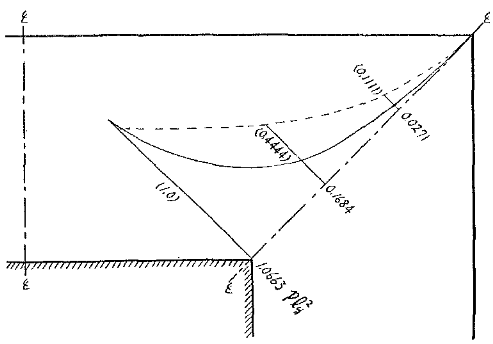


Fig. 11–A.

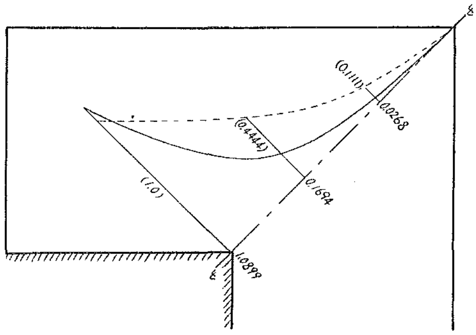


Fig. 11-B.

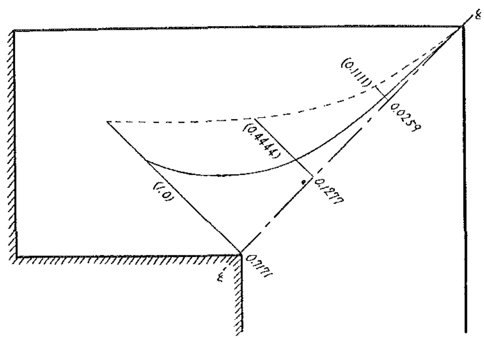


Fig. 11-C.

P.S.

Checking the sum of the reactions proved the error in calculation within the ratio of ca. 0.0002 to the unity of the total load.

Acknowledgement

The author wishes to thank Prof. Yokota for his kind interest and suggestions in this work.

References

*) Yoshizo Dobashi and Shigeyuki Miura: Studies on the Stresses of Rectangular Plates with a Thin or Open Quadrangle in the Center. TRANSACTIONS OF A.I.J. Vol. 57; MEMOIRS OF A.I.J. Vol. 42 and 45; MEMOIRS OF FACULTY OF ENGINEERING, HOKKAIDO UNIV. Vol. 19.