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Title	Stresses in Thin Plates of Cantilever Type Stretched out around an External Angle
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### Introduction

Although most problems of free edged rectangular plates have already been solved, stresses of the plates in the title remain to be solved, which are important in the practical application for the cases of pent-roofs or canopies of reinforced concrete.

As there is no information on stresses in the cases above mentioned we have no reliable methods of reinforcement of the concrete, hence sometimes the risk of cracking or collapsing of cantilever plates cannot be avoided.

The present paper discusses a solution for such plates subject to uniform load using the method of finite differences which was previously applied to the solution of the cases of rectangular plates with a thin or open quadrangle in the center<sup>\*)</sup>.

# 1. Nomenclature and the Subdivision of the Surface of the Plates

The following definitions are given here:

 $SA = \lambda A$ ,  $SA = \lambda' A$ .  $(\gamma = \lambda/\lambda')$ 

with N: Flexural rigidity of plate

- $\Psi$ : Deflection
- h: Thickness of plate
- A: Reaction
- ν: Poisson's ratio
- SA: Resultant reaction
- $\lambda$ ,  $\lambda'$ : Length of subdivision of plate

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Each surface of the plates is subdivided into equal small squares with every point of intersection numbered as shown in Fig. 1.



# 2. Setting Up the Equations

The finite difference equations at all such points derived from the governing equations for rectangular plates with a thin quadrangle in the center are shown in Table 1.

At the same time the equation for Point 31 which may come into question and the equation for determining the resultant reaction at the reentering corner point are schematized in Figs. 2-1, 2-2 and 3.

And further, these two equations are so simplified by substituting  $\nu = 0$ ,  $\lambda = \lambda'$  and  $n = \infty$  in them as shown in Fig. 4 and 5, respectively.

<b></b>	¥03	$\psi_{13}$	$\psi_{12}$	1/23	4/22	$\psi_{2l}$	¥/33	¥32	$\psi_{31}$	Ψ43	4/42	4/41	$\psi_{53}$	ψsz	Ų'51	¥63	Ψ62	¥61	
03	3	6	2	1															0.02778
13	-3	9.5	-6	-4	3		0.5											_	0.05556
12	2	-12	18	4	-16	2		2						-					0.11111
23	0.5	-4	2	8	- 6	1	-4	2		0.5									0.05556
22		3	-8	-6	21	-8	2	-8	3		1								0.11111
21			2	2	-16	20		4	-16			2						_	0.11111
33		0.5		-4	2		8	-6	11	-4.	2		0.5						0.05556
32			1	2	-8_	2	-6	19	-8	2	-8	2		1					0.11111
31					3	-8	1	-8	22.5		2	-8			1				0.11111
43				0.5			-4	2		8	-6		-4	2		0.5			0.05556
42					1		2	-8	2	-6	19	-8	2	-8	2		1		0.11111
41				·		1		2	-8	1	-8	21		2	-8			1	0.1111
53							0.5			-4	2	<u> </u>	8.5	-6	1	-4	2		0.05556
52								1		2	8	2	-6	20	-8	2	8	2	0.11111
51									1		2	-8	11	-8	22		2	-8	0.1111
63										1			-8	4		8	6	1	0.05556
62											2		4	- 16	4	-6	19	-8	0.11111
61										· · · · ·		2		4	-16	T	-8	21.	0.11111

TABLE	1–A	

# TABLE 1-B

Γ	$\psi_{03}$	$\psi_{13}$	¥12	$\psi_{23}$	W22	4/21	Ų'33	Ψ32	$\psi_{31}$	Ψ43	¥42	4/41	¥53	Ψ52	$\psi_{s_l}$	Ψ63.	¥62	¥61	
03	3	-6	2	1															002778
13	-3	9.5	-6	-4	3		0.5												005556
12	2	-12	18	4	-16	2		2					L						0.1111
23	0.5	-4	2	8	-6	1	-4	2		0.5									0.05556
22		3	- 8	-6	21	-8	2	-8	3		1								0.11111
21			.2	2	-16	20		4	~16			2							011111
33		05		-4	2		8	-6	1	-4	2		0.5						0.05556
32			1	2	-8	2	-6	19	-8	2	-8	2	L	1					0.11111
31					3	-8	1	-8	22.5		2	-8			1				0.1111
43		L		0.5			-4_	2		8	-6	1	-4	2		0.5			0.05556
42					1		2	-8	2	-6	19	-8	2	-8	2				0.11111
11						1		2	-8	1	-8	21		2	-8_				0.11111
53							0.5		· ·	-4.	2		7.5	-6	1	-3	2		0.05556
52								1		2	8	2	-6	18	-8_	2	-6	2	0.11111
51										L	2	-8	1	-8	20		2	-6_	0.11111
63										0.5			-3	2		3	-3	0.5	002778
62						[					1		2	-6	2	- 3	7.5	-4	005556
61								L	L			)		2	-6	0.5	-4	8.5	0.05556

TABLE 1-C

<b></b>	Yo3	¥13	¥12	$\psi_{23}$	$\psi_{22}$	$\psi_{2l}$	$\psi_{33}$	$\psi_{32}$	$\psi_{31}$	$\psi_{43}$	$\psi_{42}$	$\psi_{41}$	$\psi_{ss}$	$\psi_{52}$	$\psi_{sl}$	
03	3	-6	2											·		0.02778
13	-3	9.5	-6	-4	3		0.5									0.05556
12	2	-12	18	4	-16	2		2								0.11111
23	0.5	-4	2	8	-6	1	-4	2_	1	0.5			L			0.05556
22		3	-8	-6	21	-8	· 2	-8	3		1					0.1111
21	_		2	2	~16	20		4	-16			2				0.11111
33		0.5		-4	2		8	-6	1	-4	2		0.5			0.05556
32			1	2	-8	2	-6	19	-8	2	-8_	2				0.11111
31					3	-8	1	-8	22.5		2	-8		·		0.11111
43				0.5		<u> </u>	-4	2		8	-6	1	-4	2		0.05556
42					1		2	-8	2	-6	19_	-8	_2	-8	2	0.11111
41						1		2	-8	1	-8	21		2	-8	0.11111
53							0.5			-4	2_		8.5	-6	1	0.05556
52			1					T		2	-8	2	-6	20	-8	0.11111
51											2	-8	1	-8	22	0.1111



	Constant of	1/3 4/bb		
	2 Vab	$-4(\frac{1}{r}+\frac{1}{r^{3}})\psi_{b}$	$\left\{2+\frac{n-1}{n+1}\cdot\frac{\sqrt{2}}{2}\right\}/\sqrt{\psi_{bc}}$	
Y Vac	$-4(r+\frac{1}{\mu})\psi_a$	$\left(2(3j^{h}+\frac{4}{j^{h}}+\frac{3}{j^{r_{3}}})+\frac{n-1}{n+1}\frac{j}{2}\right)\psi_{0}$	-[4(+,+)+2+](+++)] 4/2	$\left\{\frac{1}{2}+\frac{n}{n+1}\right\}$ $\mathcal{Y}$ . $\mathcal{Y}_{cc}$
	2 y Vda	$-4(\frac{1}{1^{1+1}}+\frac{1}{1^{1+1}})\psi_{d}$	$\left\{2+\frac{n-1}{n+1}\cdot\frac{\sqrt{2}}{2}\right\}_{l}^{t} \psi_{cd}$	
		1/13 4/2d		$- P \gamma^{*} X = 0$

Fig. 2-1.





Fig. 2-2.





Fig. 3.





TABLE 2-A, 1	B, C Col. I Col. II Col. III Col. IV Col. V	Deflection $\Psi$ Bending Moment " Torsional Moment Reaction A	$(Pl_x^{\ 2}) \ M_y \ (\ "\ ) \ M_x \ (\ "\ ) \ M_{xy} \ (\ "\ ) \ (Pl_x)$
2 1454 0 0 0 0 0 0 0 0 0 0 0 0 0	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	. \$ 4 00 3 4 3 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

						٤
1.367/5 0 0 0	1.494.12 O O,0 7994 O.04380	1.7010 3 0 0.154 32 0.05344	2.06226 O 0.20304 0.02 <u>9</u> 19	2.62.653 0 0.13531 0.00466	3.32611 0 004765 001195	4,07334 0 0 0
0.74 199 0.11 303 0 0.04.275	0834.61 009624 0.06105 0.05471	0.98828 0.06939 0.15384 0.07361	1.29579 0.01564 0.25210 0.03944	1.85340 Q02838 Q.15394 Q06336	256895 0.04361 0.04361 0.04361 0.01677	
0,22986 028229 0 0,04631	0,27/34 0,29/93 0,032/0 0,06/57	0.34492 0.29844 0.12646 0.11530	054496 020587 036765 0.08054	1.11265 0.17506 0.17506 0.00566		-
0 0,45972 0 0 0,20/24	0 0.54268 0 0.72633	0 0.68 984 0 0 0.58 212	0 1.08992 108992 0 6.18351	В		

						×.
0	0.25239	0.67/53	1.17527	1.74.982	2.33393	2.884.54
0.50478 0 2.45484	0,16675 0,12716	0.0 <b>8</b> 460 0.14270	0.07081 0.10904	0.00956 0.06420	0.03350 0.03911	000
0 0 0.29244 0 0.600/5	0.14.622 001159 0.12477 0.12724	0.41721 000032 0.09550 0.15399	0.78370 003358 0.12723 0.11398	1.27742 0.02006 0.04282 0.05995	1.81396 0.01657 0.01657 0.04247	
0 Q 0,10328 0 0.04.101	0.05164 0.04294 0.05929 0.10430	0.76257 0.09207 0.08505 0.75937	0.35855 0.06660 0.23043 0.12542	0.78496 0.06605 0.06605 0.06605		
0 0 0 0.22650	0,/0328 0 0 0.00804	0 0325/4 0 0 035559	0 07/7/0 07/7/0 0 502/94	C	   	

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# 3. Solution of the Equations

The equations are solved simultaneously by the method of elimination and stresses are found as respectively shown in Tables 2–A, B, C.

# 4. Stress Diagrams

The distributions of bending moments  $M_x$  and  $M_y$ , torsional moment  $M_{xy}$ , reaction A and principal stresses are diagramed below. (Figs. 6–1, 7–1, 8–1)







Fig. 6-2.

show  $M_x$  and  $M_y$ , Figs. 6–2, 7–2, 8–2  $M_{xy}$  and A, Figs. 9–A, 9–B, 9–C the principal stress diagram of each plate, respectively.)









Fig. 8-1.











Fig. 9-A.



### Conclusions

The results for each plates are examined and respectively summarized as follows:

- Plate A: Bending moments  $M_x$ ,  $M_y$  and reaction A are highly concentrated at the reentering corner of the plates. Especially  $M_y$  is larger than twice the value for the corresponding cantilever and almost two thirds of the total load is concentrated at that corner.
- Plate B: The effect of the transition when the line of symmetry of Plate A is made free is appreciable by the fact that stresses are concentrated at the reentering corner slightly higher than in case of Plate A, so that  $M_{\nu}$  along such free edge is made rather smaller than that of the corresponding cantilever.

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Plate C: Stress concentration at the corner is smaller than in case of Plate A or B since stresses are distributed along the side edge, induced by the transition from free (Plate B) to built-in edges. The characteristics of plate C are discriminated from those of Plate A or B by the fact that the distributions of the stresses in Plate C are considerably similar to those in a rectangular plate with two adjacent edges built in and the others free. Figs 10-1 and 10-2 explain the concentration of  $M_x$ ,  $M_y$  along the extension line of the reentering built-in edges down the corner.



In Figs. 11–A, B and C is compared the bending moment Mn (principal bending moment) of each plate with that of the corresponding cantilever which diagonally spans from its reentering corner to external corner.



Fig. 11-A.



Fig. 11-C.

P.S.

Checking the sum of the reactions proved the error in calculation within the ratio of ca. 0.0002 to the unity of the total load.

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### References

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