Bending Stresses in Beamless Stair Slabs

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Introduction

In case of stair slabs, as shown in Fig. 1 (a) and (b), with subbeams and bridgeboards, B_1 and B_2, respectively, landings and flights are usually designed as rectangular plates with all edges fixed and in case of having only B_1 (without B_2) are designed as rectangular plates with three edges fixed and one free. It is noted that stairs are often designed without B_1 and B_2.

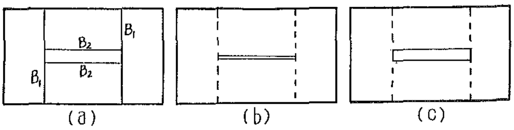


Fig. 1.

The difficulty in analyzing the actions of these slab members has brought about conventional solutions, developed by regarding flights as plates with three edges fixed or mere cantilevers and by regarding landings as beams with a sufficiently large width to be substituted for the corresponding slabs or as all-edge fixed slabs.

These assumptions may be inevitable for lack of closer approximations.

The modes of stress distributions in stair slab (a) may be assimilated to those in slab (b) or (c) in Fig. 1. In both cases flights should be treated as having two different rigidities in londitudinal and transversal directions.

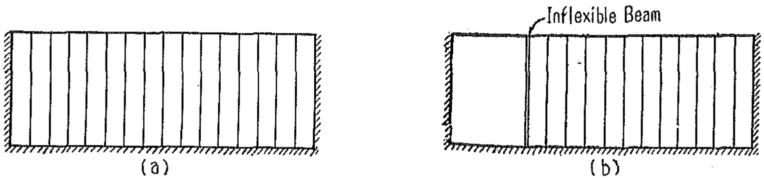


Fig. 2.

Approaches from this viewpoint seem not to have been made except by Y. Yokoo[1] who presented a solution for such a type as shown in Fig. 2 (a) and 2 (b) by the method of difference.

The present paper presents solutions for a few cases of stair slab as shown

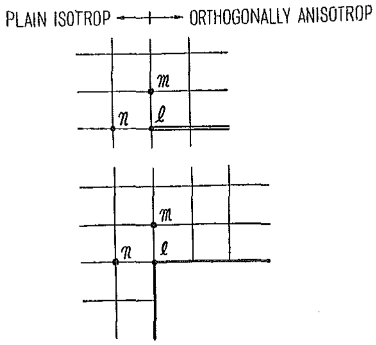


Fig. 3.

in Fig. 1 (b). The solutions are obtained from simultaneous systems of difference equations concerning isotropic (landing) and anisotropic (flights) parts and including the equations at the points in question relative to both these parts (shown in Fig. 3).

The results of the experiment help those equations to be set up on the assumption that a stair slab be approximately treated as a horizontal plane body which is simply supported at the end of its slit insted of as a system of horizontal and sloping members.

1. Subdivision of Slab Surface and Notation

Each slab surface is plotted out into equal small squares with all intersecting points numbered as shown in Fig. 4.

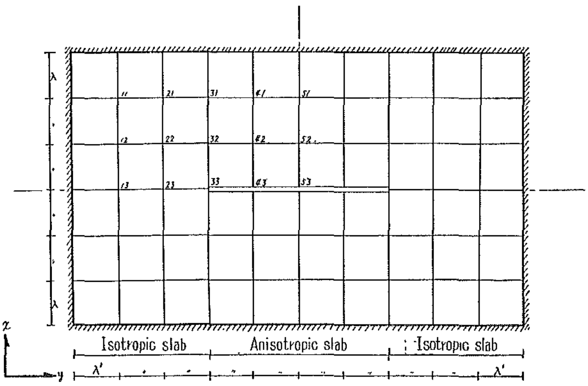


Fig. 4.

$\psi = \frac{N}{\lambda^2} w$	w:	deflection
	λ:	width of a square
$N = \frac{Eh^3}{12(1-\nu^2)}$	:	flexual rigidity of normal slab
$N_x = \frac{(Eh^3)_x}{12(1-\nu_x\nu_y)}$	:	flexual rigidity of anisotropic slab in x-direction
$N_y = \frac{(Eh^3)_y}{12(1-\nu_x\nu_y)}$	:	do. in y-direction

with E: Young's modulus of normal and anisotropic slab in x and y-directions.
h: thickness of slab in x and y-direcions.
ν: Poisson's ratio.
SA: reaction resultant.
A: reaction.

2. Governing Differential Equations of Stair Slabs

The governing differential equation of an orthogonally anisotropic slab is

$$N_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + N_y \frac{\partial^4 w}{\partial y^4} = p \quad \cdots\cdots \text{(A)}$$

with $2H = 4C - (N_x \nu_y + N_y \nu_x)$

$2C = (1 - \sqrt{\nu_x \nu_y}) \sqrt{N_x N_y}$

In this equation N_x, N_y and H must be decided in order to carry out computations. Strictly speaking, these values should be experimentally established but here they are conventionally adopted. An approximate relation, $H^2 = N_x N_y$, is assumed here as is generally the case with computative studies of this kind.

Accordingly, Eq. (A) is

$$N_x \frac{\partial^4 w}{\partial x^4} + 2\sqrt{N_x N_y} \frac{\partial^4 w}{\partial x^2 \partial y^2} + N_y \frac{\partial^4 w}{\partial y^4} = p \quad \cdots\cdots \text{(B)}$$

Putting $N_x = N_y = N$ in Eq. (B) the equation of isotropic slabs is obtainable.

And further, stresses and reactions are expressed as follows:

$$\left.\begin{aligned}
M_x &= -N_x \left(\frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} \right) \\
M_y &= -N_y \left(\frac{\partial^2 w}{\partial y^2} + \nu_x \frac{\partial^2 w}{\partial x^2} \right) \\
M_{xy} &= -2C \frac{\partial^2 w}{\partial x \partial y} \\
Q_x &= -N_x \frac{\partial^3 w}{\partial x^3} - (N_x \nu_y + 2C) \frac{\partial^3 w}{\partial x \partial y^2} \\
Q_y &= -N_y \frac{\partial^3 w}{\partial y^3} - (N_y \nu_x + 2C) \frac{\partial^3 w}{\partial x^2 \partial y} \\
A_x &= -N_x \left\{ \frac{\partial^3 w}{\partial x^3} + \left(\frac{4C}{N_x} + \nu_y \right) \frac{\partial^3 w}{\partial x \partial y^2} \right\} \\
A_y &= -N_y \left\{ \frac{\partial^3 w}{\partial y^3} + \left(\frac{4C}{N_y} + \nu_x \right) \frac{\partial^3 w}{\partial x^2 \partial y} \right\}
\end{aligned}\right\} \quad \cdots\cdots \text{(C)}$$

For simiplicity putting $\nu=0$ in Eq. (C) we obtain

$$\left.\begin{aligned}
M_x &= -N_x \frac{\partial^2 w}{\partial x^2} \\
M_y &= -N_y \frac{\partial^2 w}{\partial y^2} \\
M_{xy} &= -\sqrt{N_x N_y} \frac{\partial^2 w}{\partial x \partial y} \\
Q_x &= -\left(N_x \frac{\partial^3 w}{\partial x^3} + \sqrt{N_x N_y} \frac{\partial^3 w}{\partial x \partial y^2}\right) \\
Q_y &= -\left(N_y \frac{\partial^3 w}{\partial y^3} + \sqrt{N_x N_y} \frac{\partial^3 w}{\partial x^2 \partial y}\right) \\
A_x &= -\left(N_x \frac{\partial^3 w}{\partial x^3} + 2\sqrt{N_x N_y} \frac{\partial^3 w}{\partial x \partial y^2}\right) \\
A_y &= -\left(N_y \frac{\partial^3 w}{\partial y^3} + 2\sqrt{N_x N_y} \frac{\partial^3 w}{\partial x^2 \partial y}\right)
\end{aligned}\right\} \cdots\cdots\cdots\cdots \text{(D)}$$

And substituting $N_x/N_y=k^4, N_y/N=n$ in Eq. (B) we obtain

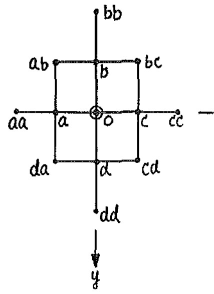
$$k^4 \frac{\partial^4 w}{\partial x^4} + 2k^2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \mathrm{p}/nN \cdots\cdots\cdots\cdots \text{(E)}$$

Thus the corresponding stresses and reactions are indicated as

$$\left.\begin{aligned}
M_x &= -nk^4 N \frac{\partial^2 w}{\partial x^2} \\
M_y &= -nN \frac{\partial^2 w}{\partial y^2} \\
M_{xy} &= -nk^2 N \frac{\partial^2 w}{\partial x \partial y} \\
Q_x &= -nk^2 N\left(k^2 \frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2}\right) \\
Q_y &= -nN \left(\frac{\partial^3 w}{\partial y^3} + k^2 \frac{\partial^3 w}{\partial x^2 \partial y}\right) \\
A_x &= -nk^2 N\left(k^2 \frac{\partial^3 w}{\partial x^3} + 2 \frac{\partial^3 w}{\partial x \partial y^2}\right) \\
A_y &= -nN \left(\frac{\partial^3 w}{\partial y^3} + 2k^2 \frac{\partial^3 w}{\partial x^2 \partial y}\right)
\end{aligned}\right\} \cdots\cdots\cdots\cdots \text{(F)}$$

Eqs. (E) anp (F) are the governing differential equations of a stair slab composed of isotropic and anisotropic parts.

If these parts are joined together in y-direction the following relation stands between them[15].



$\psi_b' =$	$\frac{2\eta}{\eta+1}\psi_b$	$-2\frac{\eta-1}{\eta+1}\psi_0$	$\frac{\eta-1}{\eta+1}\psi_d$
$\psi_d' =$	$\frac{2}{\eta+1}\psi_d$	$2\frac{\eta-1}{\eta+1}\psi_0$	$-\frac{\eta-1}{\eta+1}\psi_b$

… (G)

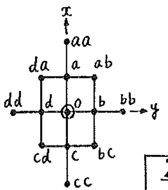
Fig. 5.

3. Finite Difference Equations at the Points in Question

According to the previous paper[3] such equations are schematized as under.

Developing into Difference Expression

The development and schematization of the fundamental differential equations into difference expressions produce Fig. 6.



		$\eta k^4 \gamma \psi_{aa}$		
	$\frac{2\eta k^2}{\gamma}\psi_{da}$	$-\left(4\eta k^4\gamma + \frac{4\eta k^2}{\gamma}\right)\psi_a$	$\frac{2\eta k^2}{\gamma}\psi_{ab}$	
$\frac{\eta}{\gamma^3}\psi_{dd}$	$-\left(\frac{4\eta k^2}{\gamma}+\frac{4\eta}{\gamma^3}\right)\psi_d$	$\left(6\eta k^4\gamma + \frac{8\eta k^2}{\gamma} + \frac{6\eta}{\gamma^3}\right)\psi_0$	$-\left(\frac{4\eta k^2}{\gamma}+\frac{4\eta}{\gamma^3}\right)\psi_b$	$\frac{\eta}{\gamma^3}\psi_{bb}$
	$\frac{2\eta k^2}{\gamma}\psi_{cd}$	$-\left(4\eta k^4\gamma + \frac{4\eta k^2}{\gamma}\right)\psi_c$	$\frac{2\eta k^2}{\gamma}\psi_{bc}$	
		$\eta k^4 \gamma \psi_{cc}$		

$-p\gamma\lambda^2 = 0$

Fig. 6.

Resultant reaction $SA_x = \lambda' A_x$ and $SA_y = \lambda A_y$ become, from $A_x = -nk^2N(k^2\partial^3 w/\partial x^3 + 2\partial^3 w/\partial x \partial y^2)$,

$SA_x =$

	$-\frac{nk^4r}{2}\psi_{aa}$	
$-\frac{nk^2}{r}\psi_{da}$	$(nk^4r+\frac{2nk^2}{r})\psi_a$	$-\frac{nk^2}{r}\psi_{ab}$
$\frac{nk^2}{r}\psi_{cd}$	$-(nk^4r+\frac{2nk^2}{r})\psi_c$	$\frac{nk^2}{r}\psi_{bc}$
	$\frac{nk^4r}{2}\psi_{cc}$	

Fig. 7.

and from $A_y = -nN(\partial^3 w/\partial y^3 + 2k^2\partial^3 w/\partial x^2\partial y)$,

$SA_y =$

	$-\frac{nk^2}{r}\psi_{da}$		$\frac{nk^2}{r}\psi_{ab}$	
$-\frac{n}{2r^3}\psi_{dd}$	$(\frac{n}{r^3}+\frac{2nk^2}{r})\psi_d$		$-(\frac{n}{r^3}+\frac{2nk^2}{r})\psi_b$	$\frac{n}{2r^3}\psi_{bb}$
	$-\frac{nk^2}{r}\psi_{cd}$		$\frac{nk^2}{r}\psi_{bc}$	

Fig. 8.

Concentrated Reaction F become from F

$F = 2M_{xy} =$

$-\frac{nk^2}{2r}\psi_{ab}$		$\frac{nk^2}{2r}\psi_{bc}$
$\frac{nk^2}{2r}\psi_{da}$		$-\frac{nk^2}{2r}\psi_{cd}$

Fig. 9.

Now, if a point on the slab is regarded as the junction of slab A, B, C and D and the resultant reaction for each slab is considered

$$SA_{\text{corner}} = (I)\times 1/4 + (J)\times 1/2 + (K)\times 1/2 + (F)$$

then the resultant reaction at each corner point becomes

$$SA_{\text{cor.,er}} = (I)\times 1/4 + (J)\times 1/2 + (K)\times 1/2 + (L)$$

The conditions of equilibrium at this corner point are:

$$\sum M_x = 0 \qquad SM_A + SM_B - SM_C - SM_D = 0$$
$$\sum M_y = 0 \qquad SM_A + SM_D - SM_B - SM_C = 0$$
$$\sum Z = 0 \qquad SA_A + SA_B + SA_C + SA_D = 0$$

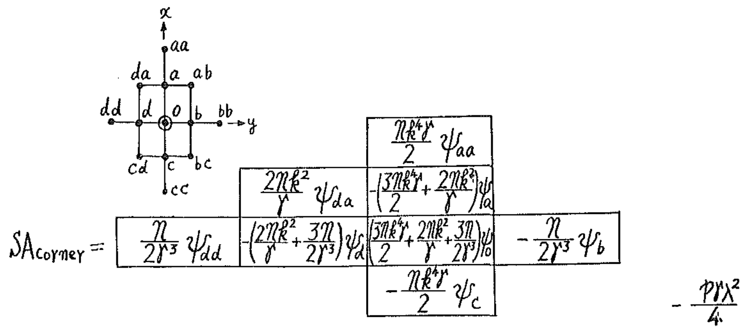


Fig. 10.

For the points adjacent to the end of the slit the respective equations are set up by eliminating the terms due to external points availing the condition $M_x\lambda/2=0$.

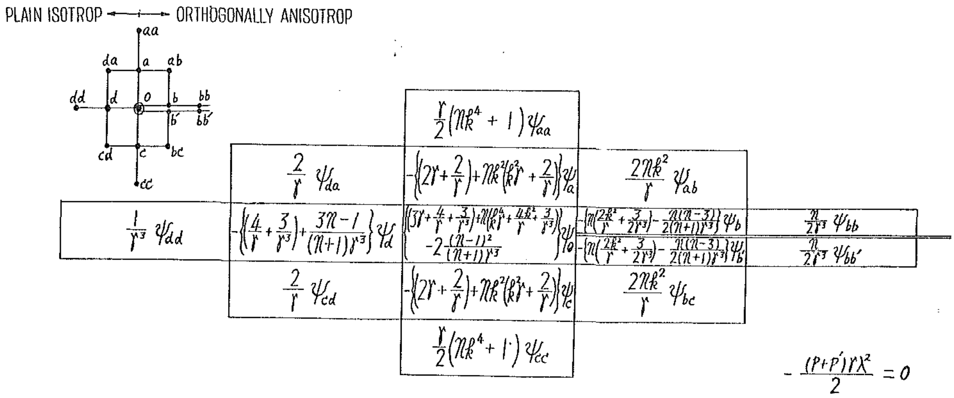


Fig. 11.

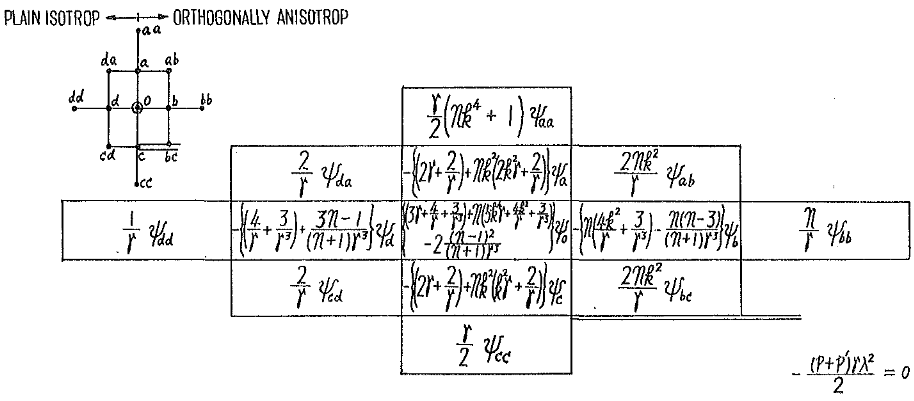


Fig. 12.

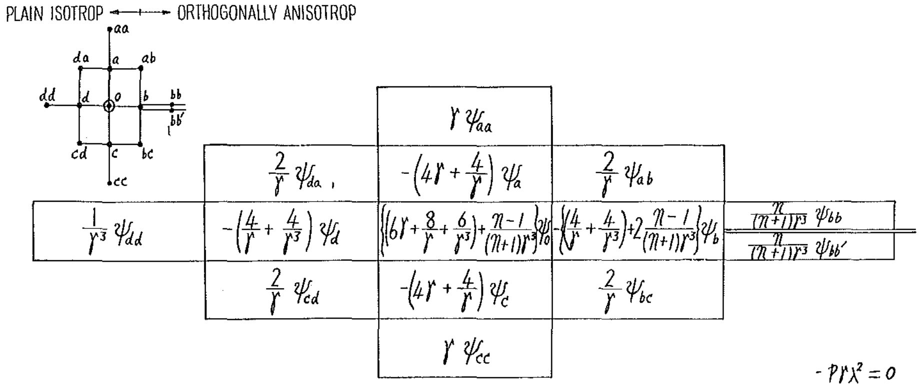


Fig. 13.

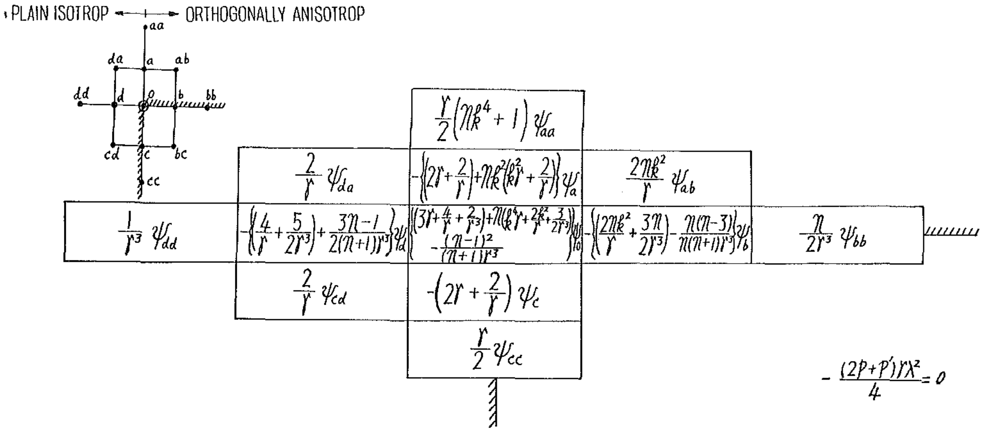


Fig. 14.

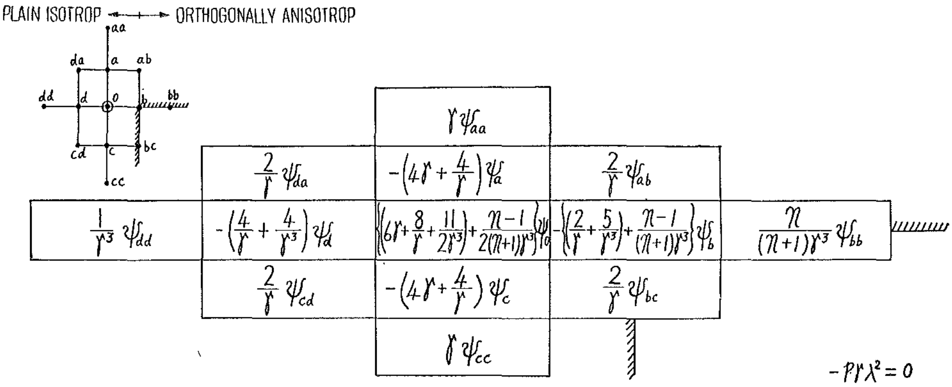


Fig. 15.

4. Solution of Equations and Stress Tables

As the flexural rigidity of a flight for transversal direction is computable

TABLE 1.

Riser 18cm Tread 27cm

Thickness of Flight

P	10	11	12	13	14
J_x	606	695	796	906	1027

Thickness of Landing

P_0	12	13	14	15
J_0	144	183	227	282

P/P_0	10	11	12	13	14
12cm	4.21	4.83	5.54	6.30	7.13
13	3.02	3.80	4.35	4.95	5.61
14	2.62	3.04	3.48	3.96	4.43
15	2.15	2.46	2.82	3.22	3.64

In case $J_x/J_0 = 4$

$\therefore n = 4/3$

by Eq. (B) the value of N_x/N_y may be determined by experiment. The following experimental and computative values of N_x/N_y and N_x/N, respectively, were reported by Y. Yokoo, as shown in Fig. 16 and Table 1. Hence these values

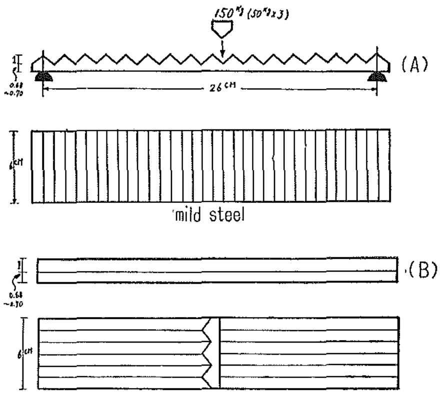


Fig. 16.

Deflection of Strip (A) Average by Four Trials 72.125 — Deflection of Strip (B) Average by Ten Trials 22.61 (unit 1/100 mm)

$N_x/N_y = 72.125/22.61 = 3.19 \fallingdotseq 3$

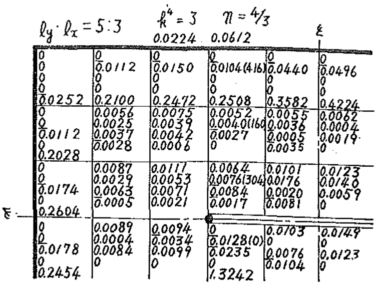
are replaced in the respective equation at each point so that it is set up again as shown in Table 2 (where $r=1$ and point support at point 33 are assumed). The equations are solved by elimination and stresses are found as shown in Table 3 a, 3 b, 3 c and 3 d. In the following are plotted the stress diagrams for each slab.

TABLE 2.

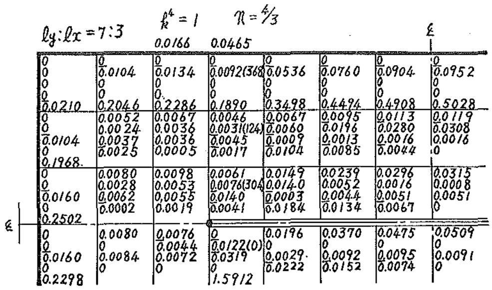
$\eta = 4/3 \quad k^4 = 3 \quad l_y : l_x = 5 : 3$

	ψ_{11}	ψ_{12}	ψ_{13}	ψ_{21}	ψ_{22}	ψ_{23}	ψ_{31}	ψ_{32}	ψ_{41}	ψ_{42}	ψ_{43}	ψ_{51}	ψ_{52}	ψ_{53}	Load Team
11	22	−8	1	−8	2		1								0.02778
12	−8	22	−8	2	−8	2		1							0.02778
13	2	−16	21		4	−8									0.02778
21	−8	2		21.14286	−8	1	−8.28571	2	1.14286						0.02778
22	2	−8	2	−8	21.14286	−8	2	−8.28571		1.14286					0.02778
23		4	−8	2	−16	20.14286		4			1.14286				0.02778
31	1			−8.28571	2		30.92824	−16.61880	−14.18998	4.61880		1.33333			0.02778
32		1		2	−8.28571	2	−16.61880	33.64235	4.61880	−14.18998	4.61880		1.33333		0.02778
41				0.85714			−10.64253	3.46412	41.71362	−18.92824	3	−10.92824	3.46412		0.02084
42					0.85714		3.46412	−10.64253	−18.92824	35.71362	−12.92824	3.46412	−10.92824	3.46412	0.02084
43						0.42857		3.46412	3	−12.92824	13.35681		3.46412	−5.46412	0.01042
51							2		−21.85648	6.92824		40.85648	−18.92824	3	0.02084
52								2	6.92824	−21.85648	6.92824	−18.92824	34.85648	−12.92824	0.02084
53										6.92824	−10.92824	3	−12.92824	12.92824	0.01042

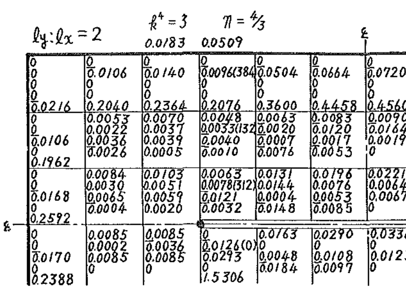
TABLE 3.



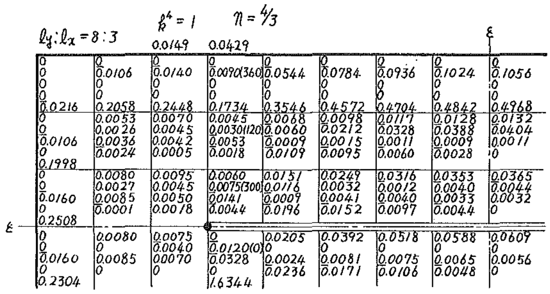
(a)



(c)



(b)



(d)

Col. I	Index of Deflection	Ψ	(Pl_x^2)
Col. II	Bending Moment	M_x	"
Col. III	"	M_y	"
Col. IV	Torsional Moment	M_{xy}	"
Col. V	Reaction	A	(Pl_x)

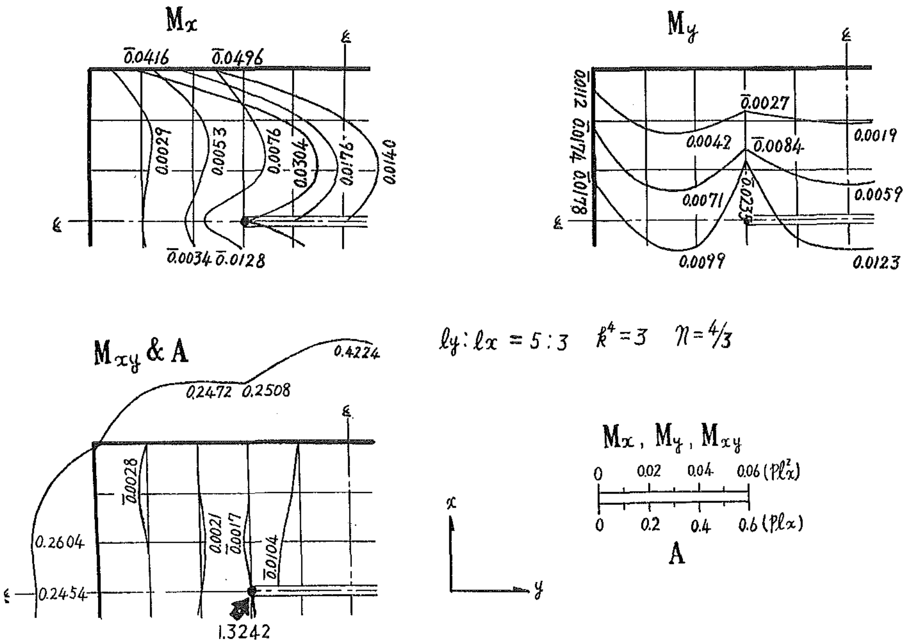


Fig. 17-A.

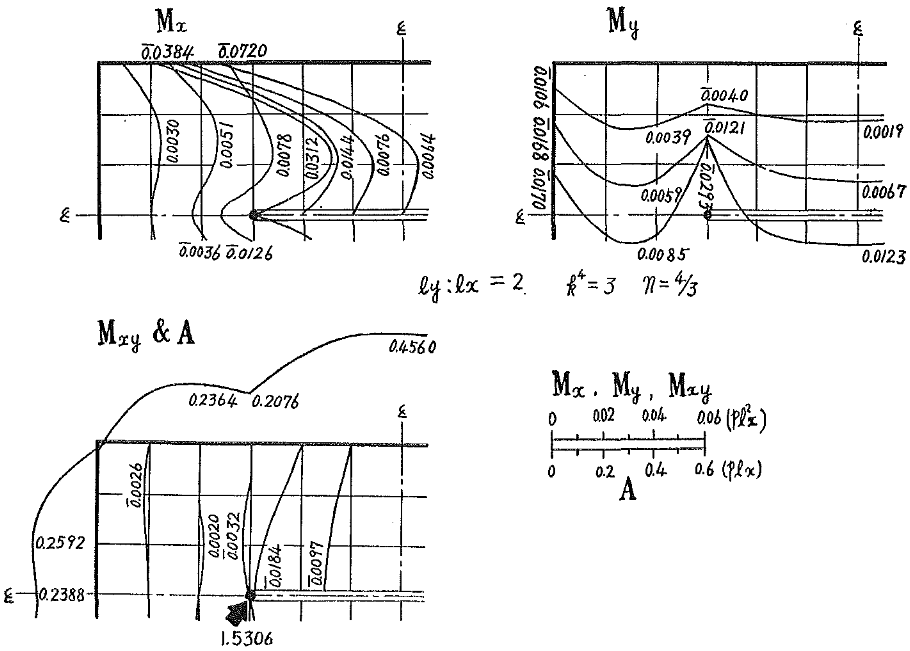


Fig. 17-B.

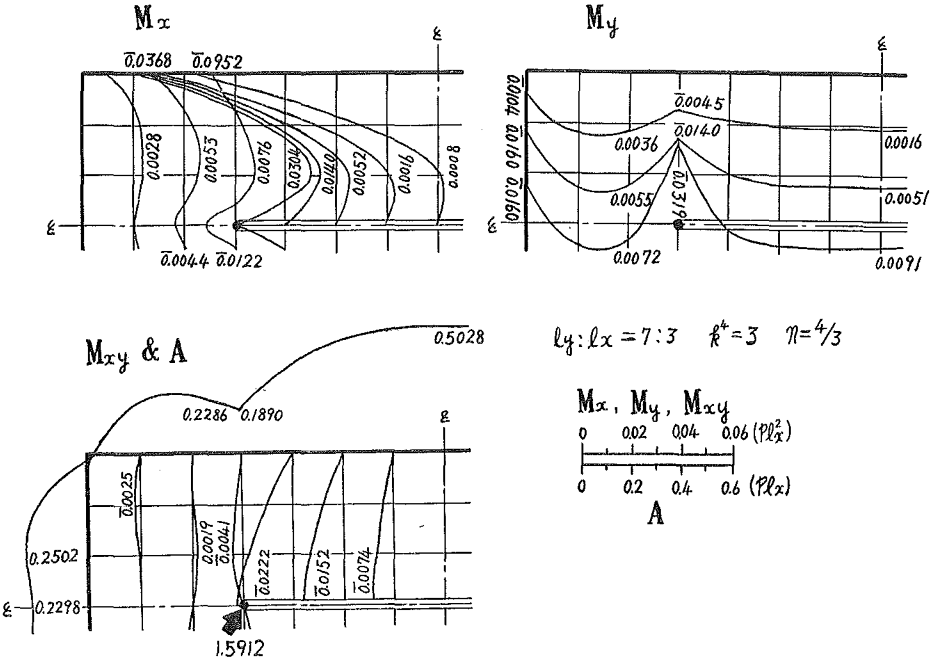


Fig. 17-C.

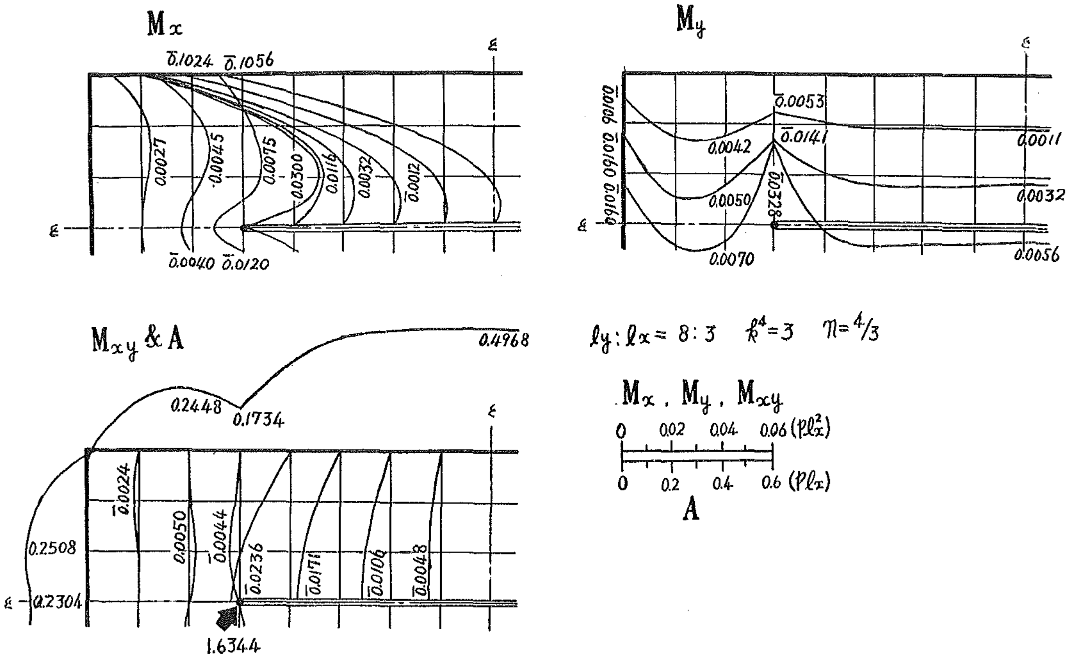


Fig. 17-D.

5. Stress Diagrams

Table 4 and Fig. 18 are a stress table and a stress diagram respectively plotted from the solution for a stair slab with $l_x/l_y=5/3$, for reference to the corresponding anisotropic state of flight, on the assumption that its flight be a flat isotropic slab with the same depth as its landing.

TABLE 4.

$l_y:l_x=5:3$ $k^4=1$ $\eta=1$

0 0 0 0 $\bar{0}$.0300	0 $\bar{0}$.0120 0 0 0.1986	0 $\bar{0}$.0196 0 0 0.2634	0 $\bar{0}$.0228 0 0 0.2478	0 $\bar{0}$.0304 0 0 0.3222	0 $\bar{0}$.0344 0 0 0.3636
0 0 $\bar{0}$.0120 0 0.2022	0.0060 0.0028 0.0082 $\bar{0}$.0034	0.0098 0.0060 0.0022 $\bar{0}$.0012	0.0114 0.0090 $\bar{0}$.0022 $\bar{0}$.0028	0.0152 0.0058 0.0018 $\bar{0}$.0041	0.0172 0.0043 0.0040 0
0 0 $\bar{0}$.0184 0 0.2670	0.0092 0.0037 0.0048 0.0002	0.0136 0.0083 0.0042 0.0035	0.0138 0.0162 $\bar{0}$.0106 $\bar{0}$.0016	0.0246 0.0132 0.0053 $\bar{0}$.0062	0.0301 0.0123 0.0110 0
0 0 $\bar{0}$.0174 0 0.2256	0.0087 $\bar{0}$.0010 0.0083 0	0.0091 $\bar{0}$.0090 0.0095 0	0 $\bar{0}$.0276 $\bar{0}$.0299 0 1.4688	0.0208 $\bar{0}$.0076 0.0109 $\bar{0}$.0107	0.0307 0.0012 0.0198 0

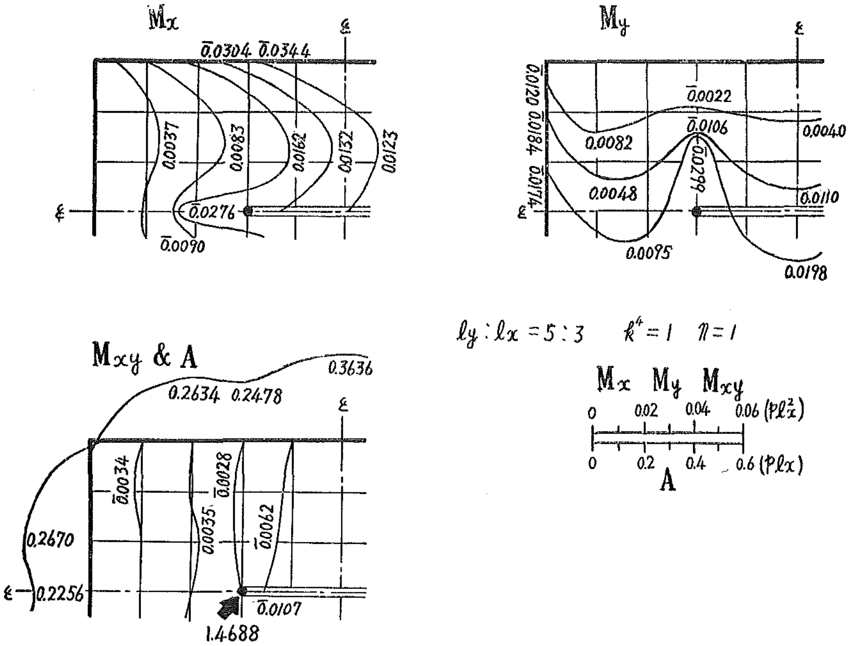


Fig. 18.

6. Examination of Stress Distribution

Reaction at Point Support

Table 5 shows the reaction at the point support relative to the total load. The increase in the ratio of longer to shorter edge of the flight when that ratio of the landing is constant causes the decrease in the share of the resultant reaction, acting on the point support. This may be adequately explained as follows. The resultant reaction over the cantilevered part near the center is concentrated in the fixed part near the edge. It is also made known by the resultant reaction derived from the assumption that landing and flight be both orthogonally isotropic.

TABLE 5.

l_y / l_x	5/3	2.0	7/3	8/3
$\sum A\ (Pl_x^2)$	10	12	14	16
${}_{\text{pin}}A_{33}$ 〃	1.3242 (1.4688)	1.5306	1.5912	1.6344
${}_{\text{pin}}A_{33}/\sum A(\%)$	13.2	12.8	11.4	10.2

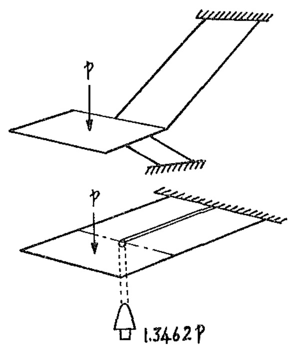


Fig. 19.

The fact is already revealed in another paper that the ratio of resultant reaction to total load at the point support at the slit end is 134.6%. Though a relatively large normal force acts on the actually sloping stair slab which closely resembles the above mentioned slab in the mode of bending, the bending is still decisively large among design factors compared with normal force as apparently seen in Fig. 19.

Hence the reaction at the point support for a flat slab with all edges fixed and with point support at slit edge may be negligibly small since its bending actions should resemble that of a beamless stair slab.

Change in Stresses Due to Elongation of Flight

In Fig. 20 are shown stress diagrams for a stair slab with l_x/l_y variable from 5/3 to 8/3, by flight length. It is noted in this diagram that the bending in cantilever direction (x) of the flight shows a remarkable variation along its center line (in direction y, parallel to slit). It is, however, a matter of course as long as it is treated as a cantilever.

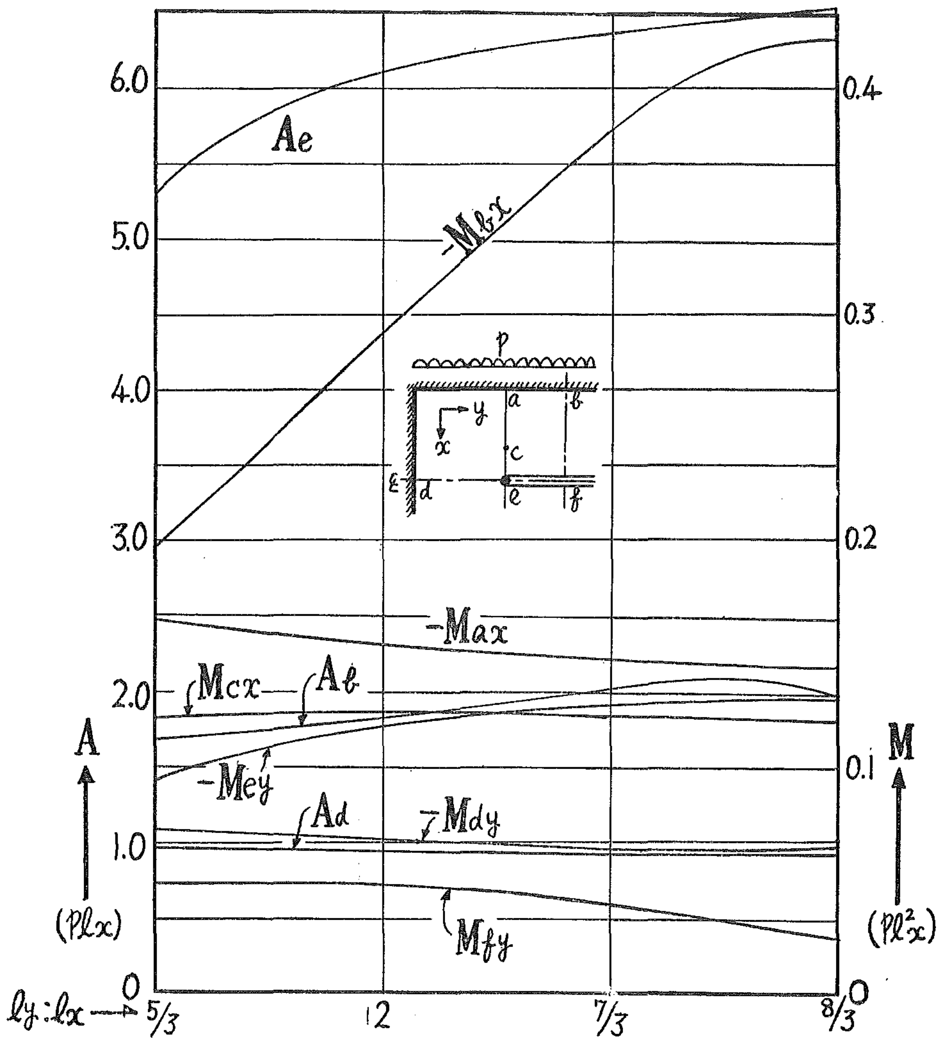


Fig. 20.

Comparison of Author's Values with Other Workers

In Fig. 21 values at major points over the flight by his analysis are compared with those over the three-edge fixed, one-edge free slabs by their analysis.

The values by the author and those by Yokoo are fairly close to each other except in bending moment in y-direction, M_{ay}. This gap in the value may presumably be caused by the author's gross subdivision of the landing into only three segments for the shorter direction. Therefore, the assumption of regarding the flight as an anisotropic slab with three edges fixed may sufficiently close.

Further, bending moment M_{ay} at the tip of free edge is always larger than bending moment M_{cx} for three-edge fixed slabs, but for stair slabs a larger rigidity in x-direction than y brings about $M_{cx} > M_{by}$.
And M_{cx} for orthogonally isotropic slabs and that for anisotropic slabs are considerably different, so that the use of three-edge fixed slab design diagrams for design of flights should be carefully made.

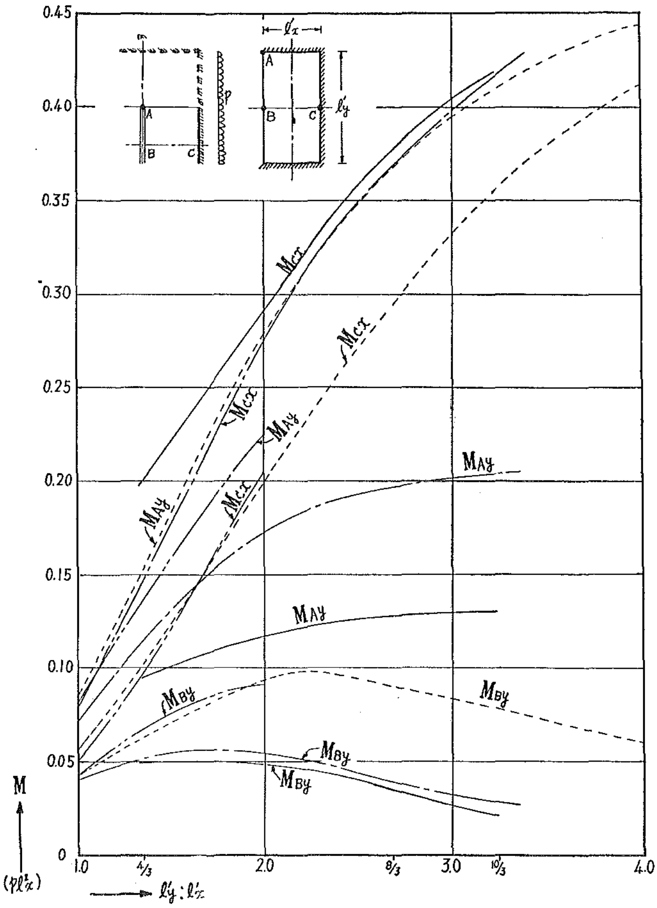


Fig. 21. $k^4=3$ $n=4/3$ { Dobashi — Four-edge fixdanisotrop in fught Anisotrop.
{ Yokoo
$k^4=1$ $1n=1$ { Yokota — Tree-edge fixedone-edge free } Plainisotrop.
{ Higashi
All under uniform load.

Comparison of Bending Moment for Anisotropic Flight Slabs with that for Three-Edge Fixed Isotropic Slabs with $l_x/l_y=1$. Four-Edge Fixed Isotropic Slabs with $l_x/l_y=2$ and for Two-Edge Fixed Beams

In Fig. 22 bending moment along the border line of flight and landing and that along the extention line of slit are respectively compared with bending moment for three-edge fixed slabs along center line of shorter edge (derived by Yokota) and that for all-edge fixed slabs with $l_x/l_y=2$ or two-edge fixed beams, all under a uniform load.

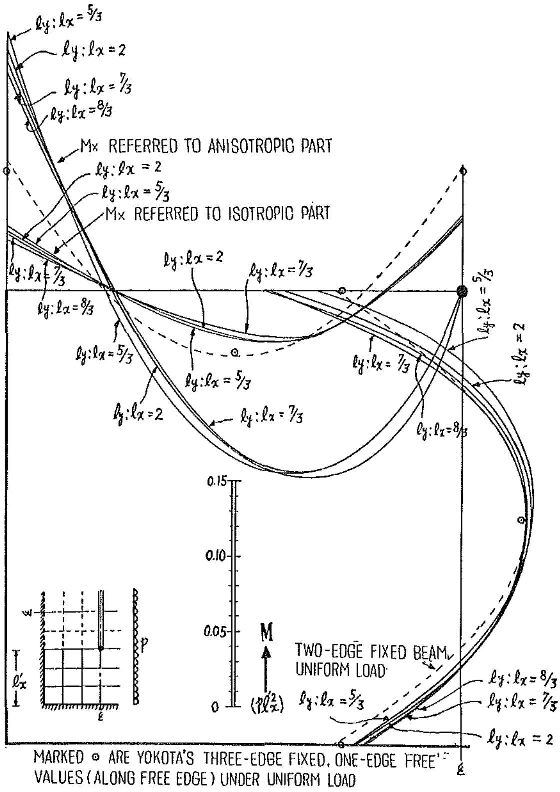


Fig. 22.

This comparison obviously shows that the conventional assumption which has it that the longer fibres of a landing be a beam with unit breadth, is inadequate at least for the longer direction. Field cracks over landings near the end of slit which are encounterd frequently, are considered to be caused by this bending (principal moment).

Bending moment for this slab along the shorter center line fairly resembles that for all-edge fixed slab along that line and also that for two-edge fixed beam spanning along that line. So our design of landings should be refered to as 'beamed' value.

Further, as shown in Fig. 22 the positive moment for x-direction for flight is notably large near the border line of landing and flight (ca. 1/8 wℓ^2).

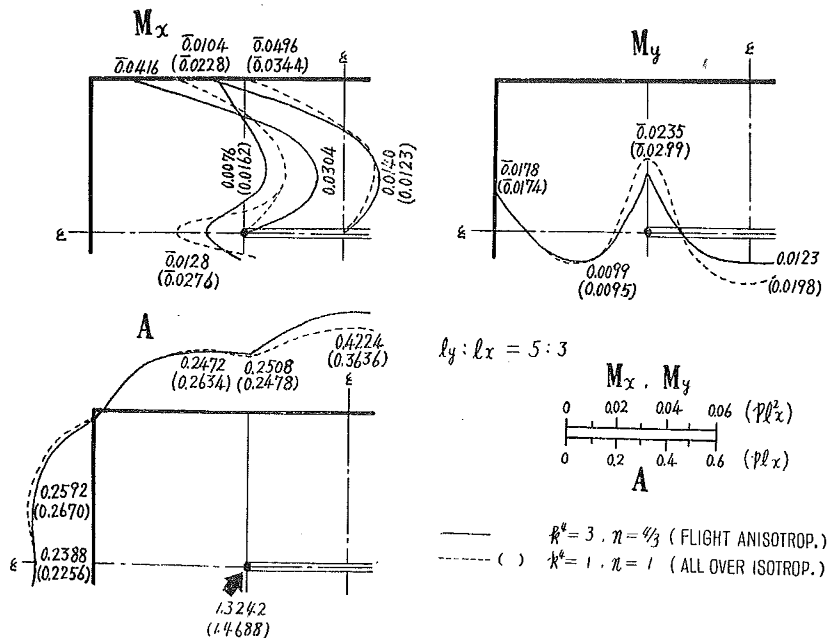


Fig. 23.

Comparison at Main Points of Isotropic Slitted S'abs With Point Support at Slit End With the Same Demensioned Beamless Stair Slabs, All Under Uniform Load

In Fig. 22 stresses at main points are compared between a stair slab and a plain slab with an identical dimension and load condition. In the former both bending moment and reaction are concentrated in transversal direction of the flight and in the latter stresses are prevailing for sloping direction of the flight and for both directions of landing.

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