The Longitudinal Turbulent Motion of a River Flow and its Role on the Diurnal Variation of Water Temperature

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Contents

Abstract

The influence of turbulence on the diurnal variation of water temperature of a river is mainly discussed in this paper.

At the upper reaches of a river, the diurnal variation of water temperature is controlled by the apparent air temperature, while the influence of longitudinal turbulent motion is first introduced when the condition $\frac{v^2}{4\eta}+K\doteqdot\sigma$ is satisfied. On the other hand, at the lower reaches where the discharge increases the heat is transported only with the flow of water mass, and it may be seen that the amplitude of variation of water temprature decreases with the flow in contrast to that at the upper reaches by the influence of turbulence except when the condition $\frac{v^2}{4\eta}\gg\sigma$ is satisfied.

At the Etanbetsu River, the influence was confirmed by the observations of water temperature and other meteorological and hydrological elements.

1. Introduction

The diurnal variation of water temperature of a river is controlled by thermal conditions of the environment such as

(1) solar radiation,

(2) air temperature near the river surface,
(3) evaporation from the river surface,
(4) long-wave radiation from the river surface,
(5) heat transfer to the river bed,

and also by mechanical conditions of the flow.

Because the river flow is a turbulent flow, the diurnal variation of water temperature must be influenced by turbulence.

In this paper, the water temperature is assumed to be constant in a cross section of the flow by the vertical and horizontal turbulent diffusions. And the influence of the longitudinal turbulent motion on the amplitude and the phase of the diurnal variation of water temperature at the upper and lower reaches of a river may be discussed.

2. The Fundamental Equations

In terms of the following notations

θ : the temperature of river water,
c, ρ : the specific heat and the density of water,
h, v : the mean depth and the mean velocity of the flow,
k : the heat transfer coefficient between water and air near the river surface,
η : the coefficient of longitudinal turbulent diffusion,

the differential equation of the variation of water temperature is given by

$$\left.\begin{aligned} \frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial x}&=K(T-\theta)+\eta\frac{\partial^2\theta}{\partial^2 x} \\ K&=\frac{k}{c\rho h} \end{aligned}\right\} \qquad (1)$$

where T is the apparent air temperature[1].

Therefore, the term $k(T-\theta)$ means the quantity of heat which the water received from the air and other thermal sources through unit area per unit time.

If it is assumed that the diurnal variations of θ and T can be represented by sine functions, Eq. (1) gives the following relation by denoting the mean values of θ and T as $\bar{\theta}$ and $\bar{T}$:

$$v\frac{d\bar{\theta}}{dx}=K(\bar{T}-\bar{\theta})+\eta\frac{d^2\bar{\theta}}{dx^2} \qquad (2)$$

The solution of Eq. (2) can easily be obtained:

$$\bar{T}-\bar{\theta}=(\bar{T}-\theta_0)\,e^{\left(\frac{v}{2\eta}-\sqrt{\left(\frac{v}{2\eta}\right)^2+\frac{K}{\eta}}\right)x} \qquad (3)$$

where θ_0 is the mean water temperature at $x=0$, and $\bar{T}$ is assumed to be constant. This is the relation between the mean water temperature and the mean apparent air temperature.

When T and θ are represented as

$$T = \bar{T} + Te^{i\sigma t}$$
$$\theta = \bar{\theta}(x) + X(x)e^{i\sigma t}$$
$$\sigma = \frac{2\pi}{24}\ (1/hr)$$

the differential equation (1) can be solved with the boundary condition

$$\theta = \theta_0 + \Theta_0 e^{i(\sigma t + \Psi_0)} \qquad at \quad x = 0\,,$$

where Θ_0 is the amplitude of θ and Ψ_0 is the phase lag of T and θ at a station $x=0$.

The real part of the solution of Eq. (1) gives the diurnal variation of water temperature:

$$\left.\begin{aligned}
&\theta - \bar{\theta} = \Theta' \cos(\sigma t - \Psi') + \Theta_0 e^{\left(\frac{v}{2\eta} - \sqrt{P}\right)x} \cos(\sigma t - \sqrt{Q}\,x + \Psi_0) \\
&\Theta' = T\cos\varphi \sqrt{\{\cos\varphi - e^{Px}\cos(Qx+\varphi)\}^2 + \{\sin\varphi - e^{Px}\sin(Qx+\varphi)\}^2} \\
&\Psi' = \tan^{-1}\frac{\sin\varphi - e^{Px}\sin(Qx+\varphi)}{\cos\varphi - e^{Px}\cos(Qx+\varphi)} \\
&\text{where} \quad P = \frac{v}{2\eta} - \sqrt{\frac{\sqrt{\left(\frac{v^2}{4\eta}+K\right)^2 + \sigma^2} + \left(\frac{v^2}{4\eta}+K\right)}{2\eta}} \\
&Q = \sqrt{\frac{\sqrt{\left(\frac{v^2}{4\eta}+K\right)^2 + \sigma^2} - \left(\frac{v^2}{4\eta}+K\right)}{2\eta}} \\
&\varphi = \cos^{-1}\frac{K}{\sqrt{K^2+\sigma^2}}
\end{aligned}\right\} \quad (4)$$

The water temperature θ in Eq. (4) can easily be represented in the following form:

$$\theta = \bar{\theta} + \Theta\cos(\sigma t + \Psi)$$

then X and Ψ mean the amplitude and the phase lag.

If the influence of turbulent motion is neglected, the differential equation of the variation of water temperature becomes

$$\frac{\partial\theta}{\partial t} + v\frac{\partial\theta}{\partial x} = K(T - \theta)$$

$$K=\frac{k}{c\rho h}\ .$$

In this case, the relation between $\bar{T}$ and $\bar{\theta}$ is represented as

$$\bar{T}-\bar{\theta}=(\bar{T}-\theta_0)\,e^{-\frac{k}{c\rho vh}x} \tag{5}$$

and the water temperature can be given by

$$\left.\begin{aligned}
&\theta-\bar{\theta}=\Theta'\cos(\sigma t-\Psi')+\Theta_0 e^{-\frac{k}{c\rho vh}x}\cos\left(\sigma t-\frac{\sigma x}{v}+\Psi_0\right)\\
&\Theta'=T\cos\varphi\sqrt{\left\{\cos\varphi-e^{-\frac{k}{c\rho vh}x}\cos\left(\frac{\sigma x}{v}+\varphi\right)\right\}^2+\left\{\sin\varphi-e^{-\frac{k}{c\rho vh}x}\sin\left(\frac{\sigma x}{v}+\varphi\right)\right\}^2}\\
&\Psi'=\tan^{-1}\frac{\sin\varphi-e^{-\frac{k}{c\rho vh}x}\sin\left(\frac{\sigma x}{v}+\varphi\right)}{\cos\varphi-e^{-\frac{k}{c\rho vh}x}\cos\left(\frac{\sigma x}{v}+\varphi\right)}\\
&\varphi=\cos^{-1}\frac{k}{\sqrt{k^2+(c\rho\sigma h)^2}}
\end{aligned}\right\} \tag{6}$$

Eqs. (5) and (6) correspond to the special case of $\frac{v^2}{4\eta}+K\gg\sigma$ in Eqs. (3) and (4).

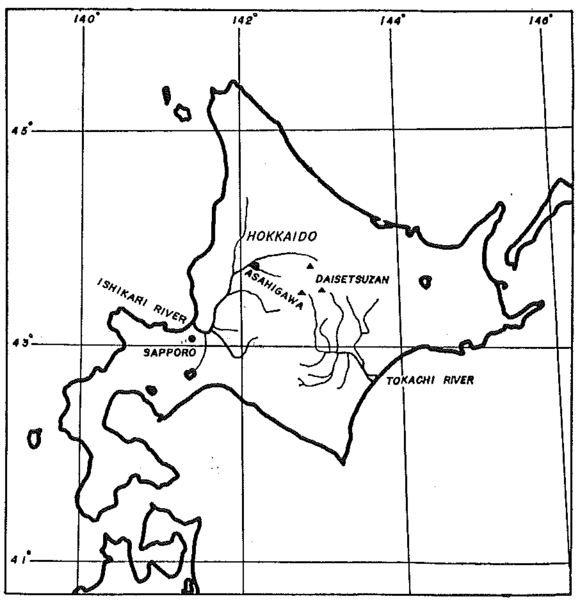

Fig. 1 (a).

Therefore, it can be seen that the influence of turbulent motion on the diurnal variation of water temperature may first be introduced when the following condition is satisfied:

$$\frac{v^2}{4\eta} + K \doteqdot \sigma .$$

3. Observational Results and Discussion

In 1960 the author and others observed water and air temperature and other meteorological elements necessary to determine the apparent air temperature T_r at four stations along the Etanbetsu River shown in Fig. 1. The current velocity of flow and the water level were also measured together.

The observations were carried out at intervals of one hour consecutively from 12h on August 25 to 16h on August 26, 1960. The longitudinal turbulent diffusion coefficient η, the mean velocity v and the mean depth of flow h at sections St. I~St. A and St. III~St. IV were measured by salt-water method[2] one time per one section during the term of observations.

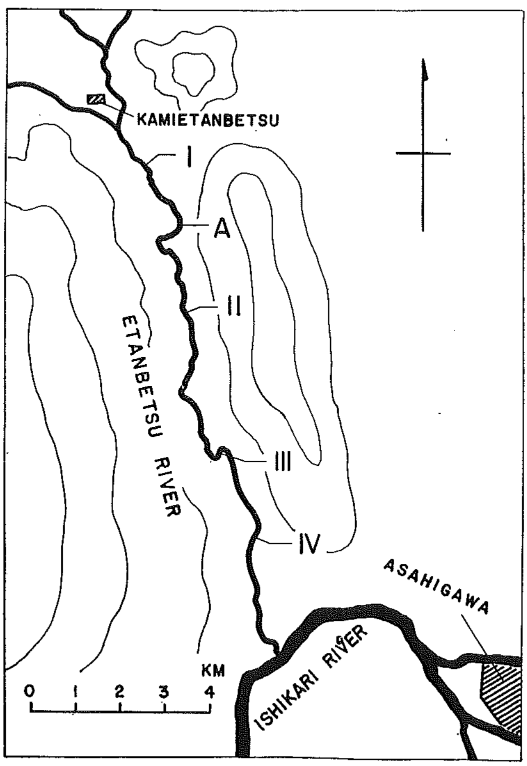

Fig. 1 (b).

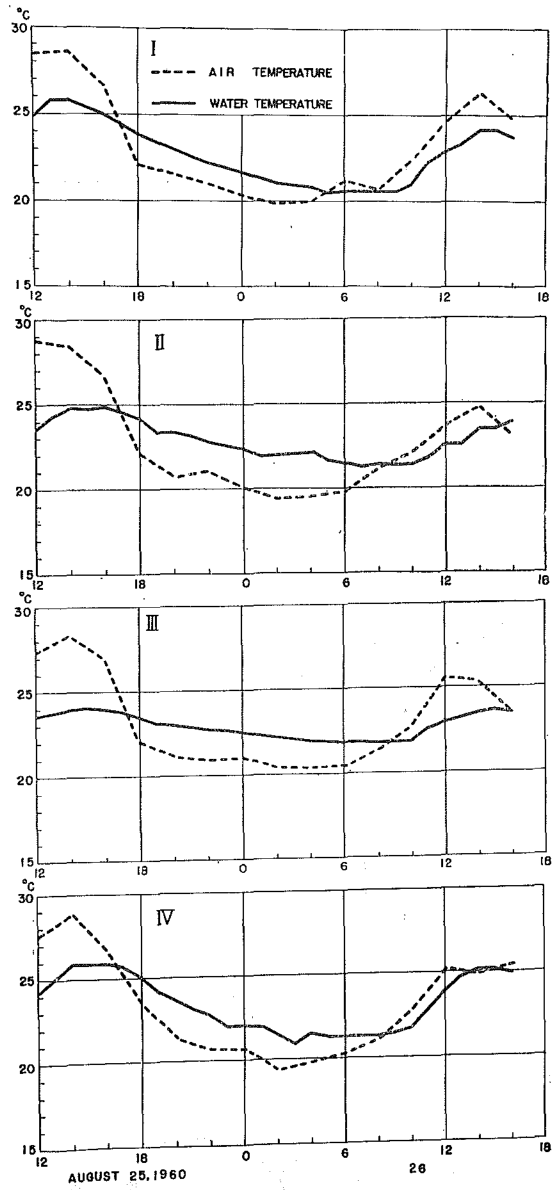
°C
30
25
20
15
I
AIR TEMPERATURE
WATER TEMPERATURE
12
18
0
6
12
18
II
III
IV
AUGUST 25, 1960
26

Fig. 2.

Some of the observed results are shown in Fig. 2 and Table. 1.

From Fig. 2 and Table 1, it is seen that the mean water temperature becomes higher as the water flows down from St. I to St. IV, and the amplitude decreases from St. I to St. III and increases contrary wise from St. III to St. IV.

TABLB 1 (a).

Station	Distance (km)	Mean Discharge (m^3/sec)	$\bar{\theta}$ (°C)	Θ (°C)	Ψ (hr)
I	0	0.26	22.2	1.8	2.0
II	5.1	0.38	22.4	1.3	4.0
III	9.3	0.31	22.6	0.9	3.0
IV	11.9	0.34	22.9	2.1	3.0

TABLE 1 (b).

Section	Distance (km)	Mean Velocity (cm/sec)	Mean Depth (cm)	η (C. G. S.)	$\frac{v^2}{4\eta}+K$ (C. G. S.)
St. I–St. A	1.8	6.5	52	2.8×10^5	5.3×10^{-5}
St. III–St. IV	2.6	18	23	3.6×10^5	27×10^{-5}

Because the water of flow was stagnated here and there between St. I and St. III, the mean velocity was small and the mean depth was deep there. At the section St. I–St. A, the condition $\frac{v^2}{4\eta}+K\fallingdotseq\sigma$ is satisfied because $\frac{v^2}{4\eta}+K=5.3\times10^{-5}$ (C.G.S.) and $\sigma=7.3\times10^{-5}$ (C.G.S.).

On the other hand, the condition $\frac{v^2}{4\eta}+K\gg\sigma$ is satisfied at the section St. III–St. IV as shown in Table 1 (b).

By the use of Eqs. (3) and (4), the mean water temperature, the amplitude and the phase lag at station A and IV can be estimated with the observed results of water temperature at stations I and III respectively.

When the observed values of k and T at station III are used, c, ρ, σ, k, $\bar{T}$ and T are given as

$$c=1\ (\text{cal/g°C}),\quad \rho=1\ (\text{g/cm}^3),\quad \sigma=\frac{2\pi}{24\times60\times60}(1/\text{sec})$$

$$k=0.78\times10^{-3}\ (\text{C.G.S.}),\quad \bar{T}=21.9°\text{C},\quad T=6.4°\text{C}.$$

The estimated results are listed in the columns (a) of Table 2 as compared against the observed values at the station IV.

If the influence of turbulent motion is neglected, the mean water temperature, the amplitude and the phase lag are given by Eqs. (5) and (6).

The values $\bar{\theta}$, Θ and Ψ at Stations A and IV computed by Eqs. (5) and (6) with the same values of c, ρ, σ, k, $\bar{T}$, T, h, v, θ_0, Θ_0 and Ψ_0 that are used above are shown in column (b) of Table 2. By comparing the values in the columns (a) and (b) of Table 2, it may be seen that the amplitudes and the phase lags are not equal at Station A.

On the other hand, at Station IV the satisfactory coincidences between the values in columns (a) and (b) can be seen.

Therefore, as will be predicted by Eqs. (4) and (6) under the conditions $\frac{v^2}{4\eta}+K \doteqdot \sigma$ and $\frac{v^2}{4\eta}+K \gg \sigma$, the influence of turbulent motion on the diurnal variation of water temperature were recognized at station A and were not at Station IV.

TABLE 2.

Station	$\bar{\theta}$ (°C)			Θ (°C)			Ψ (hr)		
	obs.	cal.		obs.	cal.		obs.	cal.	
		(a)	(b)		(a)	(b)		(a)	(b)
A	—	22.1	22.1	—	1.9	2.2	—	5.0	5.8
IV	22.9	22.3	22.3	2.1	2.5	2.6	3.0	2.7	2.7

4. The Diurnal Variation of Water Temperature at the Lower Reaches of a River

At the lower reaches of a river where the width of flow is extremely large compared with the depth, it may be seen that the value of longitudinal turbulent diffusion coefficient is large and the influence of turbulence on the diurnal variation of water temperature is remarkable.

At the lower reaches, the flow has a large heat capacity and may hardly be influenced by the heat transfer because the discharge of flow increases by gathering the waters from some branch rivers and the depth is deep. Therefore, Eq. (1) may approximately be written as

$$\frac{\partial\theta}{\partial t}+v\frac{\partial\theta}{\partial x} \doteqdot \eta\frac{\partial^2\theta}{\partial x^2} \tag{7}$$

If it is assumed that θ is represented

by $\quad \theta=\bar{\theta}(x)+X(x)e^{i\sigma t}$

and

$$\theta=\theta_0+\Theta_0 e^{i(\sigma t+\Psi_0)} \qquad at \quad x=0\,,$$

Eq. (7) can similarly be solved in the same way as in Eq. (1):

$$\bar{\theta} = \theta_0 \tag{8}$$

and

$$\theta = \bar{\theta} + \Theta_0 e^{\left\{\frac{v}{2\eta} - \sqrt{\frac{\sqrt{\left(\frac{v^2}{4\eta}\right)^2 + \sigma^2} + \frac{v^2}{4\eta}}{2\eta}}\right\} x} \times \cos\left\{\sigma t - \sqrt{\frac{\sqrt{\left(\frac{v^2}{4\eta}\right)^2 + \sigma^2} - \frac{v^2}{4\eta}}{2\eta}}\, x\right\} \tag{9}$$

From Eqs. (8) and (9), it is seen that the mean water temperature is constant and the amplitude of the diurnal variation of water temperature decreases and the phase becomes larger with the flow.

If the condition $\frac{v^2}{4\eta} \gg \sigma$ is satisfied, Eq. (9) becomes

$$\theta = \bar{\theta} + \Theta_0 \cos\left(\sigma t - \sigma \frac{x}{v}\right) \tag{10}$$

This equation corresponds to the solution of Eq. (7) in the special case of $\eta = 0$. From Eq. (10), it can be seen that when the influence of turbulence is neglected the ampltude is constant with the flow and the phase lag of the variations of water temperature at two stations is equal to the time required to flow down from one station to the other.

The relations between η and the decreasing coefficient of the amplitude

$$\frac{v}{2\eta} - \sqrt{\frac{\sqrt{\left(\frac{v^2}{4\eta}\right)^2 + \sigma^2} + \frac{v^2}{4\eta}}{2\eta}}$$

are shown in Fig. 3 (a) for several representative mean velocities of 80, 90, 100, 110, 120, 130 and 140 cm/sec.

And the ratio of Ψ to $\frac{\sigma x}{v}$ for the respective mean velocity is plotted as a function of η in Fig. 3 (b).

It can be seen from Fig. 3 that the amplitude and the phase lag decrease by the influence of turbulent motion compared with the case of $\eta = 0$.

As shown in Fig. 3, the influence of turbulence increases as the mean velocily of flow becomes smaller when the flows of the same values of η are considered.

The influence decreses rapidly with the decrease of the value of η for the flows of the same mean velocities.

By the use of Fig. 3, the mean value of the coefficient of longitudinal turbulent diffusion between two stations of a river can be obtained when the

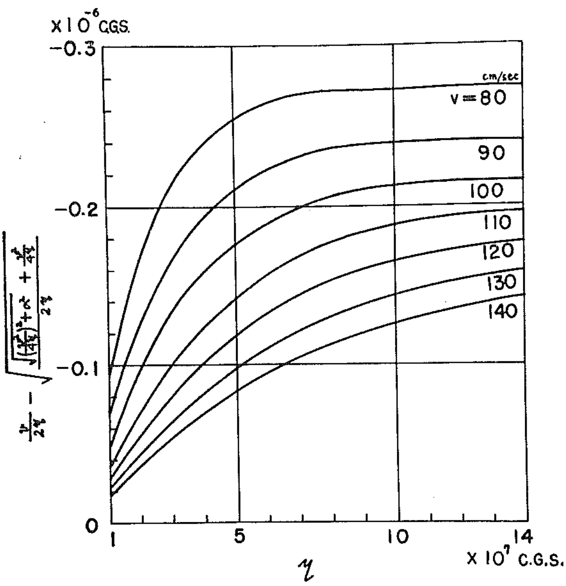
X 10^-6 C.G.S.
-0.3
-0.2
-0.1
0
cm/sec
V=80
90
100
110
120
130
140
1
5
10
14
X 10^7 C.G.S.

Fig. 3 (a).

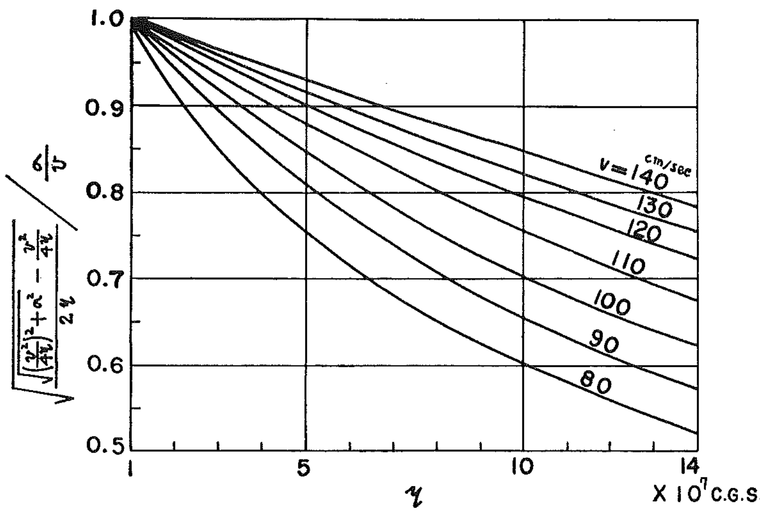
1.0
0.9
0.8
0.7
0.6
0.5
cm/sec
V=140
130
120
110
100
90
80
1
5
10
14
X 10^7 C.G.S.

Fig. 3 (b).

diurnal variations of water temperature and the mean velocities of the flow are observed at the two stations.

For example, the values of η are estimated from the observed results[3] at four stations along the Tokachi River as shown in Fig. 4 and Table 3. The observed values on May 9, 1952 and on May 14, 1953 are used.

The estimated values of η are listed in Table 3. From Table 3, it can be seen that the values of η are different even at the same section corresponding

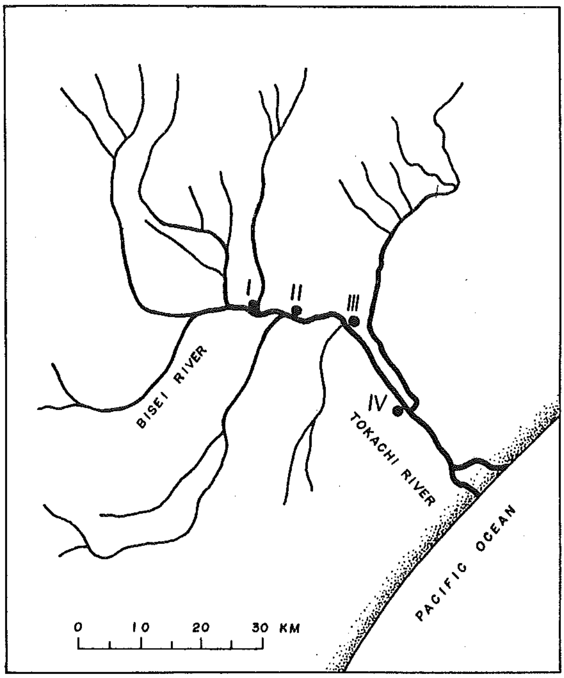

Fig. 4 (a).

TABLE 3.

Date	Section	Mean Velocity (m/sec)	Mean Discharge (m³/sec)	$\frac{v}{2\eta}-\sqrt{\frac{\sqrt{\left(\frac{v^2}{4\eta}\right)^2+\sigma^2}+\frac{v^2}{4\eta}}{2\eta}}$ (C.G.S.)	η (C.G.S.)	$\sqrt{\frac{\sqrt{\left(\frac{v^2}{4\eta}\right)^2+\sigma^2}-\frac{v^2}{4\eta}}{2\eta}}\Big/\frac{\sigma}{v}$
May 9, 1952	Shimoshihoro-Chiyoda	1.41	210	-0.93×10^{-7}	5.8×10^{7}	0.92
May 9, 1952	Chiyoda-Moiwa	1.05	217	-0.75×10^{-7}	1.8×10^{7}	0.97
May 14, 1953	Chiyoda -Moiwa	1.27	406	-0.68×10^{-7}	3.1×10^{7}	0.98

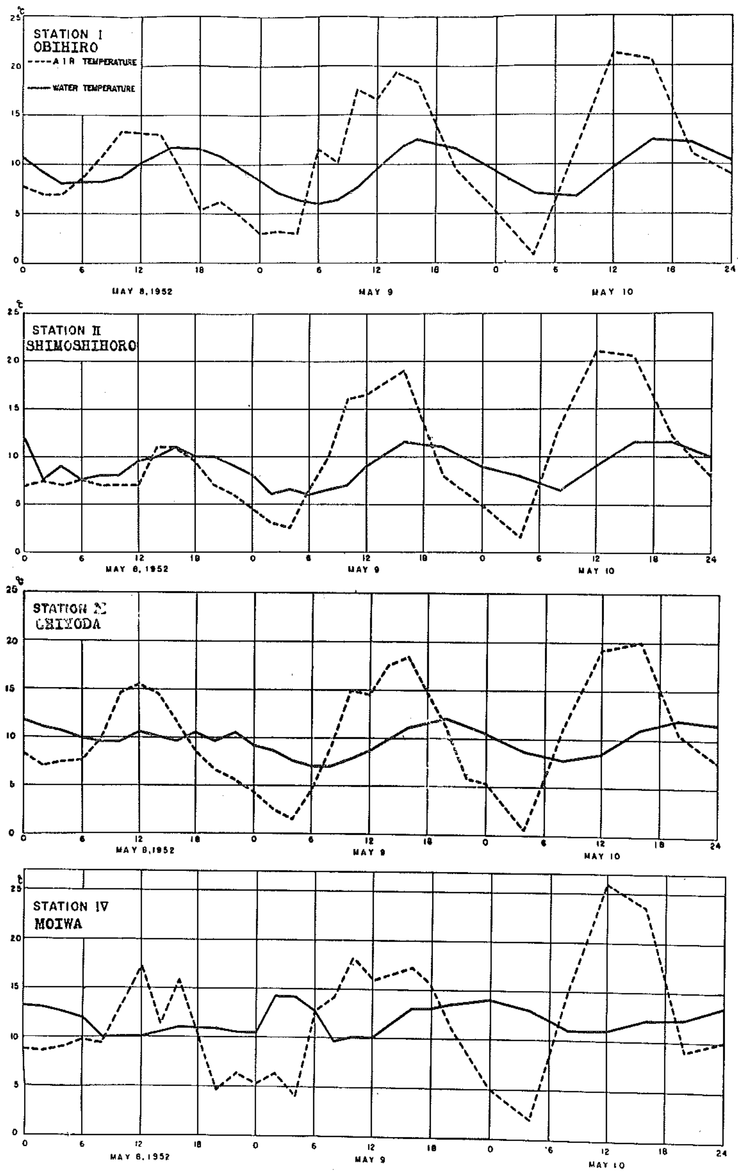

Fig. 4 (b).

to the change of the discharge. At the mid-stream of the Tokachi River where the discharge is 200~400 m³/sec, the coefficients of longitudinal turbulent diffusion are $(1.8\sim5.8)\times10^7$ C.G.S. and are much larger than $(2.8\sim3.6)\times10^5$ C.G.S. observed at the Etanbetsu River where the discharge was 0.3 m³/sec.

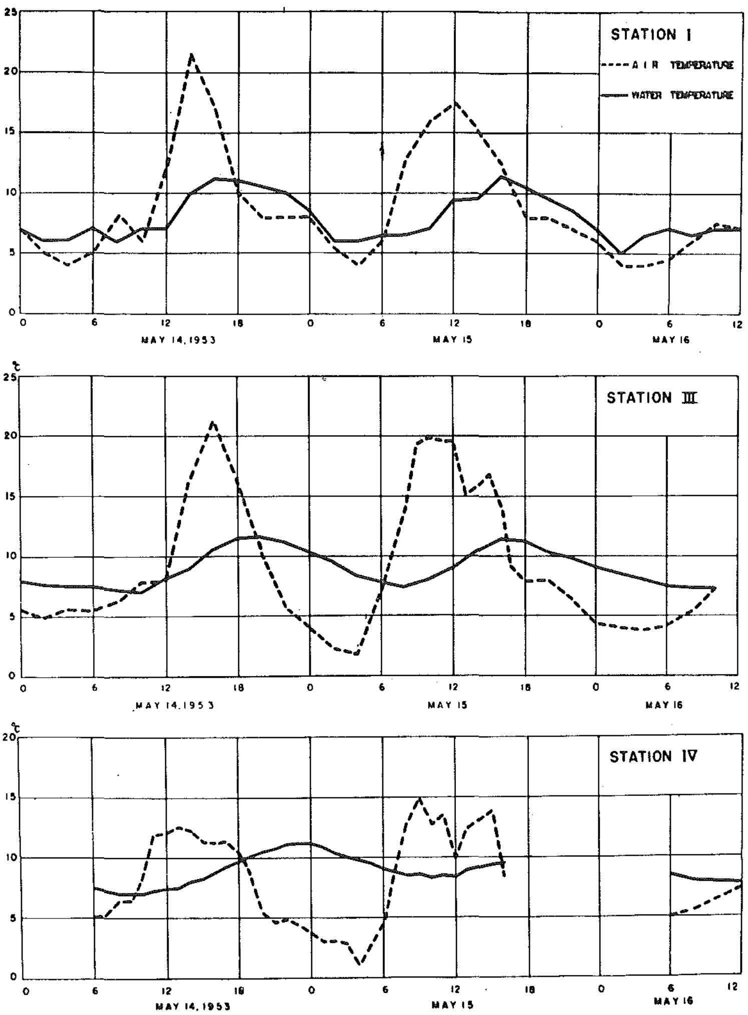

Fig. 4 (c).

5. Conclusion

The water temperature in the upper reaches of a river is controlled by the thermal condition of the external environment because the discharge is comparatively small and the depth is shallow at that point. Therefore, the mean water temperature and the amplitude of the diurnal variation of water temperature increase with the flow.

And the influence of turbulent motion on the diurnal variation of water temperature may first be introduced when the following condition is satisfied:

$$\frac{v^2}{4\eta} + K \doteqdot \sigma$$

On the other hand, at the lower reaches of a river where the discharge increases and the depth is deep, the mean water temperature and the amplitude are constant and the phase lag of water and apparent air temperature becomes $\frac{\sigma x}{v}$ because the heat is transported only with the flow of water mass at that point.

When the condition $\frac{v^2}{4\eta} \doteqdot \sigma$ is satisfied, the diurnal variation of water temperature is mainly influenced by the turbulence of the flow. Therefore, the amplitude decreases with flow in contrast with the case at the upper reaches, and the phase lag is smaller compared to $\frac{\sigma x}{v}$ as shown in Fig. 3.

Acknowledgement

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