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Frequency Pushing in O-Type Backward-Wave Oscillators*

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Contents

Summary

The Johnson backward-wave tube theory was modified to apply to frequency pushing. Theoritical dependence upon space charge and circuit loss was calculated. Good experimental confirmation was obtained with backward-wave oscillators.

1. Introduction

The backward-wave oscillator has several usefull properties as well khown; it tunes fully electronically and rapidly over a wide band of frequencies; it has an efficiency comparable to that of a reflex klystron; its frequency is rather independent of load impedance; it can be designed to be unusually stable in frequency.

Frequency pushing in O-type backward-wave oscillators is the variation obtained in the frequency of oscillator when an electron beam current is changed, the voltage of slow wave circuit being kept on constant. Several experimental results of frequency pushing appear in literature[1~10], but no analyses have been given. It was assumed that this variation of frequency was caused by the space charge of the electron beam current. This means

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that the Pierce' velocity parameter changes with the beam current, as the space charge in the tube increases with increased beam current. Now, a general formula[11~13] of frequency pushing in the O-type backward-wave oscillator can be obtained after some modifications of Johnson's theory[3]. This paper describes an analysis depending upon both space charge and circuit loss, based on the following assumptions; the phase-velocity conditions are satisfied; the value of the interaction parameter C is small and the tube acts as a linear device.

2. The Variation of Frequency for Small Space Charge

Let u be the average electron velocity. We have then by definition

$$u = v_p(1+bC), \tag{1}$$

where

v_p = phase velocity of a backward-wave
b = Pierce' velocity parameter
C = interaction parameter

When the electron beam current is increased and the voltage of slow wave circuit being kept constant, from equation (1) the following was obtained

$$v_{pst}(1+b_{st}C_{st}) = v_{p0}(1+b_0C_0) \tag{2}$$

A subscript st refers to a quantity when $I=I_{st}$ and a subscript 0 refers to quantity when $I>I_{st}$. I_{st} is the value of beam current to start oscillation and I is the beam current operating value. Let f and f_{st} be the frequency for the beam current operating value and the frequency when $I=I_{st}$, respectively. If $|f-f_{st}| \ll f_{st}$, it may be assumed that the velocity parameter, b, is independent of C in the case of small space charge ($QC \leqq 0.25$) and circuit loss, $L \geqq 0$, as shown in Fig. 1. This figure shows the theoritical curves of b_{st} vs QC at start oscillation, where QC and L are space charge parameter and the distributed loss of the circuit, respectively. Here, it was assumed that space charge parameter and guide wavelength on circuit are not the function of frequency. This approximation leads to an accuracy of the order C in the theoritical value of frequency pushing. Under these conditions the difference of

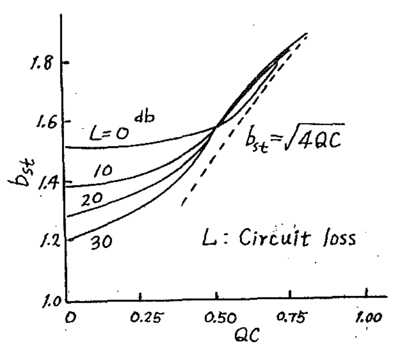

Fig. 1. Theoritical curves of b_{st} vs QC at start oscillation

phase velocities may be written as

$$v_{pst}-v_{p0} \doteqdot v_{pst}b_{st}\,(C_0-C_{st}), \tag{3}$$

and the interaction parameter when $I>I_{st}$ can be given as

$$C_0 = C_{st}\,(I/I_{st})^{1/3}. \tag{4}$$

From equations (3) and (4) the variation of phase velocities for $QC\leqq 0.25$ and $L\geqq 0$ is derived

$$v_{pst}-v_{p0} = v_{pst}b_{st}C_{st}\,[(I/I_{st})^{1/3}-1] \tag{5}$$

On the other hand, the phase velocity of backward-wave for frequency close to f_{st} is approximated by the first two terms of a power-series expansion about f_{st},

$$v_p \doteqdot (v_p)_{\omega=\omega_{st}} + (\omega-\omega_{st})\left(\frac{\partial v_p}{\partial\omega}\right)_{\omega=\omega_{st}} \tag{6}$$

where $\omega_{st} = 2\pi f_{st}$.

Remembering that

$$\frac{\partial v_p}{\partial\omega} = \frac{v_p}{\omega}\left(1+\left|\frac{v_p}{v_g}\right|\right) \tag{7}$$

where v_g is the group velocity of backward-wave. The equation (6) becomes

$$v_{p0}-v_{pst} \doteqdot \frac{v_{pst}}{\omega_{st}}\left(1+\left|\frac{v_{pst}}{v_{gst}}\right|\right)(\omega_0-\omega_{st}). \tag{8}$$

From equations (5) and (8), it is calculated that

$$\begin{aligned}
\omega_{st}-\omega_0 &\doteqdot \frac{b_{st}}{\dfrac{1+|v_{pst}/v_{gst}|}{C_{st}\omega_{st}}}[(I/I_{st})^{1/3}-1] \\
&\doteqdot \frac{b_{st}}{\dfrac{\left(\dfrac{l}{u}+\dfrac{l}{|v_{gst}|}\right)}{C_{st}\dfrac{\omega_{st}}{u}l}}[(I/I_{st})^{1/3}-1] \\
&= \frac{2\pi\,(CN)_{st}b_{st}}{(T_b+T_c)_{st}}[(I/I_{st})^{1/3}-1] \\
&= \frac{2\pi\,(CN)_{st}b_{st}}{(T_{bc})_{st}}[(I/I_{st})^{1/3}-1]
\end{aligned}$$

Using these calculations, the difference of frequency when $I>I_{st}$ can easily be obtained,

$$f_{st}-f_0=\frac{(CN)_{st}b_{st}}{(T_{bc})_{st}}[(I/I_{st})^{1/3}-1] \tag{9}$$

hence

l = length of active portion of beam and circuit

$T_b = l/u$, transit time of an electron beam down the length, l

$T_c = l/|v_g|$, transit time of r. f. energy down the length, l

$T_{bc} = T_b + T_c$

$N = \omega l/2\pi u$, number of guide wavelength on circuit

$(CN)_{st}b_{st}$ = value of parameter when $I=I_{st}$, as shown in Fig. 2 and Table 1.

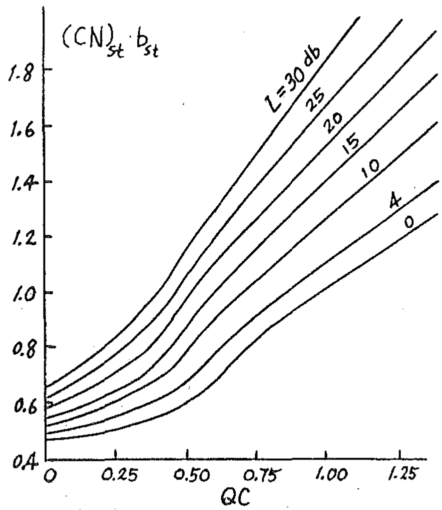

Fig. 2. Conditions at start oscillation

TABLE 1 Theoritical $(CN)_{st}\cdot b_{st}$

L \ QC	0	0.25	0.50	0.75	1.00	1.50
0	0.4781	0.5154	0.6117	0.8546	1.0182	1.3799
2	0.4873	0.5304	0.6466	0.8927	1.0642	1.4398
4	0.4974	0.5453	0.6841	0.9309	1.1145	1.5025
6	0.5074	0.5620	0.7238	0.9665	1.1698	1.5676
10	0.5290	0.5974	0.8034	1.0457	1.2828	1.7134
15	0.5574	0.6472	0.8969	1.1559	1.4192	1.8959
20	0.5881	0.7027	0.9865	1.2761	1.5564	2.0808
25	0.6204	0.7627	1.0755	1.3973	1.7018	2.2772
30	0.6548	0.8270	1.1686	1.5221	1.8539	2.4758

This is the formula of frequency pushing for small space charge ($QC \leqq 0.25$) and $L \geqq 0$. Conditions at start of oscillation, $(CN)_{st} b_{st}$ can easily be calculated by results of Johnson[3]. $(T_{bc})_{st}$ can readily be obtained from dispersion curves of slow wave circuit and operating conditions of oscillator, likewise.

3. The Variation of Frequency for Large Space Charge

3. 1 Lossless Circuit

As for the case of large space charge ($QC \geqq 0.75$) and no loss ($L=0$), it can be shown that in the evaluation of the charactristic of the oscillator, only two waves are of importance. In this region, it can be given that $b=\sqrt{4QC}$. (see Fig. 1) If $|f_0-f_{st}| \leqq f_{st}$, $(QC)_0$ can be approximated by

$$(QC)_0 = (QC)_{st} \, (I/I_{st})^{1/3} \tag{10}$$

Here subscripts have the same meaning of Section 2. Then the difference of velocity parameter is given

$$b_0 - b_{st} = 2\, (QC)_{st}^{1/2} \, [(I/I_{st})^{1/3} - 1] \tag{11}$$

Next, from equation (1)

$$b = (u - v_p)/v_p C$$

and hence

$$\frac{db}{dv_p} = -\frac{u}{Cv_p^2} \doteqdot -\frac{1}{Cv_p}$$

$$\therefore \quad \left(\frac{dv_p}{db}\right)_{v_p = v_{pst}} \doteqdot -\frac{1}{Cv_{pst}} \tag{12}$$

where the phase velocity, v_p, is approximated by u.

If the variation of frequency is very small, it can be assumed that the parameters except b are independent of the frequency. In other words, the tube parameters C, N and QC are not considered to be functions of frequency. The only parameter considered to vary with the frequency is the phase velocity parameter, b. The variation of b is determined by the dispersion charactristics of the space harmonic. However, C and QC are functions of I, making the frequency pushing term a function of I. In this manner the beam current affects the frequency of the backward-wave oscillator. The approximation which conditions C, N and QC to be independent of frequency leads to an accuracy of the order C in the value of frequency pushing.

Expanding b about ω_{st} in power-series of $(v_p - v_{pst})$ and approximating by the first two terms, we obtain

$$b \doteqdot (b)_{v_p=v_{pst}} + (v_p - v_{pst}) \left(\frac{db}{dv_p} \right)_{v_p=v_{pst}} \tag{13}$$

From equations (12) and (13) the velocity parameter b_0 as $I > I_{st}$ is

$$b_0 - b_{st} \doteqdot \frac{v_{pst} - v_{p0}}{C_0 v_{pst}} \tag{14}$$

Using equations (8), (11) and (14), it is calculated that

$$\omega_{st} - \omega_0 \doteqdot \frac{\dfrac{2\,(QC)_{st}^{1/2}}{1 + |v_{pst}/v_{gst}|}}{C_0 \omega_{st}} [(I/I_{st})^{1/6} - 1]$$

$$\doteqdot \frac{\dfrac{b_{st}}{(T_{bc})_{st}}}{2\pi\,(CN)_{st}\,(I/I_{st})^{1/3}} [(I/I_{st})^{1/6} - 1]$$

From these calculations, the frequency pushing for large space charge and zero loss can readily be obtained

$$f_{st} - f_0 = \frac{(CN)_{st} b_{st}}{(T_{bc})_{st}} [(I/I_{st})^{1/2} - (I/I_{st})^{1/3}] \tag{15}$$

Remembering that

$$(CN)_0 = (CN)_{st}\,(I/I_{st})^{1/3} \tag{16}$$

and

$$(CN)_{st} = \frac{1}{2} (QC)_{st}^{1/4} \tag{17}$$

We can derive from equation (15) for frequency pushing as a function of $(CN)_{st}$,

$$f_{st} - f_0 = \frac{8\,(CN)_{st}}{(T_{bc})_{st}} [(I/I_{st})^{1/2} - (I/I_{st})^{1/3}] \tag{18}$$

Theoritical values of $(CN)_{st}$ and b_{st} are given in Fig. 2 and Table 1.

3. 2 Loss in Circuit

With largd space charge ($QC \geqq 0.75$) and loss ($L > 0$) two waves are of importance in the characteristics of the oscillator. In this case the value of b_{st} is approximated by $\sqrt{4QC}$ in the same manner of large space charge and zero loss, as shown in Fig. 1. According to the dervation described in Section 3. 1 the frequency pushing for $QC \geqq 0.75$ and $L > 0$ is given by equation (15) given above.

4. The Variation of Frequency for Middle Space Charge

The formulas of frequency pushing that $QC \leqq 0.25$ and $QC \geqq 0.75$ were shown in equations (9) and (15), respectively. For middle space charge and $L \geqq 0$, we will take the average between equations (9) and (15),

$$f_{st}-f_0=\frac{(CN)_{st}b_{st}}{2\,(T_{bc})_{st}}\left[(I/I_{st})^{1/2}-1\right] \tag{19}$$

The conditions at start oscillation are shown in Fig. 2 and Table 1.

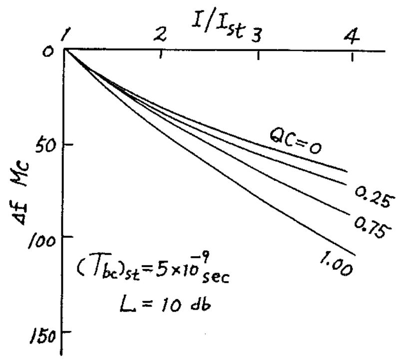

Fig. 3. I/I_{st} vs Δf with QC as parameter

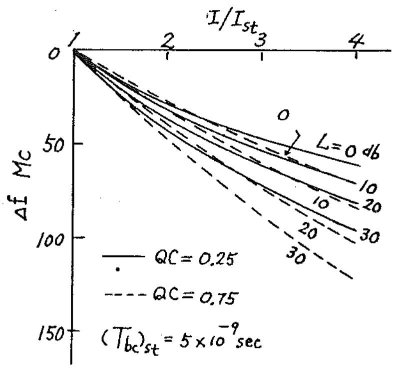

Fig. 4. I/I_{st} vs Δf with L as parameter

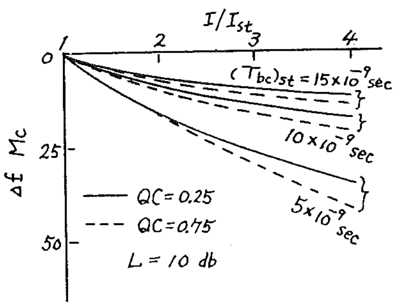

Fig. 5. I/I_{st} vs Δf with $(T_{bc})_{st}$ as parameter

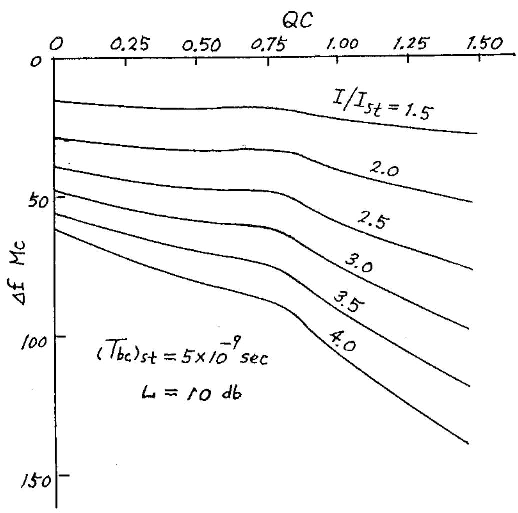

Fig. 6. QC vs Δf with I/I_{st} as parameter

5. Further Considerations of Frequency Pushing

The variation of frequency considering both loss and space charge were given by equations (9), (15) and (19). The conditions of start oscillation in various of QC and L were shown in Fig. 2 and Table 1, using Johnson's paper[3]. From these results it is seen that in order to have as small a frequency pushing as possible, the electron beam, the circuit loss and the space charge should be made as small as possible, but the transit time should be made as long as possible.

When $(T_{bc})_{st}=5\times 10^{-9}$ sec. and $L=10$ db, the theoritical curves of I/I_{st} vs the decrease of frequency, Δf, with QC as parameter are shown in Fig. 3. For $(T_{bc})_{st}=5\times 10^{-9}$ sec. and $QC=0.25$, 0.75, theoritical values of I/I_{st} vs Δf with L parameter are given in Fig. 4. In the case where $L=10$ db and $QC=0.25$, 0.75, plots of Δf, the decrease of frequency, as a function of I/I_{st} with parameter $(T_{bc})_{st}$, are plotted in Fig. 5. In the region where $L=10$ db and $(T_{bc})_{st}=5\times 10^{-9}$ sec. curves of Δf as a function of QC are presented in Fig. 6. In this case the decrease of frequency where $QC\leqq 0.25$, $0.25<QC<0.75$, and $QC\geqq 0.75$, were calculated by equations (9), (19) and (15), respectively.

6. A Comparison of Theoritical and Experimental Values

The Formulas described above indicate that as a result of induced frequency pushing higher beam currents at fixed slow wave circuit to cathode voltage, produce a drop in oscillation frequency. This is in conflict with data reported by Kompfner and Williams[1], but in agreement with those reported by Sullivan[2], Johnson[3], Chang[4], Menke[5], Beaver[6], Pallul and Goldberger[7], Okamura[8], Higuchi[9] and Kamihara[10].

Because the dimensions of tube were known in detail the comparison of theoritical and experimental values was made with backward-wave oscillators of Sullivan[2] and Okamura[8]. First, let us compare the theoritical values with the experimental data of Sullivan's oscillator[2]. Experimental values of frequency pushing at 10000 Mc were shown in his paper. The circuit voltage was 1150 v at that frequency. In the first place it was assumed that the electron beam was spaced out at zero in. from the circuit and the thickness of electron beam was 0.0118 in., which was equal to a gap of defining aperture. From $I_{st}=0.62$ ma, the theoritical value of starting current calculated by him, an impedance parameter, K, and interaction parameter, C, are

$$K=7.2\,\Omega\,; \qquad Q=2.38$$
$$C_{st}=0.99\times 10^{-2}\,; \qquad (QC)_{st}=0.0235.$$

The values of CN and b, presented below, were calculated from conditions at start oscillation,

$$(CN)_{st} = 0.3165$$
$$b_{st} = 1.5197$$

where it was assumed that the loss in circuit was zero as seen in his paper. Using values calculated

$$(T_b)_{st} = 3.196 \times 10^{-9}\ \text{sec}\ ; \qquad (T_c)_{st} = 1.988 \times 10^{-9}\ \text{sec},$$

the drop in frequency is calculated by the following equation

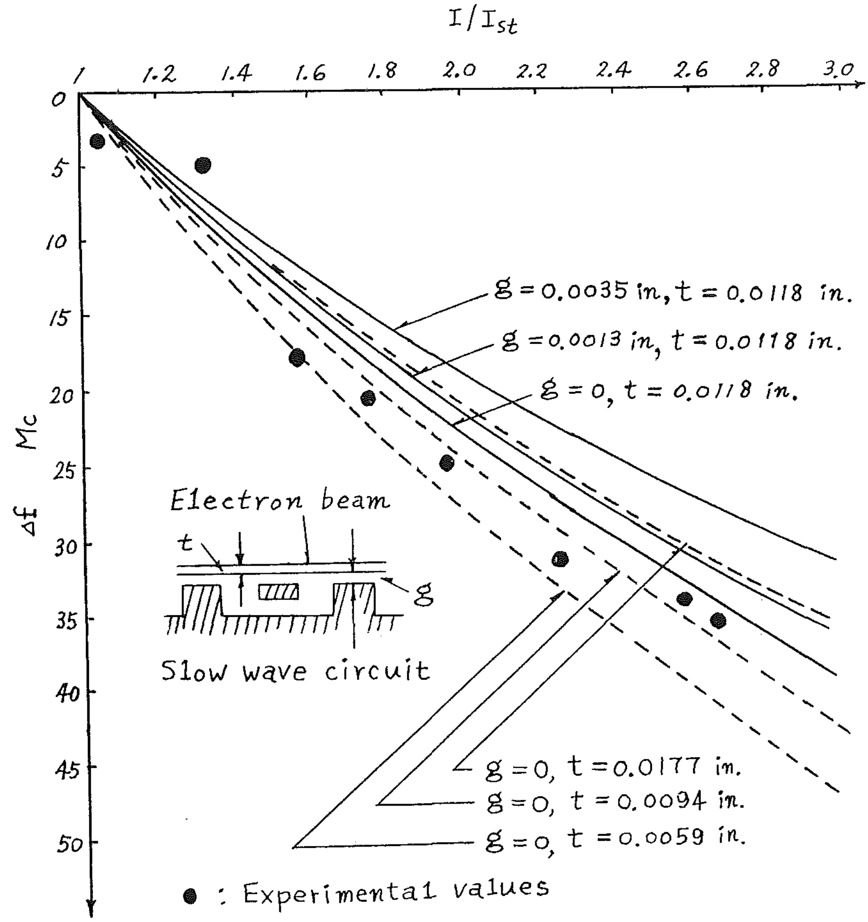

Fig. 7. Check of theory of frequency pushing by Sullivan's experimental data

$$f_{st}-f_0=\Delta f=\frac{(CN)_{st}b_{st}}{(T_{bc})_{st}}[(I/I_{st})^{1/3}-1]$$
$$=90.05\,[(I/I_{st})^{1/3}-1]\quad \text{Mc}$$

Theoritical values calculated from this equation are plotted in Fig. 7 with Sullivan's experimental data. In the case where $t=0.0118$ in., $g=0.0013$, 0.0035, in. and in the case where $g=0$, $t=0.0059$, 0.0094, 0.0177, in., theoritical values are also shown in the same figure, where t is the thickness of beam and g is the gap width between the beam and the circuit. As seen from Fig. 7 good experimental confirmation is obtained with Sullivan's oscillator.

Secondly, let us compare the theoritical values with data of Okamura's tube[8]. This was a helix type backward-wave oscillator and frequency pushing measured at 9000 Mc band was shown in his paper. Theoritical values of frequency pushing were calculated from the following conditions ;

l = 7.9 in., length of active portion of beam and circuit

f_{st} = 9750 Mc at V_h = 1500 v

f_{st} = 9600 Mc at V_h = 1400 v

which V_h is a circuit-to-cathod voltage. These comparisons are presented in

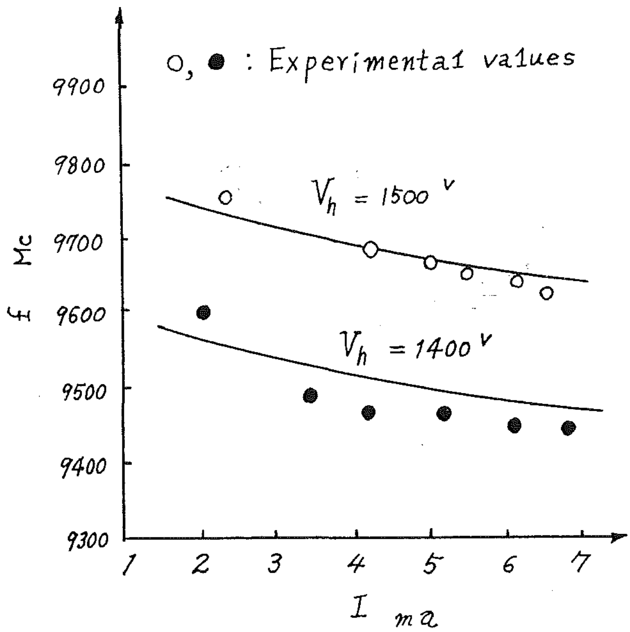

Fig. 8. Check of calculated values of frequency pushing by Okamura's experimental data

Fig. 8. As seen from this figure, the correlation between theory and experiment was in resonably accordance.

7. Conclusions

The frequency pushing depending upon space charge and cicuit loss were calculated under the assumptions where the phase-velocity conditions are satisfied and the tube acts as alinear device. The dervation was based on the usual backward-wave oscillator model of Johnson.

The general charactristic of frequency pushing could be summarized as follows ;

1) In the region of small space charge and $L \geqq 0$,

$$f_{st}-f_0=\frac{(CN)_{st}b_{st}}{(T_{bc})_{st}}[(I/I_{st})^{1/3}-1], \tag{9}$$

2) In the region of middle space charge and $L \geqq 0$,

$$f_{st}-f_0=\frac{(CN)_{st}b_{st}}{2\,(T_{bc})_{st}}[(I/I_{st})^{1/2}-1], \tag{19}$$

3) In the region of large space charge and $L \geqq 0$,

$$f_{st}-f_0=\frac{(CN)_{st}b_{st}}{(T_{bc})_{st}}[(I/I_{st})^{1/2}-(I/I_{st})^{1/3}], \tag{15}$$

where values of $(CN)_{st}b_{st}$ were shown in Fig. 2 and Table 1. Good experimental confirmation was obtained with backward-wave oscillators.

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