Percentage Breakdown Characteristics of Electric Spark of Parallel Plane Gap and Sphere Gap

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Introduction

The percentage breakdown characteristic of high voltage apparatuses or various kinds of spark gaps is a very important characteristic in high voltage engineering when impulse breakdown is considered. But hitherto the percentage breakdown characteristic has been dealt with mainly experimentally and theoretical considerations are scarce. In fact there are some difficulties when this characteristic is considered theoretically. But if some assumptions are made the curves of the characteristic (percentage breakdown vs. impulse voltage) can be calculated from the general equation for statistical time lag proposed by the author and some results may be obtained. The theory, the assumptions and the general calculating procedure, the applications of the procedure for the percentage breakdown voltage characteristics of plane parallel gap and sphere gap, the results, the discussion and the conclusion are in the following.

Theory, assumptions and general calculating procedure

It is well known that a spark discharge may or may not occur even when a impulse voltage of the same height is applied to a spark gap. That is to say, the spark discharge has a statistical nature in itself. This nature appears, when a spark discharge occurs, as a time lag. There are two kinds of time lags, the statistical time lag and the formative time lag though both of them have been handled together in theory in recent years. The statistical time lag is a lag in which the appearance of an initial electron is concerned during the application of an impulse voltage and the formative time lag is a lag between the appearance of an initial electron and accomplishment of spark formation. In one condition both statistical time lag and formative time lag take comparative values but in another condition the statistical time lag is larger than the formative time lag and vice versa. Here, only the condition when the statistical time lag is larger than the formative time lag is considered. This condition may be obtained when the irradiation to spark gap is not so intense and the overvoltage is not so small.

The basic equation for statistical time lag is

$$dp(t) = \left\{1 - p(t)\right\} \lambda(t)\, dt \qquad (1)$$

where $p(t)$ is the probability of the occurrence of spark discharge between the time impulse application (time zero) and the time t, $dp(t)$ is the probability of the occurrence of spark discarge between the time t and $t+dt$. $\lambda(t)$ acts as a kind of proportional constant for the occurrence of spark discharge between the time t and $t+dt$ and can be written as

$$\lambda(t) = W(t)\, n_0(t)$$

where $W(t)$ is the breakdown probability[1] function and $n_0(t)$ is the electron liberation rate from the cathode.

Equation (1) can be integrated under the initial condition $p(0)$ equals zero at the time t is zero.

$$q = 1 - p(t) = e^{-\int_0^t \lambda(t) dt} \qquad (2)$$

where q is the probability of non-occurrence of the spark. If the lateral irradiation is constant, that is, if the liberation rate of initial electron is constant and the applied impulse voltage has constant height and infinite length, then $n_0(t)$ and $W(t)$ are constant with respect to time and $\lambda(t)$ also becomes a constant. Hence equation (2) can be written as

$$q = 1 - p(t) = e^{-n_0 W t} \qquad (3)$$

Equation (3) is the Laue equation for statistical time lag.

When the equation (1) is set up, we implicitly assume the one dimensional electric field distribution, that is met, for instance, in the parallel plane electrode gap or the concentric cylinder gap. In this case the electron liberated from any part of the cathode can contribute to the occurrence of the spark with equal probability. But in the case of the two dimensional electric field distribution as in the sphere gap the electron from different part of the cathode may have different probability for the occurrence of the spark. Equation (1) can not be applied to this case directly.

Even in the case of the two dimensional field distribution, if a small area is taken on the cathode, the electrons from the area may be considered to have same probability for the occurrence of the spark. Then the equation (1) can be applied for the small area, and the equation is

$$dp(t, \theta) = \left\{1 - p(t, \theta)\right\} W(t, \theta)\; n_0(t, \theta)\; ds(\theta)\, dt \qquad (4)$$

where θ is the independent variable that indicates position of the small area on the cathode, $ds(\theta)$ is the small area of the position on the cathode under consideration and $n_0(t, \theta)\ ds(\theta)$ is the number of the electrons liberated from the small area on the cathode per unit time at the time t.

Equation (4) is integrated with the initial condition that the probability of the occurrence of the spark is zero at the time t being equal to zero.

$$q(\theta) = 1 - p(t, \theta) = e^{-\int_0^t W(t, \theta)\, n_0(t, \theta)\, ds(\theta)\, dt} \tag{5}$$

Equation (5) gives the probability of the occurrence of the spark by the electrons that are liberated from the small area.

The probability of the spark over the whole cathode is obtained as follows. The probability of non-occurrence of the spark over the whole cathode is the multiplication of the probabilities of non-occurrence of the spark in each area on the assumption that the process of the spark formation of each area can be regarded independent. The equation is

$$Q = \lim \prod q(\theta_i) = e^{-\int_0^t \int_{(\theta)} W(t, \theta)\, n_0(t, \theta)\, ds(\theta)\, dt} \tag{6}$$

Thus the probability of spark occurrence over the whole cathode $P(t)$ is obtained as

$$P(t) = 1 - Q \tag{7}$$

This is the general equation from which the calculation on the phenomenon due to the statistical time lag is derived.

Application of the general calculating procedure for the percentage breakdown characteristics of parallel plane gap and sphere gap

From equation (7) the percentage breakdown characteristic when impulse voltage is applied can be obtained. As is shown in the following, the percentage breakdown curve takes a different feature according to the form of voltage impulse applied. Generally the standard impulse wave is used in the impulse test. But it is very difficult to calculate the percentage breakdown characteristic when the standard impulse voltage is applied which consists of the difference of two exponentially decreasing time function with different time constant. So the term that has a smaller time constant is neglected in the calculation, i.e. the standard impulse wave is approximated by exponentially decreasing impulse wave. To compare the result with this, the calculation when the rectangular pulse wave is applied is also performed.

(1) The case of the rectangular pulse application

Here the percentage breakdown characteristic is calculated when a rectangular pulse wave with the height V_0 and the duration T is applied.

1 a) Parallel plane gap

The static spark voltage of the parallel plane gap is assumed V_s. $W(t, \theta)$ in the equation (6) becomes $W(t)$ regardless of θ because the electric field distribution is one dimensional. The breakdown probability function $W(t)$ is known to be the function of percentage overvoltage Δv[1]. That is to say,

$$W(t) = f(\Delta v) \tag{8}$$

where

$$\Delta v = \frac{V_0 - V_s}{V_s} \tag{9}$$

Now Δv is a constant with respect to time, and $W(t)$ is also a constant with respect to time. The breakdown probability function W takes zero value when percent overvoltage Δv is zero and at small Δv W increases directly proportoinal to Δv, but it gradually saturates with increasing Δv and slowly approaches to one as Δv continues to increase[1]. The author assumed the following relation between W and Δv to make calculation easy.

$$W(\Delta v) = L\Delta v \tag{10}$$

where L is a constant.

The relation of direct proportion between W and Δv does exist until the value of Δv attains to about 0.1. If external irradiation for the gap is constant, the liberation rate of the electron $n_0(t)$ can be regarded constant. The equation (6) becomes

$$Q = e^{-n_0 s L T \Delta v} \tag{11}$$

where s is the area of the cathode and n_0 is the electron liberation rate per unit area. This is equal to the conventional equation by Laue.

1 b) Sphere gap

Usually the static breakdown voltage of the sphere gap means the spark voltage across the smallest distance between the two spheres. The smallest distance lies on the line connecting the centers of the two spheres. In other words, sparks of static breakdown occur on this line. But as far as the impulse breakdown is concerned, sparks can occur in the region outof the line that connects two sphere centers as well as on the line. This fact is frequently seen in the experiment.

Thus the static breakdown voltage other than on the axis line must be known in order to calculate percentage breakdown characteristics. But this is not known and so an assumption is made as follows. If the radius of the sphere electrode is R, the gap length d and the static spark voltage along the axis line V_{sd}, the static breakdown voltage of a position on the cathode is assumed to be

$$V_s(\theta) = V_{sd} + 2R(1-\cos\theta)\,a \qquad (12)$$

where θ is the angle between the center axis line and the line that connects the position on the cathode and the center of the sphere and a is a proportional constant.

Equation (12) means that the static breakdown voltage at the position θ is the sum of the static breakdown voltage of the sphere gap V_{sd} and the voltage that is proportional to the difference between the distance that connects the positions θ on each sphere surface parallel to the center axis line and gap length d. This assumption may be allowed as a simplest one. In this case $\Delta v\,(t, \theta)$ is

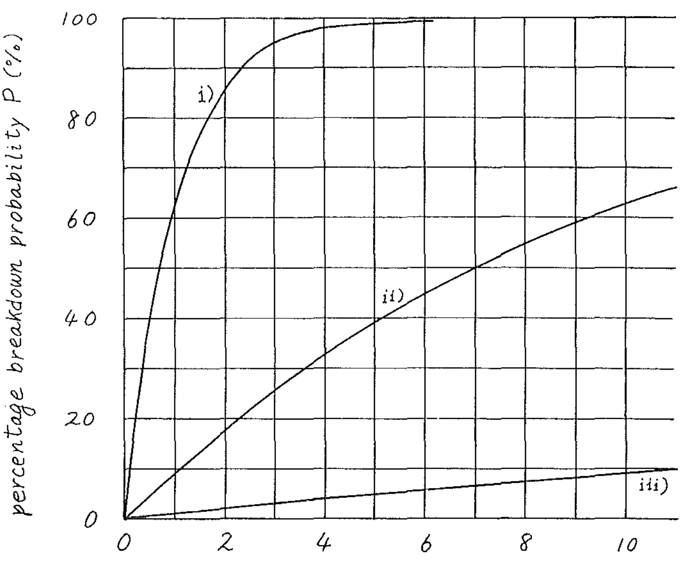

Fig. 1. Showing P to Δv relationship of parallel plane gap with rectangular pulse applied
i) $n_{0}sLT=10^2$, ii) 10^1, iii) 10^0

$$\Delta v\,(t,\,\theta) = \Delta v\,(\theta) = \frac{V_0 - V_s(\theta)}{V_s(\theta)} \tag{13}$$

and so

$$W(t,\,\theta) = W(\theta) = L\Delta v\,(\theta)$$

and the electron liberation rate $n_0(t,\,\theta)$ is assumed to be constant. The probability of non-occurrence of spark over the whole gap becomes

$$Q\,(\Delta v_0) = (1+\Delta v_0)^{-G(1+\Delta v_0)}\,e^{G\Delta v_0} \tag{14}$$

where

$$G = \pi L T n_0 R V_{sd}/a \tag{15}$$

$$\Delta v_0 = \frac{V_0 - V_{sd}}{V_{sd}} \tag{16}$$

and percentage breakdown curve can be obtained as

$$P(\Delta v_0) = 1 - Q\,(\Delta v_0)$$

Equations (11) and (14) are shown in figs. 1 and 2 respectively.

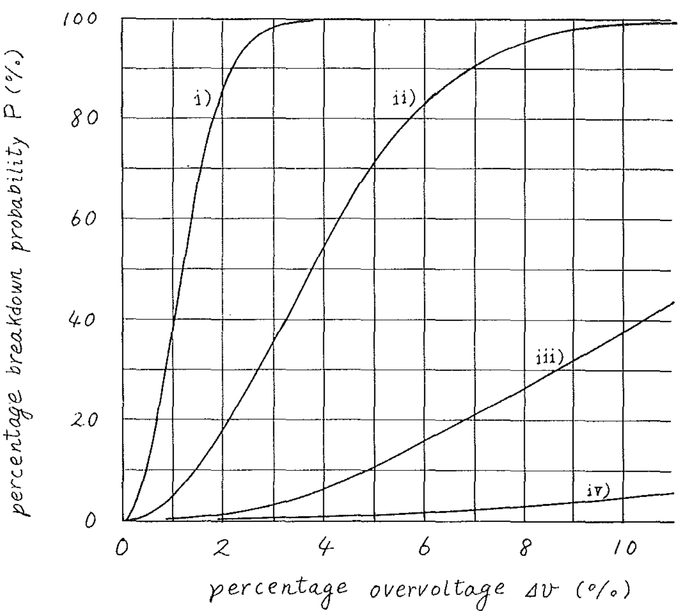

Fig. 2. Showing P to Δv relationship of sphere gap with rectangular pulse applied
i) $G=10^4$, ii) 10^3, iii) 10^2, iv) 10^1

2) The case where exponentially decreasing impulse voltage is applied

The exponentially decreasing impulse voltage is assumed to have the form of $V_0 \exp(-\gamma t)$ where V_0 is the crest value and γ is a constant, the reciprocal of which, is the time constant of the decreasing wave form.

2 a) Parallel plane gap

In this case

$$\Delta v(t, \theta) = \Delta v(t) = \frac{V_0 e^{-\gamma t} - V_s}{V_s} \tag{17}$$

where V_s is the static spark voltage of the gap. The breakdown probability function is

$$W(t, \theta) = W(t) = L\Delta v(t) \tag{18}$$

The electron liberation rate $n_0(t, \theta)$ is assumed to be constant in this case. The result is

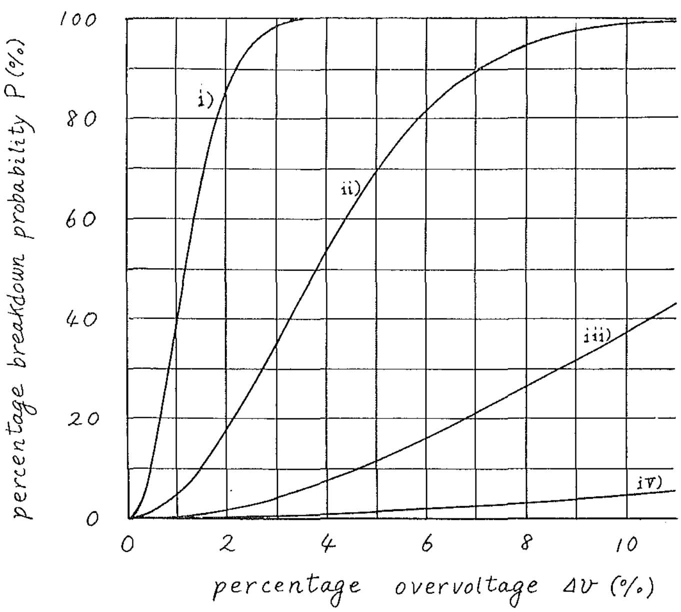

Fig. 3. Showing P to Δv relationship of parallel plane gap with exponentially decreasing impulse applied.

i) $\frac{Ln_0 S}{\gamma} = 10^4$, ii) 10^3, iii) 10^2, iv) 10^1

$$Q = (1+\Delta v_0)^{\frac{Ln_0 s}{\gamma}} \cdot e^{-\frac{Ln_0 s}{\gamma}\Delta v_0} \tag{19}$$

where Δv_0 is $\Delta v\,(0)$ and s is the area of the cathode.

2 b) Sphere gap

In this case, the percentage overvoltage is the function of both t and θ.

$$\Delta v\,(t,\,\theta) = \frac{V_0 e^{-\gamma t} - V_s(\theta)}{V_s(\theta)} \tag{20}$$

and

$$W\,(t,\,\theta) = L\Delta v\,(t,\,\theta) \tag{21}$$

The electron liberation rate $n_0(t,\,\theta)$ is again assumed to be a constant n_0. The result is

$$Q = (1+\Delta v_0)^{-\frac{\delta}{\gamma}(2+\Delta v_0)} \cdot e^{\frac{2\delta}{\gamma}\Delta v_0} \tag{22}$$

where

$$\delta = \pi L n_0 R V_{sd}/a \tag{23}$$

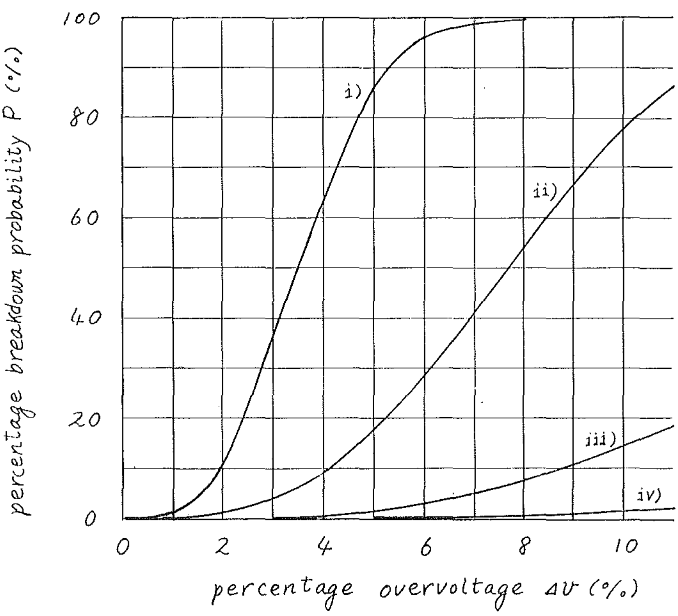

Fig. 4. Showing P to Δv relationship of sphere gap with exponentially decreasing impulse applied.

i) $\frac{\delta}{\gamma}=10^5$, ii) 10^4, iii) 10^3, iv) 10^2

and

$$\Delta v_0 = \Delta v\,(0,\ 0) = \frac{V_0 - V_{sd}}{V_{sd}} \tag{24}$$

The equations (19) and (22) are shown in the figs. 3 and 4 respectively.

Results

From the above calculations, the following facts can be pointed out.

1) The percentage breakdown characteristic of the parallel plane gap rises linearly from zero at the percentage overvoltage Δv being equal to zero when the rectangular pulse voltage is applied.

2) When the exponentially decreasing voltage $V_0\exp(-\gamma t)$ is applied to parallel plane gap, the percentage breakdown curve rises slowly as the percentage overvoltage Δv_0 increases from zero.

3) The percentage breakdown curve rises also slowly from zero when a rectangular voltage wave is applied to the sphere gap.

4) The percentage breakdown curve of sphere gap when exponentially decreasing voltage $V_0\exp(-\gamma t)$ is applied rises from zero more slowly than when rectangular pulse voltage is applied.

5) In all cases, the percentage breakdown curve approaches slowly to one (100 percent) as the percentage overvoltage increases.

6) As may be seen from the figures the percentage breakdown characteristic is influenced by the intensity of irradiation. If the intensity of irradiation is increased, i.e. n_0 is increased, the curve of the characteristics appears to shrink parallelly with abscissa, the point of zero percent overvoltage being fixed in the graph. The voltage at which the percentage breakdown is fifty per cent decreases also with increasing irradiation. The curve describing the relation between the electron liberation rate (i.e. irradiation intensity) and the fifty percent breakdown overvoltage can be obtained from the figures.

Discussion

The percentage breakdown characteristics so far studied[2] are mostly those on sphere gaps to which standard impulse voltage is applied. According to these studies, the percentage breakdown curvei ncreases slowly from zero when the pecentage overvoltage Δv is equal to zero, and attains the maximum gradient at about the magnitude of 0.5, then the gradient falls and the curve slowly approaches to one as the percentage overvoltage continues to increase. The curve of the percentage breakdown characteristic calculated for the sphere gap to which the exponentially decreasing voltage $V_0\exp(-\gamma t)$, which is the approximation of the standard impulse wave, is applied, has as same appearance as the characteristic curves by the studies.

The fact that the percentage breakdown characteristic curve is influenced by the wave form of the applied impulse voltage can easily be observed in the figures as was pointed out in the foregoing paragraph. This fact is seen in the parallel plane gap as well as in the sphere gap and the gradient at the zero overvoltage of the percentage breakdown characteristic curve when the exponentially decreasing voltage is applied is smaller than that when the rectangular pulse voltage is applied. A similar tendency of the characteristics as mentioned above also exists in the case where the impulse voltage having the same wave form is applied for both gaps. That is to say, the percentage breakdown characteristic curve of the sphere gap rises more slowly from zero than that of the parallel plane gap even when the impulse voltage which has the same wave form is applied. The effect due to the difference of the impulse wave form applied and that due to the geometrical form of the spark gap electrodes on the percentage breakdown characteristic have thus similar influence and they work additively. Hence, the case where the exponentially decreasing voltage is applied for the sphere gap has the lowest gradient at the zero overvoltage of the percentage breakdown characteristic curve among the cases.

The fact just mentioned above, i.e. the gradient at the zero overvoltage of the percentage breakdown characteristic curve of the sphere gap to which the exponentially decreasing impulse voltage is applied is very low, leads to the conclusion that the estimation of the static breakdown voltage of the sphere gap is very difficult when the standard impulse voltage is applied. There is always a chance of mis-estimation of the static breakdown voltage for the larger side. This chance may be eliminated to a certain degree by suitable irradiation. But inversely speaking, there may be the chance that the static breakdown voltage of the sphere gap is taken to be decreased by external irradiation. Of course, the irradiation can decrease the static breakdown voltage in some cases. The irradiation intensity is that which varies the static breakdown voltage. The discussion of the problem will be done later.

The assumptions set forth for the calculation will be discussed in the following. In the calculations the formative time lag is neglected throughout. The effect may appear in the case where the liberation rate of the initial electron n_0 is large, for in that case the statistical time lag is reduced comparative to the formative time lag. The error of the calculation can not be neglected then. The calculation when the formative time lag is taken into account and is assumed to be inversely proportional to the percentage overvoltage is performed[3]. The rise point of the percentage breakdown curve shifts from zero percent overvoltage to a little larger one. The magnitude of the shift is influenced by the applied impulse wave form and the formative time lag to percentage overvoltage relationship. According to this fact there is also a chance of mis-estimation of the static breakdown voltage from the standard impulse wave application when the formative time lag is comparative to the statistical time lag. The results of the calculation is rather complicated.

The assumption that the breakdown probability function $W(t, \theta)$ is directly proportional to the percentage overvoltage $\Delta v(t, \theta)$ must be examined on the limit of its application. As was described before, the breakdown probability function has zero magnitude where the percentage overvoltage is zero and increases in direct proportion as the percentage overvoltage increases. But it begins to saturate on the way and finally approaches very slowly to one. The assumption adopted is valid for the range where the percentage overvoltage is small. If the range is required to be magnified, some other approximation must be adopted which has the property of saturation. The author selected the following equation.

$$W(t, \theta) = 1 - e^{-L\Delta v(t, \theta)} \tag{25}$$

where L is a constant.

The calculations using this new assumption are made for the parallel plane gap to which the rectangular pulse wave and the exponentially decreasing wave voltages are applied respectively. The results are as follows.

$$Q = e^{-n_0 s L(1-e^{-L\Delta v})T} \tag{26}$$

$$Q = (1+\Delta v_0)^{-\frac{n_0}{\gamma}} \cdot e^{\frac{n_0}{\gamma}\left[\left\{-E_i(-L)\right\} - \left\{-E_i(-L\,\overline{1+\Delta v_0})\right\}\right]e^{L}} \tag{27}$$

where Q is the probability of non-occurrence of a spark, n_0 is the initial electron liberation rate per unit area of the cathode, s is the area of the cathode of the parallel plane gap, L is the constant that decides the relationship between the breakdown probability function and the percentage overvoltage as shown in the equation (25), Δv is the percentage overvoltage for the parallel plane gap, Δv_0 is the initial percentage overvoltage when the exponentially decreasing voltage is applied, γ is the reciprocal of the time constant of the exponentially decreasing voltage and $E_i(x)$ is the integral exponential function. Of course the equation (26) is concerned with the rectangular pulse wave application and the equation (27) is concerned with the exponentially decreasing wave application. The results of the numerical calculation of the equations (26) and (27) are shown in figs. 5 and 6 respectively. From the results the fact is known that there may be a case where the percentage breakdown pro-

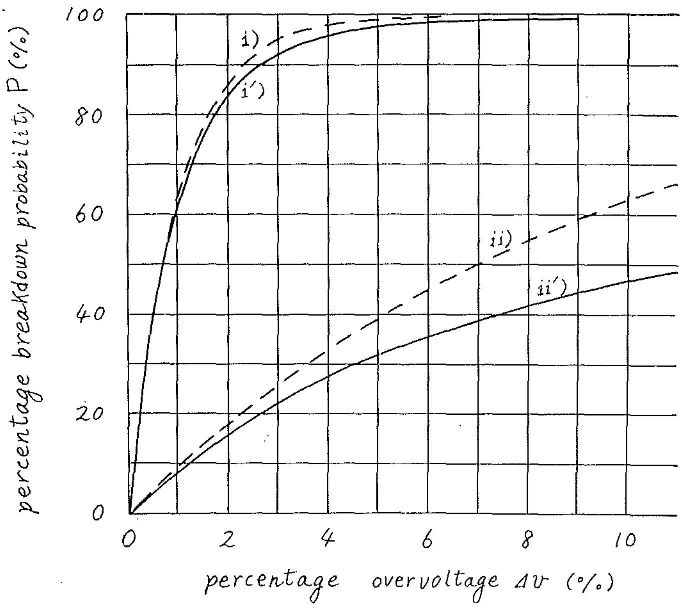

Fig. 5 a. Shows the same case in Fig. 1 but the saturation tendency of W is considered (solid lines), the dashed lines are the reproductions of i) and ii) in Fig. 1 respectively.
i') $L=10^1$, $n_0sT=10^1$, ii') $L=10^1$, $n_0sT=10^0$

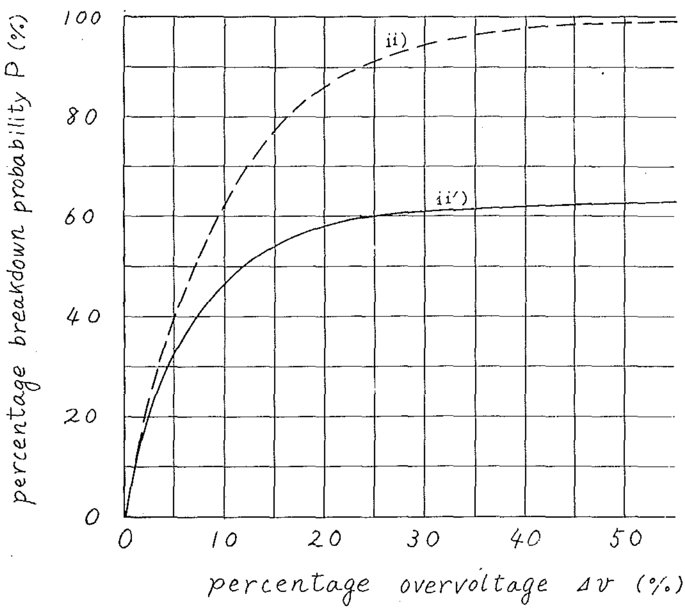

Fig. 5 b. Is the reproduction of ii) and ii') in Fig. 5 a with different scale of abscissa. ii') never reaches 100%.

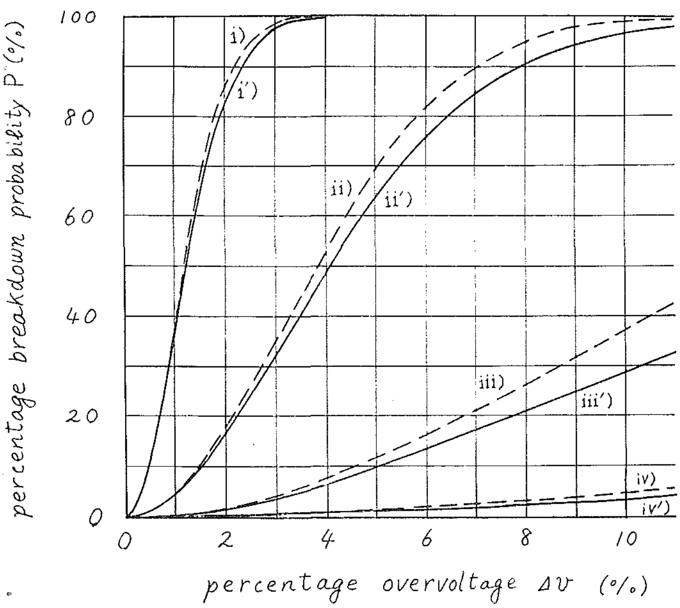

Fig. 6. Shows the same case in Fig. 3 but here the saturation tendency of W is considered (solid lines). The dashed lines are the reporductions of i), ii), iii) and iv) in Fig. 3 respectively.

i′) $L=10$, $\frac{n_0 S}{r}=10^3$, ii′) 10, 10^2, iii′) 10, 10^2, iv′) 10, 10^1

bability never attains to one. The author experienced such a case in the low pressure impulse breakdown of the air. The percentage breakdown probability never attained to one as the overvoltage was increased and was difficult to determine. So the limit of approximation of the equation (10) may firstly appear in the region where the percentage overvoltage is comparatively large. But if the initial electron liberation rate from the cathode is of suitable magnitude and the percentage overvoltage is not so large, the approximation of the equation (10) may be regarded as valid. The percentage breakdown curves of the sphere gap when the approximation of the equation (25) is adopted are under calculation, and in the case where the rectangular pulse is applied the equation closely resembles equation (27).

The problem of the relationship between the static breakdown voltage and the external irradiation intensity (the initial electron liberation rate from the cathode) is left here. In the calculations above it was always assumed that the static breakdown voltage was not influenced by the irradiation intensity. But indeed the strong irradiation can affect the static breakdown voltage[4]. This situation occurs when the irradiation intensity is too strong or a kind of inequality of the electric field intensity is too large[5]. The author is of the opinion that the strong intensity of the irradiation changes the spark mechanism. Even the decrease of the formative time lag owing to the strong irradiation can also be regarded as a kind of change of the mechanism inclusively, because of the transfer of the mechanism from the single electron spark with the weak irradiation intensity to the multiple electron spark with the strong irradiation intensity[6]. In such cases, the static spark voltage itself has a functional relationship with the irradiation intensity or the liberation rate of the initial electron. If such a situation is neglected the percentage breakdown curves under strong irradiation can not be explained by the original equation (6). An unexpected rise of the percentage breakdown curve seems to occur abruptly in the region of the comparatively large percentage overvoltage with too weak irradiation[7]. This may be regarded as a phenomenon also due to the abrupt change of the spark mechanism where the magnitude of the percentage overvoltage is larger than a certain value.

The results of further investigation and calculation on these problems will be published in the future.

Conclusion

The general equation for calculating the percentage breakdown characteristics of the impulse breakdown is set up on the statistical time lag and applied to the parallel plane gap and the sphere gap when the rectangular pulse voltage and the exponentially decreasing voltage are applied respectively. The fact that the percentage breakdown characteristic is influenced by the voltage wave form applied, the geometrical feature of the gap electrodes and the intensity of the external irradiation can be explained by the equations. The fifty per cent breakdown voltage can also be given from the calculation.

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Bibliography

1) e. g., R. A. Wijsmann: Phys. Rev. **75**, 833 (1949).
Y. Miyazoe: "Hibana no Tokeitekiseishistu ni kansuru Kenkyu (The Study on the Statistical Nature of the Spark)" (in Japanese) p. 98, (Doctor Thesis) (1961), The Faculty of Engineering Kyushu University, Fukuoka, Japan.

2) e. g., J. M. Meek: J. I. E. E. **93**, 113 (1946).
D. F. Binns and D. R. Hardy: A. I. E. E. Communication and Electronics no. 60 p. 157 (1962).
3) H. Tagashira and S. Sakamoto: "Denki Yongakkai Hokkaidoshibu Rengotaikai Yoko (The Collection of the Lectures at the Anniversary Meeting of the Hokkaido Branches of the Institutes of Electrical, Electrical Communication, Illumination and Television Engineering)" A-2, (1961) (in Japanese).
4) e. g., C. Brinkmann: Z. Phys. **111**, 737 (1939).
5) T. Ushita: "Shotokki tsuki Tokushuhodengyappu no Kichuhodentokusei ni Kansuru Kenkyu (The Study on the Characteristics of the Special Spark-gap possessing Protruding Needle)" (in Japanese) (Doctor Thesis) (Hokkaido University Library no. 4315)
6) Y. Miyazoe: p. 41 of the paper referred to 1).
7) e. g., J. M. Meek: J. I. E. E. **89**, 335 (1942).
J. M. Meek: ibid. **93**, 97 (1946)