The Helitron Waves on an E-type Filamentary Electron Beam*

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Summary

Power carried by the helitron waves on an E-type filamentary electron beam and dispersion curves were discussed. It was found that similar expressions for the charactristic of each wave component to those of O-type and M-type electron beams can be obtained by this analysis.

1. Introduction

Recently, a number of new techniques have been discussed for low-noise detection of microwave signals. The new techniques are making it possible to obtain much higher sensitvities than previously attainable. The discovery of these techniques have been timely, because simultaneously with their discovery a need for higher sensitivity has developed. The numerous applications that demand low-noise detection are intercontinental communication by means of active or passive satellite repeaters, radio astronomy and satellite-based radars and so on. At the present time there are a number of new devices capable of substantially better performance. One of them is an electron-beam parametric amplifier. Great interests have been shown in the parametric coupling on O-and M-type beams. Especially, low-noise figures have been obtained from the M-type beam of the device. The disadvantage of these types of beam is that the magnetic field requirement adds a good deal of bulk to the system, either in the form of a solenoid or a permanent magnet. An alternative is to use an

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electrostatically focused beam which has been termed "E"-type by Heffner and Watkins[1]. The electron beam of this type traces the path of a helix between two conductors which are maintained at a potential difference as shown in Fig. 1. Several theoritical and experimental results of the E-type electron beam appear in the literature[2,3,4]. Recently, waves on this type of beam in a transverse-field circuit have been pointed out by Pantell[5], who found the existence of the fast and slow space-charge waves, but did not analyze the kinetic power flow of each wave.

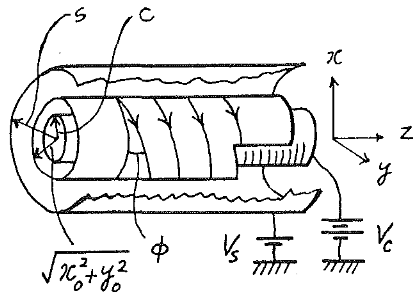

Fig. 1. Electrons form a right circular hollow beam, thin in the radial direction with an electron tracing the path of a helix between coaxial cylinders.

It is the purpose of this paper to show that the charactristics on the E-type filamentary electron beam can be expressed in a normal mode from. Especially, the author has termed these space-charge waves the "helitron waves" and the discussion applies to both dispersion curves and kinetic power flow[6,7,8].

2. Space-Charge Wave Equations

In a drift space of the E-type beam as shown in Fig. 1, the following euqations are obtained from the force equation for electrons which are traveling in the z-direction with dc velocity u_0, under electrostatic focusing conditions. It is assumed that the electron traces the path of a helix between coaxial cylinders and all electrons form a right circular hollow cylinder which is thin in the radial dimension,

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= \eta V_0 \frac{x}{x^2+y^2} \\ \frac{d^2y}{dt^2} &= \eta V_0 \frac{y}{x^2+y^2} \\ \frac{dz}{dt} &= \sqrt{\eta V_0} \tan\phi \end{aligned}\right\} \tag{1}$$

where the definitions are given as follows:

$\eta = 1.759 \times 10^{-11}$ coulombs/kg

$V_0 = (V_c - V_s)/\ln(s/c),\ \ V_c > V_s$

$V_s =$ voltage between outer cylinder and ground,

$V_c =$ voltage between inner cylinder and ground,

ϕ = electron pitch angle.

Under a small signal theory, the motion of the electron is given as a sum of a dc quantity and an RF disturbance.

$$\left.\begin{aligned} x &= x_0 + x_1 \\ y &= y_0 + y_1 \\ z &= z_0 \end{aligned}\right\} \tag{2}$$

The subscript 0 refers to the dc quantity and the subscript 1 refers to the RF motion.

Using equation (2), equation (1) becomes

$$\left.\begin{aligned} \frac{d^2x_1}{dt^2} + 2\omega_h^2 x_1 = 0 \\ \frac{d^2y_1}{dt^2} + 2\omega_h^2 y_1 = 0 \end{aligned}\right\} \tag{3}$$

where ω_h is termed "*helitron angular frequency*",

$$\omega_h = \sqrt{\eta V_0/(x_0^2 + y_0^2)}.$$

These are the fundamental equations that describe the types of waves that can propagate on the electron beam in the form of a transverse disturbance when the beam is subject to an electrostatic field. They are in a form analogous to the space-charge wave equations of the O-type beam.

Equation (3) can be rewritten by transverse velocities v_{1x} and v_{1y} as follows :

$$\left.\begin{aligned} \frac{dv_{1x}}{dt} + 2\omega_h^2 x_1 = 0 \\ \frac{dv_{1y}}{dt} + 2\omega_h^2 y_1 = 0 \end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned} \frac{dx_1}{dt} = v_{1x} \\ \frac{dy_1}{dt} = v_{1y} \end{aligned}\right\} \tag{5}$$

Now, we introduce the effective transverse current[9] K, as well as Chu's kinetic voltage U.

$$\left.\begin{aligned} 2U &= U_x + jU_y \\ 2K &= K_x + jK_y \end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned} U_x &= -eu_0v_{1x}/m \\ U_y &= -eu_0v_{1y}/m \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} K_x &= j\omega\sigma_0 x_1 \\ K_y &= j\omega\sigma_0 y_1 \end{aligned}\right\} \tag{8}$$

where σ_0 is the total charge of the electron beam per transverse unit area. The subscript x denotes a x component and the subscript y denotes a y component.

From equations (4), (5) and (6), the following space-charge wave equations are obtained:

$$\left.\begin{aligned} \left(\frac{d}{dz}+j\beta_e\right)U &= -j\frac{2\omega_h^2 m}{\omega e\sigma_0}K \\ \left(\frac{d}{dz}+j\beta_e\right)K &= -j\frac{\omega\sigma_0 e}{mu_0^2}U \end{aligned}\right\} \tag{9}$$

These expressions are very similar to those for the longitudinal beam (one dimensional)[10].

Next, we define the helitron normal mode $a_{\pm}$ using U and K,

$$a_{\pm} = \frac{1}{4\sqrt{Z_0}}(U \pm Z_0 K) \tag{10}$$

where Z_0 is the charactristic beam impedance for the small signal quontities of the helitron waves,

$$Z_0 = \frac{2V_0}{|I_0|}\frac{\sqrt{2}\,\beta_h}{\beta_e}, \tag{11}$$

where I_0 is the dc beam current ($|I_0| = -\sigma_0 u_0$), $\beta_e = \omega/u_0$, and $\beta_h = \omega_h/u_0$.

Using equations (10) and (11), equation (9) then becomes

$$\left\{\frac{d}{dz}+j\left(\beta_e \mp \sqrt{2}\,\beta_h\right)\right\} a_{\pm} = 0. \tag{12}$$

These are the normal mode forms of the helitron waves. Accordingly, for an arbitrary excitation in terms of the mode amplitudes at the input plane $z=0$, the solutions of equation (12) are

$$\left.\begin{aligned} a_{\pm(z)} &= a_{\pm(0)} \exp\left(-j\beta_{\pm} z\right) \\ &= a_{\pm(0)} \exp\left\{-j\left(\beta_e \mp \sqrt{2}\,\beta_h\right) z\right\} \end{aligned}\right\} \tag{13}$$

From this equation (13), U and K may be expressed as follows:

$$\left.\begin{aligned} U &= 2\mathrm{Re}\left\{\sqrt{Z_0}\,(a_{+(z)}+a_{-(z)})\,e^{j\omega t}\right\} \\ K &= 2\mathrm{Re}\left\{\frac{1}{\sqrt{Z_0}}\,(a_{+(z)}-a_{-(z)})\,e^{j\omega t}\right\} \end{aligned}\right\} \tag{14}$$

3. Dispersion Charactristics

The phase velocities of the helitron waves in equation (12) are given by

$$v_{p\pm} = \frac{\omega}{\beta_e \mp \sqrt{2}\,\beta_h} = \frac{u_0}{1\mp(\sqrt{2}\,\omega_h/\omega)}\,. \tag{15}$$

It is easy to see from equation (15) that the a_+ mode travels with a phase velocity slightly greater than the dc velocity, whereas the a_- mode travels with a phase velocity slightly less than the dc velocity. Accordingly, the a_+ mode is called the *fast helitron wave*, and the a_- is called the *slow helitron wave*. These phase velocities are sketched in Fig. 2.

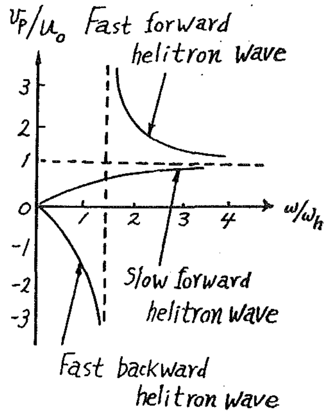

Fig. 2. The helitron mode dispersion curves.

The group velocity is given by

$$\begin{aligned} v_{g\pm} &= \left(\frac{\partial\beta_\pm}{\partial\omega}\right)^{-1} = u_0\left(1\mp\frac{\partial\sqrt{2}\,\omega_h}{\partial\omega}\right)^{-1} \\ &= u_0 \end{aligned} \tag{16}$$

since $\beta_\pm = \beta_e \mp \sqrt{2}\,\beta_h$ and β_h is independent of frequency.

It is a charactristic of waves that energy is propagated at the group velocity, which in this case is the dc beam velocity.

Let us consider the fast helitron mode. If $\omega > \sqrt{2}\,\omega_h$, then $v_{p_+} > u_0$, so that this is a fact forward helitron wave. If $0 < \omega < \sqrt{2}\,\omega_h$, v_{p_+} is negative and the wave travels backward. It is seen that the phase velocity for this mode is zero when the signal frequency is zero and approaches minus infinity as $\omega \to \sqrt{2}\,\omega_h$ from the left. As $\omega \to \sqrt{2}\,\omega_h$ from the right, the phase velocity approaches plus infinity. When $\omega \gg \sqrt{2}\,\omega_h$, the phase velocity approaches the dc beam velocity. Next we will consider the slow helitron mode in the same way. From equations (15) and (16), it is seen that the wave travels forward with a velocity less than u_0.

4. Kinetic Power Flow in Helitron Waves

The power carried by helitron waves can be given by the generalized Chu's power theorem as follows:

$$P_k = \frac{1}{2} \mathrm{Re}\,[UK^*]. \tag{17}$$

Using equation (14), power P_k then becomes

$$P_k = 2\,(|a_+|^2 - |a_-|^2). \tag{18}$$

It is seen that the a_+ mode, the fast helitron wave, carries positive kinetic power, whereas the a_- mode, the slow helitron wave, carries negative kinetic power. These correspond to those in the power flow in O-and M-type electron beams. In order to show that P_k is independent z, we differentiate equation (14) with z and equation (13) and their complex conjugate substitute into this expression. It then follows immediately that

$$\frac{dP_k}{dz} = 0 \tag{19}$$

In conclusion, the power flow on the E-type filamentary electron beam in the drift space discussed here is given as equations (14) and (19).

In order to excite a fast helitron wave, kinetic power must be given to the beam; to excite a slow helitron wave, kinetic power must be extracted from the beam, since the slow helitron wave carries negative power. Conversely, to remove a fast helitron wave from the beam, power must be extracted from the beam, and to remove a slow helitron wave power must be added to the beam. Further, from equation (18), it is seen that no net power is required from the driving source if both modes are equally excited.

5. A Comparison of Helitron and Cyclotron Waves

The analysis developed in this paper showed the dispersion curves and power flow in helitron waves. The charactristic is analogous to the cyclotron waves[10] on the M-type electron beams. Table 1 gives a comparison of cyclotron and helitron waves.

The M-type electron beam has the disadvantage in that the magnetic field requirement adds a good deal of bulk to the system. But in the E-type electron beam no magnetic field is required. Advantages for eliminating the magnet are fairly obvious: reduction in overall weight of the tube and absence of power supply for the magnet. Then this type of beam is suitable for satellite repeaters, satellite-based radars and electronic research equipment of aircrafts.

TABLE 1. Comparison of cyclotron waves and helitron waves

	Cyclotron Waves		Helitron Waves	
Equations of Motion	$d^2x/dt^2=\omega_c(dy/dt)$ $d^2y/dt^2=-\omega_c(dy/dt)$ $dz/dt=\sqrt{2\eta V_0}$		$d^2x/dt^2=\omega_h^2 x$ $d^2y/dt^2=\omega_h^2 y$ $dz/dt=\sqrt{\eta V_0}\tan\varphi$	
Small-Signal Assumption	$x=x_0+x_1,\ y=y_0+y_1,\ z=z_0$			
Space Charge Wave Equations	$d^2x_1/dt^2-\omega_c(dy_1/dt)=0$ $d^2y_1/dt^2+\omega_c(dx_1/dt)=0$		$d^2x_1/dt^2+2\omega_h^2 x_1=0$ $d^2y_1/dt^2+2\omega_h^2 y_1=0$	
Mode Amplitudes	$a_\pm=(U_x\pm jU_y)$ $Z_0=(2V_0/\|I_0\|)(\beta_c/\beta_e)$		$a_\pm=(U\pm Z_0K)/4\sqrt{Z_0}$ $U=U_x+jU_y$ $K=K_x+jK_y$ $Z_0=(2V_0/\|I_0\|)(\sqrt{2}\beta_h/\beta_e)$	
Normal Mode Form of Wave Equations	$\{d/dz+j(\beta_e\mp\beta_c)\}a_\pm=0$		$\{d/dz+j(\beta_e\mp\sqrt{2}\beta_h)\}a_\pm=0$	
Space Charge Waves	Fast Cyclotron Wave	Slow Cyclotron Wave	Fast Helitron Wave	Slow Helitron Wave
Phase Constants	$\beta_+=\beta_e-\beta_c$	$\beta_-=\beta_e+\beta_c$	$\beta_+=\beta_e-\sqrt{2}\beta_h$	$\beta_-=\beta_e+\sqrt{2}\beta_h$
Phase Velocities	$V_{p+}=u_0/1-(\omega_c/\omega)$	$V_{p-}=u_0/1+(\omega_c/\omega)$	$V_{p+}=u_0/1-(\sqrt{2}\omega_h/\omega)$	$V_{p-}=u_0/1-(\sqrt{2}\omega_h/\omega)$
Group Velocities	$V_{g+}=u_0$	$V_{g-}=u_0$	$V_{g+}=u_0$	$V_{g-}=u_0$
Sign of Kinetic Power	+	−	+	−
Remarks	ω_c = cyclotron angular frequency, ηB $\beta_e=\omega/u_0$, $\beta_c=\omega_c/u_0$, $\beta_h=\omega_h/u_0$ ω = signal frequency u_0 = drift velocity		ω_h = helitron angular frequency, $\sqrt{\eta V_0/(x_0^2+y_0^2)}$	

6. Conclusions

In this paper, a normal mode analysis of an E-type filamentary electron beam is presented. This beam was focused by introducing a potential difference between two concentric cylinders with an electron tracing the path of a helix between cylinders.

The normal mode analysis developed in this paper yielded the following interesting results :

1) The space-charge wave on this beam was termed "helitron wave".

2) Two helitron mode amplitudes were defined as follows;

$$a_{\pm} = (U \pm Z_0 K)/4\sqrt{Z_0}$$

where a_+ = mode amplitude of the fast helitron wave
a_- = mode amplitude of the slow helitron wave
U = Chu's kinetic voltage, $-(m/e)\, vu_0$
v = transverse velocity of the electrons in the beam
u_0 = drift velocity
K = effective transverse current caused by the displacement of the electron beam from dc value
Z_0 = characteristic beam impedance for the small-signal quantities of the helitron wave

3) It was shown that the a_+ mode, the fast helitron wave, travels with a phase velocity slightly greater than the drit velocity, whereas the a_- mode, the slow helitrcn wave, travels with a phase velocity slightly less than drift velocity.

4) It was shown that the helitron wave carries positive kinetic power, whereas the slow helitron wave carries negative kinetic power.

Acknowledgment

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