Settling Property Determination

Norihito TAMBO

Abstract

In this paper the author proposed to use time-accumulation curves as a fundamental property to define the settling property of a suspension. Two methods to determine time-accumulation curves were devised. One is the balance analysis method for laboratory use and the other is the pipette analysis method for field practice.

I. Settling Velocity of Discrete Particles

The settling properties of various suspensions, flocculated and otherwise, are the most important factors in the design and operation of the sedimentation processes. As is well known, the settling phenomena are classified into three categories; free settling, hindered settling and compression. In addition, free settling is divided into two categories, non-flocculent and flocculent settling. The former is the settling of discrete particles without interaction with other particles. This phenomenon is to be considered as the fundamental one. Other states of sedimentation are considered on the basic theory of discrete particle settling.

The classic laws of sedimentation of individual particles were developed by Stokes[1], Allen[2], Newton[3], Tsurumi[4] and others[5]. Under the influence of gravity, any particle having a greater density than that of water will settle in water at an accelerating velocity until the frictional drag of the water against the particle reaches the driving force, namely the effective weight of the particle. After the initial transient situation, the settling velocity is essentially constant and depends upon the size, shape and density of the particle, and the density and viscosity of the water. For theoretical computation of settling velocity, the particle is commonly assumed to be spherical. Settling velocities of other shaped particles can be analyzed in relation to spheres.

The general equation for the settling velocity of an individual particle is shown as follows:

$$w^n = \frac{4}{3} \cdot \frac{g}{k} \frac{(\rho - \rho_0)}{\rho_0^{n-1}} \mu^{n-2} d^{3-m} \qquad (1)$$

where w = the settling velocity of discrete particles (cm/sec)

n = the state of flow; i.e., $n=1$ for stream line flow, n is more than 1 but less than 2 for transitional flow, and $n=2$ for turbulent flow.
g = the acceleration of gravity (cm/sec^2)
ρ = the density of the particle (g/cm^3)
ρ_0 = the density of the liquid (g/cm^3)
μ = the viscosity of the liquid (g/cm·sec)
d = the diameter of the particle (cm)
k = a constant having the following relationship with the coefficient of drag C_D;

$$C_D = k\left(\frac{wd\rho}{\mu}\right)^{n-2} = kR^{n-2} \qquad (2)$$

R = the Reynolds number

Stokes' Law

When the particle diameter d is sufficiently small, the predominance of viscous forces results in accelerative and resistive forces proportional to the first power of the velocity, i.e. $m=1$ in Equation (1) and the drag coefficient C_D is given as follows;

$$C_D = \frac{k}{R} = \frac{24}{R} \text{ or } \frac{24}{dw} \qquad (3)$$

Then the Equation (1) becomes;

$$w = \frac{1}{18} g \frac{(\rho-\rho_0)}{\mu\rho_0} d^2 . \qquad (4)$$

Figure (1) shows that it fits experimental data exactly for values of R up to about 0.55, and offers a reasonably good fit, up to $R=1.0$.

Newton's Law

When the inertia force become so predominant that the resistive force is proportional to the second power of the velocity ($n=2$) then $C_D=k$. The experimental data in Figure 1 show that $C_D \fallingdotseq 0.4$.

Substituting $n=2$ and $k=0.4$ in Equation (1);

$$w = 1.82\sqrt{\frac{(\rho-\rho_0)}{\rho_0} dg} . \qquad (5)$$

$C_D=k=0.4$ fits the experimental data in the R range of 1.000−250,000.

Allen's Law

If the values $C_D=30/R^{0.625}$ and $n=1.375$ are substituted in Equation (1)

then the resulting equation,

$$w = 0.20\left[\frac{(\rho-\rho_0)}{\rho_0}g\right]^{0.72}\cdot\left[\frac{d^{1.18}}{\left(\frac{\mu}{\rho_0}\right)^{0.45}}\right] \qquad (6)$$

gives a reasonable degree of fit to the experimental data for all values of R from 1 to 1,000. This is called Allen's law.

Figure 2[6] shows the commonly accepted value for sand particles of specific gravity of approximately 2.65 and for flocs having a specific gravity of approximately 1.001 to 1.50. According to the curves shown in Figure 2, almost all coagulated particles are in the range of Stokes' and Allen's laws. It is possible to apply Stockes' law to rather minute particles; when the density of floc is 1.02 and water temperature is 10°C, the applicable upper limit of Stokes' law is to a floc having a diameter of 0.6 mm. Therefore, it is necessary to use Allen's law for the well flocculated particles in the calculation of settling velocity.

II. Principle of Settling Property Measurement for Discrete Particles[7]

As stated in the previous section, each discrete particle would settle according to any one of the laws stated before. However, in a practical case, in a settling basin, there are many sized particles ranging from Stokes' range to Newton's range in a suspension. For the purpose of a sedimentation basin design, it becomes essential to know and identify the settling property of the suspension. To describe the settling property of a suspension of particles, it is convenient to use settling velocity distribution. This can be found by suitable experimental measurements of a suspension and is independent of the type of governing sedimentation law for each component particle of the suspension. It is furthermore directly useful for sedimentation basin design.

Possible methods of measuring settling velocity include optical and electron microscopy, radiation scattering and transmission, light scattering and transmission, and sedimentation. Among these, the method of sedimentation is by its nature the most direct way to measure the settling velocity distribution. It is also the simplest. All of the other methods are indirect and a suitable sedimentation law must be assumed in the calculation of the settling velocity distribution from the measurements.

Gravitational sedimentation methods for measuring the settling velocity distribution can be divided into two categories, the incremental method and the cumulative method. In the first, concentrations are measured at a fixed

point below the surface of the fluid at various times or at different points at an instant of time after settling begins. In the second, or cumulative method, the accumulated sediment is measured at a fixed level until all particles between that point and the fluid's surface have settled.

Among incremental methods, the pipette method and the density method —the hydrometer method—are most common. However, for the purpose of plane sedimentation basin design in water works either method is not sufficient. The former requires a great many samples to make possible a high accuracy of measurement for such low concentration suspensions as may occur in raw water of ordinary water works. In addition, it is troublesome to evaporate the water from the sample for weighing. The latter hydrometer method does not have ample sensitivity for these dilute suspensions.

On the other hand, should the use of a sensitive automatic weighing instrument be available, the cumulative method would be the most convenient method for these purposes. The author suggests the use of cumulative weight method using a sensitive autobalance for the settling analysis of discrete particles.

In the cumulative weight method, the settling velocity distribution function of a suspension is obtained by weighing the particles as they accumulate on a pan at the bottom of the settling analysis cylinder. It will be assumed that the pan has unit area and is situated at a depth h from the surface of the suspension. The fluid through which the particles move is considered quiescent. At the initial state the particles of each size are distributed uniformly throughout the settling tube. The settling of each particle is assumed independent of the others, i.e. no hindered settling is assumed. After time t some particles suspended between the surface and the depth h at the initial stage pass through the horizontal plane at the depth h and others remain in the water body above depth h. Those particles which pass through the h plane are intercepted by the pan. The pan receives all of the particles falling down from the column having unit cross-sectional area and height h above the pan. No consolidation occurs for the particles which accumulate on the pan; then no variation of the apparent density occurs in the course of measurement of accumulation weight.

As shown in Figure 3 the accumulated weight G increases with time t and finally reaches an asymptote which is parallel to the horizontal axis of the graph. The height of the asymptote above the horizontal axis is the ultimate accumulation weight of the suspension.

The function $f(w)$ is called the frequency distribution function of particle settling velocity of a suspension, or simply in this thesis, the frequency distri-

bution function. The quantity $f(w)dw$ means the weight of particles with a settling velocity between w and $w+dw$ as shown in Figure 4. For discrete particles $f(w)$ is a fixed property of suspension.

The weight $q(t)$ of particles which settled in time t on the pan at the depth h is made up of two parts, the particles having greater settling velocity than w_i and those having a smaller velocity:

$$q(t)=\int_{w_i}^{w_{\max}} f(w)dw+\int_{w_{\min}}^{w_i}\frac{wt}{h}f(w)dw \tag{7}$$

where $q(t)$: the weight of particles settling at time t upon an accumulation pan at depth h.
w: the settling velocity of a particle
$w_{\max}$: the settling velocity of the largest suspended particle
$w_{\min}$: the settling velocity of the smallest suspended particle
w_i: the settling velocity of the particle which falls the distance h in time t; i.e., $w:=h/t$

Differentiating Equation (7) with respect to t gives;

$$\frac{dq(t)}{dt}=-f(w_i)\frac{dw_i}{dt}+\frac{w_it}{h}f(w_i)\frac{dw_i}{dt}+\int_{w_{\min}}^{w_i}\frac{wf(w)dw}{h}$$

$$=\int_{w_{\min}}^{w_i}\frac{wf(w)}{h}dw\,. \tag{8}$$

Let $Q(w)$ denote the total weight of particle having a settling velocity greater than w. Then, from Equation (7) and (8), it can be seen that

$$q(t)=Q\left(\frac{h}{t}\right)+t\frac{dq(t)}{dt} \tag{9 a}$$

and letting $w=h/t$ gives

$$Q(w)=q(t)-t\frac{dq(t)}{dt} \tag{9 b}$$

$$f(w)=-\frac{dQ(w)}{dw}=-\frac{dQ(w)}{dt}\cdot\frac{dt}{dw}$$

$$=\frac{h}{w^2}\left[\frac{dq(t)}{dt}-\frac{dq(t)}{dt}-t\frac{d^2q(t)}{dt^2}\right]=-\frac{t^2}{w}\frac{d^2q(t)}{dt^2} \tag{10}$$

where $q(t)$: the cumulative frequency distribution of particle settling velocity.

Therefore, with the use of a suitable balance, it is possible to determine the settling velocity distribution as a function of the cumulative function $q(t)$.

But commonly the function $q(t)$ is not known strictly, so that it is necessary to perform stepwise numerical differentiation using the accumulation curve which is given at n points at equal intervals t as shown in Figure 4.

Equation (9b) may be modified as follows for the purpose of stepwise calculation.

$$Q_{i-1} = G_i - i\Delta t \cdot \frac{G_i - G_{i-1}}{\Delta t} = (1-i)G_i + G_{i-1} . \tag{11}$$

As shown in Figure 4 the ordinate of the frequency distribution diagram is divided into n sections of width Δw between $w_{\min}$ and $w_{\max}$.

$$\Delta w = \frac{w_{\max} - w_{\min}}{n} \tag{12}$$

The weight of particles belonging to each increment is calculated according to the following equations:

$$\left.\begin{aligned}
F_n &= G_n - \left[(1-n)G_n + nG_{n-1}\right] = n(G_n - G_{n-1}) \\
F_{n-1} &= G_n - \left[(2-n)G_{n-1} + (n-1)G_{n-1}\right] - F_n \\
F_{n-2} &= G_n - \left[(3-n)G_{n-1} + (n-2)G_{n-1}\right] - (F_n + F_{n-1}) \\
F_i &= G_n - \left[(1-i)G_i + iG_{i-1}\right] - \sum_{i=n}^{i+1} F_i \\
F_1 &= G_n - \sum_{i=n}^{0} F_i
\end{aligned}\right\} \tag{13}$$

where F_i = the weight of the particles having the settling velocity $W_i = h/1/2 \cdot (t_i + t_{i-1})$.

It is possible to determine the frequency distribution of the settling velocity by a graphical method based on the same idea.

III. Settling Property Measurement of Flocculent Particles[8]

When two or more suspended particles collide, join and behave as a single particle thereafter, the phenomenon is called flocculation. Suspensions in which the particles have a tendency to flocculate are called flocculent suspension, and the particles are called flocculent particles.

The particles caused by chemical coagulation, such as alum floc, are typical of the phenomenon. The discussion in the previous section does not apply directly to the description of the settling of flocculent particles such as alum flocs. The flocs are put into the settling cylinder and dispersed uniformly

throughout the whole depth of the cylinder at the beginning of the analysis. Then settling begins. In the course of settling, because of the non-uniformity of the particle size in the suspension, the greater particles, having a higher settling velocity, collide and capture the smaller particles of slower velocity. Thus the frequency distribution of the settling velocity and the total number of the particles in the suspension changes momentarily. Then for the flocculent suspensions it is necessary to devise a new method for describing the settling properties to replace the steady frequency for discrete suspensions.

The quantity of paricles which accumulate on a unit horizontal area at depth h below the surface of suspension in the short period dt can be given as follows:

$$dq = \left[\int_{w_{\min}}^{w_{\max}} w \cdot f(w, h, t)\, dw\right] dt \tag{14}$$

where w: the settling velocity of a particle

f: the instantaneous frequency distribution function of particle settling velocity. That is, $f(w, h, t)dw$ is the weight of particles per unit volume at depth h, at time t, with the velocity in the range w to $w+dw$

$w_{\max}$: the largest settling velocity which can be seen in the suspension

$w_{\min}$: the smallest settling velocity which can be seen in the suspension

In actual cases it is impossible to measure $f(w, h, t)$. By the introduction of the idea of instantaneous mean settling velocity $\bar{w}$ defined by the Equation (15), we have

$$\bar{w} = \frac{\int_{w_{\min}}^{w_{\max}} w \cdot f(w, h, t)\, dw}{\int_{w_{\min}}^{w_{\max}} f(w, h, t)\, dw} \tag{15}$$

Equation (14) can be rewritten as follows:

$$\begin{aligned} dq &= \left[\bar{w}(h, t)\int_{w_{\min}}^{w_{\max}} f(w, h, t)\, dw\right] dt \\ &= \bar{w}(h, t) \cdot C(h, t)\, dt \end{aligned} \tag{16}$$

where $\bar{w}(h, t)$: the instantaneous mean settling velocity of whole particles at time t and depth h

$C(h, t)$: the total weight of whole particles in a unit volume at time t and depth h. This is called instantaneous local concentration of particles

Typical accumulation curves of flocculent and non-flocculent suspensions

are shown in Figure 5. The accumulation curves of non-flocculent suspensions have no transition point, but those of flocculent suspensions are generally S-shaped as in curve 2. The manifestation of such a transition point in accumulation curves for flocculent suspensions can be explained schematically by Figure 6.

For simplicity, consider a suspension which consists of particles of two sizes. The initial settling property of each particle can be seen by the accumulation curve $O-C_1$ for smaller particles and the curve $O-B_1$ for larger particles. Assume that at time A, flocculation happens instantly at every point of the cylinder. Actually the phenomenon occurs continuously in the whole volume of the settling tube. Now some of the larger particles of Group II will collide with some of the smaller particles of Group I. Group I then loses weight ΔC, but the settling velocity of the remaining smaller particles remains the same. Hence, the initial accumulation curve $O-C_1$ is deformed into OA_1C_1. On the other hand Group II increases its total weight from B_0-B_1 to B_0-B_2', gaining group weight ΔC, and increases in mean settling velocity. The increase in settling velocity of Group I decreases the total settling period necessary for this group from B_0 to $(B_0-\Delta t)$. Hence, curve OB_1 should be corrected to OA_2B_2. Then, by the addition of both component curves, the accumulation curve of the whole suspension is shown as $OABC$. It indicates the typical S-shaped curve which differentiates prominently flocculant suspensions from non-flocculent suspensions.

As the preceding discussion indicates the settling phenomena of flocculent suspensions are too complex to be treated by theoretical means and even by complete statistical treatment of experimental data, so that it is necessary to devise other convenient and practical methods of describing the settling properties.

For practical purposes, time accumulation curves are often compared in terms of the items (i) $t(25\%)$, $t(50\%)$, $t(75\%)$, $t(95\%)$ and so on, and (ii) t(trans.) which are defined as follows:

(i) Each index $t(25\%)$, $t(50\%)$, $t(75\%)$ and $t(95\%)$ means the time required from the beginning of the settling to the settling of 25%, 50%, 75% and 95% respectively of the suspended particles on the accumulation pan, and (ii) t(trans.) is the time when the time-accumulation curve $q(t)$ has its transition point. It also shows the time when the maximum value of $dq(t)/at$ occurs. It is possible to determine the point of the maximum sludge deposition in an ideal basin in accordance with following relations:

$$X(S\text{-max}) = t(\text{trans.}) \times U_0 \tag{17}$$

where $X(S\text{-max})$: the distance from the inlet of the ideal settling basin to the point where maximum depth of the settled sludge can be seen.

U_0: the mean displacement velocity of the ideal basin.

Curves of $\bar{w}(h, t)$ are also useful in studying settling properties of flocculent suspensions. Such curves can be determined experimentally by the following method. The concentration $c(t, h)$ adjacent to the pan can be obtained, for example, by continuous measurement of turbidity. Then from the accumulation curve and Equation (16b) it is possible to calculate the instantaneous local mean settling velocity.

$$\bar{w}(h, t) = \frac{dq(t)}{dt} \cdot \frac{1}{C(h, t)} \tag{16 b}$$

The analytical procedure which the author devised and used for these purposes in this investigation will be described in the following sections.

IV. Balance Analysis of Suspensions

As stated in the previous section, it is necessary to weigh continuously the particles which are accumulating on the pan of the settling tube with sufficient accuracy to obtain a useful time-accumulation curve. The first apparatus which the author devised and used for the investigations is a balance suspension analyzer.

(a) **Apparatus.** The apparatus is shown in Figure 7 and has been used only in the laboratory for fundamental experiments which necessitate extraordinarily high accuracy of measurement.

The apparatus consists of three parts; a settling tank, an auto-balance, and a mechanical agitator. The details of each part are described in the following:

The settling tank is a brass circular tube having a diameter of 25 cm and a total depth of 80 cm. The upper 70 cm of the tube is cylindrical and the lower 10 cm is conical in form. Glass inspection windows of 5 cm width are placed on both sides of the tube. The standard volume of the test suspension is 25.0 liters.

The auto-balance part plays the most important role in this analyzer. This part consits of a sensitive balance, capable of continuous weighing and a pan hanging from the balance in the suspension. The pan should have a diameter and weight which are in harmony with the sensitivity and capacity of the auto-balance. For this purpose an auto-balance which has a sensitivity of 0.1 mg to 0.5 mg and a maximum capacity of 50 g to 200 g is suitable. A constant

load balance, SHIMAZU Type-L-2, having a 0.1 mg sensitivity and a 200 g maximum capacity was used in the author's investigations. Variation of weight in the range of 0 mg to 100 mg is indicated directly by the auto-vernier of the balance. Hence, without touching any part of the balance, one can measure the accumulated weight on the pan up to 100 mg if one sets the auto-vernier reading at zero at the beginning of the settling analysis. The high weighing capacity, available up to 200 g, makes it possible to use a rigid and stout accumulation pan and hanger. Thus, the stability of the accumulation pan in the water can be increased markedly.

To hang the accumulation pan in the suspension, a phosphor bronz hanger, such as shown in Figure 8 was used instead of the original weighing pan. An unfavorable moment of the hanger at point A which would be caused by the assymmetry of the hanger is eliminated by attaching a counterweight at point F. The stability of the hanger is increased by a horizontal bar-balance of length 23 cm with lead weights on both ends of the bar.

The accumulation pan is made of plastic resin and has a diameter of 65 mm and a depth of 15 mm. It is suspended from the hanger mentioned above by an amilan (poly-amid resin) cord at a depth of 45 cm from the surface of the suspension. After the floc is formed by the agitation, the pan sinks in the suspension immediately and the measurement of accumulation weight begins. Absorption of water by the cord influences the weighing, so the cord should be saturated with water before the experiment begins by soaking for about 12 hours. Since even minute air bubbles adhering to the cord may also cause erronious results special care should be given to elliminate bubbles.

The mechanical agitator is a vertical rotating shaft with blades. It is driven by a 1/8 Hp electric induction motor. The speed of agitation can be varied by multiple-step reduction gears and pulleys. In a high speed range—450, 300, 150, 75 and 50 rpm and in a low speed range—30, 20, 10, 5 and 3 rpm are possible. The details of the agitator blades are shown in Figure 7 for standard experiments, but the arrangement can be modified easily. Because it is impossible to pour the flocs into the settling analyzer without any breaking prior to the tests, it is customary in the laboratory to prepare the floc in the analyzer.

(b) **Method.** The author has used the following methods and procedures in his investigations.

At first, 25 liters of tap water are placed in the settling tank, and turbid particles such as kaolinite and other clays are poured into the vessel in dry state or concentrated suspension and mixed thoroughly to make a homogeneous suspension. When natural suspensions are used this is not necessary.

Following this procedure, suitable types and amounts of coagulants and coagulant aids are added to the raw suspension in the appropriate order. Then flash mixing begins with the highest speed (300 rpm) for 2 to 4 min. Following the flash mixing a series of gentle agitations are performed to build up so-called good flocs. Then the flocculation proceeds.

As soon as possible after the floc building agitation, the weighing pan is hung in the suspension at a predetermined level and the weighing begins. The initial weight of the pan, when no floc accumulation can be seen, is simply called "initial weight" in this paper. Meanwhile the initial turbidity of the suspension is measured by a photo-turbidimeter. The floc should be broken completely before this measurement. From the beginning, the weight of the accumulation pan is measured every thirty seconds. When no noticeable change of accumulated weight occurs, the weight reading is stopped. The accumulated weight at this instant is called the total accumulation weight or simply "total weight". Thereafter, as soon as possible, the final turbidity distribution of the suspension is measured by siphoning out samples of the suspension at every 10 cm from the surface of the suspension, then the samples are shaken vigorously to break them into the original particle size, and these are measured with the photo-turbidimeter.

The next step is to draw the time-accumulation curve (cumulative settling curve) of the suspension from the results obtained above, according to the manner described in section three.

In many cases it is more convenient to draw the accumulation curve of a set of experiments in percentile fraction form. To do this, it is necessary to know the ultimate weight of accumulation. Practically, the measurement period cannot be prolonged sufficiently to reach the final state and so ordinarily there exists a slight difference between the total weight measured and the real ultimate weight. Therefore, it is necessary to estimate the ultimate weight of the experiment.

The procedure for the calculation of the ultimate weight is as follows. (Refer Figure 9).

(i) Divide the volume of the settling tube above the accumulation pan into several fractions. The fractions have volumes V_1, V_2, V_3 and so on. It is convenient to coutrol the uppermost fraction at half size of the others, because the turbidity profile curve changes abruptly near the surface of the suspension.

(ii) Calculate the initial total turbidity.

$$\text{Initial total turbidity} = T_A \cdot V \tag{18}$$

where T_A: the initial turbidity of suspension. Assuming uniform distribution throughout the whole depth.
V: the volume of the settling tube above the accumulation pan.

(iii) Calculate the final total turbidity.

$$\text{Final total turbidity} = \sum_{i=1}^{n} T_{B_i} \cdot V_i \tag{19}$$

where T_{B_i}: the turbidity which remains at depth h (a representative point of fractional volume V_i) at the final stage of the settling analysis.
V_i: the i-th fractional volume.

(iv) Then the total turbidity removed is $(VT_A - \sum V_i T_{B_i})$. This total turbidity removed is another expression of the total accumulation weight. Hence, it is possible to find a numerical conversion factor which connects the two expressions.

$$K_t = \frac{G(\text{total})}{(a/A)(T_A V - \sum T_{B_i} V_i)} \tag{20}$$

where K_t: the turbidity-accumulation weight conversion factor of the suspension.
$G(\text{total})$: the total accumulation weight on the pan.
a: the horizontal area of the accumulation pan.
A: the horizontal sectional area of the settling tube.

(v) The difference between the total weight and the ultimate weight can be calculated as follows:

$$G(\text{ult.}) - G(\text{total}) = K_t(a/A)(\sum T_{B_i} V_i - VT_c) \tag{21}$$

where $G(\text{ult.})$: the ultimate accumulation weight.
T_c: the turbidity composed of very minute particles such as colloids which do not settle by gravitational force. For convenience sake, in the author's investigations, the final turbidity of the experiment at the surface of the suspension is taken as T_c.

(vi) Hence, a percentile accumulation at time t can be calculated by Equation (22).

$$G(t)\% = \frac{G(t)}{G(\text{total}) + K_t(a/A)(\sum T_{B_i} V_i - T_c V)} \times 100 \tag{22}$$

where $G(t)\%$: the percentile accumulated fraction of flocs at time t on the

TABLE 1.

Raw Water (Artificially turbid)	Turbidity	500 ppm of kaolinite
	Water Temperature	10.3°C
	pH	7.1
	Alkalinity	
Dosage	Alum	15 ppm
	Sodium Alginate	0.2 ppm
Agitation	Rapid Mixing	300 rpm 4 min
	Slow Agitation	50 rpm 14 min
Pan	Depth from surface	45 cm
	Diameter	6.5 cm

TABLE 2.

(I) Time of Settling	(II) Reading of Balance	(III) Weight of Accumulation	(N) Per Cent Weight of Accumulation
0* min	7.2110 g	0 mg	0 %
5	7.2148	3.8	7.4
10	7.2203	9.3	18.0
15	7.2266	15.6	30.2
20	7.2328	21.8	42.2
25	7.2386	27.6	53.4
30	7.2431	32.1	62.1
35	7.2472	36.2	70.0
40	7.2492	38.2	73.9
45	7.2517	40.7	78.7
50	7.2533	42.3	81.8
55	7.2550	44.0	85.1
60	7.2556	44.6	86.3
65	7.2562	45.2	87.4
70	7.2568	45.8	88.6
75	7.2570	46.0	89.0
80	7.2574	46.4	89.7
85	7.2580	47.0	90.9
90	7.2585	47.5	91.9
95	7.2588	47.8	92.5
100	7.2589	47.9	92.7
105	7.2590	48.0	92.8
110	7.2592	48.2	93.2
115	7.2593	48.3	93.4
120	7.2594	48.4**	93.5

* Initial Reading ** Total Weight

plane at depth h, when ultimate weight is taken as 100%.

$G(t)$: the accumulated weight at time t on the pan at depth h.

(c) **Example.** An example of this measurement will be shown here. The conditions of the run are given in Table 1.

The readings of accumulated weight are listed in column II of Table 2 in 5 min. intervals. (The original data were taken every 30 sec.). Column III shows the weight of accumulation on the pan.

To calculate the percentile accumulation on the pan, an estimate of the ultimate accumulation weight must be made. The turbidity data of the run are given in Table 3.

TABLE 3.

	Turbidity		Volume of the Fraction		Total Turbidity
Initial	T_A: 55.0		V: 1/4 D^2h cm³ 22077	T_iV_i	T_A: ** 1214235 °cm³
	h=0 cm T_{B_i}: 0.4*		V_i: 2453	$T_{B_i}V_i$: 981.2	
Final	10	1.6	4906	7849.6	
	20	2.8	4906	13736.8	$T_{V_i}V_i$=
	30	4.6	4906	22567.6	80458.4
	40	7.2	4906	35323.2	

* $T_a \fallingdotseq 0.4$ ** °: Turbidity unit

The following values are substituted into Equation (16): $G(\text{total})=48.4$ mg, $a=33.17$ cm², $A=490.6$ cm², $T_AV=1214235$ °cm³ and $T_{B_i}V_i=80458.4$ °cm³. The value K_t is calculated as follows:

$$K_t = \frac{G(\text{total})}{(a/A)(T_AV-\sum T_{B_i}V_i)} \cdot \frac{\text{mg}}{\text{°cm}^3} = 0.000637 \text{ mg/°cm}.$$

Then the ultimate accumulated weight is, from Equation (21), calculated as follows,

$$G(\text{ult.})\,\text{mg} = 48.4 + 0.000637\left(\frac{33.17}{490.6}\right)(86458.4 - 0.4 \times 22077)$$
$$= 51.7 \text{ mg}.$$

Hece, the percentile weights of accumulation are calculated as in Column IV of Table 2 and shown as in Figure 10.

V. Pipette Analysis of Suspensions

As was described in the previous section, the balance analysis method is excellent for settling property determination in the laboratory but it is too delicate to apply in practical plant experiments or field measurements. Accordingly, it is necessary to devise another method by which one can oblain sufficiently accurate data without difficult handling. For this purpose the author devised a kind of multiple depth settling analyzer which is based upon McLaughlin's method[9].

Ordinarily the pipette method requires many samples to obtain highly accurate measurements in the case of dilute solution. However, weak metal coagulant floc, for example alum floc, can be easily broken into minute, nearly uniform original particles by vigorous shaking. There exists a linear relationship between the turbidity of a suspension of the finely broken particles and the weight concentration of the particles. Then, the turbidity measurements can replace troublesome and time-consuming measurements of weight concentration. With this convenient method of turbidity measurement, the pipette method can be applied to the practical determination of the settling properties of flocs.

(a) **Apparatus.** One of the multiple-depth settling analyzers which the author devised and used is shown in Figure 11.

The settling tube is a hard vinyl chloride circular tube having an inner diameter of 12.5 cm and a total length of 350 cm with a flange at the top end of the tube. Four soft-vinyl sampling tubes of 4 mm inner diameter are inserted into holes bored in the side of the settling tube at points 40, 80, 160 and 240 cm from the bottom. The end of the sampling tubes project 3 cm from the inner wall into the settling tube to avoid unfavorable wall effects on the settling property measurements. To show the depth of sampling clear marks are spaced every 5 cm from the bottom. A 3-way glass stopcock is fixed at the opposing end of each of these sampling tubes. Through one way of the stopcock, samples are withdrawn into sampling jars by a 50 cc syringe. To achieve a state of quiescent settling in the tube and to avoid the exchange of water between the inside of the settling tube and the outer water body, a rubber ball is fixed at the bottom end of the tube, and inflated with air through a vinyl tube from the surface of the water by a hand operated pump.

Several kinds of apparatus were used to fix the settling tube at any desired position. One which was used in a rectangular settling basin is shown in Figure 12.

(b) **Method.** The sampling is done by the following procedure. The

settling tube is inserted into the suspension at a desired place without expanding the rubber ball. The suspension to be measured comes into the tube. Then the rubber ball stopper is expanded with the air and the suspension is allowed to settle quiescently in the vertical cylinder. As soon as possible after the bottom is closed, the first samples are taken through the sampling tubes. The first 50 cc of water withdrawn by each syringe is the water resting in the sampling tube and must be discarded. The next sample is usable. During the settling a series of samples are taken at each of the four depths and samples are analyzed for turbidity by photo-turbidimeter after being shaken vigorously. Ordinarily sampling is done every 30 min. or every 1 hour for 6 hours from the beginning of the settling. After 12 or 24 hours, the final sampling is done. During the measurement the water temperature remains almost constant because the settling tube is fixed in a large water body having nearly a constant temperature.

The procedure for analyzing the samples for the settling property identification is as follows:

For the purpose of explanation, consider a hypothetical settling cylinder such as shown in Figure 13–a. The turbidity of the samples withdrawn at the beginning of the test simultaneously, at many depths are plotted as curve $t=0$ in the diagram of Figure 13–b. At time $t=T_i$, another set of samples are withdrawn and their turbidities are plotted as curve $t=T_i$. The curve $t=T_2$ and $t=T_3$ represent similar measurements at times T_2 and T_3, respectively. These curves are called the concentration-profile curves of the suspension at time t and the whole family of curves is called the concentration profile diagram.

This phenomenon can be treated as a one-dimensional non-steady fluid flow. The one-dimensional continuity equation of suspended particles is written as follows;

$$\frac{\partial c(h,\ t)}{\partial t}+\frac{\partial[\bar{w}(h,\ t)\cdot c(h,\ t)]}{\partial h}=0\ . \tag{23}$$

Integrating this equation from $h=0$ to $h=h_i$ with respect to depth h gives,

$$\bar{w}(h,\ i,\ t)\cdot c(h,\ t)=-\int_0^{h_i}\frac{\partial c(h,\ t)}{\partial t}dh=-\frac{\partial}{\partial t}\int_0^{h_i}c(h,\ t)dh\ . \tag{24}$$

The time accumulation curve and local mean settling velocity at the depth h_i is calculated by the following procedure which is based upon Equation (24).

The total floc which passes through the plane h_i of the cylinder between time $t=T_i$ and $t=T_{i+1}$, the shaded area in the Figure 13–b, is calculated as

TABLE 4.

I h_i (cm)	II t (min)	III t (min)	IV $C(h, t)$ (Turb. unit)	V $G(T_i)$ *	VI $\sum G(T_i)$ *	VII $G(T)$ (%)	VIII $\bar{w}$ (cm/min)
50	30	30	16.0	92.5	92.5	12.6	0.193
	60	30	11.5	250.0	342.5	46.6	0.725
	120	60	8.0	182.5	525.0	71.4	0.380
	180	60	7.5	45.0	570.0	77.6	0.100
	240	60	6.0	47.5	617.5	84.0	0.132
	300	60	5.5	35.0	652.5	88.8	0.106
	360	60	5.0	35.0	687.5	93.5	0.117
	1440	1080	4.0	47.5	735.0	100.0	—
100	30	30	16.0	177.5	177.5	12.4	0.370
	60	30	13.0	422.5	600.0	41.9	1.083
	120	60	10.0	365.0	965.0	67.4	0.608
	180	60	9.0	67.5	1032.5	72.1	0.125
	240	60	7.0	130.0	1162.5	81.2	0.301
	300	60	6.0	90.0	1252.5	87.4	0.250
	360	60	5.0	75.0	1327.5	92.7	0.250
	1440	1080	4.0	105.0	1432.5	100.0	—
150	30	30	16.5	247.5	247.5	11.7	0.500
	60	30	14.5	547.5	795.0	37.7	1.259
	120	60	12.0	520.0	1315.0	62.3	0.722
	180	60	9.5	140.0	1455.0	69.0	0.246
	240	60	8.0	217.5	1672.5	79.3	0.453
	300	60	6.0	167.5	1840.0	87.2	0.465
	360	60	5.0	125.0	1965.0	93.1	0.417
	1440	1080	4.5	145.0	2110.0	100.0	—
200	30	30	17.5	307.5	307.5	11.1	0.586
	60	30	15.5	630.0	937.5	33.8	1.355
	120	60	13.5	632.5	1570.0	56.6	0.781
	180	60	12.0	267.5	1837.0	66.2	0.372
	240	60	10.0	287.5	2125.0	76.2	0.479
	300	60	9.5	272.5	2397.5	86.4	0.478
	360	60	7.5	170.0	2567.5	92.5	0.378
	1440	1080	5.5	207.5	2775.0	100.0	—

* the reading of planimeter (the areas of figures)

follows;

$$\int_0^{h_i}\left[C(h,\ T_{i-1})-C(h,\ T_i)\right]dh=\Delta G(T_{i_{h=h_i}}) \tag{25}$$

where $\Delta G(T_i)$: the total floc which passes through the h_i plane in time $\Delta t_i=(T_i-T_{i-1})$.

From this Equation (25) the accumulated weight of the time T_i on the h_i plane is calculated as follows.

$$G(T_{i_{h=h_i}})=\sum_{j=1}^{i}\int_0^{h_i}\left[C(h,\ T_{j-1})-(h,\ T_j)\right]dh\ . \tag{26}$$

The time accumulation curve at depth h_i is obtained by plotting the values of $G(T_i)$ with respect to the increasing time of sedimentation T_i. As the area f'b'bi is $G(T_i)$, f'b'b2 is $G(T_2)$ and f'b'bi is $G(T_i)$, this plotting is easily done by measuring these areas graphically and plotting against the corresponding T_i.

The local mean settling velocity is calculated also graphically according to the following equation;

$$\bar{w}(h_i,\ T_i)=\frac{\Delta G(T_i)_{h=hi}}{\Delta t_i\cdot C(h_i,\ T_i)}\ . \tag{27}$$

(c) **Example.** An example of this measurement is shown here. The turbidity of each sample taken at a given time and depth is listed in Column II of Table 4. From these data a concentration profile diagram is drawn as in Figure 14. The equantities $G_{i_{h=h_i}}$ calculated graphically by the use of a planimeter, are shown in Column V of Table 4. In Column II are the summations of the values of Column V from the beginning of settling to a given time, which is the value of accumulation at that instant. The percentile values of accumulation, which are the ratio of the accumulation values to the 24 hours value, are shown in Column VII. The quantities $\bar{w}(h_i,\ t_i)$ calculated according to the Equation (20) are shown in Column VIII. The calculated time-accumulation curves and the instantaneous settling velocity variation with time are plotted as in Figures 15 and 16 respectively.

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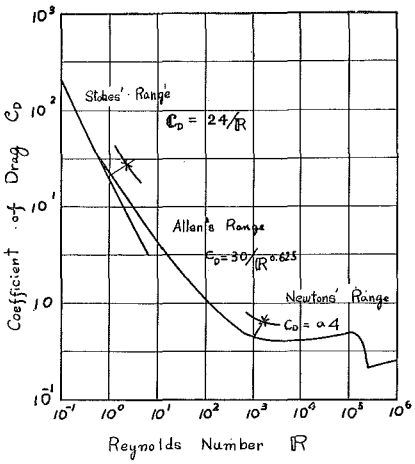


Fig. 1.

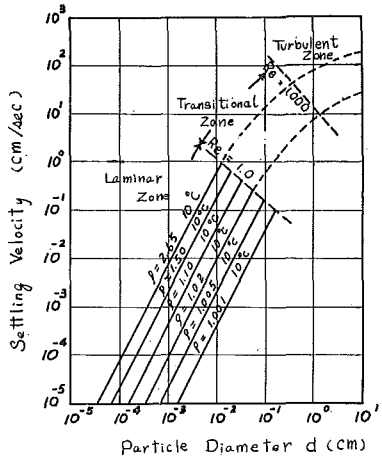


Fig. 2.

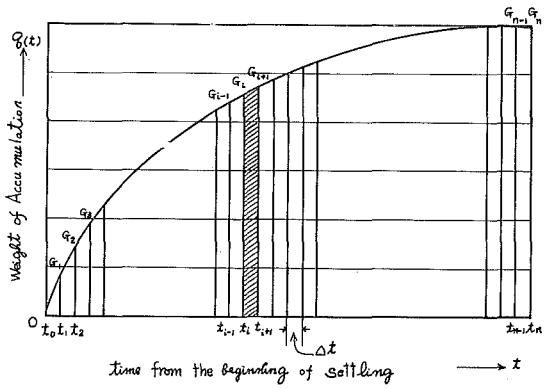


Fig. 3.

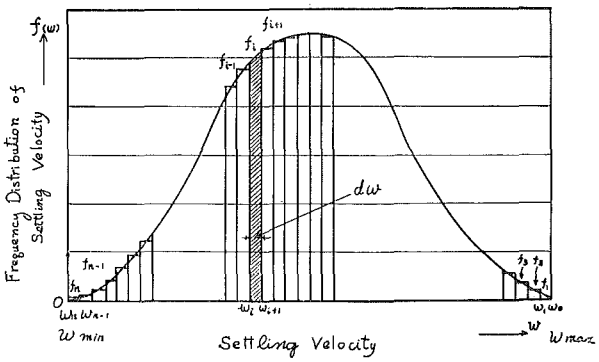


Fig. 4.

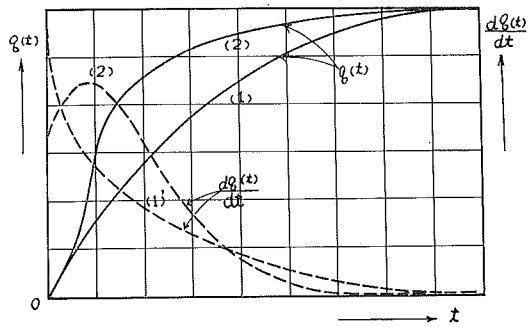


Fig. 5.

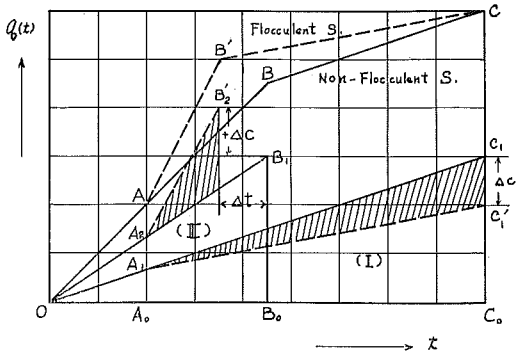


Fig. 6.

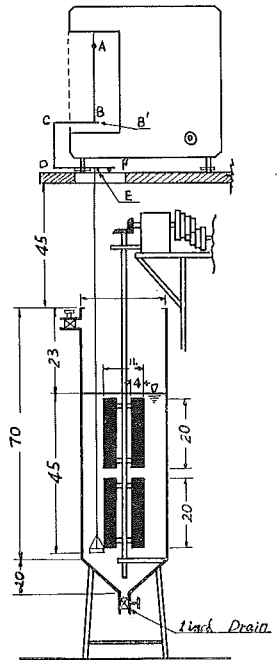
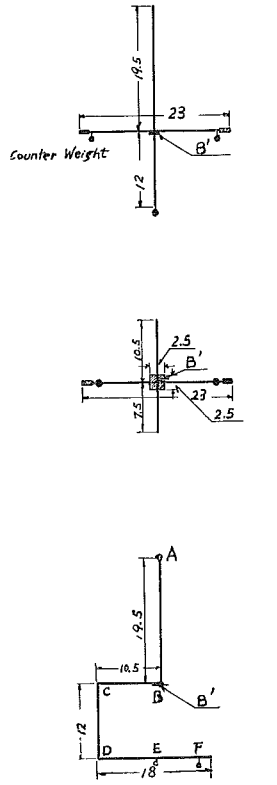


Fig. 7.



(Unit: cm)

Fig. 8.

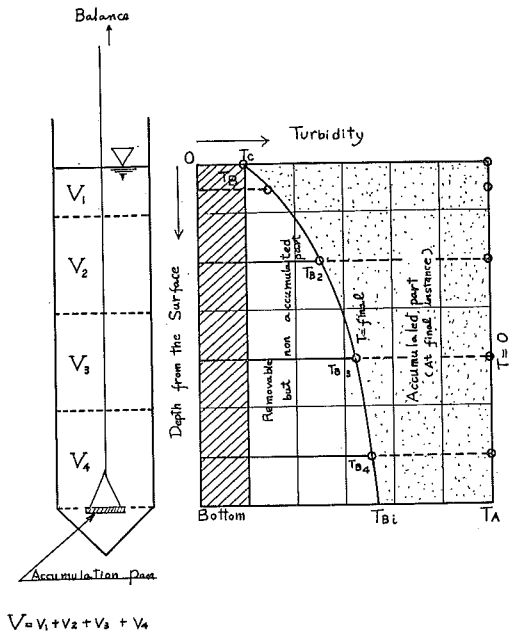

Balance
V1
V2
V3
V4
Accumulation pan
V = V1 + V2 + V3 + V4
Turbidity
0
Tc
TB
TB2
TB3
TB4
T=final
Depth from the Surface
Removable but non accumulated part
Accumulated part (At final instance)
T=0
Bottom
TBi
TA

Fig. 9.

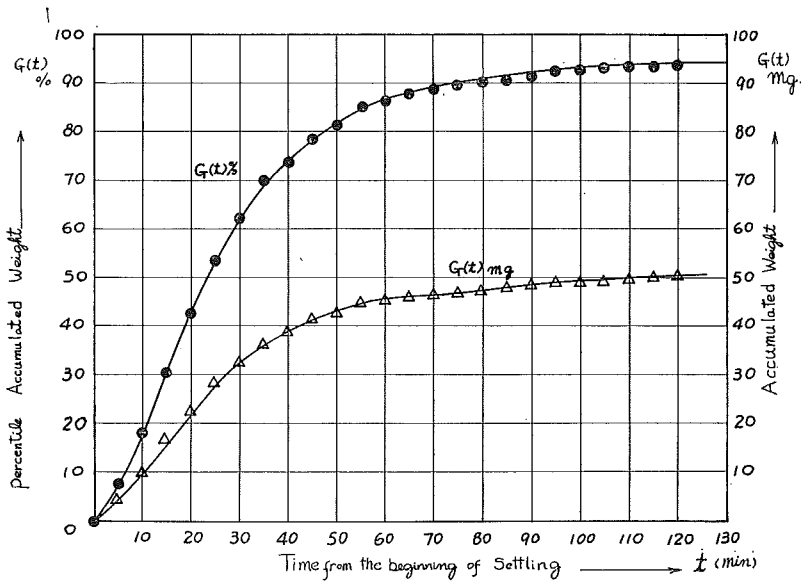

G(t) % 90
G(t) mg.
G(t)%
G(t) mg
Percentile Accumulated Weight
Accumulated Weight
Time from the beginning of Settling
t (min)
0
10
20
30
40
50
60
70
80
90
100
110
120
130

Fig. 10.

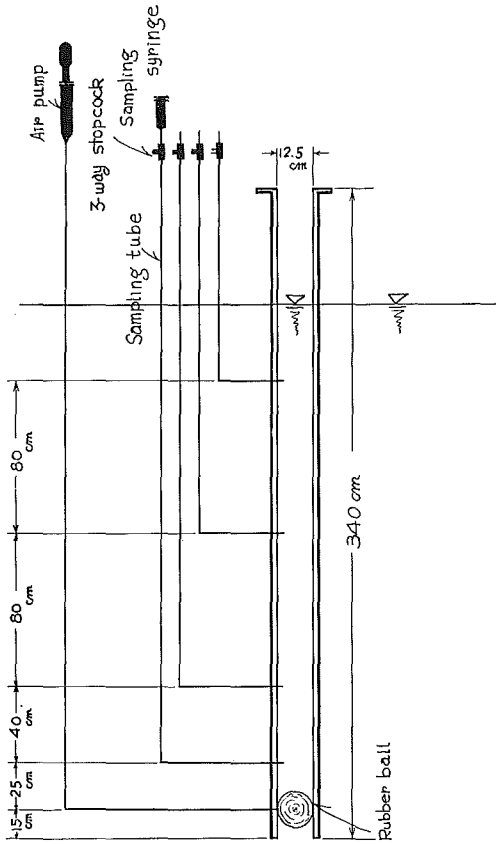

Air pump
3-way stopcock
Sampling syringe
Sampling tube
12.5 cm
80 cm
80 cm
40 cm
25 cm
15 cm
340 cm
Rubber ball

Fig. 11.

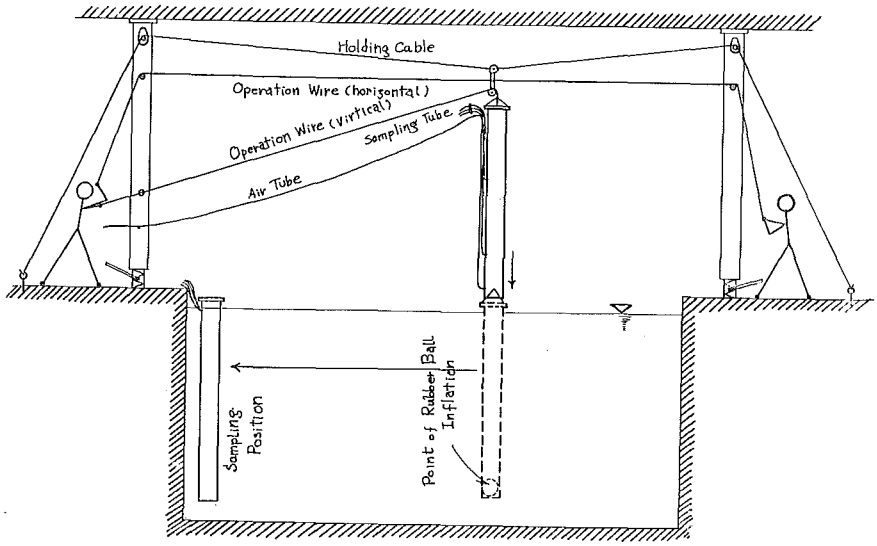

Holding Cable
Operation Wire (horizontal)
Operation Wire (Virtical)
Sampling Tube
Air Tube
Sampling Position
Point of Rubber Ball Inflation

Fig. 12.

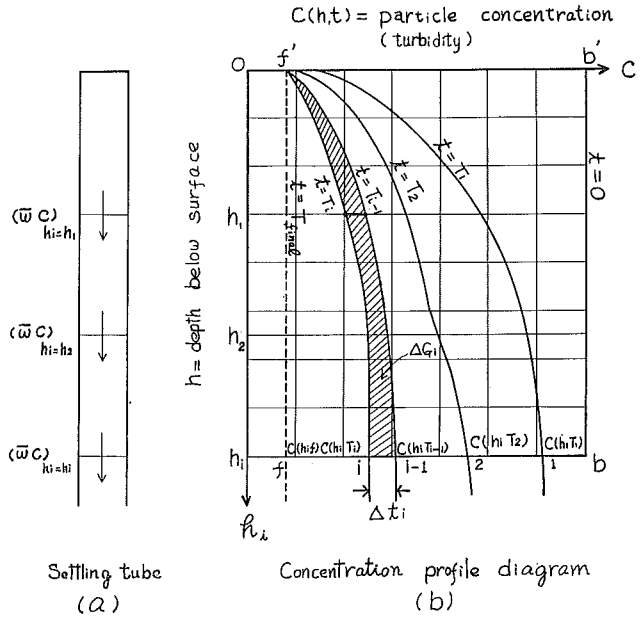

C(h,t) = particle concentration
(turbidity)
h= depth below surface
Settling tube
(a)
Concentration profile diagram
(b)

Fig. 13.

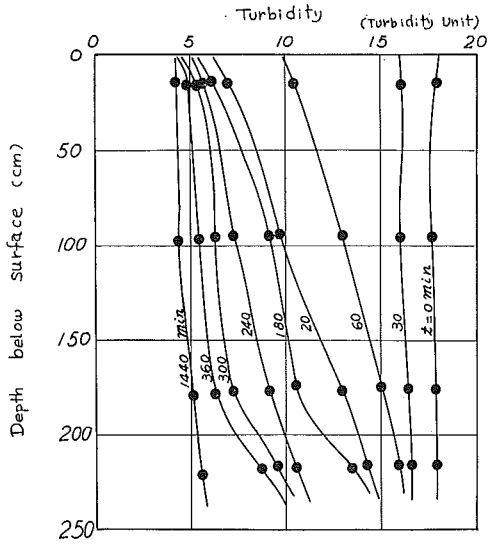

Turbidity
(Turbidity Unit)
Depth below surface (cm)
1440 min
360
300
240
180
120
60
30
t=0 min

Fig. 14.

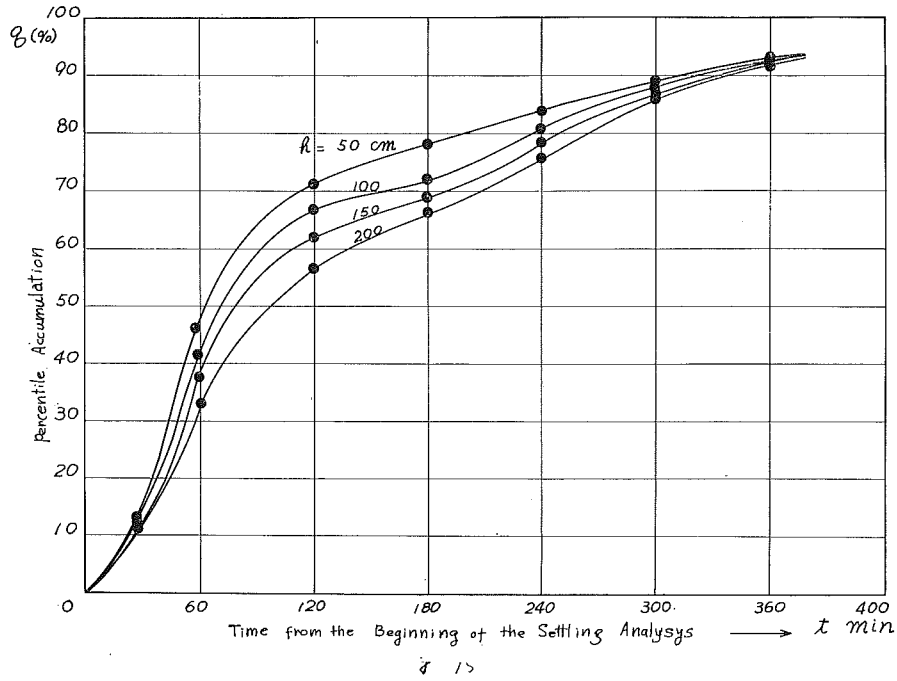

100
q (%)
90
80
70
60
50
40
30
20
10
0
h = 50 cm
100
150
200
Percentile Accumulation
60
120
180
240
300
360
400
Time from the Beginning of the Settling Analysys
t min

Fig. 15.

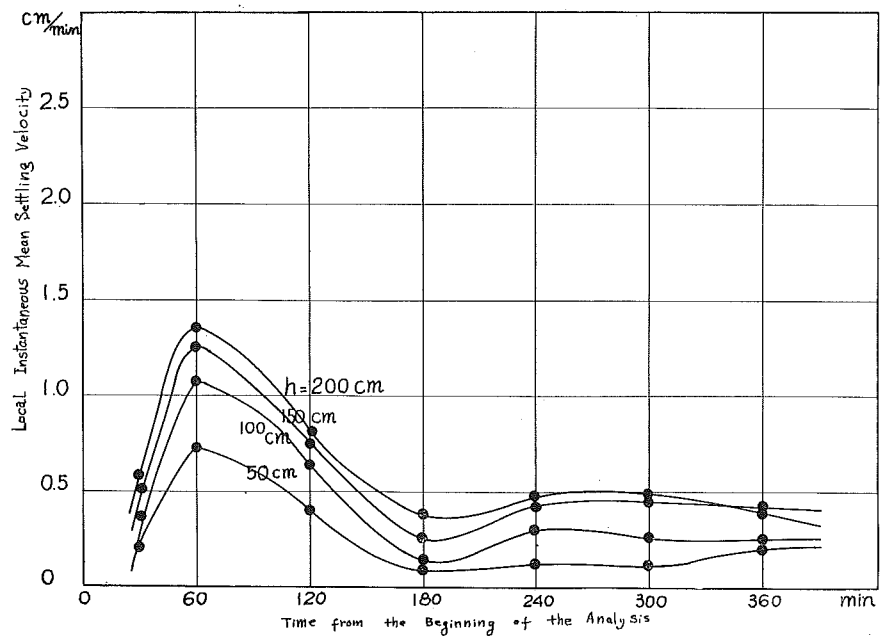

cm/min
2.5
2.0
1.5
1.0
0.5
0
Local Instantaneous Mean Settling Velocity
h=200 cm
150 cm
100 cm
50 cm
0
60
120
180
240
300
360
min
Time from the Beginning of the Analysis

Fig. 16.