Addition and Subtraction on Logarithmic Slide Rule

Y. KUMAZAWA

It has been generally accepted that in regard to the function of logarithmic slide rule, addition and subtraction cannot be performed nor can any operation involving them be done completely (1).

However, the slide rule may be manipulated far over the usual limit of application not only theoretically but also practically.

Fortunately, modern slide rules which have S, ST and T scales for trigonometric function are suitable for this purpose, while the slide rules which have the S, L and T scales on the back of the slide can not be used. This is because it is absolutely indispensable that the angle marked on S scale correspond to the value of sines read on the C or D scale.

Before dealing with the theory, the operational method for the functional addition or subtraction is described first, because simple addition or subtraction, though possible, is not practical on the slide rule.

(A) $\dfrac{1}{x}=\dfrac{1}{a}\pm\dfrac{1}{b}$

The above type of formula is most widely used by opticians, electricians and others and its computation must be performed frequently.

Nomograms or special slide rule, (2) are most commonly used in order to avoid the tedious computation of such a formula.

But, generally, nomograms or special slide rules show a larger relative error and have less accuracy than the standard slide rule.

Hence, it is preferable to use a standard slide rule if the operation can be performed with ease and sufficient rapidity.

Example 1. $\dfrac{1}{x}=\dfrac{1}{12.2}+\dfrac{1}{47}$.

Operation: (Fig. 1)

(I) Set the right-index, i.e., S 90° or T 45° of the slide against 47 on A scale.

(II) Move the runner to 12.2 on A scale, and read the angle on T scale. 27° is read under the hair-line.

(III) Move the runner to 27° on S scale and read the answer on A scale. $x=9.7$ is read.

The exact value of x is 9.69. Hence the relative error is 1 : 969 i.e.,

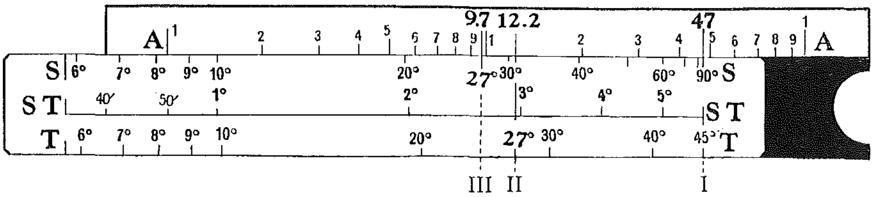

Fig. 1.

0.103%.

Remark: The larger number must be selected for the first setting.

Example 2. $\frac{1}{x}=\frac{1}{9.7}-\frac{1}{12.2}$.

At a glance it seems that the operation is so simple and is a slight modification of the above.

But at the present stage the general operational method must be done differently because the ordinary slide rules do not have a tangent scale over 45°.

Operation:

(I) Set the right-index of the trigonometric scales of the slide against 12.2 on A scale.

(II) Move the runner to 9.7 on A scale and read the angle on S scale. 63° (black) or 27° (red) is read under the hair-line.

(III) Move the slide to set 27 (black) on S scale under the hair-line. The answer is 47 on the A scale against the right-index of the slide.

This operation is performed on A and S scales, bur if the angle on S scale in the operation (II) is less than 45°, it is self-evident that the answer also can be read on A scale after the runner is moved to the same angle on T scale.

(B) $\frac{1}{x^2}=\frac{1}{a^2}\pm\frac{1}{b^2}$

The type of formula, above mentioned is encountered by special scientists or engineers. For instance, the resultant of equivalent orifice or conductance-factor in series in mine ventilation, is expressed by this type of formula.

Example 3. $\frac{1}{x^2}=\frac{1}{51^2}+\frac{1}{63^2}$.

Operation:

(I) Set the right-index of the trigonometric scales of the slide against 63 on D scale.

(II) Move the runner to 51 on D scale and read the angle on T rcale.

39° is read under the hair-line.

(III) Move the runner to 39° on S scale and read the answer on D scale. $x=39.6$ is read.

The exact value of x is 39.64. Hence the relative error is 0.101%.

Example 4. $\frac{1}{x^2}=\frac{1}{39.6^2}-\frac{1}{51^2}$.

Operation:

(I) Set the right-index of the trigonometric scale of the slide against 51 on D scale.

(II) Move the runner to 39.6 on D scale and read the angle on S scale. 51° (black) or 39° (red) is read under the hair-line.

(III) Move the slide to set 39° (black) on S scale under the hair-line. The answer is 63 on D scale against the right-index of the slide.

As above described, formula (A) is computed on A and trigonometric scales, but in formula (B), D scale, instead of A, must be employed. This is the single difference between them.

In Fig. 2, CD is a perpendicular to hypotenuse AB in a right triangle ABC. Then, there is a relation, important but not popular as follows (3):

$$\frac{1}{\overline{CD}^2}=\frac{1}{\overline{AC}^2}+\frac{1}{\overline{BC}^2}.$$

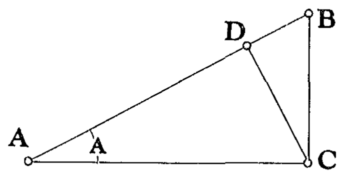

Fig. 2.

Therefore the algebraic relation of formula (B) is transformed into a geometrical relation, which can be solved as a trigonometric problem.

The operation (I) and (II) in Example 3 is a process to compute the angle A and the (III) is a manipulation of the law of sines on the rule.

In Example 4, the operation is a dual performances of the law of sines. If the number to be squared is represented on C or D scale, the value of the corresponding number must be represented on A or B scale.

On account of this nature of the logarithmic scale, for example, the area of circle is read on A or B scale when the value of diameter is set on C or D scale.

In the Operation for Examples 1 or 2, the A scale, instead of D, must be used owing to this reason.

(C) $a+b=c$ (D) $a^2+b^2=c^2$

The above formulae are modified as follows:

$$\frac{1}{\frac{1}{a}}+\frac{1}{\frac{1}{b}}=\frac{1}{\frac{1}{c}} \qquad \frac{1}{\frac{1}{a^2}}+\frac{1}{\frac{1}{b^2}}=\frac{1}{\frac{1}{c^2}}.$$

Hence, if the rules have the AI or inverted A scale and DI or inverted D scale, their operations are the same as in the previous examples, insofar as the trigonometric scales are concerned.

But generally the rules do not have AI and DI scales and so their operation may be done by inverting the slide.

The fundamental principle of addition and subtraction on the rule is divided into two types and the above described procedures all belong to one type of the two.

The other type of operation is based on a simple relation of three sides of a right triangle.

The problem of three sides of a right triangle can be solved for the computation of vectors or complex quantities as a geometrical relation on the rule (4).

But this operation has an algebraic meaning.

This operation must be performed on D and trigonometric scales and if the operation is done on A, instead of D, and trigonometric scales in the same manner, the simple addition or subtraction, i.e., $x=a+b$ may be done on the rule, though it is not practical.

The fundamental procedure of the second type of computation will be explained with the following two examples.

Example 5. $22+12.5=x$.

Operation:

(I) Set the right-index of trigonometric scales of the slide against 22 on A scale.

(II) Move the runner to 12.5 on A scale and read the angle on T scale. 37° is read under the hair-line.

(III) Move the slide to set 37° on S scale under the hair-line. The answer is 34.5 on A scale against the right-index of the slide.

Example 6. $34.5-22=x$.

Operation:

(I) Set the right-index of trigonometric scales of the slide against 34.5 on A scale.

(II) Move the runner to 22 on A scale and read the angle on S scale. 37° (red) is read under the hair-line.

(III) Move the runner to 37 (black) on S scale. The answer is 12.5 on A scale under the hair-line.

Hence, it will be understood that any type of the above mentioned formula may be computed by each of the two types of operation.

From the above description, it is recognized that the trigonometric function is the mediator or a sort of parameter in mathematics to change the idea of computation.

Hence the combination of this mediator and the K or LL scale may be used to solve the other type of addition or subtraction and also it is possible that the A, D and other scales may be employed in one operation, if necessary.

As already stated the answer of addition or subtraction on the rule is read on the logarithmic scale by a three step operation, while in multiplication or division, it is a two step operation. Consequently the addition or subtraction may be directly and wholly computed on the rule, even though combined with multiplication or division.

Example 7. $\frac{1}{F}=(1.55-1)\left(\frac{1}{6.5}+\frac{1}{9.0}\right).$

Solution: By the operation as in Ex. 1 and simple division on A and B scales,

$$\frac{1}{F}=\frac{0.55}{3.77}=\frac{1}{6.85}.$$

$F=6.85$, while, on nomogram (5), $F=6.90$.

References

1) Johnson: The Slide Rule, 1949, p. 1.
2) Mackey: Graphical Solution, 1936, p. 10.
3) Blaess: Die Strömung in Röhren, 1911, p. 36.
4) Bishop: Practical Use Of The Slide Rule, 1947, p. 59.
5) Levins: Nomography, 1948, p. 161.