Stretching of Band Saw Blade by Utilizing Thermal Strain

(Heat Tension of Band Saw Blade)

Osamu DOI*

Abstract

Tensioning is essential for wood cutting band saw blade to increase the capacity of wood working by securing (1) running stability on the wheels[1),2)] and (2) buckling rigidity against feeding force[3),4)]. Usually, tensioning is given by rolling with a pair of rollers in uniform intensity in the direction of length but in variable intensity for the function of width. Consequently the saw blade has variable plastic strains as the function of width[5),6)].

Instead of plastic elongation, local contraction with thermal strain given by heating at comparatively low temperatures (300~450°C) on the line near the front edge must have the same effect on the deformation of the blade. This new method is called "Heat Tensioning"[7)~11)] and has many advantages over "Roll Tensioning", such as (1) uniform tension, (2) short working time, (3) easy correction of overs and shorts of tension, (4) no spoiling in the property of material.

In this paper, the author proposes some theories and characteristics on heat tension obtained by his analyses and experiments[7)~11)].

1. Foreword

The author has been studying theories concerning tensioning of band saw blade (Fig. 1) and characteristics of the band saw machine (Fig. 2).

Band saw blade with tension is deformed convexly in the cross section[1),2)] when it is bent on the wheel surface and contact forces or pressures are concentrated at the front and back edge lines of the blade when it is pulled by two wheels[12),13)]. The tensile stress on the cross section of the blade in the free part is not uniform and concentrates at both edge lines[12)].

These mechanical effects are important and essential for increasing both running stability[1),2)] and buckling rigidity[3),4)] of the blade.

For the above mentioned purposes, roll tensioning and heat tensioning are both useful and the latter has many advantages compared with the former.

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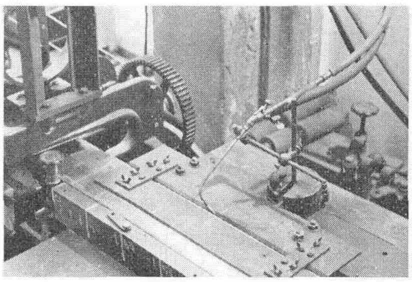


Fig. 1. Roll Stretcher and Heat Tensioning Apparatus

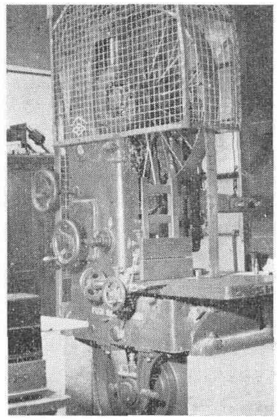


Fig. 2. Typical Wood Cutting Band Saw Machine

2. Outline of Roll Tension

Roll tension is given by a pair of rollers of the same dimensions. Rolling force is usually not more than 1000 kg and the rolled trace is about several millimeters in width (Fig. 3).

Saw-doctors adopt various distributions of plastic strain, but these varieties are not essential. Typical strain distribution is expressed as[6),12)]

$$\varepsilon_x^0 = (4/h^2)\cdot\varepsilon_0\left\{(h/2)^2 - y^2\right\}$$

where h = width of blade, ε_0 = maximum strain at the middle line of blade, y = ordinate of width, measured from the center of blade.

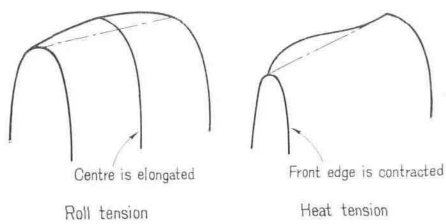


Fig. 3.

The maximum value of ε_0 is theoretically limited by the phenomena named "Jumping" of the blade which occurs when the maximum strain exceeds the critical value $\varepsilon_{0cr} = 8.53(b/h)^2$, where b = thickness of blade[6)].

The value of ε_0 can be calculated by the author's formula[5)] from the deformation curve of cross section which is measured with a tension measuring apparatus (Fig. 18) designed by the author by putting the saw on a stand with constant known curvature. On the other hand, the author measured the strain with destructive method and the two methods gave nearly the same value of $0.03\sim0.06\%$ in practical working[6)].

Futhermore, the following strain which is proportional to the distance from the front edge is added[6),12)]

$$\varepsilon_x^0 = (1/R)\left\{(h/2)+y\right\}$$

so that the back line of the blade has a curvature $(1/R)$, when it is laid on a flat plate. R is called "Back radius of curvature" and in practical use it is about 100～1000 m.

3. Outline of Heat Tension

Continuous heating of the blade along a longitudinal line near the front edge in a restricted state, as the blade is put on a flat plate, produces compressive thermal stress. If the stress exceeds the yield stress in the local position of the saw material at the elevated temperature, plastic strain remains at room temperature. When this blade is bent in the longitudinal direction, the cross section becomes convex (Fig. 3).

The value of residual strain is measured at about $-0.1 \sim -0.2\%$ by destructive and non-destructive methods[9)~12)].

It is important in heat tensioning to select a proper heating temperature. Minimum temperature must be above the temperature level at which the thermal stress exceeds the yield stress of the material. It is practical to set the minimum heating temperature at 300°C and this fact was ascertained by experiments. The maximum temperature might be restricted with tempering temperature from the view point of the cutting tool. Band saw material is made of tool steel with high carbon or alloy steel (mainly of Ni-steel) and is quenched from about 800～850°C and tempered at 400～500°C in order to maintain proper hardness and toughness.

From these two limits, the range of heating temperature should be set between 300～500°C, and for practical purposes it is sufficient to obtain proper intensity of heat tensioning to adopt the heating temperature of less than 450°C.

This heating temperature is controlled by the capacity of a torch, pressure of acetylene and oxygen gas and feeding velocity of blade.

The author adopted a neutral flame with a tip of 25 l/hr as the standard, varying feeding velocity in a range of about 1.0～2.2 m/min and adjusting oxygen gas pressure between 0.4～1.2 kg/cm^2.

4. Characteristics of Heat Tension

On the results of theories and experiments, characteristics of heat tension are compared with roll tension according to the following points[7)~11)].

(1) Working of heat tension has such merits as follows: (a) large sim-

plicity, (b) without special excellence, (c) large economy of working time, (d) uniform tension, (e) large convenience to work perforated blade.

(2) Such workings as follows are very difficult by heat tensioning. (a) local corrections of tension, (b) leveling of jointed part. So that in heat tensioning, it would be practical to adopt a band saw with (i) uniform material, (ii) uniform dimension, (iii) uniform shape.

(3) As compressive thermal strain is concentrated within a heated part of several millimeters width, it is easy to correct overs and shorts of tension. If tension is over, only a slight uniform rolling on the heated part is sufficient, and if it is short, an additional heating is necessary. It is possible to eliminate the strain of heat tension with rolling thereafter.

(4) Tension curve of blade varies according to thickness, width, position of heating and amount of strain and has an original shape. Degree of back curvature is also proportional to the intensity of tension. It is impossible to obtain various combinations of tension. If the blade is heat tensioned at the tooth filet line, a usual blade has its maximum point in its cross section curve at $1/4 \sim 1/3$ of width, hence the tension near the back line becomes poor.

(5) Tension curve and intensity of back curvature can be varied freely and easily by an addition of roll tensioning. Poor tension near the back line also can be increased by additional roll tensioning.

(6) To correct and increase the intensity of tension near the back line, secondary heat tensioning on the back edge is useful. This new method is named by the author "Double Heat Tensioning" and has considerable merits for perforated blades.

(7) Though in roll tension, the maximum strain is about 0.05%[11], thermal compressive strain reaches $-0.1 \sim -0.2\%$[11]. On account of the large amount of strain, strain decrease of the blade by repeated loading when used in sawing is slower than that of roll tension.

(8) There are two different cases, one is tooth filet line heating (edge heating) and the other is inside line heating (about $5 \sim 10$ mm inside the filet line).

Edge heating has such advantages as follows: (a) standard type of tension curve, (b) no effect of temperature difference between front and back surfaces, (c) least strain for obtaining a sufficicient degree of tension, (d) least heat loss and heat input. But intensity of tension decreases in proportion to the diminishing of strained part by regrinding.

In inside heating, on the contrary, a decrease of width by regrinding up to the strained part makes tension stronger.

(9) By heating under tempering temperature for a short time, there are

no adverse effects on properties and hardness of material.

5. Calculations of Deformations

5.1 Edge Heating

(a) Tension

To simplify the problem, the following assumptions will be made. (1) Neglect teeth of blade, (2) consider a thin plate of infinite length, (3) blade is fixed flat, and is heated and cooled sudddenly at one edge, (4) width of heated part h_0 is very narrow compared with whole width h, (5) residual part $(h-h_0)$ is kept at room temperature.

Thermal stress at heated part h_0 exceeds yield stress at this temperature; plastic deformation occurs and residual strain ε_x^0 appears in longitudinal direction when it returns to room temperature.

$$\varepsilon_x^0 = -\varepsilon_0 \qquad (\varepsilon_0 > 0) \tag{1}$$

In order to obtain the tension curve of heat tensioned blade, perpendicular deformation w of a thin, long cylinder which has an initial strain ε_x^0 in one edge part $y=0\sim h_0$ is considered as a problem of large deflection of thin plate or shell (Fig. 4).

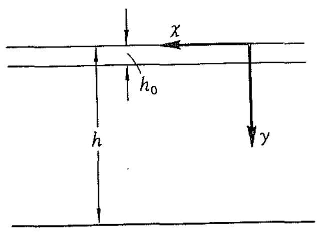


Fig. 4.

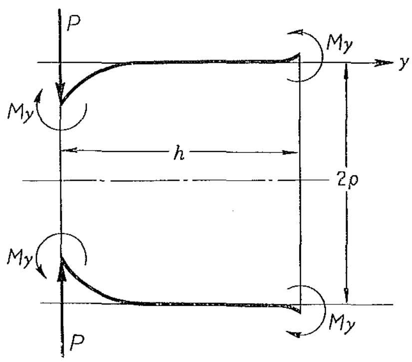


Fig. 5.

Deformation can be denoted as

$$w = x^2/2\rho + \varphi(y) \tag{2}$$

where x is the axis of length, y is the axis of width, ρ is the radius of curvature of cylinder, and the origin is taken at one edge (Fig. 5).

Tension curve of cross section is expressed by $\varphi(y)$, which should be solved. Since the exact solution is complex and has a poor value in practical use, an approximate solution is considered.

Cut off and separate the blade longitudinally into narrow elements of length $2\pi\rho$ and contract only one element of width h_0 at one edge to have a permanent strain $(-\varepsilon_0)$, bend each element and close into an elemental straight cylinder; thus each element without permanent strain should have a radius ρ. The perpendicular deflection w is expressed by uniform bending moments M_x and M_y as follows.

$$w = -\frac{M_x - \nu M_y}{2D(1-\nu^2)}x^2 - \frac{M_y - \nu M_x}{2D(1-\nu^2)}y^2 \tag{3}$$

where $b=$ thickness of blade, $\nu=$ Poisson's ratio, $D=Eb^3/12(1-\nu^2)=$ flexural rigidity of plate, $E=$ Young's modulus.

To restrict curving in y direction, it will be sufficient to put $M_y=\nu M_x$, then deformation may be expressed simply as

$$w = -(M_x/2D)x^2$$

and

$$\left.\begin{aligned} M_x &= -D(d^2w/dx^2) \\ M_y &= -\nu D(d^2w/dx^2) \end{aligned}\right\} \tag{4}$$

For elemental cylinder with initial strain $(-\varepsilon_0)$ must be extent with internal pressure $p=Eb\varepsilon_0/\rho$ to increase its length; then the internal force required may be summed up for the width h_0 and given as $P=ph_0=Ebh_0\varepsilon_0/\rho$.

Each elemental cylinder with the actions of bending moments M_x, M_y and only one of width h_0 with the actions of both moments and internal force P are built up into the original cylinder of blade and combined with each other, which has still no curvature in y direction.

Hence the tension curve to be calculated may be given by the deformation of a straight and thin walled cylinder when it is exposed to external force P at one edge and bending moment

$$M_y = \nu D(d^2w/dx^2) = \nu D/\rho \quad \text{at both edges.}$$

If the blade is comparatively wide, assuming the cylinder as semi-infinite one, the radial deformation by the external force P acting at one edge $y=0$ is written as[14)]

$$w = \left(\frac{Ebh_0\varepsilon_0}{\rho}\Big/2\beta^3D\right)e^{-\beta y}\cos\beta y \tag{5}$$

where $\beta^4=Eb/4D\rho^2$ and the deformation by the bending moment M_y acting at one edge $y=0$ is[14)]

$$w = -\left(\frac{\nu D}{\rho}\Big/2\beta^2D\right)e^{-\beta y}(\cos\beta y - \sin\beta y) \tag{6}$$

Actually the width is finite, so that deformation with bending moment M_y at the other edge $y=h$

$$w = -\left(\frac{\nu D}{\rho} \Big/ 2\beta^2 D\right) e^{-\beta(h-y)} \left\{\cos \beta(h-y) - \sin \beta(h-y)\right\} \tag{6'}$$

must be added. Then summation φ of eq. (5), (6), (6)' should satisfy approximately the boundary conditions at $y=0$ and h, and the practical solution of tension curve may be written as

$$\varphi = 2\beta\rho h_0\varepsilon_0 e^{-\beta y} \cos \beta y - \frac{\nu}{2\beta^2\rho}\Big[e^{-\beta y}(\cos \beta y - \sin \beta y) + e^{-\beta(h-y)}\left\{\cos \beta(h-y) - \sin \beta(h-y)\right\}\Big] \tag{7}$$

Putting $\nu = 0.3$

$$\varphi = 2.570\sqrt{(\rho/b)} \cdot h_0\varepsilon_0 e^{-\beta y} \cos \beta y - 0.0908 \cdot b\Big[e^{-\beta y}(\cos \beta y - \sin \beta y) + e^{-\beta(h-y)}\left\{\cos \beta(h-y) - \sin \beta(h-y)\right\}\Big] \tag{8}$$

Tension curve ψ is measured from the base line ϕ which includes two point φ_0 and φ_h at each ordinate $y=0$ and h. ϕ is written as

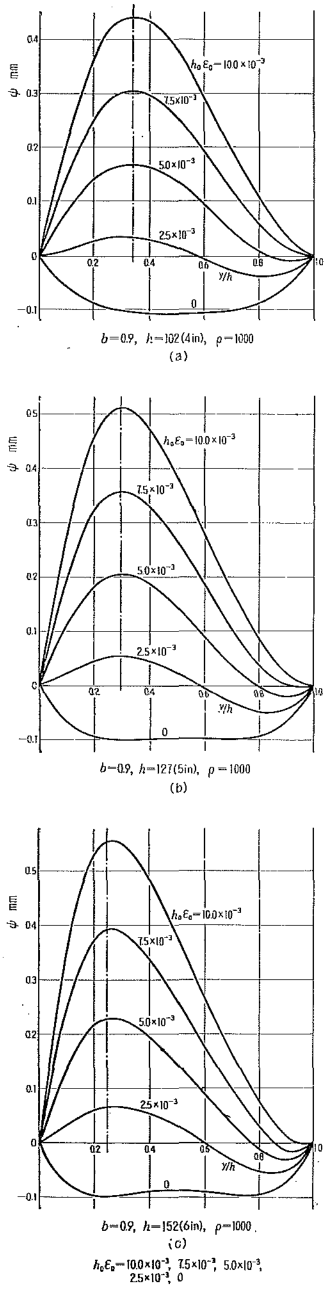
$$\begin{aligned}\phi &= \varphi_0 - (\varphi_0 - \varphi_h)\cdot(y/h) \\ &= 2.570\sqrt{\rho/b} \cdot h_0\varepsilon_0\left\{1-(y/h)+(y/h)e^{-\beta h}\cos \beta h\right\} \\ &\quad -0.0908 \cdot b\left\{1+e^{-\beta h}(\cos \beta h - \sin \beta h)\right\}\end{aligned}$$

ψ is expressed as

$$\begin{aligned}\psi &= \phi - \varphi \\ &= 2.570\sqrt{\rho/b} \cdot h_0\varepsilon_0\left\{1-(y/h)+(y/h)e^{-\beta h}\cos \beta h - e^{-\beta y}\cos \beta y\right\} \\ &\quad -0.0908 \cdot b\Big[1+e^{-\beta h}(\cos \beta h - \sin \beta h) - e^{-\beta y}(\cos \beta y - \sin \beta y) \\ &\quad - e^{-\beta(h-y)}\left\{\cos \beta(h-y) - \sin \beta(h-y)\right\}\Big]\end{aligned} \tag{9}$$

Since βh is moderately large practically, approximations $e^{-\beta h}\cos \beta h = 0$ and $e^{-\beta h}(\cos \beta h - \sin \beta h) = 0$ are permitted, and the practical formula to calculate tension curve is

$$\begin{aligned}\psi &= 2.570\sqrt{\rho/b} \cdot h_0\varepsilon_0\left\{1-(y/h) - e^{-\beta y}\cos \beta y\right\} \\ &\quad -0.0908 \cdot b\Big[1-\left\{e^{-\beta y}(\cos \beta y - \sin \beta y) \right. \\ &\quad \left. + e^{-\beta(h-y)}(\cos \beta \overline{h-y} - \sin \beta \overline{h-y})\right\}\Big]\end{aligned} \tag{10}$$



$b=0.9$, $h=102$(4in), $\rho=1000$
(a)

$b=0.9$, $h=127$(5in), $\rho=1000$
(b)

$b=0.9$, $h=152$(6in), $\rho=1000$
(c)
$h_0\varepsilon_0=10.0\times10^{-3}$, 7.5×10^{-3}, 5.0×10^{-3}, 2.5×10^{-3}, 0

Fig. 6.

Tension curves calculated from eq. (10) for the blade of 20 B.W.G. ($b=0.9$ mm) in thickness and of 4″, 5″, and 6″ ($h=102$, 127 and 152 mm) in width by varying intensity of tension ($h_0\varepsilon_0$) are shown in Fig. 6. Tension curve due to heat tension is special; the maximum point ψ appears near the front side and tension nearer to the back is rather poor.

When thermal strain is large, tension curve may be discussed approximately only by lst. term of eq. (10), so that the approximate position y_0 at which $\psi_{\max}$ appears is decided by differentiating as

$$e^{-\beta y_0}(\cos\beta y_0+\sin\beta y_0)=1/\beta h \qquad (11)$$

and values (y_0/h) for various thickness and width are calculated from eq. (11) as shown in Table 1.

TABLE 1. Values of (y_0/h) at $\rho=1{,}000$ mm

h / b mm	4″ 102 mm	5″ 127 mm	6″ 152 mm
0.90	0.35	0.30	0.25
0.45	0.27	0.23	0.20

The wider and the thinner the blade is, the nearer the maximum point approaches to the front side and the poorer the intensity of tension of back side becomes.

For this weak point, it is useful to add several lines of rolling on the back side or to heat the back edge slightly.

(b) Back Curvature

When the blade is placed flat on a flat plate, from the relation between curvature and bending moment which is needed to act the pulling force ($Ebh_0\varepsilon_0$) on heated part h_0 to straighten the curved blade, it requires the moment

$$M=Ebh_0\varepsilon_0 h/2=EI/R_F, \quad I=bh^3/12$$

and back curvature $1/R_F$ is given as follows.

$$1/R_F = 6h_0\varepsilon_0/h^2 \tag{12}$$

When the blade is bent in yz plane at a radius of curvature ρ, back radius R_B is measured as the distance between a pole and back line on the conical surface including $y=0$ and $y=h$.

From the relations

$$\rho/R_B = (\varphi_0 - \varphi_h)/h$$

$$\varphi_0 - \varphi_h = 2\beta\rho h_0\varepsilon_0(1-e^{-\beta h}\cos\beta h) \fallingdotseq 2\beta\rho h_0\varepsilon_0$$

back curvature $1/R_B$ is given as

$$1/R_B = 2\beta h_0\varepsilon_0/h = 2.570\, h_0\varepsilon_0/(h\sqrt{b\cdot\rho}) \tag{13}$$

Where two kinds of back curvature $1/R_F$ and $1/R_B$ do not coincide with each other and the relation between these back curvatures is

$$R_B/R_F = 2.34\sqrt{b\cdot\rho}/h \tag{14}$$

On the contrary, when back cuvature is given by uniformly increasing rolling load proportional to the distance from front edge, R_B coincides with R_F.

5.2 Inside Heating

(a) Tension

When heat working is given on the filet line, cracks often occur at the filet points due to large stress concentrations, so that heating inside of filet line is of practical use. In the case of inside heating, deformation can be also calculated in almost the same way as in the case of edge heating (Fig. 7).

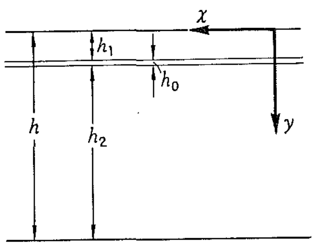


Fig. 7.

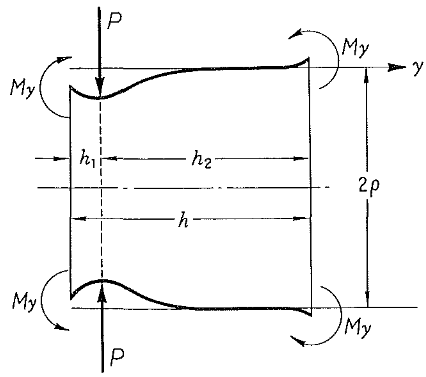


Fig. 8.

Let us consider the deformation of blade which has an initial compressive strain ($-\varepsilon_0$) in the width h_0 at a distance h_1 inside of the filet line and is bent in direction x at a radius of curvature ρ (Fig. 8).

In order to simplify the problem, it is assumed that uniform external radial force P acts on one cross section of infinite long cylinder.

Radial deformation at a distance η apart from loading point is[14)]

$$w=(P/8\beta^3D)\cdot e^{-\beta\eta}(\cos\beta\eta+\sin\beta\eta)$$

Denote the distance y from edge as

$$y=h_1-\eta \quad \text{or} \quad h_1+\eta$$

therefore

$$\eta=h_1-y \quad \text{or} \quad y-h_1$$
$$(y<h_1) \qquad\qquad (y>h_1)$$

hence

$$w=(P/8\beta^3D)\cdot e^{-\beta|y-h_1|}\{\cos\beta|y-h_1|+\sin\beta|y-h_1|\} \tag{15}$$

Effects of both edges are expressed with bending moments M_y at both edges, refering eq. (6), (6)',

$$w=-(M_y/2\beta^2D)\Big[e^{-\beta y}(\cos\beta y-\sin\beta y)$$
$$+e^{-\beta(h-y)}\Big\{\cos\beta(h-y)-\sin\beta(h-y)\Big\}\Big] \tag{16}$$

Take $P=Ebh_0\varepsilon_0/\rho\ ,\quad M_y=\nu D/\rho$

and the sum of both deformations by P and M_y is the approximate solution to be obtained and is written as φ

$$\varphi=(\beta\rho h_0\varepsilon_0/2)e^{-\beta|y-h_1|}\Big\{\cos\beta|y-h_1|+\sin|y-h_1|\Big\}$$
$$-(\nu/2\beta^2\rho)\Big[e^{-\beta y}(\cos\beta y-\sin\beta y)+e^{-\beta(h-y)}\Big\{\cos\beta(h-y)-\sin\beta(h-y)\Big\}\Big] \tag{17}$$

Putting $\nu=0.3$

$$\varphi=0.6425\sqrt{\rho/b}\cdot h_0\varepsilon_0e^{-\beta|y-h_1|}\Big\{\cos\beta|y-h_1|+\sin\beta|y-h_1|\Big\}$$
$$-0.0908\cdot b\Big[e^{-\beta y}(\cos\beta y-\sin\beta y)+e^{-\beta(h-y)}\Big\{\cos\beta(h-y)-\sin\beta(h-y)\Big\}\Big] \tag{18}$$

Tension curve ϕ is measured from the base line ϕ including both edges $y=0$ and h

$$\phi=\varphi_0-(\varphi_0-\varphi_h)(y/h)=0.6425\sqrt{\rho/b}\cdot h_0\varepsilon_0\Big[e^{-\beta h_1}(\cos\beta h_1+\sin\beta h_1)$$
$$-(y/h)\Big\{e^{-\beta h_1}(\cos\beta h_1+\sin\beta h_1)-e^{-\beta h_2}(\cos\beta h_2+\sin\beta h_2)\Big\}\Big]$$
$$-0.0908\cdot b\Big\{1+e^{-\beta h}(\cos\beta h-\sin\beta h)\Big\}$$

where $h_1+h_2=h$, then

$$\begin{aligned}\psi&=\phi-\varphi\\&=0.6425\sqrt{\rho/b}\cdot h_0\varepsilon_0\Big[e^{-\beta h_1}(\cos\beta h_1+\sin\beta h_1)-(y/h)\Big\{e^{-\beta h_1}(\cos\beta h_1\\&\quad+\sin\beta h_1)-e^{-\beta h_2}(\cos\beta h_2+\sin\beta h_2)\Big\}-e^{-\beta|y-h_1|}\Big\{\cos\beta|y-h_1|\\&\quad+\sin\beta|y-h_1|\Big\}\Big]-0.0908\cdot b\Big[1+e^{-\beta h}(\cos\beta h-\sin\beta h)-e^{-\beta y}(\cos\beta y\\&\quad-\sin\beta y)-e^{-\beta(h-y)}\Big\{\cos\beta(h-y)-\sin\beta(h-y)\Big\}\Big]\end{aligned}\tag{19}$$

Practically as βh and βh_2 are both comparatively large, the approximations

$$e^{-\beta h}(\cos\beta h-\sin\beta h)=0\,,\qquad e^{-\beta h_2}(\cos\beta h_2+\sin\beta h_2)=0$$

are recognized, then

$$\begin{aligned}\psi&=0.643\sqrt{\rho/b}\cdot h_0\varepsilon_0\Big[\Big\{1-(y/h)\Big\}e^{-\beta h_1}(\cos\beta h_1+\sin\beta h_1)\\&\quad-e^{-\beta|y-h_1|}\Big\{\cos\beta|y-h_1|+\sin\beta|y-h_1|\Big\}\Big]\\&\quad-0.0908\cdot b\Big[1-\Big\{e^{-\beta y}(\cos\beta y-\sin\beta y)\\&\quad+e^{-\beta(h-y)}(\cos\beta\overline{h-y}-\sin\beta\overline{h-y})\Big\}\Big]\end{aligned}\tag{20}$$

is the practical formula to calculate the tension curve in the case of inside heating.

But since this formula is introduced by applying deformation of infinite cylinder, if (h_1/h) is small, the error becomes comparatively large, then in these cases the formula of edge heating is more appropriate.

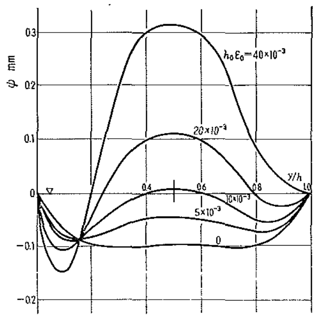
The factor of lst. term of eq. (20) is 1/4 of that of eq. (10) in the case of edge heating. Thus deformation by edge heating is larger than that by inside heating with equal thermal strain. Furthermore, as the cooling effect by means of heat transfer in inside heating is larger than that in edge heating, the maximum temperature in inside heating is lower than that in edge heating under equal heating conditions.

From these reasons, heat tension by edge heating has more intensity than that by inside heating at the same condition of heating.

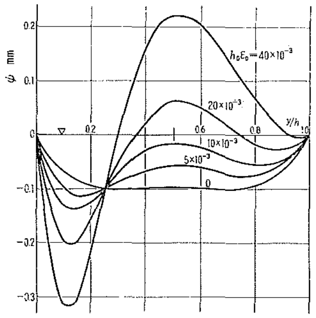
Fig. 9 shows various kinds of tension curves calculated from eq. (20) for 20 B.W.G. 5″ blade, varying $(h_1/h)=0.05$, 0.1 and 0.2.

(b) Back Curvature

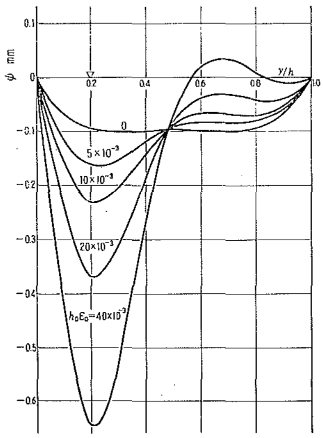
Back curvature $1/R_F$ which should be measured on the flat plate is introduced from the relation



$b=0.9$, $h=127$ (5in), $h_1/h=0.05$, $\rho=1000$
(a)



$b=0.9$, $h=127$ (5in), $h_1/h=0.1$, $\rho=1000$
(b)



$b=0.9$, $h=127$ (5in), $h_1/h=0.2$, $\rho=1000$
(c)
$h_0\varepsilon_0=40\times10^{-3}$, 20×10^{-3}, 10×10^{-3}, 5×10^{-3}, 0

Fig. 9.

$$M=Eb\cdot h_0\varepsilon_0\left[(h/2)-h_1\right]=Ebh^3/12\cdot R_F$$

to

$$1/R_F=(6h_0\varepsilon_0/h^2)\left[1-2(h_1/h)\right] \qquad (21)$$

and it is also smaller than that by edge heating.

In a bent state at a curvature $(1/\rho)$ in direction of length, back curvature $1/R_B$ is introduced from the relation

$$\rho/R_B=(\varphi_0-\varphi_h)/h$$

according to conical surface including $y=0$ and h, where

$$\varphi_0-\varphi_h=(\beta\rho h_0\varepsilon_0/2)\left[e^{-\beta h_1}(\cos\beta h_1+\sin\beta h_1)\right.$$
$$\left.-e^{-\beta(h-h_1)}\left\{\cos\beta(h-h_1)+\sin\beta(h-h_1)\right\}\right]$$

and as $h\gg h_1$ generally, the 1st. term of [ ] is near 1 and the 2nd. term is neglegible. Therefore $1/R_B$ is expressed practically as

$$1/R_B=(\beta\rho h_0\varepsilon_0/2)e^{-\beta h_1}(\cos\beta h_1+\sin\beta h_1)$$
$$\fallingdotseq 0.643\,h_0\varepsilon_0/(h\sqrt{b\cdot\rho}) \qquad (22)$$

It is also less than 1/4 of edge heating.

6. Experiments

Basic experiments are done to ascertain tension and back forseen from theories and to make a standard of working.

TABLE 2. Chemical Components (%) of Test Piece Material and Conditions of Heat Treating

C	Si	Mn	P	S	Cu	Ni	Cr
0.80	0.23	0.30	0.009	0.015	0.19	0.90	0.06

Quenching		Tempering	
Temp.	Time	Temp.	Time
890°C	2 min	485°C	50 s

6.1 Test Piece

Band saw blade of SKS–5 of 20 B.W.G. (b=0.9 mm) in thickness and 5″ (h=127 mm) in width is taken and its chemical components and conditions of heat treatments are shown in Table 2. Length is 2200 mm for measuring tension, back, temperature etc. (Fig. 10).

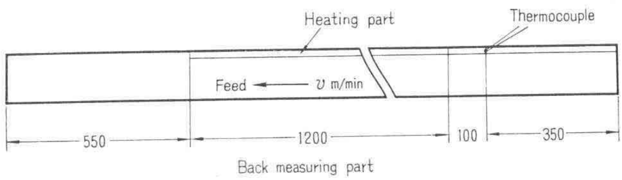


Fig. 10.

6.2 Apparatus

The main parts of heat working apparatus (Fig. 1) are as follows.

Guide (Tensioning anvil), Flame cover (Width of slit is set at 7 mm), Feeding roll (Roll stretcher, feeding velocity of which is steplessly variable for 0.41～10.3 m/min), Torch (Torch for gas welding, standard size of tip is 25 l/hr, neutral flame is used), C_2H_2 bottle (Resolved acetylene), O_2 bottle (Pressure of exit is controled for 0.4～1.2 kg/cm^2).

6.3 Heating Temperature

Thermocouple of Cu-Constantan (0.5 mmϕ, 2.5 m length) is welded in the center line of heating on the back surface and leads to a D-type vibrator of electro-magnetic oscillograph, to record the temperature change on oscillo-paper. An example of recording is shown in Fig. 11.

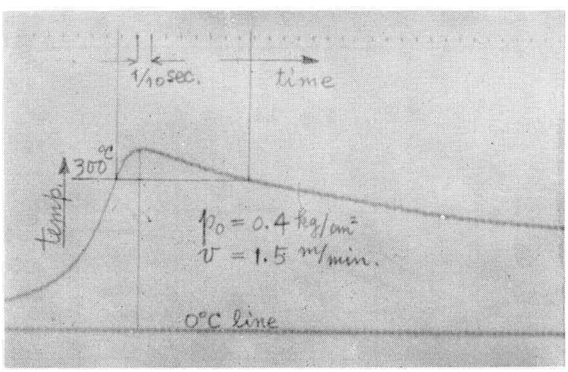


Fig. 11. Oscillo-record, Temperature-Time

Fig. 12～17 show the maximum heating temperature T_{max}, width of blue heating h_0 (which is nearly equivalent to strained width) and maintaining time τ over 300°C which corresponds to feeding verocity v and O_2 pressure p_0.

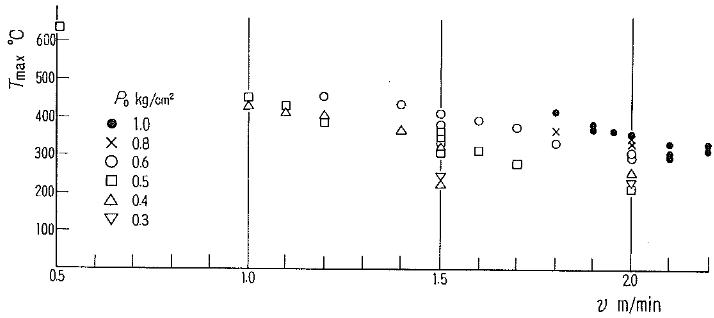


Fig. 12.

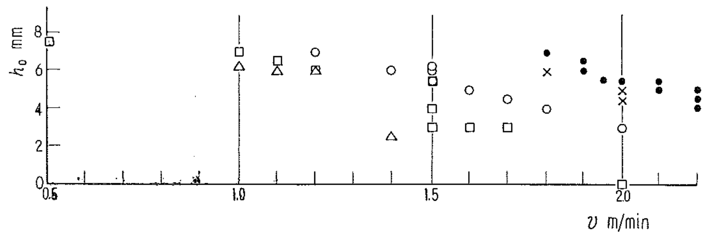


Fig. 13.

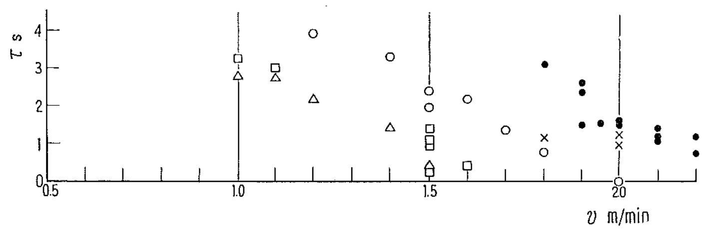


Fig. 14.

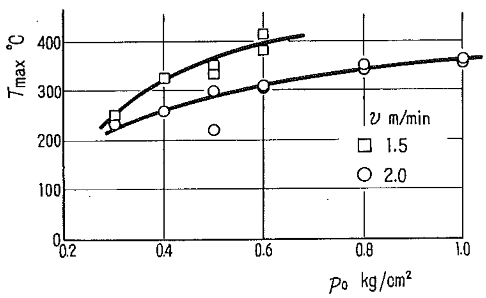


Fig. 15.

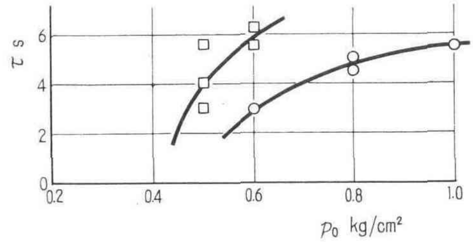


Fig. 16.

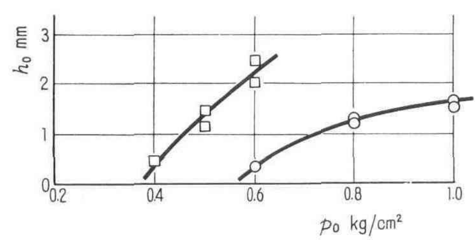


Fig. 17.

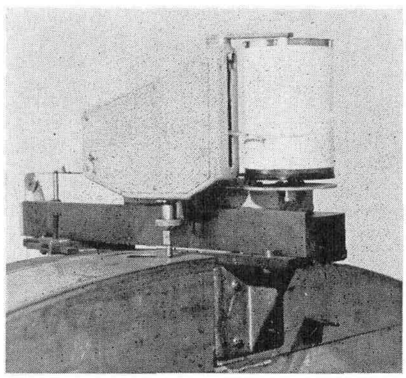


Fig. 18. Tension curve recorder

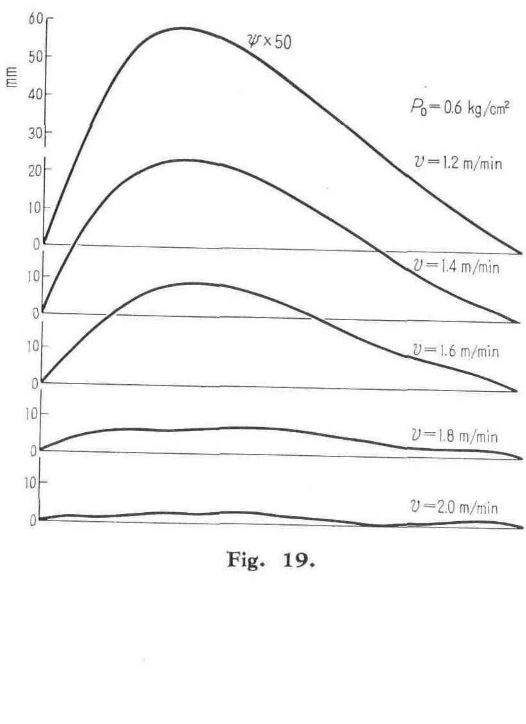


Fig. 19.

6.4 Tension

The tension curve is continuously recorded with an apparatus as shown in Fig. 18, laying the blade freely on a wooden stand with a constant radius of curvature $\rho = 1000$ mm. Vertical magnification of record is $\times 50$ and horizontal $\times 1$. Examples of tension curves measured are shown in Fig. 19.

Maximum height ψ_{max} represents the intensity of tension and depends on feeding velocity v and gas pressure p_0 as shown in Fig. 20 and 21.

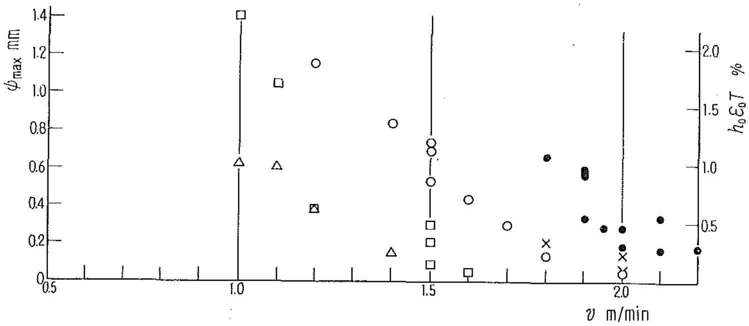


Fig. 20.

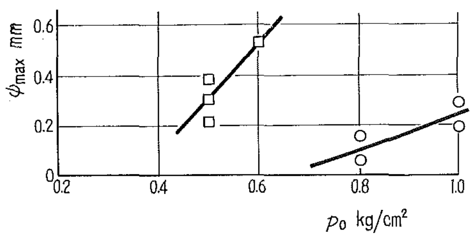


Fig. 21.

Mean value of 40 abscissas of maximum height (y_0/h) is $0.296 \fallingdotseq 0.30$ and is in good agreement with the theoretical value.

From eq. (10), the relation

$$h_0\varepsilon_0 = 0.0164\,\psi_{max} + 0.00162$$

is obtained for this (y_0/h), so that ψ_{max} curve is rewritten by the scale of $h_0\varepsilon_0^T$ (ε_0^T is strain measured from tension).

6.5 Back

Curvature of back edge line $1/R_{F'}$ on the flat plate is measured with straight ruler as shown in Fig. 22 and has the same inclinations as Fig. 23, 24. It can be rewritten also by the scale of $h_0\varepsilon_0^B$ (ε_0^B is strain measured from back) from the equaiton

$$h_0\varepsilon_0 = h^2/6R_{F'}$$

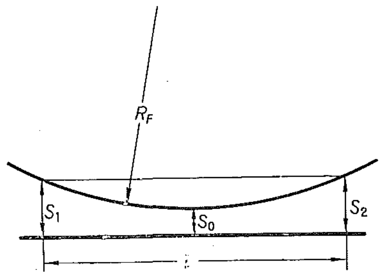


Fig. 22.

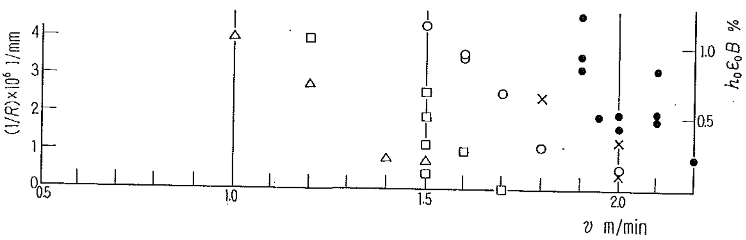


Fig. 23.

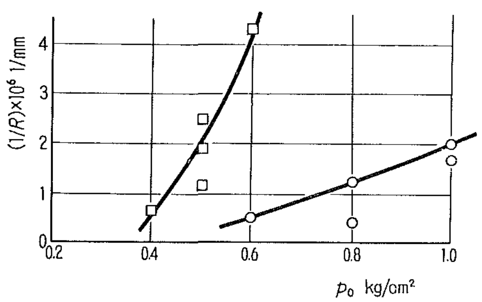


Fig. 24.

6.6 Hardness

A small piece of 20×50 is inserted tightly at the middle edge of the blade of 900 length (Fig. 25) and is heated in the same manner as before and Micro-Vicker's hardness (500 g load) is measured on both surfaces for various heating temperatures. The results are shown in Fig. 26, 27. Over 350°C on back surface, hardness decreases gradually, but there is no sudden fall under 400°C. From this curve, a temperature difference of about 50°C between front surface and back is estimated.

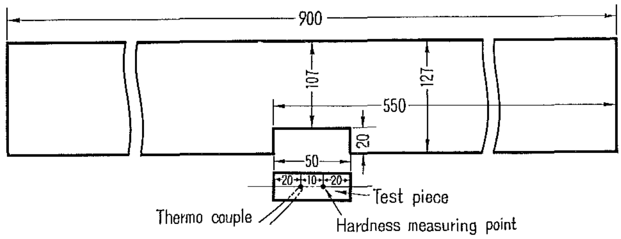


Fig. 25.

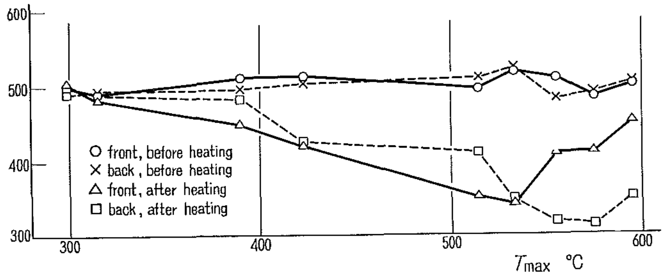


Fig. 26.

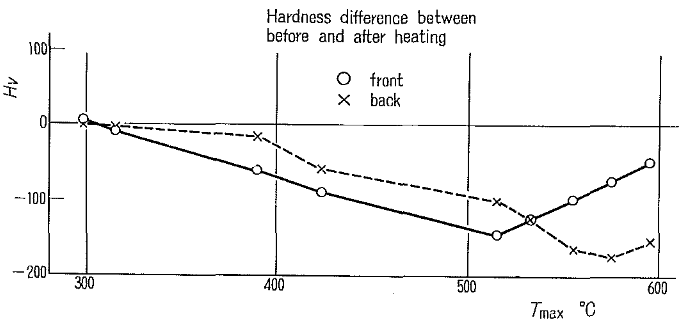


Fig. 27. Hardness differenice between before and after heating

6.7 Inside Heating

The same experiments were done for the conditions in which the position of heating h_1 is varied as $(h_1/h)=0.05$, 0.1, 0.2. The results are shown in Fig. 28, 29 and tension curves are recorded as Fig. 30.

From these results the following conclusions are obtained. (1) The larger (h_1/h) is, the nearer the tension curve approaches the theoretical one. (2) Difference appears between both tension curves measured from front and back surfaces because of temperature difference. (3) Effects of heating and tensioning

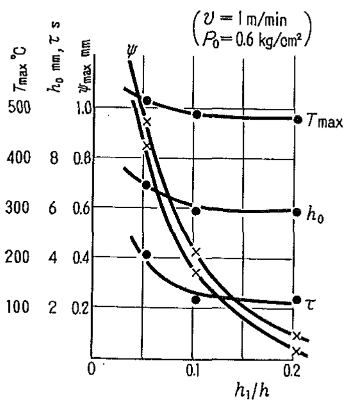


Fig. 28.

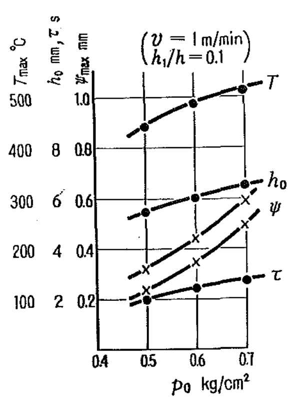


Fig. 29.

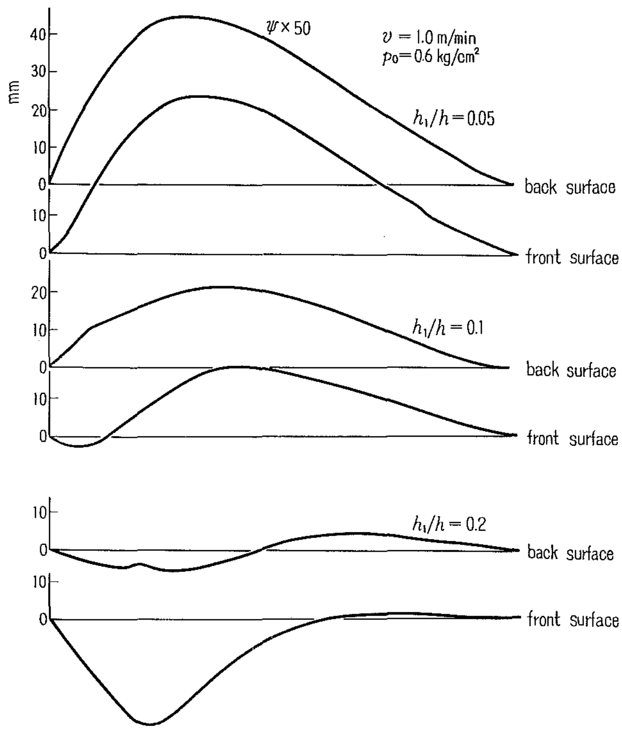


Fig. 30.

is less than that of edge heating. (4) The nearer to the edge line the position of heating is, the better the tension curve coincides with the results of edge heating.

6.8 Double Heating (Double Heat Tension) [10),15),16)]

An example of tension curve is shown in Fig. 31, when front and back edges are heated successively. If both edges are heated under the same conditions successively, the restricting force at the secondary heating is smaller

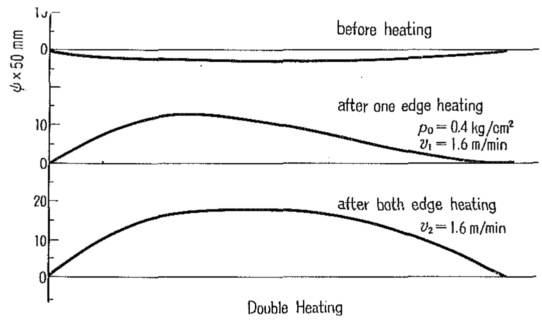


Fig. 31. Double heat tension

than that of the primary one because of deformation which has occured already and consequently tension does not occur symmetrically. To obtain a symmetrical tension curve, the secondary heating should be rather stronger.

In Fig. 32, 33, the abscisas of maximum point (y_0/h) and back curvature $(1/R)$ are shown for the combinations of feeding velocities in front v_1 and in back v_2 in the case of constant oxygen gas pressure $p_0 = 0.4$ kg/cm². These

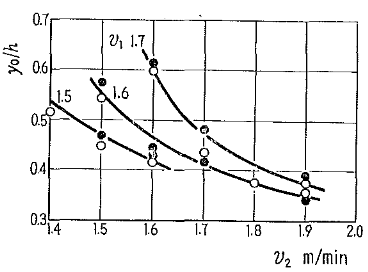


Fig. 32.

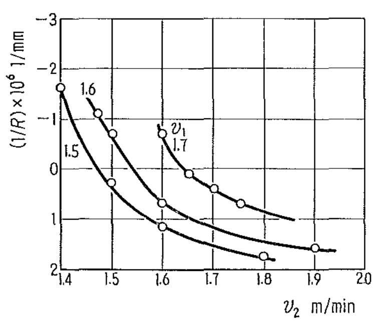


Fig. 33.

curves also do not cross the line $(y_0/h) = 0.5$ or $(1/R) = 0$ under the same heating conditions $v_1 = v_2$.

Proper combinations of heating conditions should be selected from these relations.

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