An Approximate Method to Calculate the Local Heatflux From a Nonisothermal Flat Plate Across a Turbulent Boundary Layer

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Abstract

A simple approximate method based on the Spalding function to calculate local heat flux from a nonisothermal flat plate is developed in this paper. A technique to cancel the major approximation errors is included. A diagram for the calculation covering a wide range of Prandtl number is made by using the Gardner and Kestin's solution of the Spalding function. Formulas on which the diagram is based are also attached for a desire to use a computer for the calculation. The method is checked by the Smith and Shar's solution of the Spalding's equation for a specified flux condition and it is shown that the results are sufficient for practical purposes.

Nomenclature

$C_f/2$, friction factor;
C_p, specific heat at a constant pressure of the fluid;
E, approximation error;
h, coefficient of heat transfer;
k, thermal conductivity of the fluid;
P_r, Prandtl number, $\rho C_p \nu/k$;
$q(x)$, wall heat flux at point x;
R_{ex}, Reynolds number, $u_s x/\nu$;
$R_{e\xi_i}$, Reynolds number, $u_s \xi_i/\nu$;
S, limit of integration, defined by equation (15');
S_p, Spalding function;
S_t, Stanton number, $h/\rho u_s C_p$;
t, difference in temperature between a point in the boundary layer and the mainstream;
t_w, difference in temperature between wall and mainstream;
t_s, mainstream temperature;
u, velocity in boundary layer, parallel to surface;

u_s, velocity of mainstream;
u^+, dimensionless velocity in boundary layer, defined by equation (22');
x, co-ordinate of distance along the wall;
x^+, dimensionless distance along the wall, defined by equation (3);
θ, dimensionless temperature, t/t_w;
ν, kinematic viscosity of the fluid;
ξ, unheated length of the wall;
ρ, density of the fluid;
τ_w, shear stress at the wall;

Subscripts

i, general expression of a segmented point;
0, leading point of the first segment;
n, trailing point of the final segment;

Introduction

Spalding [1] showed that heat transfer across a turbulent boundary layer can be reduced to a simple partial differential equation. Kestin and Persen [2], Gardner and Kestin [3] solved the equation with the boundary condition of isothermal surfaces for a wide range of Prandtl number and named the solutions the Spalding function. Smith and Shar [4] solved the equation with a boundary condition of a specified flux condition.

The problem of heat transfer across a turbulent boundary layer of a flat plate was previously solved by Rubesin [5], Seban [6], Reynolds et al [7], Tribus and Klein [8]. However, these are based on both mathematical and physical approximations, and one of the major shortcomings is said that neglect of the laminar sub-layer.

In many engineering applications, problems for the calculation of local heat flux from a nonisothermal wall temperatuer are interested. In such cases, it is well known that Eq. (1) in the following paragraph is used in which $h(x, \xi)$ is the heat transfer coefficient for an isothermal surface. But the evaluation of the integral of Eq. (1) becomes frequently difficult inasmuch as the temperature can be completely arbitrary. Hartnett, Eckert, Birkebak and Sampson [9] developed a method in which the plate is subdivided in a stream-wise direction and the temperature is approximated to a linear function of the stream-wise co-ordinate in each segment, in such a way that the Seban's or the Rubesin's formula can be applied to each segment.

A utilization of the Spalding function for the same purpose of approximation is attempted in this paper. The technique to approximate the temperature distribution is changed to suit the different nature of the Spalding function from the Seban's or the Rubesin's formula, and the method is developed in this paper. At some peculiar points, un-negligible errors may possibly appear by the approximation and an additional calculation to cancel the errors is necessary. The method of the cancellation is also developed. Finally, some results by the present method are compared with known heat flux distributions and the reliability is proved to be sufficient for practical purposes.

The Approximation

1. Basic calculation;

The local heat flux from a nonisothermal flat plate exposed to steady flow is calculated by, [10],

$$q(x)=\int_0^x h(x,\xi)\frac{dt_w(\xi)}{d\xi}d\xi \tag{1}$$

In this equation $h(x,\xi)$ denotes the heat-transfer coefficient at location x for an isothermal flat plate in which the first part with length ξ is not heated and has the same temperature as the free stream.

The Spalding function, S_p, is defined by

$$S_p=\frac{S_t\cdot P_r}{\sqrt{C_f/2}} \tag{2}$$

and the numerical solutions are given by the function of x^+ and P_r by Gardner and Kestin [3]. Here, the variable x^+ is a reduced length co-ordinate and it is

$$x^+=\int_\xi^x \frac{\sqrt{\tau_w/\rho}}{\nu}dx \tag{3}$$

Having Eq. (2), the heat-transfer coefficient, $h(x,\xi)$, can be expressed by the Spalding function.

$$h(x,\xi)=\frac{k}{\nu}u_s S_p(x^+,P_r)\sqrt{C_f/2} \tag{4}$$

For a turbulent boundary layer the friction factor, $C_f/2$, is, [11],

$$C_f/2=0.0296\left(\frac{u_s x}{\nu}\right)^{-0.2} \tag{5}$$

Eq. (1), Eq. (4), and Eq. (5) gives

$$q(x)=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\int_{\xi=0}^{x} S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}d\xi \tag{6}$$

or

$$q(x)=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\int_{S_0}^{0} S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}\,\frac{d\xi}{dx^+}dx^+ \tag{7}$$

Here, $S_0 = 0.1912\, R_{ex}^{0.9}$ (8)

The form of Eq. (7) is more preferable than Eq. (6) for the following calculations, because the solutions of the Spalding function are given by the functions of x^+ and are not given by the functions of ξ.

The integral in Eq. (7) may be a form which can not be evaluated without some tedious numerical calculations. The approximate method is essentially a technique to avoid the tedious calculation procedures. The idea is that the co-ordinate is subdivided in many segments and $(dt_w(\xi)/d\xi)\cdot(d\xi/dx^+)$ is assumed to be a constant in each segment (in other words, in each corresponding x^+ segment). The usual technique for this type of approximation assume that $(dt_w(\xi)/d\xi)$ is a constant as being seen in the paper by Hartnett et al [9].

A comparison of those two conditions will be clear by integrations of both assumptions. The former is

$$\frac{dt_w(\xi)}{d\xi}\,\frac{d\xi}{dx^+} = \text{constant} \tag{9}$$

From Eq. (3) and Eq. (5)

$$x^+ = \int_\xi^x \frac{\sqrt{\tau_w/\rho}}{\nu}\,dx = \frac{\sqrt{0.0296}}{0.9}\left(\frac{u_s}{\nu}\right)^{0.9}(x^{0.9}-\xi^{0.9}) \tag{10}$$

$$\frac{d\xi}{dx^+} = \frac{-\xi^{0.1}}{\sqrt{0.0296}\left(\dfrac{u_s}{\nu}\right)^{0.9}} \tag{11}$$

$$\frac{dt_w(\xi)}{d\xi}\,\frac{d\xi}{dx^+} = \frac{-\xi^{0.1}}{\sqrt{0.0296}\left(\dfrac{u_s}{\nu}\right)^{0.9}}\,\frac{dt_w(\xi)}{d\xi}$$

$$= \frac{-0.9}{\sqrt{0.0296}\left(\dfrac{u_s}{\nu}\right)^{0.9}}\,\frac{dt_w(\xi)}{d(\xi^{0.9})} \tag{12}$$

Finally, the conditions of Eq. (9) can be written as

$$-\frac{0.9}{\sqrt{0.0296}\left(\dfrac{u_s}{\nu}\right)^{0.9}}\,\frac{dt_w(\xi)}{d(\xi^{0.9})} = \text{constant} \tag{13}$$

An integration of Eq. (13) gives the form of

$$t_w(\xi) = C_1\xi^{0.9} + C_2 \tag{14}$$

where C_1 and C_2 are constants.

On the other hand, the later condition is

$$\frac{dt_w(\xi)}{d\xi} = \text{constant}$$

$$t_w(\xi) = C_1'\xi + C_2'$$

Although the assumption of Eq. (14) looks somewhat peculiar, it is not much different from the later assumption for a small segment size and for practical purposes.

Eq. (7) can be expanded with Eq. (12).

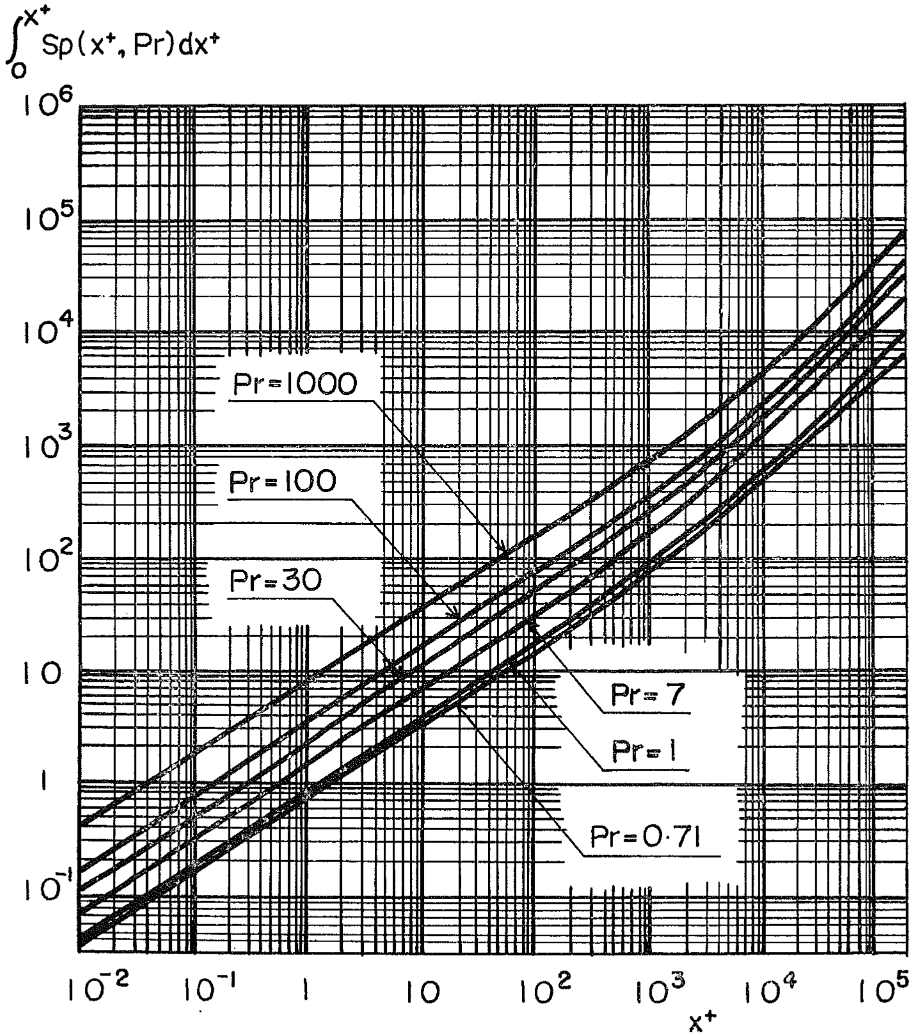

Fig. 1. Integrated values of the Spalding function

$$q(x) = -0.9\frac{k}{x^{0.1}}\left\{\left(\frac{t_w(\xi_1)-t_w(\xi_0)}{\xi_1^{0.9}-\xi_0^{0.9}}\right)\right\}\int_{S_0}^{S_1} S_p(x^+, P_r)dx^+$$
$$+\left(\frac{t_w(\xi_2)-t_w(\xi_1)}{\xi_2^{0.9}-\xi_1^{0.9}}\right)\int_{S_1}^{S_2} S_p(x^+, P_r)dx^+ + \cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\left(\frac{t_w(\xi_i)-t_w(\xi_{i-1})}{\xi_i^{0.9}-\xi_{i-1}^{0.9}}\right)\int_{S_{i-1}}^{S_i} S_p(x^+, P_r)dx^+ + \cdots$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\left(\frac{t_w(\xi_n)-t_w(\xi_{n-1})}{\xi_n^{0.9}-\xi_{n-1}^{0.9}}\right)\int_{S_{n-1}}^{S_n} S_p(x^+, P_r)dx^+ \tag{15}$$

In this equation ξ_i denotes the location where i-th segment ends. By the same token, ξ_n denotes the end point of n-th segment. As the n-th segment is the last segment, ξ_n denotes the same location where x is located. The $t_w(\xi_i)$ is the wall temperature at $\xi=\xi_i$ and S_i is

$$S_i = x^+(x, \xi_i) = 0.1912(R_{ex}^{0.9} - R_{e\xi_i}^{0.9}) \tag{15'}$$

As ξ is on the same axis of x and they both have the same origin, one can rewrite Eq (12) to be

$$q(x) = -0.9\frac{k}{x^{0.1}}\sum_{i=1}^{n}\left(\frac{t_w(\xi_i)-t_w(\xi_{i-1})}{x_i^{0.9}-x_{i-1}^{0.9}}\right)\int_{S_{i-1}}^{S_i} S_p(x^+, P_r)dx^+ \tag{16}$$

If $\int_0^{x^+} S_p(x^+, P_r)dx^+$ is tabulated for a wide range of x^+, $q(x)$ can be calculated for any distribution of wall temperature. The numerically integrated values of the Spalding function are shown in Fig. 1. The method of the integration is explained in the appendix.

2. Cancellation of Errors;

For the purpose of checking the reliability of the method, $q(x)$'s are calculated by Eq. (16) in which the wall temperature distribution is theoretically led from the Smith and Shar's solution [4] of the Spalding's equation corresponding to a known heat flux condition. The given flux distribution, the wall temperature distribution, and the calculated $q(x)$ are plotted in Fig. 2. The stream is an air flow, having the free stream velocity of 100 ft/sec and the free stream temperature is 80°F.

As may be seen in this figure the calculated $q(x)$'s resulted in close approximations of the given heat flux condition except for the regions where a big heat flux jump occur at. These errors originate thus in such regions as $(dt_w(\xi)/d\xi)$ changes so drastically that $(dt_w(\xi)/d\xi)\cdot(d\xi/dx^+)$ can not be considered

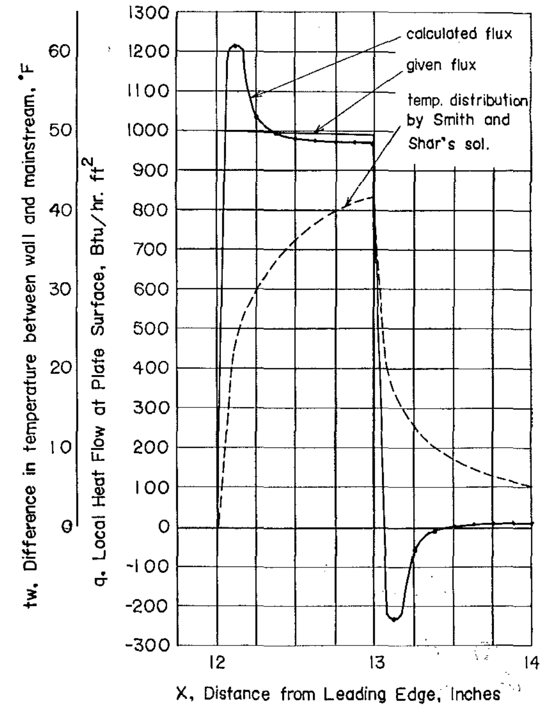

Fig. 2. An example of t_w with a given flux and $q(x)$ by Eq. (16)

as a constant even for a interval of small segment size Δx.

An additional calculation is required to cancel this kind of error. The procedure is as follows;

$$q(x)=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\int_{\xi=0}^{x}S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}\,\frac{d\xi}{dx^+}dx^+ \tag{7}$$

$$\begin{aligned}
&q(x-\Delta x)= \\
&\quad\sqrt{0.0296}\,\frac{k}{x-\Delta x}\left(\frac{u_s(x-\Delta x)}{\nu}\right)^{0.9}\int_{\xi=0}^{x-\Delta x}S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}\,\frac{d\xi}{dx^+}dx^+ \\
&=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}\left\{\int_{\xi=0}^{x}S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}\,\frac{d\xi}{dx^+}dx^+\right. \\
&\quad\left.-\int_{\xi=x-\Delta x}^{x}S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}\,\frac{d\xi}{dx^+}dx^+\right\} \\
&=\left(1-\frac{\Delta x}{x}\right)^{-0.1}q(x) \\
&\quad-\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}\int_{\xi=x-\Delta x}^{x}S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}\,\frac{d\xi}{dx^+}dx^+
\end{aligned}$$

17)

$$\Delta q(x)=q(x)-q(x-\Delta x)=\left\{1-\left(1-\frac{\Delta x}{x}\right)^{-0.1}\right\}q(x)$$
$$+\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}\int_{S_{n-1}}^{0}S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}\frac{d\xi}{dx^+}dx^+ \tag{18}$$

If the condition of Eq. (9) is applied for the same formulation as above, the resulting $\Delta q(x)$ will include the errors which are desired to be cancelled here.

$$\Delta q(x)+E=\left\{1-\left(1-\frac{\Delta x}{x}\right)^{-0.1}\right\}q(x)$$
$$+\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}\left(\frac{dt_w(\xi)}{d\xi}\frac{d\xi}{dx^+}\right)\int_{S_{n-1}}^{0}S_p(x^+,P_r)dx^+ \tag{19}$$

Substract Eq. (18) from Eq. (19).

$$E=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}\left[\int_0^{S_{n-1}}S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}\frac{d\xi}{dx^+}dx^+\right.$$
$$\left.-\left(\frac{dt_w(\xi)}{d\xi}\frac{d\xi}{dx^+}\right)\int_0^{S_{n-1}}S_p(x^+,P_r)dx^+\right]$$
$$=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.9}\left[\int_0^{S_{n-1}}S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}\frac{d\xi}{dx^+}dx^+\right.$$
$$\left.-\mathrm{const}_1\int_0^{S_{n-1}}S_p(x^+,P_r)dx^+\right] \tag{20}$$

The const_1 in Eq. (20) can be calculated.

$$\frac{dt_w(\xi)}{dx^+}=\mathrm{const}_1$$
$$t_w(\xi)=\mathrm{const}_1\cdot x^+ +\mathrm{const}_2$$
$$\mathrm{const}_1=\frac{t_w(\xi_{n-1})-t_w(\xi_n)}{S_{n-1}} \tag{21}$$

$(dt_w(\xi)/d\xi)\cdot(d\xi/dx^+)$ in the first integral of Eq. (20) will be formulated from the definition of the Spalding function. The definition is, [3],

$$S_p(x^+,P_r)=-\left(\frac{\partial\theta}{\partial u^+}\right)_{u^+=0} \tag{22}$$

where $\theta=t/t_w$

and $\quad u^+=u/\sqrt{\tau_w/\rho}$ (22′)

$$\left(\frac{\partial\theta}{\partial u^+}\right)_{u^+=0}=\left(\frac{\partial\theta}{\partial y}\cdot\frac{\partial y}{\partial u}\cdot\frac{\partial u}{\partial u^+}\right)_{u^+=0}=\frac{\nu}{u_s}\frac{1}{\sqrt{C_f/2}}\frac{\left(\dfrac{\partial t}{\partial y}\right)_{y=0}}{t_w} \tag{23}$$

Eq. (22) can be

$$S_p(x^+,P_r)=\frac{-\dfrac{\nu}{u_s}}{\sqrt{C_f/2}}\frac{\left(\dfrac{\partial t}{\partial y}\right)_{y=0}}{t_w} \tag{24}$$

A local heat flux is calculated by

$$q(\xi)=-k\left(\frac{\partial t}{\partial y}\right)_{y=0} \tag{25}$$

From Eq. (24) and Eq. (25),

$$q(\xi)=\frac{C_p\rho u_s}{P_r}\sqrt{C_f/2}\cdot S_p(x^+,P_r)t_w(\xi)$$

$$t_w(\xi)=\frac{q(\xi)}{C_p\rho u_s\sqrt{C_f/2}}\frac{P_r}{S_p(x^+,P_r)} \tag{26}$$

It means that if a heat flux jump $\Delta q(\xi)$ occurs at a point $\xi=x-\Delta x$ and the $(\Delta q(\xi)/C_p\rho u_s\sqrt{C_f/2})$ is maintained at a constant value in the region from $\xi=x-\Delta x$ to $\xi=x$, then the wall temperature increase (or decrease) in the region is

$$t_w(\xi_n)-t_w(\xi_{n-1})=\frac{\Delta q(\xi)}{C_p\rho u_s\sqrt{C_f/2}}\frac{P_r}{[S_p(x^+,P_r)]_{x,x-\Delta x}} \tag{27}$$

where $[S_p(x^+,P_r)]_{x,x-\Delta x}$ is the value of the Spalding function corresponding to the value of x^+ from the point $\xi=x$ to $\xi=x-\Delta x$. For a small x^+, the Spalding function is well approximated by the asymptotic solution of $x^+\to 0$.

$$S_p(x^+,P_r)=\frac{0.53835}{(x^+)^{1/3}}P_r^{1/3} \tag{28}$$

The segment size, Δx, is usually small enough to use Eq. (28) to evaluate $[S_p(x^+,P_r)]_{x,x-\Delta x}$. And Eq. (27) becomes

$$\frac{\Delta q(\xi)}{C_p\rho u_s\sqrt{C_f/2}}=\left\{t_w(\xi_n)-t_w(\xi_{n-1})\right\}\frac{[S_p(x^+,P_r)]_{x,x-\Delta x}}{P_r}$$

$$=\left\{t_w(\xi_n)-t_w(\xi_{n-1})\right\}\frac{0.53835}{S_{n-1}^{1/3}P_r^{2/3}} \tag{29}$$

This result is substituted in Eq. (26).

$$t_w(\xi)=\left\{t_w(\xi_n)-t_w(\xi_{n-1})\right\}\frac{0.53835}{S_{n-1}^{1/3}P_r^{2/3}}\frac{P_r}{S_p(x^+,P_r)}$$

With Eq. (28),

$$t_w(\xi)=\left\{t_w(\xi_n)-t_w(\xi_{n-1})\right\}\frac{(x^+)^{1/3}}{S_{n-1}^{1/3}} \tag{30}$$

$$\frac{dt_w(\xi)}{d\xi}=\frac{t_w(\xi_n)-t_w(\xi_{n-1})}{S_{n-1}^{1/3}}\frac{1}{3}\left\{\frac{\sqrt{0.0296}}{0.9}\left(\frac{u_s}{\nu}\right)^{0.9}\left(\xi^{0.9}-(x-\Delta x)^{0.9}\right)\right\}^{-2/3}$$
$$\times\left\{\sqrt{0.0296}\left(\frac{u_s}{\nu}\right)^{0.9}\xi^{-0.1}\right\} \tag{31}$$

Eq. (11) makes Eq. (31) to be

$$\frac{dt_w(\xi)}{d\xi}\cdot\frac{d\xi}{dx^+}$$
$$=-\frac{t_w(\xi_n)-t_w(\xi_{n-1})}{3S_{n-1}^{1/3}}\left\{\frac{\sqrt{0.0296}}{0.9}\left(\frac{u_s}{\nu}\right)^{0.9}\left(\xi^{0.9}-(x-\Delta x)^{0.9}\right)\right\}^{-2/3}$$
$$=-\frac{t_w(\xi_n)-t_w(\xi_{n-1})}{3S_{n-1}^{1/3}}(S_{n-1}-x^+)^{-2/3} \tag{32}$$

The error can be calculated with with Eq. (20), Eq (21), and Eq. (32).

$$E=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}$$
$$\times\left[\int_0^{S_{n-1}}S_p(x^+,P_r)\frac{t_w(\xi_{n-1})-t_w(\xi_n)}{3S_{n-1}^{1/3}}(S_{n-1}-x^+)^{-2/3}dx^+\right.$$
$$\left.+\frac{t_w(\xi_n)-t_w(\xi_{n-1})}{S_{n-1}}\int_0^{S_{n-1}}S_p(x^+,P_r)dx^+\right] \tag{33}$$

By applying Eq. (28) again,

$$E=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}\left(t_w(\xi_{n-1})-t_w(\xi_n)\right)(0.53835\,P_r^{1/3})$$
$$\times\left\{\frac{1}{3}\int_0^{S_{n-1}}(x^+)^{-1/3}S_{n-1}^{-1/3}(S_{n-1}-x^+)^{-2/3}dx^+-\frac{1}{S_{n-1}}\int_0^{S_{n-1}}(x^+)^{-1/3}dx^+\right\} \tag{34}$$

Put $x^+=S_{n-1}\cdot X$ into the first integral of Eq. (34).

$$\int_0^{S_{n-1}}(x^+)^{-1/3}\cdot(S_{n-1})^{-1/3}\cdot(S_{n-1}-x^+)^{-2/3}dx^+=S_{n-1}^{-1/3}\int_0^1X^{-1/3}(1-X)^{-2/3}dX$$
$$=S_{n-1}^{-1/3}\cdot\beta(2/3,1/3)=3.6273\,S_{n-1}^{-1/3} \tag{35}$$

Accordingly,

$$E=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}\left\{t_w(\xi_{n-1})-t_w(\xi_n)\right\}(0.5385\,P_r^{1/3})$$
$$\times\left\{\frac{3.6273\,S_{n-1}^{-1/3}}{3}-\frac{3S_{n-1}^{-1/3}}{2}\right\} \tag{36}$$

From Eq. (18)

$$\begin{aligned}\Delta q(x)-&\left\{1-\left(1-\frac{\Delta x}{x}\right)^{-0.1}\right\}q(x)\\ &=\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}\int_{\xi=x-\Delta x}^{x}S_p(x^+,P_r)\frac{dt_w(\xi)}{d\xi}d\xi\\ &=-\sqrt{0.0296}\,\frac{k}{x}R_{ex}^{0.9}\left(1-\frac{\Delta x}{x}\right)^{-0.1}\\ &\quad\times\left\{t_w(\xi_{n-1})-t_w(\xi_n)\right\}(0.53835\,P_r^{1/3})\frac{3.6273\,S_{n-1}^{-1/3}}{3}\end{aligned} \tag{37}$$

Divide Eq. (36) by Eq. (37).

$$\frac{E}{\Delta q(x)-\left\{1-\left(1-\frac{\Delta x}{x}\right)^{-0.1}\right\}q(x)}=\frac{\frac{3S_{n-1}^{-1/3}}{2}}{\frac{3.6273\,S_{n-1}^{-1/3}}{3}}-1=0.2406 \tag{38}$$

Looking at the left hand side of Eq. (38), the numerator is

$$E=q(x)_{\mathrm{cal.}}-q(x)$$

and the denominator is

$$\begin{aligned}\Delta q(x)-&\left\{1-\left(1-\frac{\Delta x}{x}\right)^{-0.1}\right\}q(x)\\ &=-q(x-\Delta x)+\left(1-\frac{\Delta x}{x}\right)^{-0.1}q(x)\end{aligned} \tag{39}$$

Here, $q(x)_{\mathrm{cal.}}$ denote the results by Eq. (16).
With Eq. (39) and Eq. (38), the corrected heat flux is

$$q(x)=\frac{q(x)_{\mathrm{cal.}}+0.2406\,q(x-\Delta x)}{1+0.2406\left(1-\frac{\Delta x}{x}\right)^{-0.1}} \tag{40}$$

Except for cases in which the correction is desired to be done near the leading edge of the plate, $\Delta x/x$ is usually small enough to be neglected from the denominator of Eq. (40).

$$q(x) = \frac{q(x)_{\text{cal.}} + 0.2406\, q(x - \Delta x)}{1.2406} \tag{41}$$

3. Practical procedure of the calculation;

If a temperature distribution corresponding to an unknown heat flux distribution is given, the x co-ordinate must be divided into small segments, each of which is Δx. This division should begin from the leading point where the wall temperature departs from the free stream temperature. The preferable size of a segment (i. e. the vaue of Δx) will be discussed in a later paragraph. Then calculate $q(x)$'s of every end point of segments by Eq. (16). An example of the procedure is the calculated $q(x)$ curve in Fig. 2.

The next step of the procedure is to correct every $q(x)$ by Eq. (40) or Eq. (41). This correction should begin from a point where the $q(x-\Delta x)$ is known with certainty. Such a point can always be obtained where he heating (or cooling) starts. An example of this is the corrected flux curve in Fig. 3.

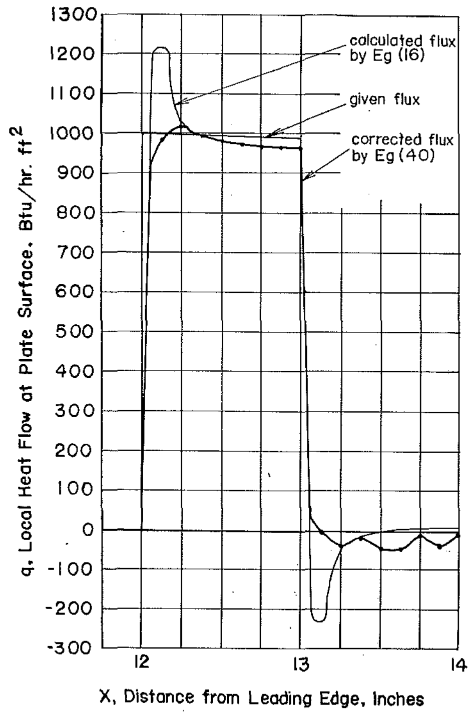

Fig. 3. A given flux, $q(x)$ by Eq. (16) and the corrected $q(x)$ by Eq. (40)

Discussion and Conclusion

The temperature distribution in Fig. 2 is a Smith and Shar's solution for a plate which is in an air flow of 100 ft/sec and 80°F. The plate is partly heated from a location of 12 inches downstream of the leading edge to 13 inches. The heat flux is 1000 Btu/h·ft² at the beginning of the heating section and is continued at the condition of $[q(x)/\rho u_s C_p \sqrt{C_f/2}]$=constant until the end point of the heating section. In Fig. 3 the given heat flux, calculated $q(x)$ by Eq. (16), and corrected $q(x)$ by Eq. (40) are plotted.

Comparing the corrected $q(x)$ with the given flux, the biggest error on the heating section is only less than 2% of the given flux. The accuracy on the unheated section can not be discussed by the percentage of error, because

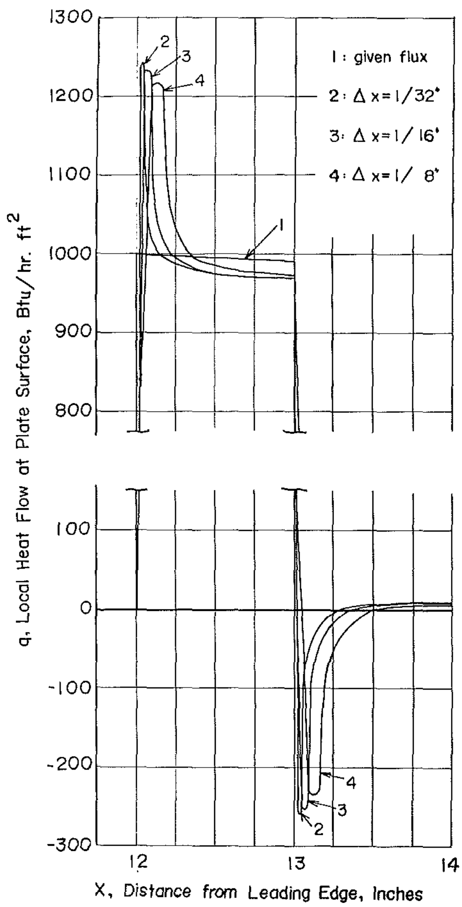

Fig. 4. Effect of the segment size on Eq. (16)

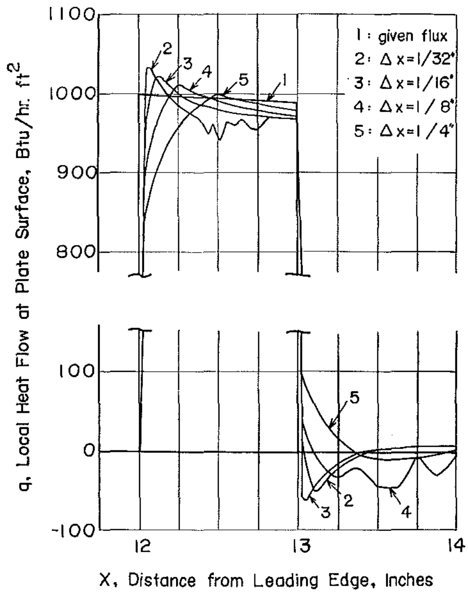

Fig. 5. Effect of the segment size on Eq. (40)

the given flux is zero. However, it may be recognized from the figure that the errors are in the same order of the heated section.

For the purpose of determining the effects of segment size Δx, $q(x)$'s by Eq. (16) and the corrected $q(x)$'s by Eq. (40) for several segment size are plotted in Fig. 4 and Fig. 5, respectively. As stated in the previous paragraph, the errors in a segment which has a big heat flux jump, are originating by that the $(dt_w(\xi)/d\xi)$ changes so drasticly that one can not approximate $(dt_w(\xi)/d\xi)\cdot(d\xi/dx^+)$ to be a constant. Accordingly, the segment size becomes smaller, and the error is larger.

As seen in Fig. 5, the errors are almost completely removed by the procedure of the cancellation. And better results are get by finer segment size. However, the error at the next segment of a big flux jump becomes comparatively large for a finer segment size. This also can be corrected by the same idea of the first segment if no flux jump occurs between the first and the second segment, but this procedure is too tedious for the purpose of the approximation. The best way seems to be in the selection of a segment size in which the second segment errors are in the same order of the first segment errors by the expense of the first one. According to the author's numerical trials, such a size are obtained by selecting the Δx which makes S_{n-1} to be arround 250.

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Appendix

The Spalding function by Gardner and Kestin is expressed by the following equation.

$$S_p(x^+, P_r) = a_0 + a_1 \log_e x^+ + a_2(\log_e x^+)^2 + a_3(\log_e x^+)^3 + a_4(\log_e x^+)^4 + a_5(\log_e x^+)^5 + a_6(\log_e x^+)^6 \qquad (\alpha)$$

In this equation $a_0 \sim a_6$ denote constants and those values are shown in Table 1. Average departure of the Spalding function by Eq. (α) from the Gardner and Kestin's numerical values was looked to be less than 0.5%. Eq. (α) also gives very close values for small x^+ to the values by Eq. (28), which is the asymptotic solution for $x^+ \to 0$.

The integrated Spalding function is obtained by integrating Eq. (α).

$$\int_0^{x^+} S_p(x^+, P_r)\,dx^+ = \{c_0 + c_1 \log_e x^+ + c_2(\log_e x^+)^2 + c_3(\log_e x^+)^3 + c_4(\log_e x^+)^4 + c_5(\log_e x^+)^5 + c_6(\log_e x^+)^6\}\,x^+ \qquad (\beta)$$

In this equation $c_0 \sim c_6$ denotes constant which values are also shown in Table 1. Fig. 1 is made from Eq. (β) for the convenience of the caluclation.

TABLE 1.

P_r	0.71	1.00	7.00	30.00	100.00	1000.00
a_0	$+4.79583\times10^{-1}$	$+5.37717\times10^{-1}$	$+1.03029$	$+1.67454$	$+2.50085$	$+5.38379$
a_1	-1.59709×10^{-1}	-1.79129×10^{-1}	-3.43143×10^{-1}	-5.56759×10^{-1}	-8.30628×10	-1.78636
a_2	$+2.68254\times10^{-2}$	$+3.00194\times10^{-2}$	$+5.67714\times10^{-2}$	$+9.19087\times10^{-2}$	$+1.37635\times10^{-1}$	$+2.98293\times10^{-1}$
a_3	-3.02140×10^{-3}	-3.37464×10^{-3}	-6.41328×10^{-3}	-1.04983×10^{-2}	-1.58345×10^{-2}	-3.45639×10^{-2}
a_4	$+2.57458\times10^{-4}$	$+2.90721\times10^{-4}$	$+5.80052\times10^{-4}$	$+9.50482\times10^{-4}$	$+1.40320\times10^{-3}$	$+2.94493\times10^{-3}$
a_5	-1.41127×10^{-5}	-1.61920×10^{-5}	-3.36835×10^{-5}	-5.39714×10^{-5}	-7.62916×10^{-5}	-1.49540×10^{-4}
a_6	$+3.34684\times10^{-7}$	$+3.88516\times10^{-7}$	$+8.21612\times10^{-7}$	$+1.27518\times10^{-6}$	$+1.72388\times10^{-6}$	$+3.15297\times10^{-6}$
c_0	$+7.19185\times10^{-1}$	$+8.06325\times10^{-1}$	$+1.54401$	$+2.50831$	$+3.74583$	$+8.06501$
c_1	-2.39602×10^{-1}	-2.68608×10^{-1}	-5.13720×10^{-1}	-8.33772×10^{-1}	-1.24498	-2.68122
c_2	$+3.99463\times10^{-2}$	$+4.47396\times10^{-2}$	$+8.52887\times10^{-2}$	$+1.38507\times10^{-1}$	$+2.07175\times10^{-1}$	$+4.47431\times10^{-1}$
c_3	-4.37365×10^{-3}	-4.90673×10^{-3}	-9.50575×10^{-3}	-1.55327×10^{-2}	-2.31800×10^{-2}	-4.97128×10^{-2}
c_4	$+3.38062\times10^{-4}$	$+3.83021\times10^{-4}$	$+7.73118\times10^{-4}$	$+1.25859\times10^{-3}$	$+1.83637\times10^{-3}$	$+3.78722\times10^{-3}$
c_5	-1.61208×10^{-5}	-1.84601×10^{-5}	-3.86132×10^{-5}	-6.16225×10^{-5}	-8.66349×10^{-5}	-1.68458×10^{-4}
c_6	$+3.34684\times10^{-7}$	$+3.88516\times10^{-7}$	$+8.21612\times10^{-7}$	$+1.27518\times10^{-6}$	$+1.72388\times10^{-6}$	$+3.15297\times10^{-6}$