Theorems on Characteristic Curves of Fan in Mine Ventilation*

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Extended characteristic curve

(a) for fan

Fig. 1 shows a general arrangement to measure h (head or depression) and Q (quantity) for plotting the curve.

It is a significant feature that another fan is installed at the point where the damper has been placed hitherto.

Fan, A, to be tested, of course revolves constantly and depression (h) and air quantity (Q) are measured by usual methods.

Fan, B, is capable of changing r.p.m. and direction of rotation freely.

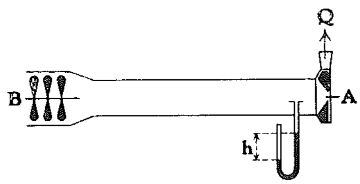

Fig. 1.

If fans A and B are acting in opposition to each other to move the air and if the head of B is equal to the head of A, there should be no movement of air, i. e., $Q=0$. By increasing r.p.m. of B still further, the current of air will be reversed, even though fan A continues to have constant revolution. Hence a characteristic curve of a fan in II-quadrant must exist.

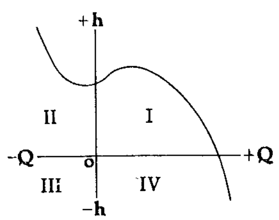

Fig. 2.

When fan B is operating in such a way as to assist fan A and its r.p.m. is increased gradually, the manometer reading h will be reversed at a certain point.

Thus, a characteristic curve in IV-quadrant must exist.

Fig. 2 shows the extended characteristics of fan.

(b) for resistance

The characteristics of resistance, $R=\frac{h}{Q^2}$, was restricted hitherto in I-quadrant. But the extended characteristics are shown in Fig. 3.

If the head or depression, $-h$, acts at a resistance, a quantity in an opposit

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* Synoptic description of the author's paper in the Japanese language; Jour. of the Mining Institute of Hokkaido, Vol. 8, No. 1, No. 2 and No. 4, 1952.

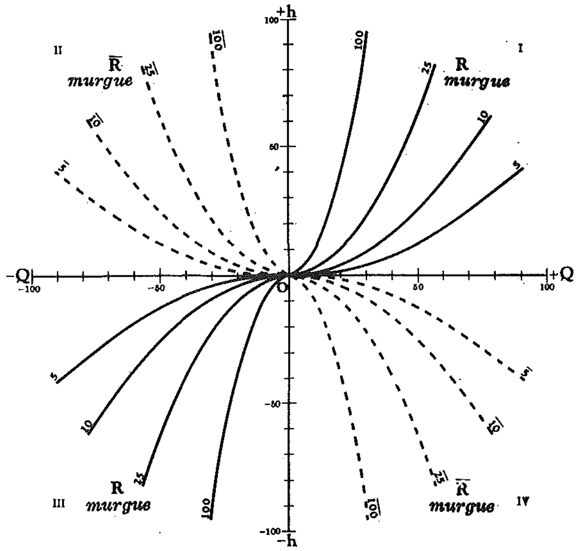


Fig. 3.

direction must exist, i. e., $-Q$.

Therefore the characteristics in III-quadrant must exist for a resistance.

The dotted line curve in II- and IV-quadrant is named "characteristics of imaginary resistance" which is a mirror image of the ordinary resistance which is named "real resistance" by the author.

The imaginary resistance is especially important when the fan-characteristics is reversed, as described later.

Graphical combination of specific resistance or equivalent orifice

Any value of resistance or equivalent orifice may be plotted in $h \cdot Q$ coordinate system readily and speedily with a special slide rule for mine ventilation which was invented in 1940 by the present author and has been widely used in Japan.

The combination of resistances may be considered similar to the combination of fans.

Fig. 4 and Fig. 5 represent the processes of combination of two resistances in series and in parallel respectively.

The graphical and superpositional combination of resistance or conductance does not have a practical importance in itself, but it lays a foundation on which a new theorem of transformation of fan-characteristics may be estab-

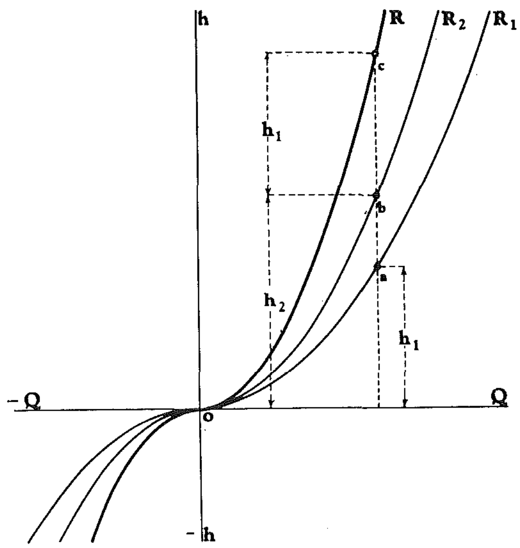


Fig. 4.

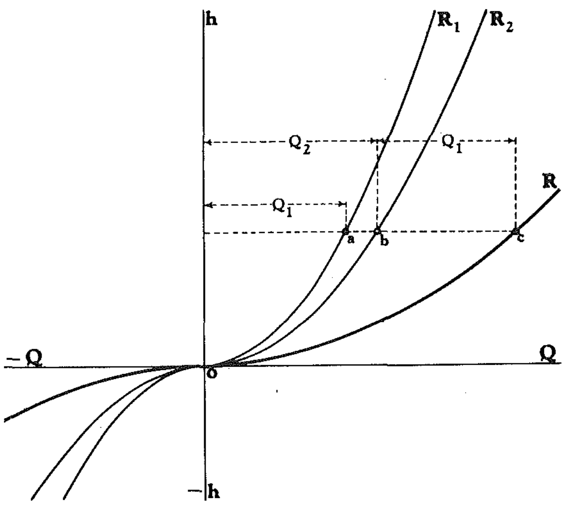


Fig. 5.

lished.

Transformation of Fan-characteristics

Any point on the characteristic curve of fan has a fixed value of h and Q. The fixed value of h and Q must correspond to a fixed value of resis-

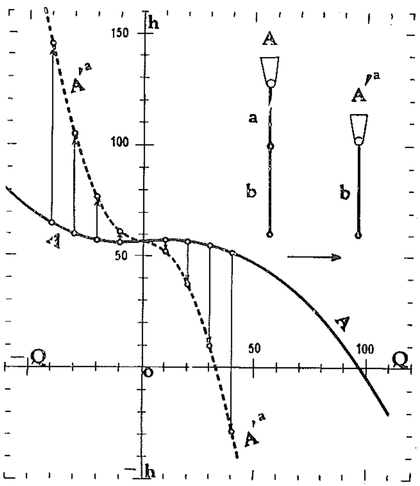


Fig. 6.

tance.

In other words, the characteristic curve of fan represents all values of external resistance.

Consequently, the fan-characteristics may be transformed in compensation for the omitted resistance which is connected to the fan in series or in parallel.

The new theorem is named "Equivalent depression source" by the author.

This theorem is divided into two types-one, in series, and other, in parallel.

(a) Transformation in series

In Fig. 6, Fan A is connected to two resistances, a and b, in series.

The dotted line curve represents the transformed characteristics when resistance a is omitted from the original circuit.

In this example, the value of resistance a is assumed to be $R_a = 0.05$ or $M_a = 50\,\mu$. Then the corresponding values of Q and h are read with the special slide rule or nomogram as follows,

$M = 50$	Q	10	20	30	40
	h	5	20	45	80

The transformed characteristics are plotted by subtraction, according to the values of h, from the original one, while in II-quadrant the value of h must be added.

The large letter is a symbol of the fan and the small letter expresses resistance.

A^a is a symbol of transformed characteristics of fan A with omitting resistance a in series.

Sometimes, the author merely uses A′ as a symbol, which implies the transformation in series.

(b) Transformation in parallel

In Fig. 7 Fan A is connected to two resistances, a and b, in parallel with each other.

The dotted line curve represents the transformed characteristics, when

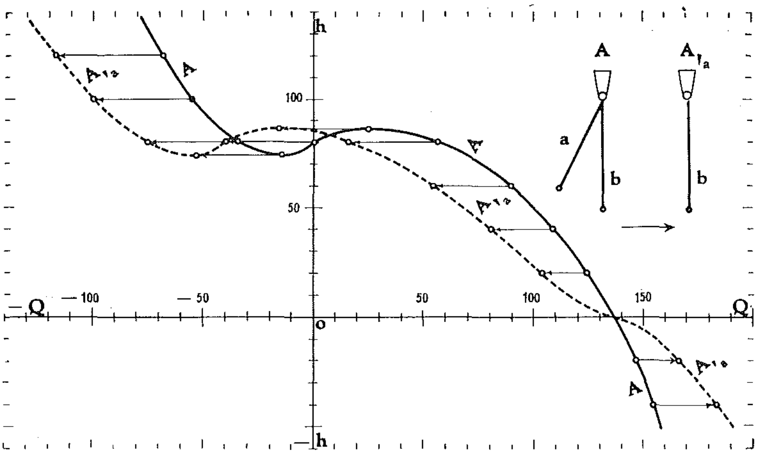


Fig. 7.

resistance a, one of the parallel, is omitted from the original circuit.

In this example, the value of resistance a is assumed to be $R_a=0.05$ or

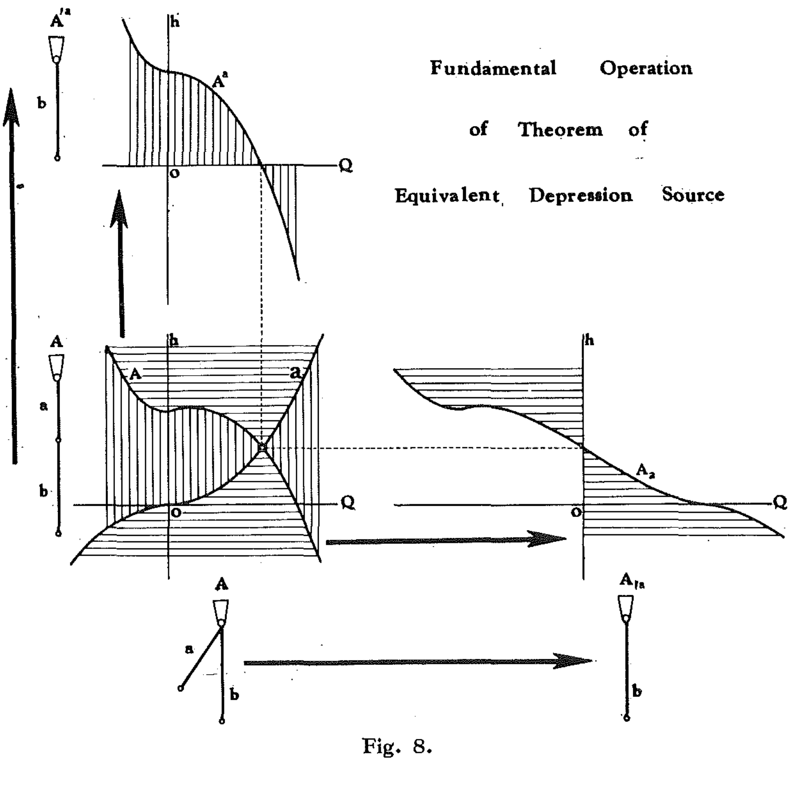


Fig. 8.

$M_a=50\ \mu$. Then the corresponding value of Q and h are as follows,

$M=50$	h	40	60	74	80	86	100	120
	Q	28.3	34.6	38.5	40	41.4	44.7	49

Transformed characteristics is plotted by subtraction, according to the values of Q from the original one, while in IV-quadrant the value of Q must be added.

A_a is a symbol of transformed characteristics of fan A with omitting resistance a in parallel.

Sometimes, the author merely uses A, as a symbol, which implies the transformation in parallel.

Fig. 8 shows a fundamental operation of theorem of equivalent depression source.

To obtain a transformed characteristic curve, the length of hatch-line which is drawn between curves A and a, is translated in a new $h \cdot Q$ coordinate system, taking the meaning of + or − as a sign of the length.

Combination of Fans

Ordinary methods of combination of fans are shown in Fig. 9 and Fig. 10.

A and B are symbols of individual fans and $\left.\frac{A}{B}\right|$ and $\overline{AB}$ are symbols of combined fans, in series and in parallel, respectively.

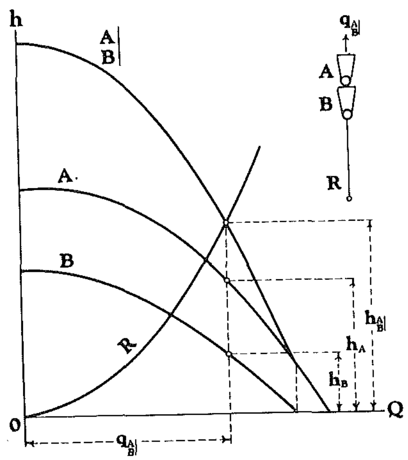


Fig. 9.

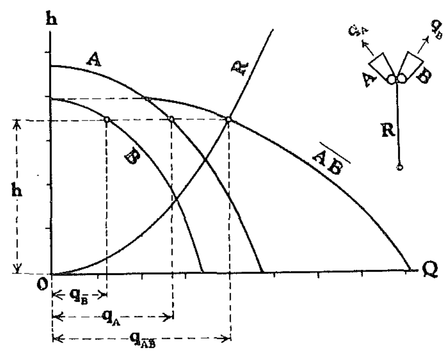


Fig. 10.

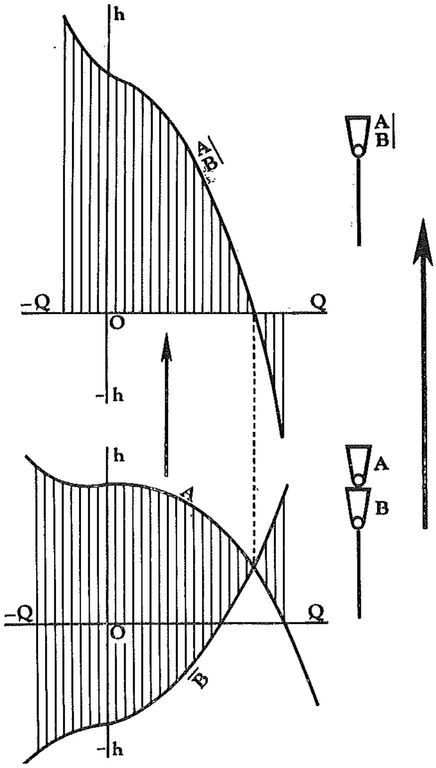


Fig. 11.

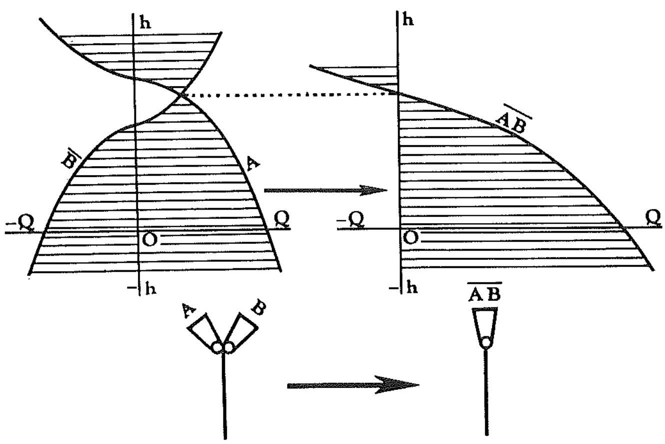


Fig. 12.

Fig. 11 and Fig. 12 represent the other type of methods used to obtain the same results of combination, as in Fig. 9 and Fig. 10. In the latter method, the characteristic curve of one of the fans must be inverted to the adjacent quadrant and the length of the hatch-line is translated to the new coordinate system.

Fig. 13 shows a combination method of fans which are acting oppositely. One of the fans must be in III-quadrant because the fan acts in $(-h)$ and $(-Q)$.

But for the translation of the length of hatch-line, the characteristics must be inverted to the adjacent quadrants, i. e. II-quadrant or IV-quadrant, owing to whether they are in series or in parallel, respectively.

The inverted characteristics of fan B is represented as $\overline{\text{B}}$.

Thus, $\left.\begin{matrix}\text{A}\\\text{B}\end{matrix}\right|$ and $\overline{\text{AB}}$ are easily plotted.

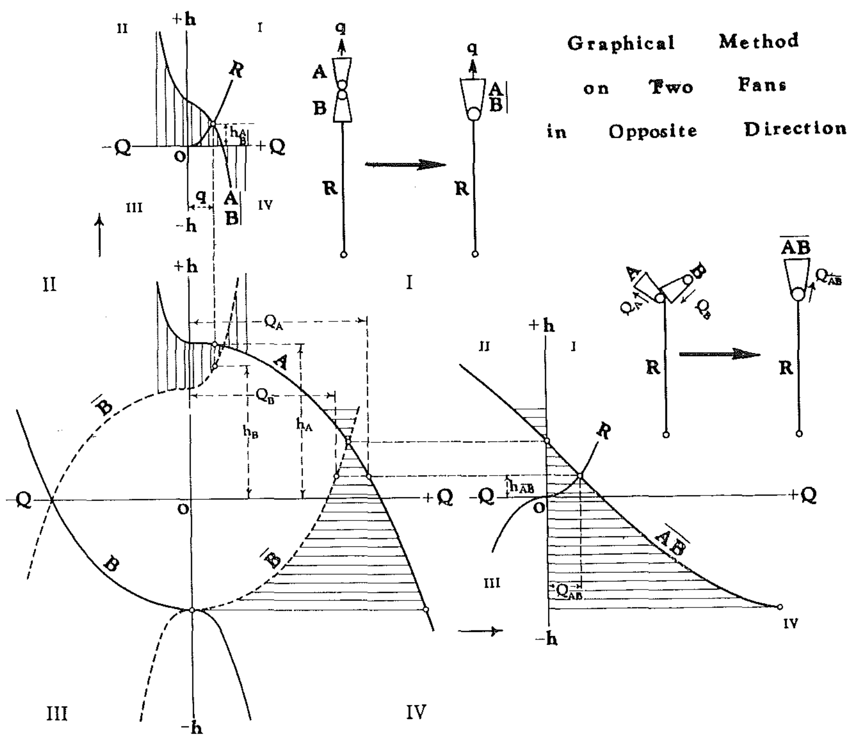


Fig. 13.

Application

In Fig. 14, two fans are instaled separately and the transforming processes are explained in it.

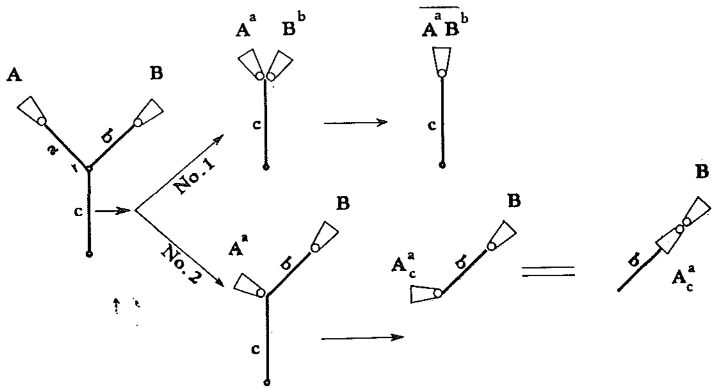


Fig. 14.

In Fig. 15, the characteristic curves of fan A and B are plotted in I-quadrant in the ordinary way and the transformation in series is applied to both characteristics, and ordinary fan combination in parallel is done.

a $=5\,\mu$, b $=12\,\mu$ and c $=4\,\mu$, $6.5\,\mu$ and $28\,\mu$ are assumed.

At the intersectional point of characteristics c $=4$ and $\overline{A'B'}$, $Q_c=94$ is read and the horizontal line through that point, meets with A′ or B′ at which $Q_A=65$ and $Q_B=29$ are read.

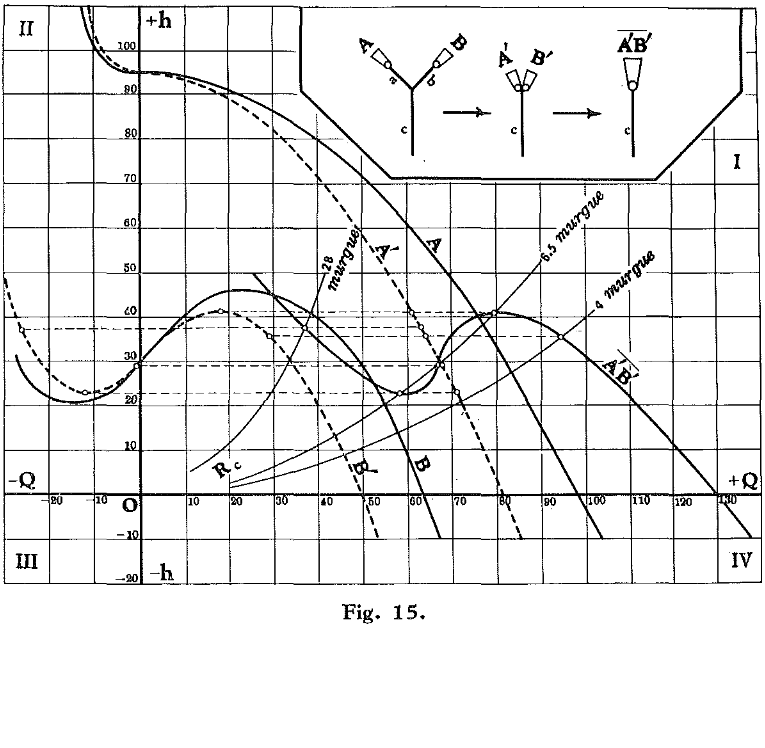


Fig. 15.

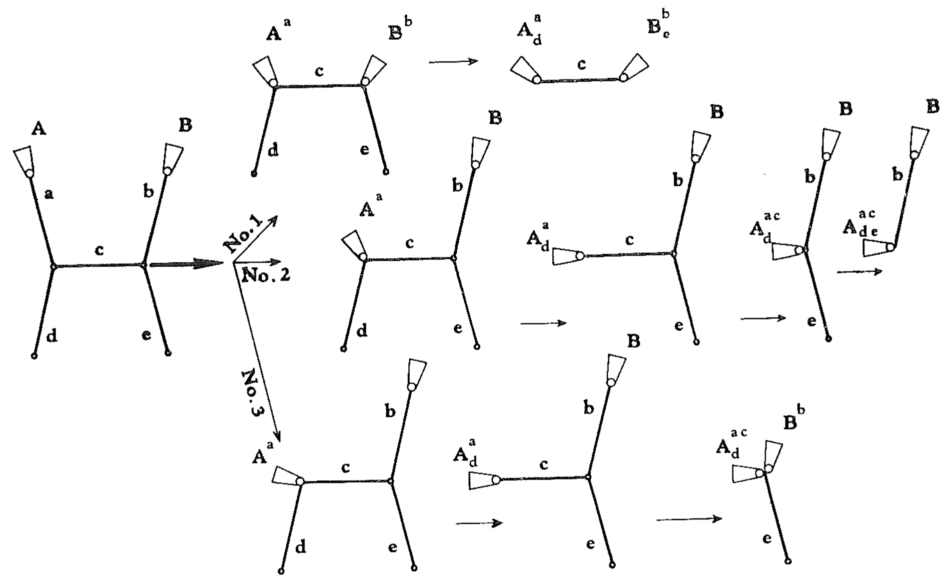


Fig. 16.

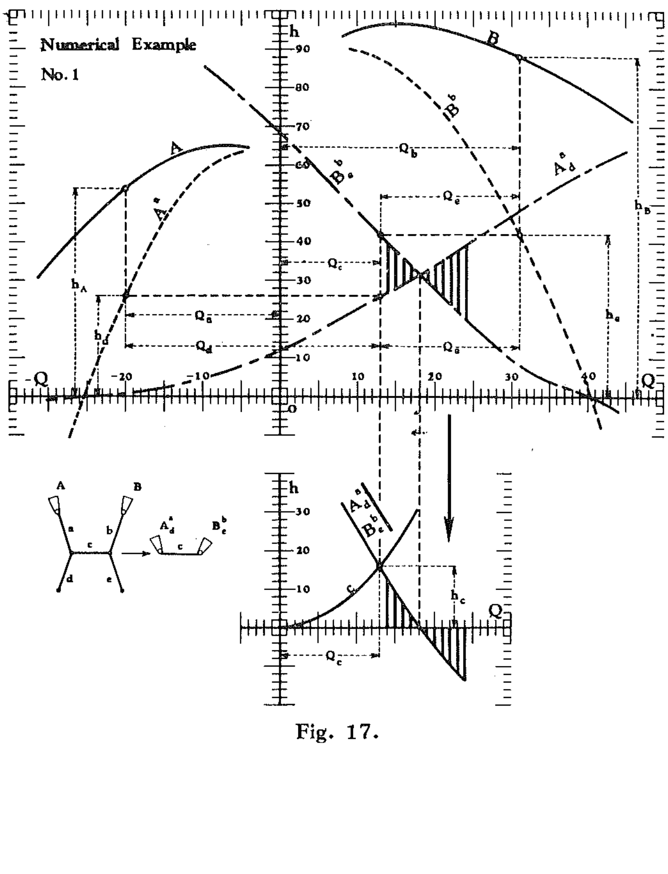


Fig. 17.

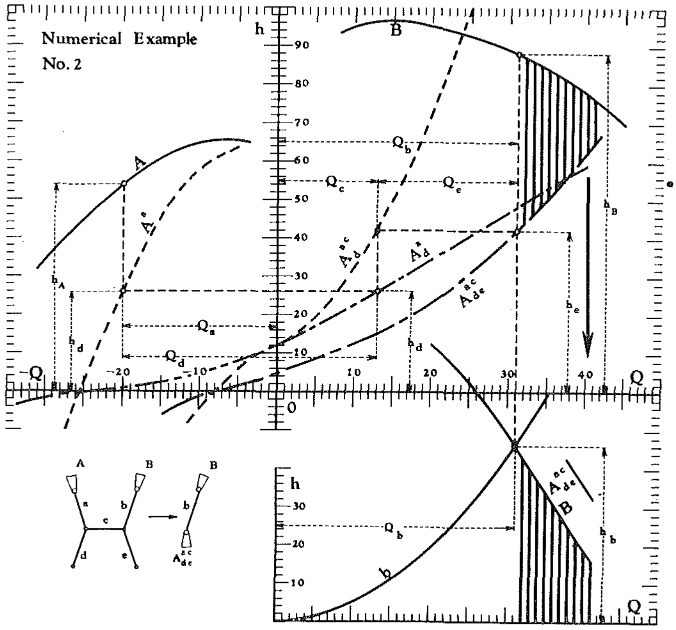


Fig. 18.

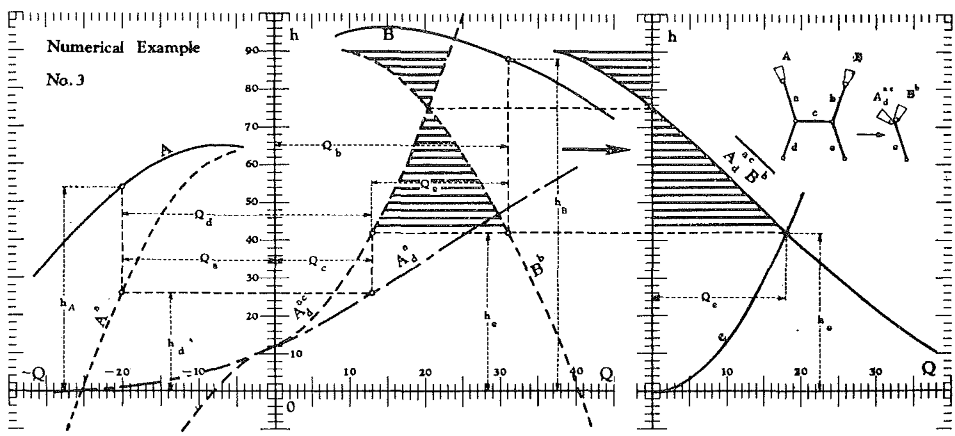


Fig. 19.

Similarly, by the curve of $c=6.5$ and $\overline{A'B'}$, three points of intersection are found as the operating points, which mean an unstable condition of ventilation.

At $Q_c=79$, both fans are acting normally, but at $Q_c=58$, $Q_A=+71$ and $Q_B=-13$ and the fan B has a counter current.

At $Q_c=67$, the fan B hardly works, $Q_B \doteqdot 0$, though it is continuing to

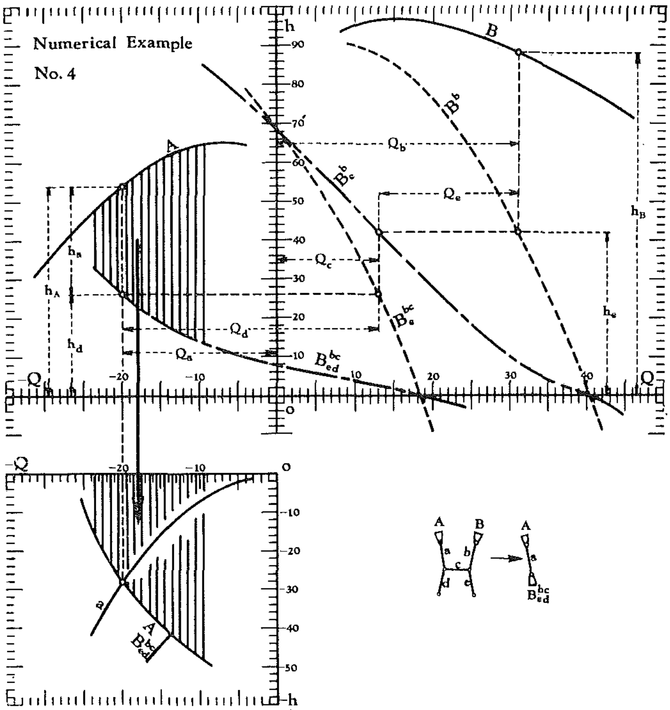


Fig. 20.

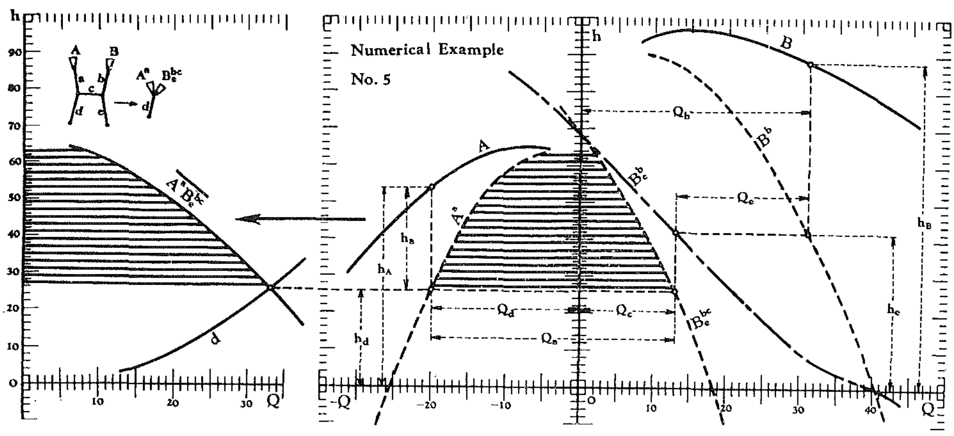


Fig. 21.

run.

By the curve c=25, $Q_c=37$ and $Q_A=63$ and $Q_B=-26$ are read. In this case fan B is stable but has a counter current.

In Fig. 16, the two fan problem is treated and transforming processes are explained.

Fig. 17, Fig. 18, Fig. 19, Fig. 20 and Fig. 21 show the same answers in the five processes, in which the characteristic curve of fan A is inverted to II-quadrant.

To obtain the answers, as shown below, the vertical lines and horizontal lines are drawn through the points of intersection, responding to the type of transformation in parallel or in series.

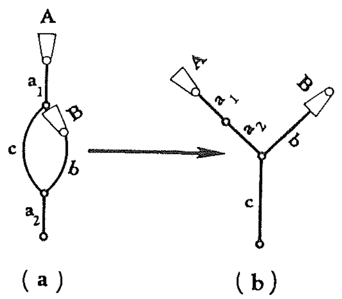


Fig. 22.

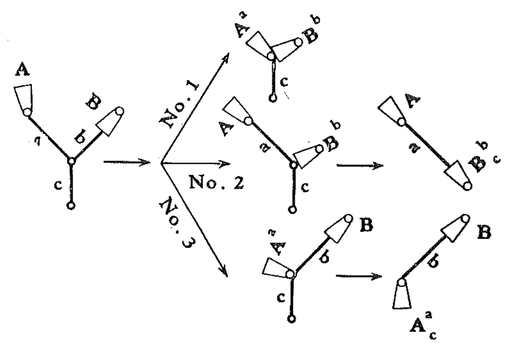


Fig. 23.

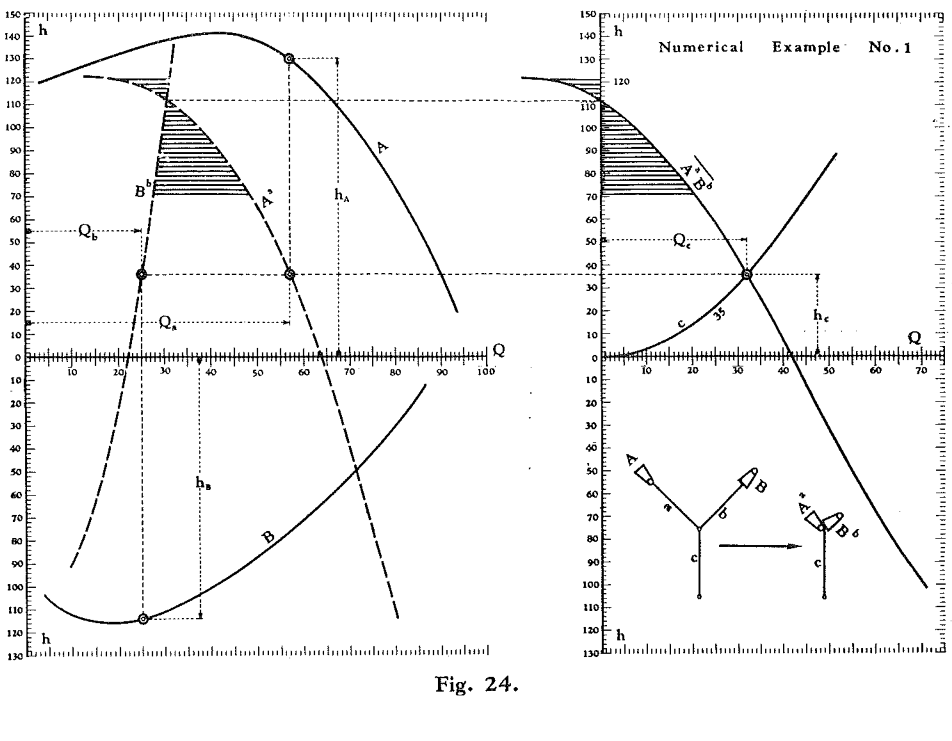


Fig. 24.

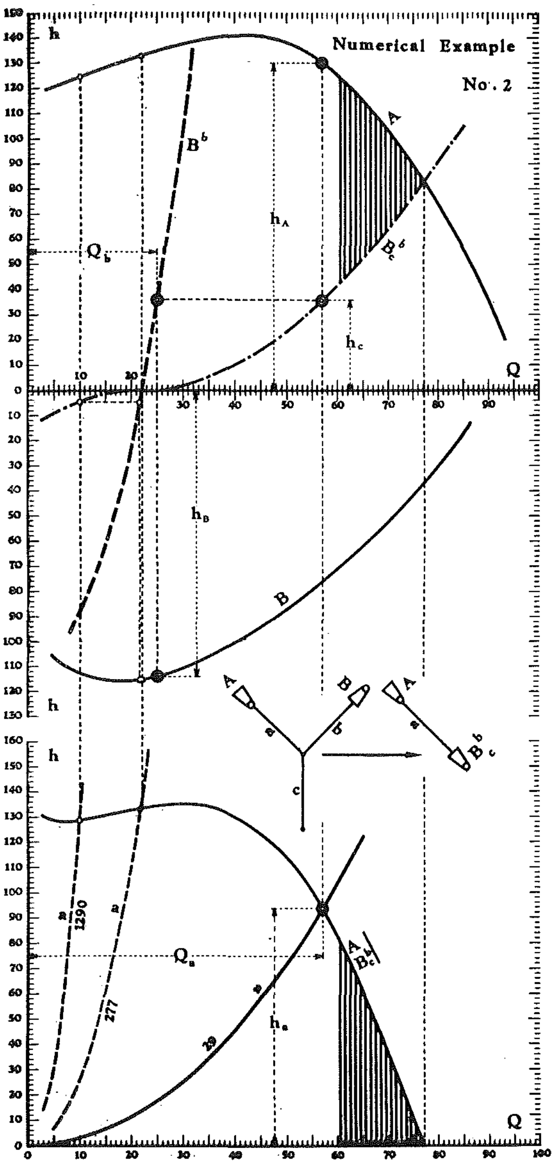

Numerical Example
No. 2
h
Q
A
B
a
b
c
hA
hB
hc
ha
Qb
Qa
1290
277
29

Fig. 25.

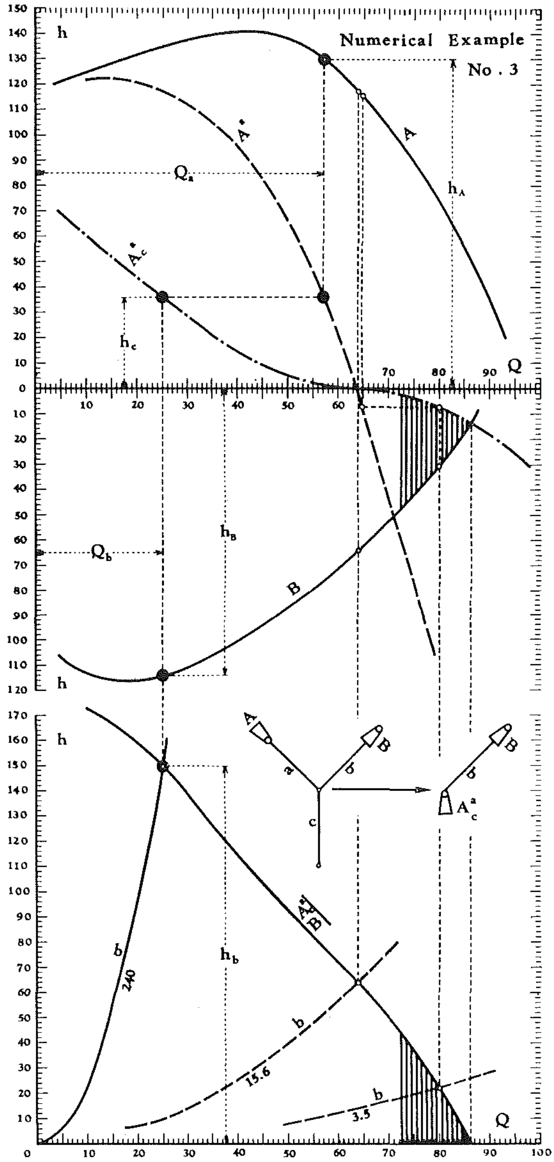

Numerical Example
No. 3
h
Q
A
B
a
b
c
hA
hB
hc
hb
Qa
Qb
240
15.6
3.5

Fig. 26.

a=70, b=48, c=95, d=24, e=130 and the characteristics of the two fans are given and the answers are as follows;

$$Q_a = 20 \quad Q_b = 31 \quad Q_c = 13 \quad Q_d = 33 \quad Q_e = 18$$
$$h_a = 28 \quad h_b = 46 \quad h_c = 16 \quad h_d = 26 \quad h_e = 42$$

Also $$h_A = h_a + h_d = 54 \quad \text{and} \quad h_B = 88$$

Fig. 22 (a) shows an underground booster fan problem, (b) modification of the circuit.

Fig. 23 shows the transforming processes.

Fig. 24, Fig. 25 and Fig. 26 are actual examples in which a=29 μ, b=240 μ and c=35 μ.

Characteristics of fan B are inverted to IV-quadrant and the answers are as follows;

$$Q_a = 57 \quad Q_b = 25 \quad Q_c = 32$$
$$h_a = 94 \quad h_b = 150 \quad h_c = 36$$

The dotted lines indicate the transformation in series and the chain lines indicate the transformation in parallel.

In Fig. 26 it is explained that the critical value of resistance b is 16.5 for the recirculation.

Theorem of Equivalent resistance

In the above description, single fan and single resistance are considered at the final step of the graphical solution, as in Fig. 27 (a). But, in Fig. 27 (b), the final resistance z has vanished and the characteristics A is transformed to the A^z, and it is obvious that the same result is obtained by both operations.

Fig. 28 (column 1) shows four cases of combination of two fans.

In (column 2), final resistance has vanished for which one of the fans is

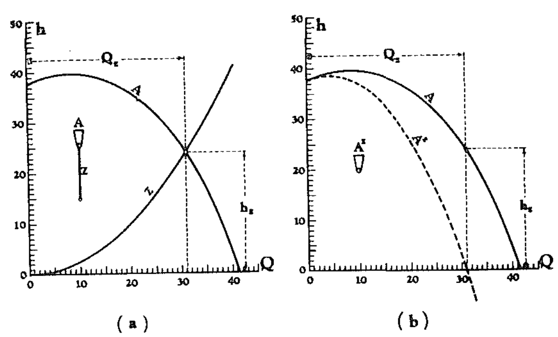


Fig. 27.

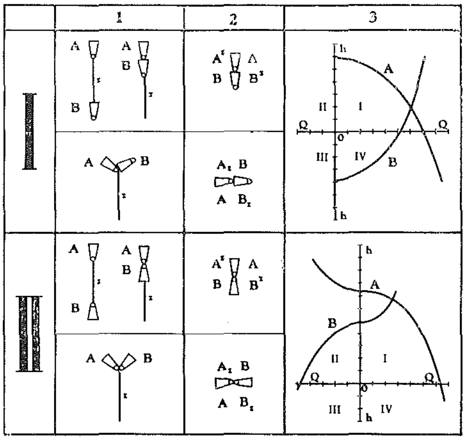


Fig. 28.

transformed. Thus the fans are arranged in one of two kinds of directions, same or opposite.

(column 3) shows two kinds of arrangement of the characteristics of fans, owing to the arrangement of fans.

In the transformation on inverted characteristics, its operation on the graph is addition but not subtraction.

Consequently the inverted characteristics of fan may be considered as the base on which superpositional addition of resistance or conductance may be done.

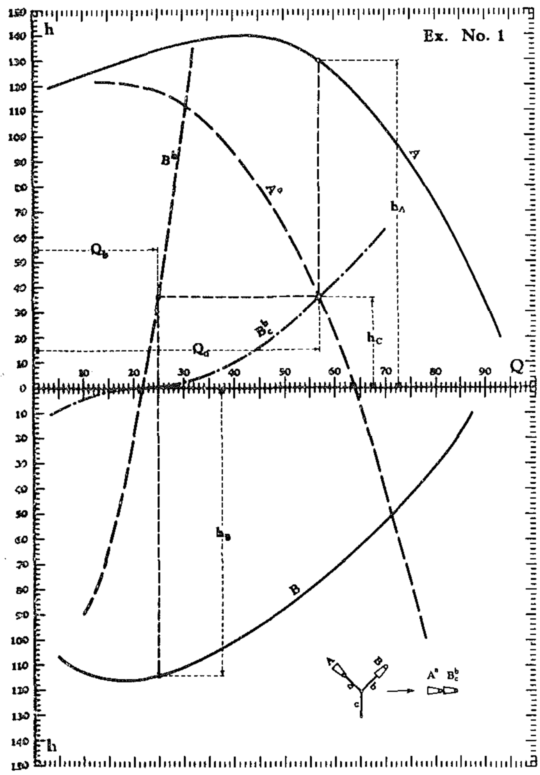


Fig. 29.

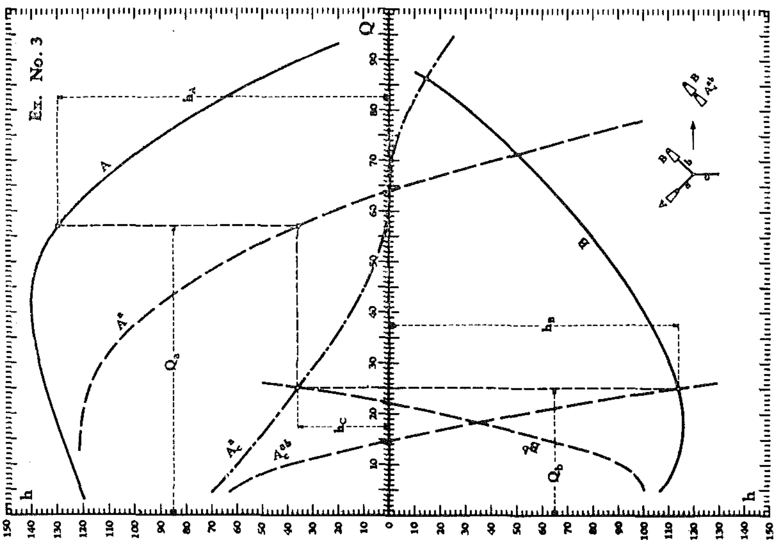


Fig. 31.

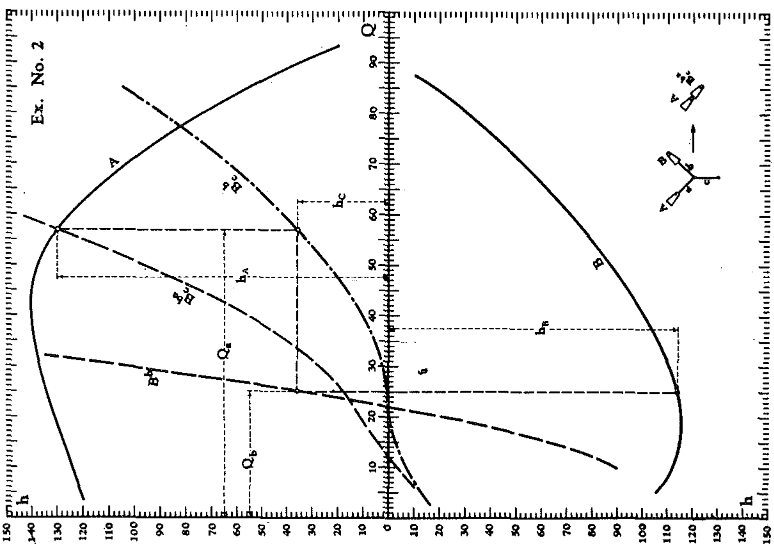


Fig. 30.

In other words, the inverted characteristics are equivalent resistance. Of course, the characteristics plotted in I-quadrant are transformed by the theorem of equivalent depression source but the inverted characteristics are superposed by the theorem of equivalent resistance.

Hence, the translation of length of hatch-line is not necessary.

Fig. 29, Fig. 30 and Fig. 31 show the new solution of the same problem, as in Fig. 24, Fig. 25 and Fig. 26.

Also, Fig. 32, Fig. 33 and Fig. 34 correspond to the Fig. 17, Fig. 18 and Fig. 19.

Fig. 35 shows a perfect and direct solution of diagonal system which is

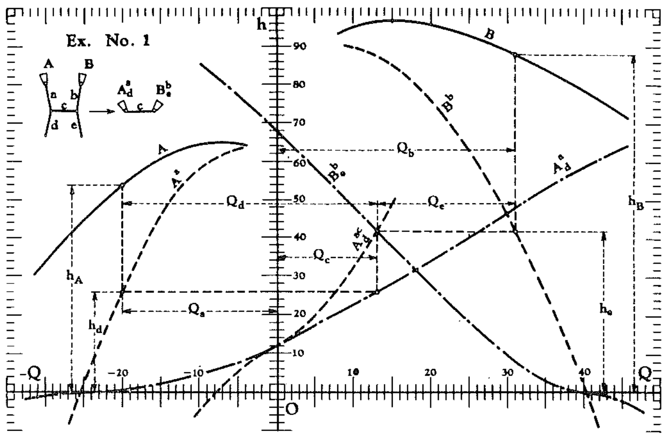


Fig. 32.

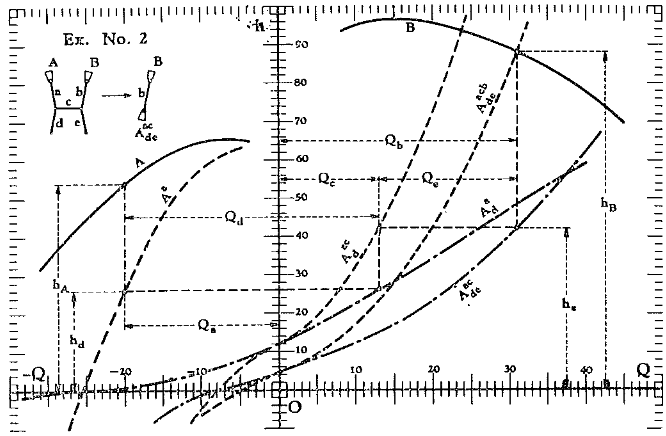


Fig. 33.

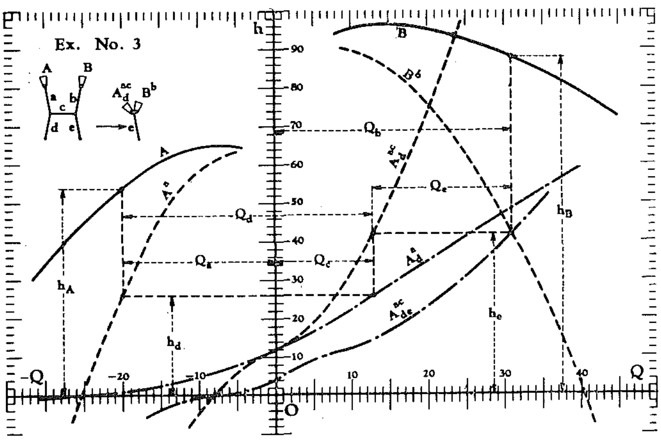


Fig. 34.

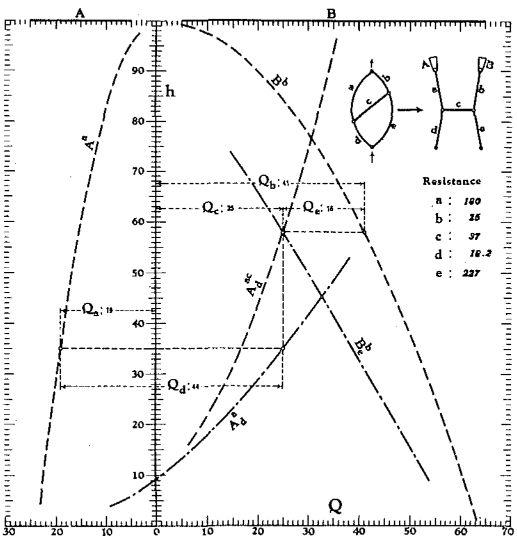


Fig. 35.

same fundamentally as the above example.

In this case, the constant depression of imaginary fans which act separately, must be considered.

In this example, a=180, b=25, c=37, d=18.2, e=227, are given, and answers are as follows.

$$Q_a = 19 \quad Q_b = 41 \quad Q_c = 25 \quad Q_d = 44 \quad Q_e = 16$$

Also, $$h = 100 \quad Q_{a+b} = 60$$

Total resistance $M = 28\ \mu$

Fig. 36 shows the principle for solving more complicated circuits. Of course, A, B and C are the imaginary fans which act at the constant depression.

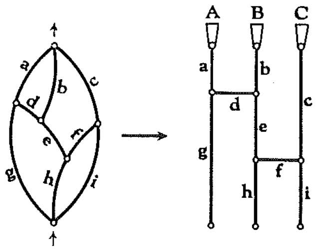


Fig. 36.

To solve the network of ventilating circuit it is necessary that the network must be simply reduced to the connections, in series and in parallel.

Fortunately, the triangular transformation for mine ventilation was reported by the same author in 1942 and it has been used also in U. S. S. R.

Diagram

$\Delta \rightleftarrows \curlywedge$

in Mine Ventilation

R: resistance A: conductance

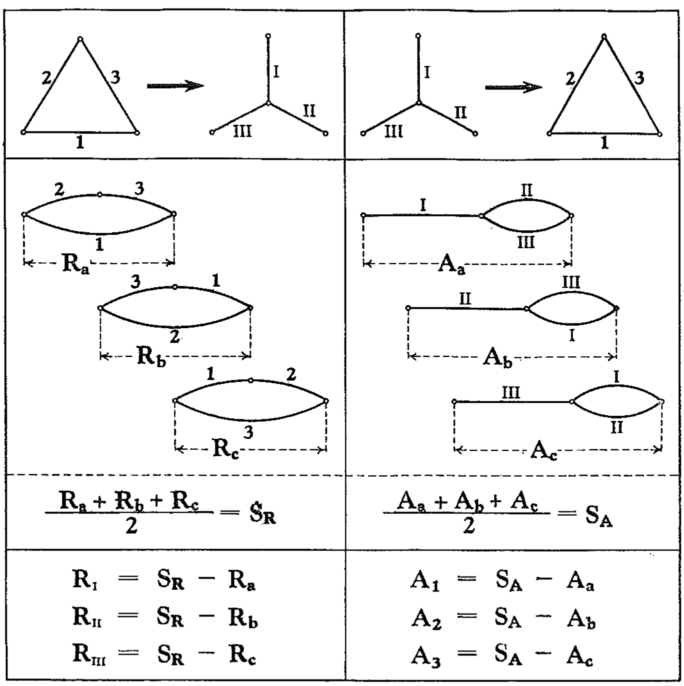


Fig. 37.

since 1957.

Fig. 37 is a diagram of Delta-Star (triangular transformation) in mine ventilation, as a method of direct approximation.

Conclusion

The author's opinions are as follows;

(1) The characteristic curve of fan may be plotted but cannot be expressed perfectly as a mathematical formula.

Hence the mathematical treatment of mine ventilation may come to a deadlock.

The graphical solution, above described, unilizes the actual characteristics of the fan itself and so it is a perfect method of solution.

(2) Various types of electrical analogue calculators have been developed but the characteristic curve of fan cannot be represented verbatim by electrical devices.

In many mines, several fans are acting simultaneously in one system of ventilation.

Hence, contrary to our expectation, these analogue machines do not have a practical meaning.

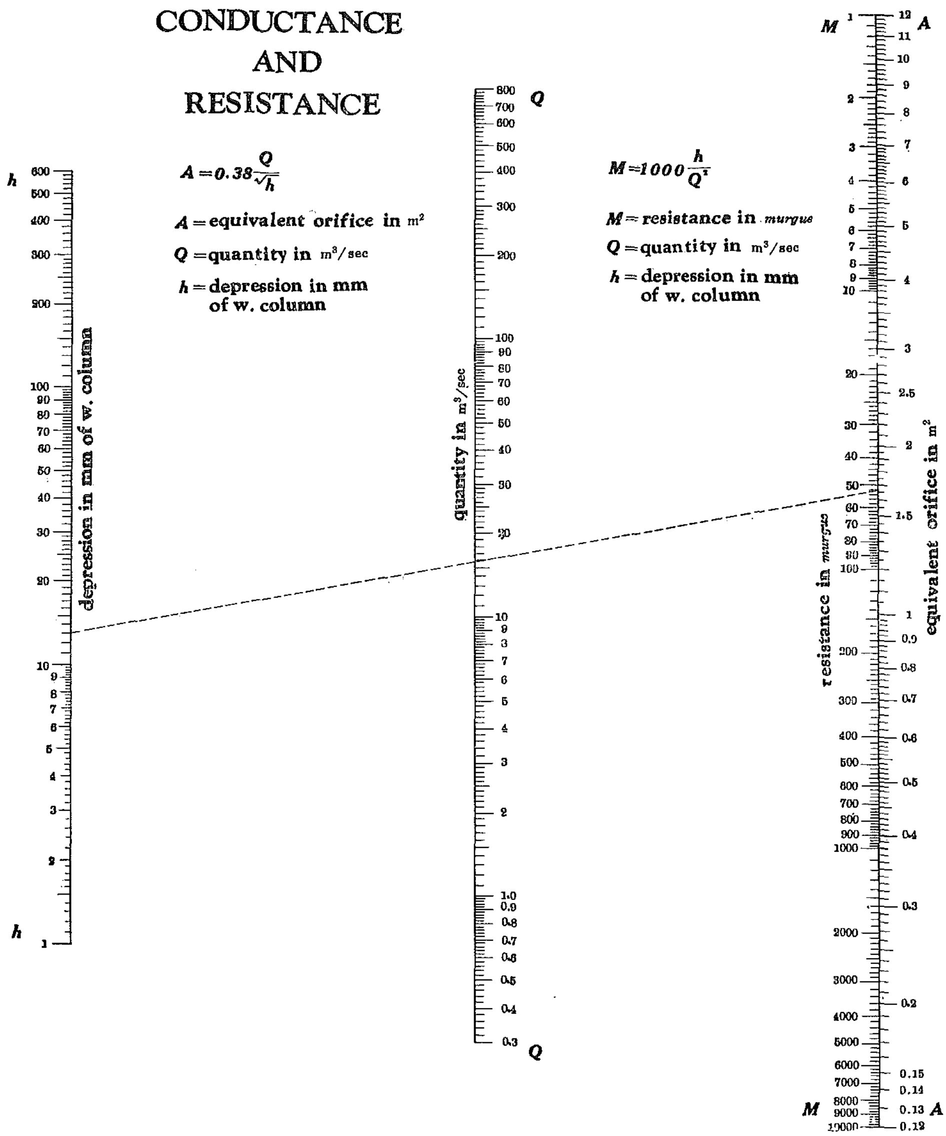
(3) The theoretical and practical importance of the Kirchhoff's law has been forgotten generally without adequate consideration.

The method of successive approximation has been proposed to solve the complex network of ventilation but its accuracy or percentage of error could not be checked rigorously because numerical examples have not been derived theoretically in advance.

There are also many cases in which Kirchhoff's law exercises its power for the actual planning of ventilation.

(4) Nomograms or special slide rule should be designed for the formulae on mine ventilation, at least, for "Resistance and Conductance".

A nomogram is attached for reference in which the calculation of two formulas is done simultaneously and the adjacent scales of A and M are useful for the mutual conversion of them.


CONDUCTANCE
AND
RESISTANCE
$A=0.38\frac{Q}{\sqrt{h}}$
A = equivalent orifice in m^2
Q = quantity in m^3/sec
h = depression in mm of w. column
$M=1000\frac{h}{Q^2}$
M = resistance in murgue
Q = quantity in m^3/sec
h = depression in mm of w. column
h
depression in mm of w. column
Q
quantity in m^3/sec
M
resistance in murgue
A
equivalent orifice in m^2