Dominant Zeros in Circuit Determinants

Teiichi KUROBE*

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Abstract

The responses of gain functions in the frequency domain and time domain are characterized by the zeros of circuit determinants. It is very difficult to solve higher order equations. However, the author obtained approximate expressions for dominant zeros and deduced a means by which to determine whether the dominant zeros are real or complex when the condition of dominant zero is satisfied. Some examples are presented which have complex dominant zeros and real dominant zero.

1. Introduction

The responses of gain function in the frequency domain and time domain are characterized by its poles which are the zeros of circuit determinants, however, since the circuit determinants are of a higher order in complicated electronic circuits using high frequency transistor models, it is very difficult to obtain zeros. If the condition of dominant zero is satisfied, the minimum zero and the sum of reciprocal of other zeros are necessary[1]. The former is the dominant zero and the latter is the excess phase coefficient. In the case of real dominant zero a simple expression is known[1], however, in the case of

* Department of Electronic Engineering, Faculty of Engineering, Hokkaido University, Sapporo, Japan.

complex dominant zeros it is not known. The author deduced a method by which to determine whether the dominant zero is real or complex and obtained the approximate expressions of dominant zeros and excess phase coefficient in the case of complex dominant zeros. More accurate expression than that known was also deduced by a different method in the case of real dominant zero.

2. Discrimination of Dominant Zeros

Let a circuit determinant be of the nth order,

$$D = H(1 + b_1 p + b_2 p^2 + b_3 p^3 + \cdots + b_n p^n) \tag{1}$$

where H is constant.

From the relations between zeros and coefficients, we obtain

$$\left.\begin{aligned} b_1 &= -\sum_{i=1}^{n} \frac{1}{p_i} \\ b_2 &= \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{1}{p_i} \frac{1}{p_j} \\ b_3 &= -\sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=j+1}^{n} \frac{1}{p_i} \frac{1}{p_j} \frac{1}{p_k} \end{aligned}\right\} \tag{2}$$

p_i, p_j, p_k are the zeros of D and their real parts are negative in stable circuits. Let $|p_1| \leq |p_2| \leq |p_3| \cdots \leq |p_n|$, the condition of dominant zero is $|p_1| \ll |p_2| \leq |p_3| \leq \cdots$ (in the case of real dominant zero) or $|p_1| = |p_2| \ll |p_3| \leq \cdots$ (in the case of complex dominant zeros).

From equation (2), b_1^2/b_2 is

$$\begin{aligned} \frac{b_1^2}{b_2} &= \frac{(\tau_1 + \tau_2 + t_0)^2}{\tau_1 \tau_2 + (\tau_1 + \tau_2) t_0 + \varDelta} \\ &= 2\left\{ \frac{1 + \dfrac{\tau_1 + \tau_2}{\tau_1 \tau_2} t_0 + \dfrac{t_0^2}{2\tau_1 \tau_2} + \dfrac{\tau_1^2 + \tau_2^2}{2\tau_1 \tau_2}}{1 + \dfrac{\tau_1 + \tau_2}{\tau_2 \tau_2} t_0 + \dfrac{\varDelta}{\tau_1 \tau_2}} \right\} \end{aligned} \tag{3}$$

where

$$\left.\begin{aligned} &\tau_1 = -\frac{1}{p_1} \qquad \tau_2 = -\frac{1}{p_2} \qquad t_0 = -\sum_{i=3}^{n} \frac{1}{p_i} \\ &\varDelta = \sum_{i=3}^{n} \sum_{j=i+1}^{n} \frac{1}{p_i} \frac{1}{p_j} \end{aligned}\right\} \tag{4}$$

In the case where the minimum zero is real, $(\tau_1^2 + \tau_2^2)/2\tau_1 \tau_2 \geq 1$ and in the case where the minimum zeros are complex, $(\tau_1^2+\tau_2^2)/2\tau_1\tau_2 \leq 1$. When $(\tau_1^2+\tau_2^2)/2\tau_1\tau_2=1$, the bracket component of equation (3) is

$$A_0=\frac{2+\dfrac{\tau_1+\tau_2}{\tau_1\tau_2}t_0+\dfrac{t_0^2}{2\tau_1\tau_2}}{1+\dfrac{\tau_1+\tau_2}{\tau_1\tau_2}t_0+\dfrac{\Delta}{\tau_1\tau_2}} \tag{5}$$

In the case of real minimum zero,

$$\frac{b_1^2}{b_2} \geq 2A_0 \tag{6}$$

In the case of complex minimum zeros,

$$\frac{b_1^2}{b_2} \leq 2A_0 \tag{7}$$

Since Δ in the equation (5) is unknown, A_0 is not decided. In the case of $\Delta=0$, A_0 is $A_{0\max}$ and the values of $A_{0\max}$ are shown in Table 1 where t_0 is the parameter. In the case of $\Delta=$max., A_0 is $A_{0\min}$. The condition of $\Delta=$max is as follows:

$$\Delta=\tau_3(\tau_4+\tau_5+\cdots)+\tau_4(\tau_5+\tau_6+\cdots)+\tau_5(\tau_6+\tau_7+\cdots)+\cdots$$

$$\tau_3+\tau_4+\cdots+\tau_n=t_0$$

Therefore

$$\Delta=\tau_3(t_0-\tau_3)+\tau_4(t_0-\tau_3-\tau_4)+\tau_5(t_0-\tau_3-\tau_4-\tau_5)+\cdots$$

The conditions of $\Delta=$max. are

$$\frac{\partial\Delta}{\partial\tau_3}=t_0-2\tau_3-\tau_4-\tau_5-\cdots\cdots\cdots=0$$

$$\frac{\partial\Delta}{\partial\tau_4}=t_0-\tau_3-2\tau_4-\tau_5-\cdots\cdots\cdots=0$$

$$\frac{\partial\Delta}{\partial\tau_5}=t_0-\tau_3-\tau_4-2\tau_5-\cdots\cdots\cdots=0$$

$$\vdots$$

$$\frac{\partial\Delta}{\partial\tau_n}=t_0-\tau_3-\tau_4-\tau_5-\cdots-2\tau_n=0$$

From above relations, we obtain

$$\tau_3=\tau_4=\tau_5=\cdots\cdots=\tau_n=\frac{t_0}{n-2} \tag{8}$$

The value of Δ which satisfies equation (8) is

$$\Delta_{\max}=\frac{t_0^2}{(n-2)^2}\left(\frac{n^2}{2}-\frac{5}{2}n+3\right) \qquad (9)$$
$$n\geq 4$$

The values of $\Delta_{\max}$ are $0.25\,t_0^2$ ($n=4$), $0.33\,t_0^2$ ($n=5$), $0.38\,t_0^2$ ($n=6$), $0.42\,t_0^2$ ($n=8$) and $0.5\,t_0^2$ ($n=\infty$). The values of $A_{0\min}$ in the case of $n=\infty$ are shown in Table 1. In the case of $n=2$, $A_0=2$ and in the case of $n=3$, A_0 is equal to $A_{0\max}(n\geq 4)$ as shown in Table 1. $A_{0\min}$ of $n=4\sim\infty$ are the values between $A_{0\max}$ and $A_{0\min}$ ($n=\infty$).

TABLE 1. Values of $A_{0\max}$ and $A_{0\min}$

A_0 \ n \ t_0		$0.1\ b_1$	$0.2\ b_1$	$0.3\ b_1$
A_0	3	1.72	1.56	1.50
$A_{0\max}$	≥ 4			
$A_{0\min}$	∞	1.69	1.47	1.33

Using $A_{0\max}$ and $A_{0\min}$ instead of A_0 in equations (6) and (7), we have

$$\left.\begin{array}{lll} \dfrac{b_1^2}{b_2}\geq & 2A_{0\min} & \text{(real minimum zero)} \\ \dfrac{b_1^2}{b_2}\leq & 2A_{0\max} & \text{(complex minimum zero)} \end{array}\right\} \qquad (10)$$

If the real minimum zero is dominant zero, $b_1^2/b_2>2A_{0\max}$ which will be clear later, therefor we can discriminate the dominant zero by equation (11).

$$\left.\begin{array}{lll} \dfrac{b_1^2}{b_2}> & 2A_{0\max} & \text{(real dominant zero)} \\ \dfrac{b_1^2}{b_2}\leq & 2A_{0\max} & \text{(complex dominant zeros)} \end{array}\right\} \qquad (11)$$

TABLE 2. Relations between $A_{\max}$, $A_{\min}|_{n=\infty}$ and ω_1/σ_1

ω_1/σ_1 \ A \ t_0	$0.1\ b_1$		$0.2\ b_1$		$0.3\ b_1$	
	$A_{\max}$	$A_{\min}$	$A_{\max}$	$A_{\min}$	$A_{\max}$	$A_{\min}$
0	1.72	1.69	1.56	1.47	1.50	1.33
1	1.02	1.00	1.06	1.00	1.13	1.00
2	0.46	0.45	0.54	0.51	0.65	0.58
4	0.14	0.14	0.18	0.17	0.24	0.21

If the complex dominant zeros are p_1, $p_2=\sigma_1\pm j\omega_1$, the bracket component of equation (3) changes due to the value of ω_1/σ_1. Let the bracket component be A_{max} where $\Delta=0$ and A_{min} where $\Delta=\Delta_{max}$, the relations between A_{max}, $A_{min}(n=\infty)$ and ω_1/σ_1 are shown in Table 2. A_{min} of $n=4\sim\infty$ are the values between A_{max} and $A_{min}(n=\infty)$. $b_1^2/b_2=2A_{min}\sim 2A_{max}$ and differences between A_{max} and A_{min} are very small.

3. Approximate Expressions of Complex Dominant Zeros

If the dominant zeros are complex conjugate,

$$b_1=\tau_1+\tau_2+t_0 \tag{12}$$

$$b_2=\tau_1\tau_2+\Delta_1 \tag{13}$$

$$b_3=\tau_1\tau_2 t_0+\Delta_2 \tag{14}$$

where

$$\tau_1=-\frac{1}{p_1}=-\frac{1}{\sigma_1+j\omega_1} \qquad \tau_2=-\frac{1}{p_2}=-\frac{1}{\sigma_1-j\omega_1}$$

$$\Delta_1=(\tau_1+\tau_2)t_0+\Delta$$

$$\Delta_2=(\tau_1+\tau_2)\sum_{i=3}^{n}\sum_{j=i+1}^{n}\tau_i\tau_j+\sum_{i=3}^{n}\sum_{j=i+1}^{n}\sum_{k=j+1}^{n}\tau_i\tau_j\tau_k$$

t_0 and Δ are equal to equation (4).

From equations (12)$\sim$(14), we obtain

$$\sigma_1=-\frac{1}{2}\frac{\tau_1+\tau_2}{\tau_1\tau_2}=-\frac{1}{2}\left\{\frac{b_1}{b_2-\Delta_1}-\frac{b_3-\Delta_2}{(b_2-\Delta_1)^2}\right\} \tag{15}$$

$$t_0=\frac{b_3-\Delta_2}{b_2-\Delta_1} \tag{16}$$

The values of Δ_1 and Δ_2 are unknown, so that the approximate expressions should be derived. Since $(\tau_1+\tau_2)t_0\gg\Delta$ under the condition of dominant zero, we assume $\Delta=0$ in the first term of equation (15) and since the second term of equation (15) is considerablly smaller than the first term, we assume that the second term is $b_3/b_2\{b_2-\Delta_1|_{\Delta=0}\}$. Then σ_1 is

$$\sigma_1\simeq-\frac{1}{2}\frac{b_1}{b_2}\left\{\frac{1-K_1}{1-K_1K_2(1-K_1)}\right\} \tag{17}$$

where

$$K_1=\frac{b_3}{b_1b_2} \qquad K_2=\frac{b_1^2}{b_2}$$

Since $(1-\varDelta_2/b_3)/(1-\varDelta_1/b_2)$ is nearly equal to 1 under the condition of dominant zero, t_0 will be

$$t_0 \simeq \frac{b_3}{b_2} \tag{18}$$

The value of ω_1 is

$$\omega_1^2 = \frac{1}{b_2-\varDelta_1} - \sigma_1^2 \simeq \frac{1}{b_2\{1-K_1K_2(1-K_1)\}} - \sigma_1^2 \tag{19}$$

When $\omega_1=0$, the dominant zeros are real and of a second order. When $\omega_1^2<0$, the minimum zero is real and dominant if $|\omega_1|$ is near $|\sigma_1|$.

Since the above approximate expressions were assumed $\varDelta=0$ or $\varDelta_1=\varDelta_2=0$, their errors must be examined. For this purpose we compare equations (17) and (18) with equations (15) and (16) where $\varDelta=\varDelta_{\max}$, $\varDelta_1=\varDelta_{1\max}$, $\varDelta_2=\varDelta_{2\max}$. $\varDelta_{\max}$ is equal to equation (9) and $\varDelta_{1\max}$ is equal to $\{(b_1-t_0)t_0+\varDelta_{\max}\}$.

From $\dfrac{\partial\varDelta_2}{\partial\tau_3}=\dfrac{\partial\varDelta_2}{\partial\tau_4}=\cdots=\dfrac{\partial\varDelta_2}{\partial\tau_n}=0$, the condition of $\varDelta_2=\max$ is

$$\tau_3=\tau_4=\cdots=\tau_n=\frac{t_0}{n-2} \tag{20}$$

Then $\varDelta_{2\max}$ is

$$\begin{aligned}\varDelta_{2\max} = {} & (b_1-t_0)\left(\frac{t_0}{n-2}\right)^2\underset{n\geq 4}{\left(\frac{n^2}{2}-\frac{5n}{2}+3\right)} \\ & +\left(\frac{t_0}{n-2}\right)^3\left\{\underset{n\geq 5}{\left(\frac{n^2}{2}-\frac{7n}{2}+6\right)}+\underset{n\geq 6}{\left(\frac{n^2}{2}-\frac{9n}{2}+10\right)}\right. \\ & \left.+\underset{n\geq 7}{\left(\frac{n^2}{2}-\frac{11n}{2}+15\right)}+\underset{n\geq 8}{\left(\frac{n^2}{2}-\frac{13n}{2}+21\right)}+\cdots\right\}\end{aligned} \tag{21}$$

As σ_1 and t_0 of equations (15) and (16) where $\varDelta_1=\varDelta_{1\max}$, $\varDelta_2=\varDelta_{2\max}$ $(n=\infty)$ are assumed to be true, the errors of equations (17) and (18) are shown in Table 4 where $b_1^2/b_2=2A_{\min}|_{n=\infty}$. The errors of ω_1 are of the same order as σ_1. The true σ_1 and t_0 are larger than the values of equations (17) and (18) and smaller than the values of equations (15) and (16) where $\varDelta_1=\varDelta_{1\max}$, $\varDelta_2=\varDelta_{2\max}$. Since the errors in Table 3 are considered to be maximum, the true errors will be smaller than these.

TABLE 3. Maximum Errors of Equations (17) and (18) (%)
$b_1^2/b_2 = 2A_{min}|_{n=\infty}$

ω_1/σ_1 \ Eq. \ t_0	0.1 b_1		0.2 b_1		0.3 b_1	
	(17)	(18)	(17)	(18)	(17)	(18)
0	< 0.5	18.3	1.6	33.6	18.0	46.0
1	< 0.5	9.9	< 0.5	20.0	3.4	30.1
2	< 0.5	4.3	< 0.5	9.1	< 0.5	14.5
4	< 0.5	1.4	< 0.5	2.9	< 0.5	4.8

4. Approximate Expression of Real Dominant Zero

If the dominant zero is real,

$$b_1 = -\frac{1}{p_1} + t_0 \tag{22}$$

$$b_2 = -\frac{t_0}{p_1} + \Delta \tag{23}$$

where

$$t_0 = -\sum_{i=2}^{n} \frac{1}{p_i} \qquad \Delta = \sum_{i=2}^{n} \sum_{j=i+1}^{n} \frac{1}{p_i} \frac{1}{p_j}$$

From equations (22) and (23), we obtain

$$p_1 = -\frac{b_1 - \sqrt{b_1^2 - 4(b_2 - \Delta)}}{2(b_2 - \Delta)} \tag{24}$$

$$t_0 = \frac{1}{2}\left\{b_1 - \sqrt{b_1^2 - 4(b_2 - \Delta)}\right\} \tag{25}$$

Since $\Delta \ll b_2$ under the condition of dominant zero,

$$p_1 \simeq -\frac{b_1}{2b_2}\left\{1 - \sqrt{1 - \frac{4b_2}{b_1^2}}\right\} \tag{26}$$

$$t_0 \simeq \frac{1}{2}\left\{b_1 - \sqrt{b_1^2 - 4b_2}\right\} \tag{27}$$

If $b_1^2 \gg 4b_2$,

$$p_1 \simeq -\frac{1}{b_1} \tag{28}$$

$$t_0 \simeq \frac{b_2}{b_1} \tag{29}$$

From $\frac{\partial \varDelta}{\partial \tau_2} = \frac{\partial \varDelta}{\partial \tau_3} = \cdots = \frac{\partial \varDelta}{\partial \tau_n} = 0$, the condition of $\varDelta = \max.$ is $\tau_2 = \tau_3 = \cdots = \tau_n = t_0/(n-1)$.

Then $\varDelta_{\max}$ is

$$\varDelta_{\max} = \left(\frac{t_0}{n-1}\right)^2 \left(\frac{n^2}{2} - \frac{3n}{2} + 1\right) \tag{30}$$

As p_1 and t_0 of equations (24) and (25) where $\varDelta = \varDelta_{\max}$ are assumed to be true, the errors of equations (26) and (27) are shown in Table 4 where $n = \infty$. The smaller n is the smaller the errors are. The relations between errors and n are shown in Table 5 where $b_1^2 = 4.5\,b_2$. The true p_1 and t_0 are smaller than the values of equations (26) and (27) and larger than the values of equations (24) and (25) where $\varDelta = \varDelta_{\max}$.

TABLE 4. Maximum Errors of Equations (26) and (27) (%) $n = \infty$

b_1^2/b_2	t_0 ($\varDelta=0$) \ Eq.	(26)	(27)
12	$0.092\,b_1$	0.55	5.75
8	$0.146\,b_1$	1.41	8.95
6	$0.211\,b_1$	3.30	14.10
5	$0.276\,b_1$	7.13	22.90
4.5	$0.336\,b_1$	11.60	30.40

TABLE 5. Maximum Errors of Equations (26) and (27) (%) $b_1^2/b_2 = 4.5$

n \ Eq.	(26)	(27)
3	7.73	18.3
4	9.23	22.7
5	9.79	24.5
6	10.25	25.9
∞	11.60	30.4

5. Examples

5.1 Shunt-peaked Amplifier

Fig. 1 (a) is the shunt peaked amplifier and its equivalent circuit is shown

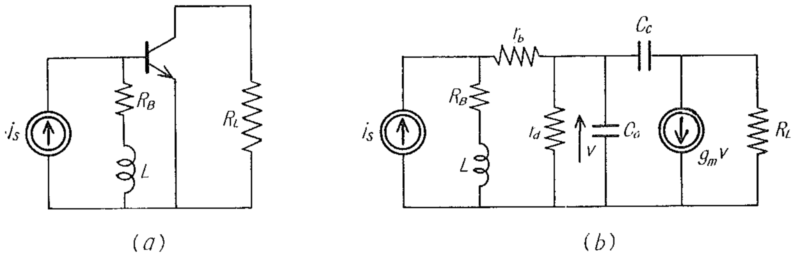


Fig. 1. Shunt-peaked Amplifier

as Fig. 1 (b). Solving this circuit equation, we obtain the circuit determinant as follows:

$$D = \begin{vmatrix} g_b + \dfrac{1}{R_B + pL} & -g_b & 0 \\ -g_b & g_b + g_d + p(C_c + C_d) & -pC_c \\ 0 & g_m - pC_c & G_L + pC_c \end{vmatrix} \tag{31}$$

where

$$g_b = \frac{1}{r_b} \qquad g_d = \frac{1}{r_d} \qquad G_L = \frac{1}{R_L}$$

From equation (31), we obtain

$$b_1 = R_\pi(C_c + C_d) + \frac{L + C_c r_d R_L (g_m r_b - g_b r_d)}{r_b + r_d + R_B} \tag{32}$$

$$b_2 = C_c C_d R_\pi R_L + \frac{L r_d \{C_d + C_c(1 + g_m R_L - g_d R_L)\}}{r_b + r_d + R_B} \tag{33}$$

$$b_3 = \frac{L C_c C_d R_\pi R_L}{r_b + R_B} \tag{34}$$

where

$$R_\pi = \frac{r_d(r_b + R_B)}{r_d + r_b + R_B}$$

Let $r_b = 50\,\Omega$ $r_d = 250\,\Omega$ $g_m = 0.2\,\mho$ $C_c = 5\,\text{pF}$ $C_d = 100\,\text{pF}$ $R_B = 75\,\Omega$ $R_L = 50\,\Omega$ $L = 0.3\,\mu\text{H}$.

Calculating b_1, b_2 and b_3, we obtain

$b_1 = 1.038 \times 10^{-8}$, $b_2 = 3.288 \times 10^{-17}$, $b_3 = 5 \times 10^{-27}$

Since $t_0 \simeq b_3/b_2 = 1.52 \times 10^{-10} = 1.47 \times 10^{-2} b_1$ and $b_1^2/b_2 = 3.28$,

we have complex dominant zeros.

Since $K_1 = b_3/b_1b_2 = 0.0147$, $K_2 = b_1^2/b_2 = 3.28$, $K_1K_2(1-K_1) = 0.0475$,

$$\sigma_1 \simeq -\frac{1}{2}\frac{b_1}{b_2} = -1.57\times10^8$$

$$\omega_1 \simeq \sqrt{\frac{1}{b_2}-\left(\frac{b_1}{2b_2}\right)^2} = 7.55\times10^7$$

$$t_0 \simeq \frac{b_3}{b_2} = 1.52\times10^{-10}$$

5.2 Transformer-coupled Tuned Amplifier

Fig. 2 (a) is the transformer-coupled tuned amplifier and its equivalent circuit is shown as Fig. 2 (b) where G_1 and C_1 are input conductance and capacitance of transistor Q_2 and G_2 and C_2 are output conductance and capacitance of transistor Q_1 which include the external capacitance, the losses and the capacitances of transformer, L_l is the leakage inductance and L_m is the magnetizing inductance of transformer and $n_1 : n_2$ is the turn ratio of an ideal transformer. Therefore Fig. 2 (b) turns into Fig. 2 (c) where

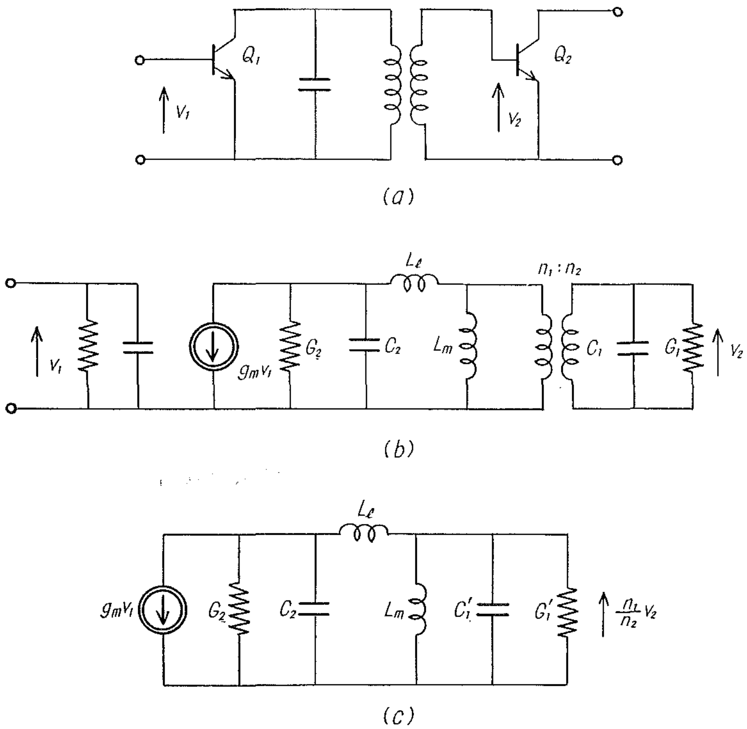


Fig. 2. Transformer-coupled Tuned Amplifier

$$C_1' = \left(\frac{n_2}{n_1}\right)^2 C_1 \qquad G_1' = \left(\frac{n_2}{n_1}\right)^2 G_1$$

Solving this circuit equation, we obtain the gain function v_2/v_1 which denominator is shown as equation (35). Therefore its zeros are the poles of gain function and correspond to the zeros of circuit determinant.

$$D = \frac{L_m}{L_l}\{L_l L_m C_1' C_2 p^4 + (L_l L_m C_2 G_1' + L_l L_m C_1' G_2) p^3 + (L_l L_m G_1' G_2 + L_l C_2 + L_m C_1' + L_m C_2) p^2 + (L_m G_1' + L_m G_2 + L_l G_2) p + 1\} \tag{35}$$

where $L = L_l + L_m$

Let $L_l = 0.1\,\mathrm{L}$ $L_m = 0.9\,\mathrm{L}$ $G_2 = G_1'$ $L = 0.12\,\mu\mathrm{H}$
$C_1 = 100\,\mathrm{pF}$ $C_2 = 100\,\mathrm{pF}$ $R_1 = 200\,\Omega$ $R_2 = 630\,\Omega$ $g_m = 0.04\,\mho$
$(n_1/n_2)^2 = 3.15$ $C_1' = 31.8\,\mathrm{pF}$.

Calculating b_1, b_2 and b_3, we obtain

$b_1 = 3.36 \times 10^{-10}$ $b_2 = 2.69 \times 10^{-16}$ $b_3 = 4.70 \times 10^{-27}$

Since $t_0 \simeq b_3/b_2 = 1.75 \times 10^{-11} = 0.0522\,b_1$ and $b_1^2/b_2 = 4.27 \times 10^{-4}$, we have the complex dominant zeros. Since $K_1 = b_3/b_1 b_2 = 0.052$, $K_2 = b_1^2/b_2 = 4.27 \times 10^{-4}$, $K_1 K_2 (1 - K_1) = 2.1 \times 10^{-5}$,

$$\sigma_1 \simeq -\frac{1}{2}\frac{b_1}{b_2} = 6.25 \times 10^5$$

$$\omega_1 \simeq \sqrt{\frac{1}{b_2} - \left(\frac{b_1}{2b_2}\right)^2} = 6.10 \times 10^7$$

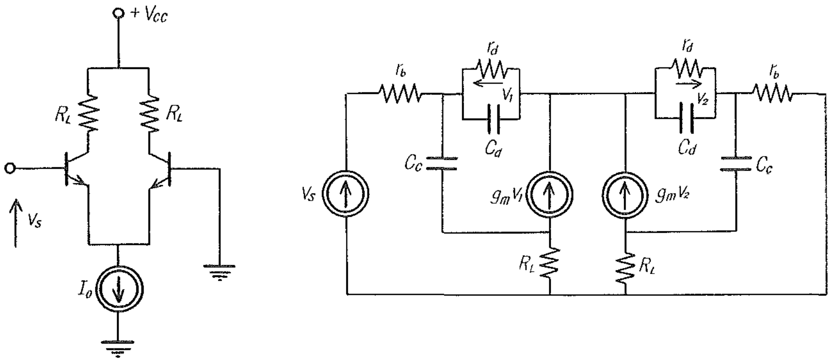
$$t_0 \simeq \frac{b_3}{b_2} = 1.75 \times 10^{-11}$$

The tuned frequency is

$$\omega_0 \simeq \frac{1}{\sqrt{b_2}} = 6.1 \times 10^7$$

5.3 Current Mode Logic Circuit

Fig. 3 (a) is the current mode logic circuit (CML) where I_0 is the constant current source. Since the transistors operate in the active region at switching transient state, its equivalent circuit is shown as Fig. 3 (b). Solving this circuit equation, we obtain the circuit determinant as follows:



(a) (b)

Fig. 3. Current Mode Logic Circuit

$$D=\begin{vmatrix} g_b+g_d+p(C_c+C_d) & -(g_d+pC_d) & 0 & -pC_c & 0 \\ -(g_d+g_m+pC_d) & 2(g_d+g_m+pC_d) & -(g_m+g_d+pC_d) & 0 & 0 \\ 0 & -(g_d+pC_d) & g_d+g_b+p(C_c+C_d) & 0 & -pC_c \\ g_m-pC_c & -g_m & 0 & G_L+pC_c & 0 \\ 0 & -g_m & g_m-pC_c & 0 & G_L+pC_c \end{vmatrix}$$

$$=\begin{vmatrix} g_b+pC_c & -(g_d+pC_d) & 0 & -pC_c & 0 \\ 0 & 2(g_d+g_m+pC_d) & -(g_m+g_d+pC_d) & 0 & 0 \\ 0 & 0 & g_d+g_b+p(C_c+C_d) & pC_c & -\mathrm{p}C_c \\ -pC_c & -g_m & 0 & G_L+pC_c & 0 \\ -pC_c & -g_m & g_m-pC_c & 0 & G_L+pC_c \end{vmatrix} \tag{36}$$

where $g_b=\dfrac{1}{r_b}\quad g_d=\dfrac{1}{r_d}\quad G_L=\dfrac{1}{R_L}$

From equation (36), we have

$$b_1=\frac{C_c}{g_b}+\frac{C_d}{g_m+g_d}+\frac{C_c+C_d}{g_b+g_d}+\frac{2C_c}{G_L}+\frac{g_mC_c}{G_L(g_b+g_d)} \tag{37}$$

$$\begin{aligned} b_2=&\frac{C_cC_d}{g_b(g_m+g_d)}+\frac{C_d(C_c+C_d)}{(g_m+g_d)(g_b+g_d)}+\frac{C_c(C_c+C_d)}{g_m(g_b+g_d)} \\ &+\frac{C_c^2}{G_L^2}+\frac{2C_c}{G_L}\left(\frac{C_c}{g_b}+\frac{C_d}{g_m+g_d}+\frac{C_c+C_d}{g_b+g_d}\right) \\ &+\frac{(g_mG_L-g_mg_d-g_bG_L)C_c^2}{G_L^2g_b(g_b+g_d)}+\frac{g_mC_cC_d}{G_L(g_m+g_d)(g_b+g_d)} \end{aligned} \tag{38}$$

Let $r_b = 50\,\Omega$ $r_d = 250\,\Omega$ $g_m = 0.2\,\mho$ $C_e = 5\,\text{pF}$ $C_d = 100\,\text{pF}$ $R_L = 1K\Omega$. Calculating b_1 and b_2, we obtain

$b_1 = 5.68 \times 10^{-8}$ $b_2 = 6.77 \times 10^{-17}$

Since $b_1^2/b_2 = 47.4$, we have the real dominant zero. Therefore

$$p_1 \simeq -\frac{1}{b_1} = -1.76 \times 10^7$$

$$t_0 \simeq \frac{b_2}{b_1} = 1.19 \times 10^{-9}$$

6. Conclusion

When the zeros of circuit determinant satisfy the condition of dominant zero, the author deduced a means by which to determine whether the dominant zero is real or complex and obtained approximate expressions of complex dominant zeros, real dominant zero and the excess phase coefficient. The higher the dominancy of zero is, the smaller the errors of approximate expressions are. In the case of $t_0 \leq 0.1\, b_1$ their errors are very small. From Tables 3~5, the utility limit of approximate expressions is determined. The author showed some examples in which the dominancy is very high.

Reference

1) Pederson, D. O.: Electronic Circuits, preliminary edition (1965), pp. 74~80 McGraw-Hill.