The Impulse Response of a Kompfner Null Coupler

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Contents

Abstract

The impulse response was derived for a Kompfner null coupler in which a forward wave and an O-type fast space-charge wave were passively coupled. The derivation of the equations is based on Barnes' analysis of the coupled-mode systems.

It was found that if the forward wave is driven with an impulse, the response of the fast space-charge wave consists of a sharply defined RF pulse, of constant maximum amplitude, in which the length increases linearly with both time and distance of propagation.

1. Introduction

The impulse response of active contraflow systems of infinite and finite lengths were given by Bobroff and Haus[1]. Their results describe the buildup of oscillation in various types of backward wave oscillators[2,3] and the buildup of stimulated Brillouin scattering[4,5]. The impulse response of passive forward wave system of an infinite length was shown by Barnes[6]. In this paper Barnes' analysis was modified to a Kompfner null coupler. While the present work is related to that of Barnes, new parameters are required because of the uncoupled $\omega-\beta$ characteristics of the fast space-charge wave.

2. The Dip Condition in the Frequency Domain

Consider two modes with time dependence $e^{j\omega t}$ which are weakly coupled. One is a forward circuit wave, and the other is a fast space-charge wave (see Fig. 1). If the reduced space-charge force in a finite beam is large, coupling

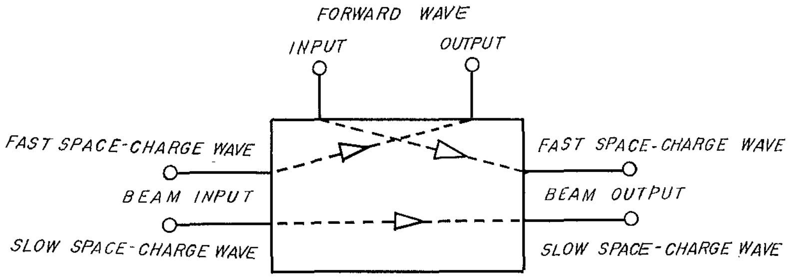

Fig. 1. Block diagram of a Kompfner null coupler.

to the slow space-charge wave may be neglected. In this case, the type of coupling is characterized by the frequency-domain coupled-mode equations[6,7]

$$\frac{\partial a_1(z,\omega)}{\partial z} = -j\beta_e(1+Cb)a_1(z,\omega)+c_{12}a_2(z,\omega)\,, \tag{1}$$

$$\frac{\partial a_2(z,\omega)}{\partial z} = c_{21}a_1(z,\omega)-j(\beta_e-\beta_q)a_2(z,\omega)\,, \tag{2}$$

where $a_1(z,\omega)$ is the frequency-domain amplitude of the forward circuit wave, $a_2(z,\omega)$ is the frequency-domain amplitude of the fast space-charge wave, β_e is electronic propagation constant, C is the gain parameter, b is the velocity parameter which measures the deviation of the uncoupled circuit phase velocity from the d–c beam velocity, defined by

$$b=\frac{u_0-v_p}{Cv_p}\,, \tag{3}$$

where u_0 is the d–c beam velocity and v_p is the phase velocity, β_q is the reduced plasma propagation constant, defined by

$$\beta_q \cong \beta_e C\sqrt{4QC}\,, \tag{4}$$

where QC is the space-charge parameter and $4QC \cong 1$ in actual devices, and c_{12} and c_{21} are the complex coupling coefficients per unit length between the two waves. The coupling is assumed to be uniform over the length of the coupler, hence mutual coupling coefficients are independent of the length. The modes are assumed to be lossless. In the absence of coupling, $\beta_e(1+bC)$ and $\beta_e-\beta_q$

are positive and the phase velocities of both waves are in the positive z-direction. For weak coupling it is assumed that the mutual coupling coefficients are small compared with $\beta_e(1+bC)$ and $\beta_e-\beta_q$ and, furthermore, it is assumed that $\beta_e(1+bC)\cong\beta_e-\beta_q$. The group velocities are in the same directions for the two modes, and it follows that

$$c_{12}=-c_{21}^{*}\,, \tag{5}$$

$$c_{12}=-jk\,, \tag{6}$$

where k is given by

$$k=\sqrt{\beta_e^3C^3/2\beta_q}=\beta_e C/\sqrt{2}\,(4QC)^{1/4}\,. \tag{7}$$

Solutions of the coupled mode equations (1) and (2) are of the form $e^{\gamma z}$ the propagation constants are

$$\gamma_{1,2}=-j(\beta_a\pm\beta_b)\,, \tag{8}$$

where

$$2\beta_a=\beta_e(1+Cb)+\beta_e-\beta_q=2\beta_e+\beta_e C(b-\sqrt{4QC}\,)\,, \tag{9}$$

$$\beta_b=+(|c_{12}|^2+\beta_d^2)^{1/2}=\beta_e C\left[\frac{1}{2\sqrt{4QC}}+\frac{b+\sqrt{4QC}}{4}\right]^{1/2}, \tag{10}$$

$$2\beta_d=\beta_e(1+Cb)-(\beta_e-\beta_q)=\beta_e C(b+\sqrt{4QC}\,)\,, \tag{11}$$

$$|c_{12}|^2=\beta_e^3C^3/2\beta_q=\beta_e^2C^2/2(4QC)^{1/2}\,. \tag{12}$$

It should be noted that γ_1 and γ_2 are purely imaginary, and there can be no exponentially growing or decaying solutions. This coupling is called "passive mode coupling or co-flow hermitian coupling"[6,7]. The behavior of this type of a coupled-mode system, for an input that varies sinusoidally with time, consists of the familiar periodic interchange of power between the two modes.

Now, provided that $a_1(0,\omega)$ and $a_2(0,\omega)$ are the frequency-domain amplitudes at the beginning of the coupling region, the complete solutions of Eqs. (1) and (2) are given by

$$a_1(z,\omega)=e^{-j\beta_a z}\left[\left(\cos\beta_b z-j\frac{\beta_d}{\beta_b}\sin\beta_b z\right)a_1(0,\omega)+\left(\frac{-jk}{\beta_b}\sin\beta_b z\right)a_2(0,\omega)\right], \tag{13}$$

$$a_2(z,\omega)=e^{-j\beta_a z}\left[\left(-\frac{jk}{\beta_b}\sin\beta_b z\right)a_1(0,\omega)+\left(\cos\beta_b z+j\frac{\beta_d}{\beta_b}\sin\beta_b z\right)a_2(0,\omega)\right]. \tag{14}$$

Assume initially that all the power P_{in} is on the forward wave circuit, hence

$$2\,|a_1(0,\,\omega)|^2 = P_{\text{in}}\,, \tag{15}$$

$$a_2(0,\,\omega) = 0\,. \tag{16}$$

It is seen that the average power on the forward wave is given by

$$P_1(z) = P_{\text{in}}(1 - F\sin^2\beta_b z)\,, \tag{17}$$

whereas the average power on the fast space-charge wave is given by

$$P_2(z) = P_{\text{in}} - P_1(z)\,. \tag{18}$$

Figure 2 shows a sketch of $P_1(z)$ and $P_2(z)$ in this case, in which

$$F = \left[1 + \frac{\beta_q(\beta_e Cb + \beta_q)^2}{2\beta_e^3 C^3}\right]^{-1} = \left[1 + \frac{(b + \sqrt{4QC})^2(4QC)^{1/2}}{2}\right]^{-1}, \tag{19}$$

is the maximum fraction of power transfered.

Consider that, before coupling, the propagation constants of two modes were identical at the frequency ω_0, i. e.,

$$b_0 = -\beta_{q0}/\beta_{e0}C\,, \tag{20}$$

so that

$$F \quad \text{at} \quad \omega_0 \equiv F_0 = 1\,, \tag{21}$$

$$\beta_{a0} = \beta_{e0} - \beta_{q0} = \beta_{e0}(1 - C\sqrt{4QC})\,, \tag{22}$$

$$\beta_{b0} = k\,, \tag{23}$$

and

$$\beta_{d0} = 0\,. \tag{24}$$

From Eqs. (17) and (18), the power carried by the two modes is found to be

$$P_1(z) = P_{\text{in}}\cos^2 kz\,, \tag{25}$$

$$P_2(z) = P_{\text{in}}\sin^2 kz\,. \tag{26}$$

It is seen that complete power transfer takes place in a length given by

$$kl_{\text{dip}} = \frac{\pi}{2}(2n+1) = \frac{\beta_e Cl}{\sqrt{2}\,(4QC)^{1/4}}\,, \qquad n = 0,\, \pm 1,\, \pm 2,\, \cdots\,. \tag{27}$$

In this length, l_{dip}, all the power introduced into the circuit is transformed to the fast space-charge waves on the beam. Furthermore, transfer takes place in exactly the same way if power is initially introduced on the fast space-charge wave, and complete power transfer will take place in the same length. The length of slow-wave circuit required for this to occur is called the Kompfner dip length, and such a device is the so-called Kompfner null coupler. It

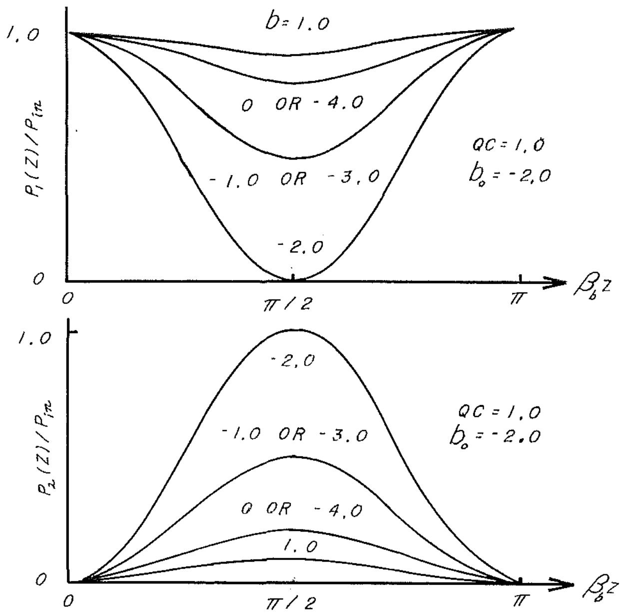

Fig. 2. Power division between two waves in the case where $QC=1.0$ and $b_0=-2.0$.

should be noted that for uniform coupliug, which has been treated here, complete power transfer is possible only for the synchronous cases (see Fig. 2).

3. The Impulse Response in the Time Domain

Now, consider the response that this coupled system shows when the input to one of the modes is an impulse in time. In particular, examine the response of the fast space-charge wave when the forward wave circuit is driven with an impulse. For weak coupling, the nature of the interaction will depend only upon the characteristics of the system at frequencies in the neighborhood of the frequency ω_0 at which the two modes equal phase velocities. Thus we shall be able to derive some rather general results without being compelled to make any severely restrictive assumptions about the nature of the modes.

Two modes of the forward circuit wave and the fast space-charge wave whose uncoupled $\omega-\beta$ characteristics intersect are shown in Fig. 3. If the coupling between two waves is weak, then the only important interaction occurs at a frequency in the neighborhood of ω_0, the frequency at which the two uncoupled waves have equal phase velocities. Assuming that the phase

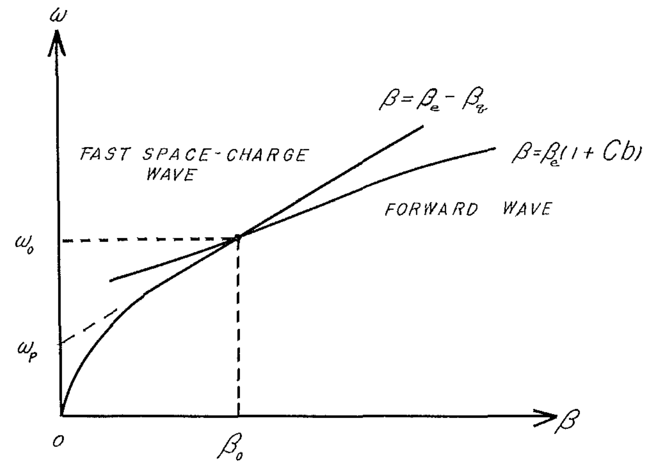

Fig. 3. $\omega-\beta$ characteristics of the forward wave and the fast space-charge wave.

constant of the forward wave can be approximated by the use of the first two terms of a power series expansion about ω_0, we have

$$\beta_e(1+Cb)\cong\left[\beta_e(1+Cb)\right]_{\omega=\omega_0}+\left[\frac{d}{d\omega}\beta_e(1+Cb)\right]_{\omega=\omega_0}(\omega-\omega_0)$$
$$=\beta_{e0}(1+Cb_0)+\left[\frac{1}{u_0}(1+Cb_0)+\beta_{e0}C\left(\frac{db}{d\omega}\right)_{\omega=\omega_0}\right](\omega-\omega_0), \tag{28}$$

where β_{e0} is the electronic propagation constant at the frequency ω_0, and b_0 is the velocity parameter at the frequency ω_0. It was assumed that the gain parameter C is independent of the frequency in the neighborhood of ω_0[8,9,10]. By definition of b,

$$\frac{db}{d\omega}=\frac{db}{dv_p}\cdot\frac{dv_p}{d\omega}, \tag{29}$$

and hence

$$\left(\frac{db}{dv_p}\right)_{\omega=\omega_0}\cong-\frac{u_0}{Cv_{p0}^2}, \tag{30}$$

where v_{p0} is the phase velocity of the forward wave at the frequency ω_0. The phase velocity of the circuit for frequencies close to ω_0 is approximated by the first two terms of a power-series expansion about ω_0 and it follows that

$$\left(\frac{dv_p}{d\omega}\right)_{\omega=\omega_0}\cong\frac{v_{p0}}{\omega_0}\left(1-\frac{v_{p0}}{v_{g0}}\right), \tag{31}$$

where v_{g0} is the circuit group velocity when $\omega=\omega_0$.

From Eqs. (28), (30) and (31), the approximation for phase constant of the forward wave, for frequencies about ω_0, can be written as

$$\beta_e(1+Cb)\cong\beta_{e0}(1+Cb_0)+\frac{\omega-\omega_0}{v_{g0}}. \tag{32}$$

In the same way, the approximation for phase constant of the fast space-charge wave can be shown as

$$\begin{aligned}\beta_e-\beta_q &\cong(\beta_e-\beta_q)_{\omega=\omega_0}+\left[\frac{d}{d\omega}(\beta_e-\beta_q)\right]_{\omega=\omega_0}(\omega-\omega_0)\\ &=\beta_{e0}-\beta_{q0}+\frac{\omega-\omega_0}{u_0}. \end{aligned}\tag{33}$$

The time-domain mode amplitudes are related to the frequency-domain mode amplitudes by the Fourier transform relation

$$a(z,t)=\frac{1}{2\pi}\int_0^\infty a(z,\omega)e^{j\omega t}d\omega+\text{c.c.}, \tag{34}$$

where c.c. indicates the complex conjugate. By writing the Fourier transform relation in the form given by Eq. (34), rather than the more familiar double-sided form, we avoided the necessity of explicitly considering the electron-wave interactions at negative frequencies; the negative frequencies are dealt with automatically by the addition of the complex conjugate term[6].

Now consider the case where boundary conditions at $z=0$ are given by

$$a_1(0,t)=a_0\delta(t), \tag{35}$$

and

$$a_2(0,t)=0, \tag{36}$$

or

$$a_1(0,\omega)=a_0, \tag{37}$$

and

$$a_2(0,\omega)=0, \tag{38}$$

where $\delta(t)$ is the unit impulse and a_0 is a real constant.

If we substitute into Eqs. (9), (10) and (11) the expressions of phase constants given by Eqs. (32) and (33), then β_a, β_b and β_d can be written in the form

$$\beta_a=\beta_{e0}-\beta_{q0}+\frac{1}{2}(\omega-\omega_0)\left(\frac{1}{v_{g0}}+\frac{1}{u_0}\right), \tag{39}$$

$$\beta_b=\left[k^2+\frac{1}{4}(\omega-\omega_0)^2\left(\frac{1}{v_{g0}}-\frac{1}{u_0}\right)^2\right]^{1/2}, \tag{40}$$

and

$$\beta_d=(\omega-\omega_0)\left(\frac{1}{v_{g0}}-\frac{1}{u_0}\right). \tag{41}$$

Further assuming that the coupling coefficient k is a sufficiently slowly varying function of frequency for the weak coupling case, we can approximate it in the neighborhood of ω_0 by a constant, namely

$$k=k_0+\left(\frac{dk}{d\omega}\right)_{\omega=\omega_0}(\omega-\omega_0)\approx k_0\,, \tag{42}$$

where k_0 is the coupling coefficient at the frequency ω_0.

If we substitute into Eq. (14) the frequency conditions, given by Eqs. (37) and (38), the expressions for phase constants, given by Eqs. (32) and (33), and the expressions for β_a, β_b and β_d, given by Eqs. (39), (40) and (41), we find the frequency-domain amplitude of the fast space-charge wave can be written in the form

$$a_2(z,\omega)=-jA_2(z,\omega-\omega_0)\exp\left\{-j\left[(\beta_{e0}-\beta_{q0})z+\frac{1}{2}(\omega-\omega_0)\left(\frac{1}{v_{g0}}+\frac{1}{u_0}\right)z\right]\right\}, \tag{43}$$

where

$$A_2(z,\omega-\omega_0)=a_0\left[1+\frac{(\omega-\omega_0)^2}{\omega_1^2}\right]^{-1/2}\sin\left\{k_0z\left[1+\frac{(\omega-\omega_0)^2}{\omega_1^2}\right]^{1/2}\right\}, \tag{44}$$

and

$$\omega_1=2k_0\Big/\left(\frac{1}{v_{g0}}-\frac{1}{u_0}\right). \tag{45}$$

Note that the parameter ω_1, which is directly proportional to the magnitude of the coupling coefficient k_0, is a measure of the angular frequency bandwidth of the electron-wave interaction. In terms of ω_1, the criterion for the weak-coupling case is, therefore,

$$\omega_1\ll\omega_0\,. \tag{46}$$

The time-domain amplitude of the fast space-charge wave is now obtained by substituting Eq. (43) into Eq. (34). The evaluation of the integral in Eq. (34) is greatly simplified if we note that for the weak coupling case where $\omega_1\ll\omega_0$, we can extend the lower limit of the integral to $-\infty$ without introducing important errors ; upon doing this, we find the time-domain amplitude is given by[11]

$$a_2(z,\ t)=F(z,\ t)\sin\left[\omega_0 t-(\beta_{e0}-\beta_{q0})z\right],\tag{47}$$

where

$$F(z,\ t)=a_0\omega_1 J_0\left\{\omega_1\left[\left(t-\frac{z}{v_{g0}}\right)\left(\frac{z}{u_0}-t\right)\right]^{1/2}\right\},\tag{48}$$

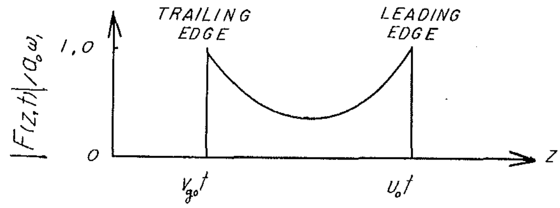

Fig. 4. Typical form of pulse envelope on the fast space-charge wave in response to an impulse input of magnitude a_0 on the forward wave.

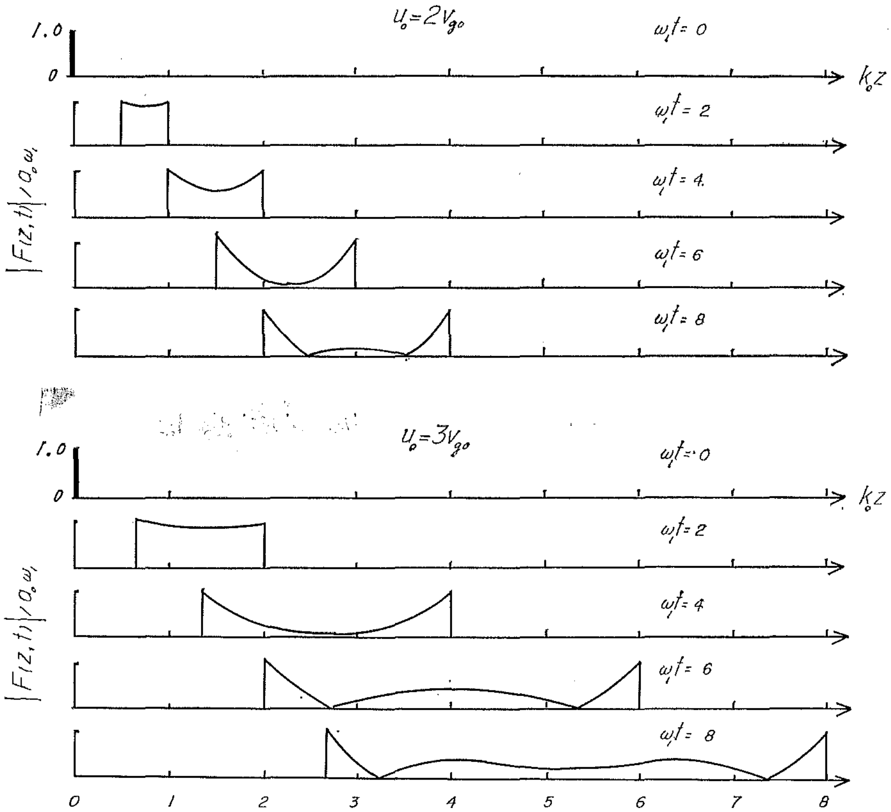

Fig. 5. A time sequence of pulse envelopes on the fast space-charge wave for the case where $u_0=2v_{g0}$ and $u_0=3v_{g0}$.

for

$$\frac{z}{u_0} < t < \frac{z}{v_{g0}},$$

and

$$F(z, t) = 0, \tag{49}$$

for

$$t < \frac{z}{u_0} \quad \text{or} \quad t > \frac{z}{v_{g0}}.$$

It is seen that the response of the fast space-charge wave to an impulse input on the forward circuit wave consists of a wave of the form $\sin[\omega_0 t - (\beta_{e0} - \beta_{g0})z]$ that is modulated in amplitude by a single pulse. A sketch of a typical form of the pulse envelope, as a function of z, is shown in Fig. 4. The pulse envelope has a sharp leading edge that travels with velocity u_0 and a sharp trailing edge that travels with velocity v_{g0}. The maximum amplitude of the pulse envelope is independent of both z and t. The pulse envelope length, measured along the z-axis, increases linearly with time. The pulse envelope length, measured along the t-axis, increases linearly with z. Figure 5 shows a normalized time sequence for the pulse envelopes for the case where $u_0 = 2v_{g0}$ and $u_0 = 3v_{g0}$.

4. Conclusions

A derivation of the time-domain response of a Kompfner null coupler was presented. It is hoped that this derivation will give new insights into the electron-wave interaction phenomena.

It is necessary that further theoretical investigations be carried out to determine the total energy carried by the pulse on the fast space-charge wave. When the $\omega - \beta$ characteristics deviate from linearity outside of the frequency range $\omega_0 - \omega_1$ to $\omega_0 + \omega_1$, the resulting effect on the sharp edges of the pulse on the fast space-charge wave should be investigated[12].

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References

1) Bobroff, D. L. and Haus, H. A.: "Impulse response of active coupled wave systems," J. Appl. Phys., vol. 38, pp. 390–403, January 1967.

2) Bobroff, D. L.: "Coupled-mode analysis of the phonon-photon parametric backward-wave oscillator," J. Appl. Phys., vol. 37, pp. 1760–1769, May 1965.

3) Bobroff, D. L.: "The buildup of oscillations in an electron beam backward-wave oscillator," IEEE, Trans. vol. ED-12, pp. 307–312, June 1965.

4) Chiao, R. Y., Townes, C. H. and Stoicheff, B. P.: "Stimulated Brillouin scattering and coherent generation of intense hypersonic waves," Phys. Rev. Lett., vol. 12, pp. 592–595, May 25, 1964.

5) Kroll, N. M.: "Excitation of hypersonic vibration by means of photoelastic coupling of high-intensity light waves to elastic waves," J. Appl. Phys., vol. 36, pp. 34–43, January 1965.

6) Barnes, C. W.: "On the impulse response of a coupled-mode system," IEEE, Trans. MTT, vol. MT-13, pp. 432–435, July 1965.

7) Louisell, W. H.: Coupled mode and parametric electronics, New York, Wiley, 1960.

8) Sakuraba, I.: "Analysis of the O-type backward-wave oscillator frequency pushing," J. Inst. Elect. Commun. Engrs. Japan, vol. 42, pp. 810–815, September 1959 (in Japanese).

9) Sakuraba, I.: "The backward-wave oscillator frequency pushing," Proc. IRE, vol. 50, pp. 2493–2494, December 1962.

10) Sakuraba, I.: "Frequency pushing in O-type backward-wave oscillators," Memoirs of Faculty of Engineering, Hokkaido University, vol. XI, pp. 447–458, 1963.

11) Korn, G. and Korn, T.: Mathematical handbook for scientists and engineers, McGraw Hill, p. 776, 1961.

12) Sakuraba, I.: "The impulse response of a Kompfner coupler," IEECE of Japan, Trans. on Microwave Theory and Techniques, No. MW 68-14, pp. 1–12, June 1968 (in Japanese).