Signal Power Output and Wavefront Curvature in Optical Heterodyne Detection Processes

Ichiro SAKURABA

Department of Electronic Engineering
Faculty of Engineering
Hokkaido University
Sapporo, Japan

(Received August 4, 1969)

Contents

Abstract

In optical heterodyne detection processes, the directional characteristic for illumination by spherical wavefronts was calculated. The derivation was based on Corcoran's analysis for a one-dimensional photosensitive surface.

The directivity factor of a strip detector of width L, $D^2(\beta_c, \varepsilon, L)$ is given by $\left\{C\left[\tau(z=L)\right]-C\left[\tau(z=0)\right]\right\}^2+\left\{S\left[\tau(z=L)\right]-S\left[\tau(z=0)\right]\right\}^2$. When the curvature is zero, $D^2(\beta_c, 0, L)$ is shown by $\mathrm{sinc}^2(\beta_c L/2)$, which is the well-known directivity factor of plane waves. In exactly parallel and normal incidence beams the ratio of power output obtained for spherical wavefronts to that for plane waves decreases as the curvature of wavefronts increases. In normal incidence but with non-parallel beams, the directional pattern is narrowed as the wavefronts become progressively more curved. In the case where $\sqrt{2|\varepsilon|/\pi}\,L=0.2$, $\theta_1=0$, $\theta_2=\delta\theta$, and $L=3$ mm at 6000 Å, patterns of converging and diverging wavefronts are much the same with that of plane waves.

1. Introduction

In optical heterodyne detection processes strict alignment tolerances necessary to keep signal and local oscillator wavefronts in phase, over the detector surface have been pointed out by Stroke[1], Siegman, Harris, McMurtry[2], Sakuraba, Chida[3] and DeLange[4] and directions for apparently relaxing these alignment tolerances have been offered by Read, Fried and Turner[5),6], and the antenna properties of optical heterodyne receivers have also been pointed out by Siegman[7]. Corcoran[8] showed the directional characteristics of a one-dimensional strip detector in the case of unfocused light beams, focusing light beams with a single aperture and focusing beams with multiple apertures. The directional characteristics of two-dimensional detectors have recently been given by Sakuraba and Tsubo[9].

These directional properties arise since the surface or detector volume is generally large compared to the optical wave length. As a result, the phase difference between two light beams and thus the phase of the difference frequency signal, can vary widely over the complete area of coincidence. When the detector is in the near-field of a laser, or when a lens is used to direct the radiation on to it, the detector surface is illuminated by spherical light waves. One of conditions for maximum beat signal is that the wavefronts must have the same curvature. In work done hitherto directional problems on spherical wavefronts have been mentioned but no quantitative results have been given in detail.

This paper deals with the directional characteristics for illumination by spherical wavefronts in optical heterodyne detection processes. The derivation will be based on Corcoran's analysis for one-dimensional detectors and information on output power and curvature of wavefronts will be added by extending his analysis to illumination by spherical wavefronts.

2. General Solution

Analyses of the photodetection mechanism for incoherent and coherent radiation showed that the current output of the detector is proportional to the square of the total electric field at each point on the detection surface[10),11]. This was summed up over the entire area, and temporally averaged over a short period of time (10^{-10} seconds for photoelectric material)[12]. Then the output current of the detector process can be expressed as

$$i(t)=\frac{1}{T}\int_{t-T}^{t}\int_{A}\left[V^{(r)}(P,t')\right]^{2}dA\,dt'$$
$$=\frac{1}{2}\int_{A}<VV^{*}>dA+\frac{1}{4}\int_{A}<V^{2}+V^{*2}>dA\,, \qquad (1)$$

where $V^{(r)}(P,t)$ is the real disturbance at point P and time t proportional to a component of the electric field[13] and $V(P,t)$ is the complex function associated with $V^{(r)}(P,t)$. For simplicity an unessential constant factor has been dropped and in actual problems this constant might be a suitably averaged function of angle of incidence and polarization of the light. The brackets indicate the time average over an interval T around t. When the field at the detector surface is a superposition of monochromatic waves:

$$V(P, t) = \sum_n E_n(P) e^{-j\omega_n t}. \tag{2}$$

the output current is

$$i(t) = \frac{1}{2}\int_A < \sum_n E_n(P)\cdot E_n^*(P) > dA + \frac{1}{2}\int_A < \sum_{\substack{mn \\ m\neq n}} E_n(P)\cdot E_m^*(P) e^{-j(\omega_n-\omega_m)t} > dA$$
$$+ \frac{1}{4}\int_A <(\sum_n E_n(P)e^{-j\omega_n t})^2 + (\sum_n E_n^*(P)e^{j\omega_n t})^2 > dA. \tag{3}$$

The first term shows the *d–c* current and the third term has optical frequency terms. It is noted that optical frequency terms are presumably completely suppressed by the averaging process. Even if they were not, we have no practical means of coupling or measuring an optical-frequency-modulated electron current. Then the output current becomes

$$i(t) = \sum_{\substack{m,n \\ m\neq n}} e^{-j(\omega_n-\omega_m)t}\int_A E_n(P)\cdot E_m^*(P) dA, \tag{4}$$

where an average was taken over a period so that

$$\omega_n - \omega_m < \frac{1}{T} < \omega_n,\ \omega_m \tag{5}$$

and the average value of the current, the unessential constant factor and the angular brackets were ignored. In particular when the field at P is due to the superposition of two waves,

$$i(t) = e^{j(\omega_2-\omega_1)t}\int_A E_1(P)\cdot E_2^*(P) dA. \tag{6}$$

It is conventional to relate the peak ac photocurrent i_1 to the power output through a fictitious equivalent resistance R_{eq} defined by the equation

$$P_{out} = \frac{1}{2}|i_1|^2 R_{eq}. \tag{7}$$

The equivalent resistance depends on the circuit characteristics of the optical devices and its output connections[14)~18)]. From Eqs. (6) and (7) the expression for the output can be written as

$$P_{out} = \frac{1}{2}R_{eq}\left|\int_A E_1(P)\cdot E_2^*(P) dA\right|^2. \tag{8}$$

With a knowledge of the spatial variation of the electric field across the detection surface, the power output for the problems of interest can be calculated from Eq. (8).

3. Signal Power Output for Illumination by Spherical Wavefronts

A schematic representation of the problem of detecting two spherical wavefronts is shown in Fig. 1. It is assumed that the wavefronts are not too strongly divergent or convergent on to the active length of the detector, and any $1/r$ dependence of amplitudes is neglected by the above approximation. Thus the field at the photosensitive surface is given by

$$V(P, t) = E_1(P)e^{-j\omega_1 t} + E_2(P)e^{-j\omega_2 t}, \tag{9}$$

where

$$E_1(P) = A_1 e^{\mp jk_1 r_1'}, \qquad E_2(P) = A_2 e^{\mp jk_2 r_2'}, \tag{10}$$

and k_1 and k_2 are propagation constants of the two light waves. The upper and lower signs refer to converging and diverging wavefronts, respectively. Figure 1 refers to diverging wavefronts but the results are exactly analogous to converging wavefronts, for which L_1 and L_2 can be envisaged as to the right of the origin.

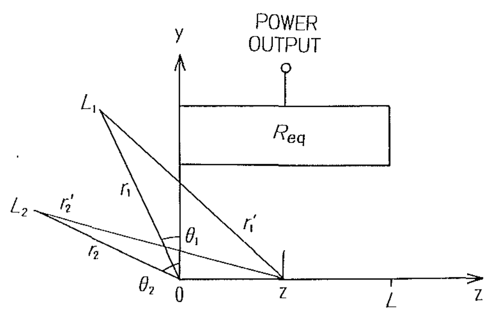

Fig. 1. Schematic diagram of the detector showing illumination by radially propagating waves in optical heterodyne detection processes.

The quantity r_n represents the distance from the center of divergence L_n of the component of the radiation to any arbitrary point on the detector. For convenience we take this as the origin $z=0$, see Fig. 1. The distance from a point P to a point L_n is r_n'. θ_n is the angle of incidence of the ray through the origin with respect to the outward normal from the photosurface. If the active length of the detector, L, is small compared with r_n, then by the binominal expansion

$$r_n' \approx r_n \mp z \sin\theta_n + \frac{z^2}{2r_n}, \qquad (n=1,2,) \tag{11}$$

The simplest example we can consider is a photosensitive surface which is a strip of width L in the z-direction and uniform in the x-direction. Through use of Eqs. (8) and (11) it follows that

$$\begin{aligned}\int_0^L E_1(z)\cdot E_2^*(z)dz &= \int_0^L A_1A_2^* e^{\mp j(k_1r_1-k_2r_2)}e^{-j\beta_c z}e^{\mp j\left(\frac{k_1}{2r_1}-\frac{k_2}{2r_2}\right)z^2}dz,\\ &= \int_0^L A_1'A_2'^* e^{-j[\beta_c z+\varepsilon z^2]}dz,\\ &= \int_0^L A_1'A_2'^* e^{-j\varepsilon\left(z+\frac{\beta_c}{2\varepsilon}\right)^2}e^{+j\varepsilon\left(\frac{\beta_c}{2\varepsilon}\right)^2}dz,\end{aligned} \tag{12}$$

where

$$\beta_c = k_2\sin\theta_2 - k_1\sin\theta_1, \tag{13}$$

$$\varepsilon = \pm\left(\frac{k_1}{2r_1}-\frac{k_2}{2r_2}\right), \tag{14}$$

and the constant phase term has been absorbed into $A_1'A_2'^*$. In terms of the Fresnel integrals

$$C(x) = \int_0^x \cos\left(\frac{\pi}{2}t^2\right)dt, \qquad S(x) = \int_0^x \sin\left(\frac{\pi}{2}t^2\right)dt,$$

Eq. (12) becomes

$$\begin{aligned}\int_0^L E_1(z)\cdot E_2^*(z)dz &= A_1'A_2'^*\left(\frac{\pi}{2|\varepsilon|}\right)^{1/2}e^{j\varepsilon\left(\frac{\beta_c}{2\varepsilon}\right)^2}\\ &\times\left(\left\{C\left[\tau(z=L)\right]-C\left[\tau(z=0)\right]\right\}\mp j\left\{S\left[\tau(z=L)\right]-S\left[\tau(z=0)\right]\right\}\right),\end{aligned} \tag{15}$$

where

$$|\varepsilon|\left(z+\frac{\beta_c}{2\varepsilon}\right)^2 = \frac{\pi}{2}\tau^2. \tag{16}$$

The expression for the signal power output is therefore given by

$$P(\varepsilon, L) = \frac{1}{2}R_{\text{eq}}\left(\frac{\pi}{2|\varepsilon|}\right)|A_1'A_2'^*|^2 D^2(\beta_c, \varepsilon, L), \tag{17}$$

where

$$D^2(\beta_c, \varepsilon, L) = \left\{C\left[\tau(z=L)\right]-C\left[\tau(z=0)\right]\right\}^2+\left\{S\left[\tau(z=L)\right]-S\left[\tau(z=0)\right]\right\}^2. \tag{18}$$

$D(\beta_c, \varepsilon, L)$ is the directivity factor. This refers to either diverging or converging wavefronts through the value of ε.

4. Characteristics of Power Output

Now consider the power output in the case where $|\varepsilon|=0$. Equation (12) becomes

$$\int_0^L E_1(z)E_2^*(z)dz=\int_0^L A_1'A_2'^* e^{-j\beta_c z}dz$$

Therefore

$$P(0, L)=\frac{1}{2}R_{eq}L^2|A_1'A_2'^*|^2D^2(\beta_c, 0, L), \tag{19}$$

where

$$D^2(\beta_c, 0, L)=\mathrm{sinc}^2(\beta_c L/2). \tag{20}$$

$P(0, L)$ is the well-known power output obtained for plane wave illumination and $D(\beta_c, 0, L)$ is the directivity factor and the notation sinc $x=(\sin x)/x$ is used. It is apparent that $D(\beta_c, 0, L)\geqq 1/\sqrt{2}$ for $\beta_c L\leqq\frac{8}{9}\pi$. In a case in which $\theta_1=\theta_2=\theta$, and $\omega_2=\omega_1+\Delta\omega$, the required condition for negligible reduction is

$$\lambda\geqq 2.25\,L\sin\theta$$

where λ is the wavelength of the difference frequency. This condition is well satisfied in microwave frequency bands, since it would be very unusual to have a detector whose dimension L approached a difference frequency wavelength. In the second case where two beams are incident at nearly equal but non-normal angles, $\theta_1=\theta$ and $\theta_2=\theta+\delta\theta$, the requirement for $D(\beta_c, 0, L)\geqq 1/\sqrt{2}$ becomes

$$\delta\theta\leqq\lambda_2/2.25\,L\cos\theta$$

where λ_2 is the optical wavelength. Since the detector dimension is much larger than the optical wavelength λ_2, this is a very stringent angle limitation.

When $|\varepsilon|$ increases, $D^2(\beta_c, \varepsilon, L)$ tends to a limit of 1/2. Then the power output becomes

$$P(\varepsilon, L)=\frac{1}{4}R_{eq}\left(\frac{\pi}{2|\varepsilon|}\right)|A_1'A_2'^*|^2 \tag{21}$$

This is then independent of length L. It is the power output at the beat frequency that would be obtained from convergent beams which are focused almost to a spot on the detector.

Next consider the power output in the case where $\theta_1 = \theta_2 = 0$ as the curvature of wavefronts increases. In this case it follows that

$$\left.\begin{aligned} & C\left[\tau(z=0)\right] = S\left[\tau(z=0)\right] = 0\,, \\ & \mathrm{sinc}\,(\beta_c L/2) = 1\,. \end{aligned}\right\} \tag{22}$$

Therefore, the ratio of $P(\varepsilon, L)$ to $P(0, L)$ becomes

$$\frac{P(\varepsilon, L)}{P(0, L)} = \frac{1}{\tau^2(z=L)} \left\{ C^2\left[\tau(z=L)\right] + S^2\left[\tau(z=L)\right] \right\}. \tag{23}$$

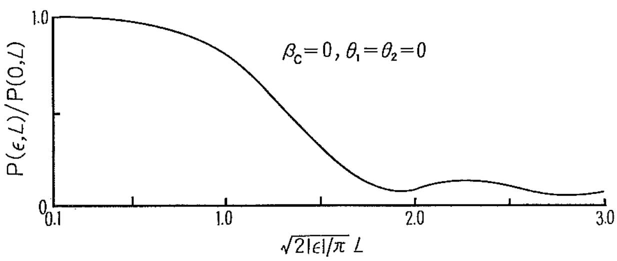

Fig. 2. Reduction in power output as the curvature of wavefronts increases. This is normalized to the power output obtained for plane wave illumination. It was assumed that $\beta_c = 0$ and $\theta_1 = \theta_2 = 0$.

A plot of Eq. (23) as a function of $\tau^2(z=L)$ is shown in Fig. 2. This case is that in which $\theta_1 = \theta_2 = 0$, namely $\beta_c = 0$. This means exactly parallel and normal incidence beams. Figure 2 shows the reduction factor in normalized power output as the curvature of wavefronts increases. It is apparent that the required condition for negligible reduction in detection can be calculated by $|\varepsilon| L^2 \leqq 0.87\pi$. For example if two light sources transmit at 6000 Å and impinge at angles $\theta_1 = \theta_2 = 0$ on a detector surface whose effective mixing length L is 3 mm, it follows that

$$|1/r_1 - 1/r_2| \leqq 58$$

When $\theta_1 = 0$, $\theta_2 = \delta\theta, L = 3$ mm and the wavefronts are convergent at 6000 Å, plots of normalized directivity factor with $\sqrt{2|\varepsilon|/\pi}\,L$ as the parameter are presented in Fig. 3. This result shows that the directivity increases when the wavefronts become progressively more curved. In the case where $\sqrt{2|\varepsilon|/\pi}\,L = 0.2$, $\theta_1 = 0$, $\theta_2 = \delta\theta, L = 3$ mm at 6000 Å, the normalized directivity factors for converging and diverging wavefronts are given in Fig. 4. The normalized directivity factor of plane waves, namely $|\varepsilon| = 0$, is also shown in Fig. 4. As seen from this figure, the theoretically expected curves of spherical wavefronts are almost the same with that of plane waves.

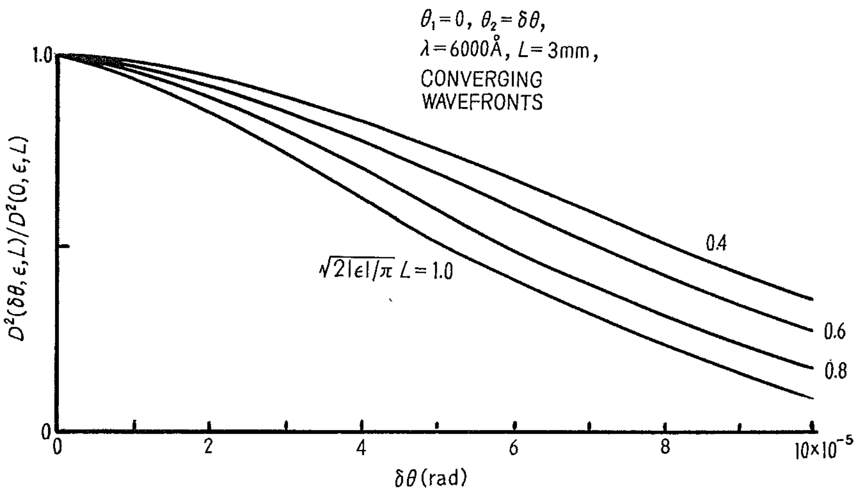

Fig. 3. $D^2(\delta\theta, \varepsilon, L)/D^2(0, \varepsilon, L)$ of converging wavefronts with $\sqrt{2|\varepsilon|/\pi}\,L$ as the parameter in the case where $\theta_1=0$, $\theta_2=\delta\theta$ and $L=3$ mm at 6000 Å. The directivity increases as the wavefronts become more curved.

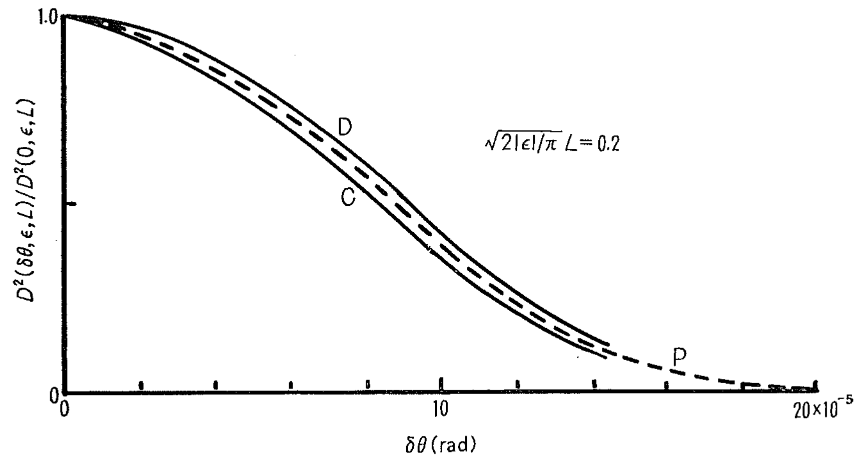

Fig. 4. The effect of beam tilt on signal power output; (C) converging wavefronts, (D) diverging wavefronts, and (P) plane waves, in the case where $\sqrt{2|\varepsilon|/\pi}\,L=0.2$, $\theta_1=0$, $\theta_2=\delta\theta$ and $L=3$ mm at 6000 Å and $\varepsilon=0$ for plane waves.

5. Conclusions

In optical heterodyne detection processes, the directional characteristics for illumination of spherical wavefronts was calculated. The derivation was based on Corcoran's analysis for a one-dimensional photosensitive surface.

The general characteristic of directivity factors could be summarized as follows:
1) The directivity factor of a strip detector of width L, $D^2(\beta_c, \varepsilon, L)$ is given by $\{C[\tau(z=L)]-C[\tau(z=0)]\}^2+\{S[\tau(z=L)]-S[\tau(z=0)]\}^2$. This refers to either diverging or converging wavefronts through the value of ε.
2) In the case where $|\varepsilon|=0$, the directivity factor, $D^2(\beta_c, 0, L)$ is shown by $\mathrm{sinc}^2(\beta_c L/2)$. This is the well-known directivity factor for plane waves.
3) When $|\varepsilon|$ increases, the directivity factor $D^2(\beta_c, \varepsilon, L)$ tends to a limit of 1/2. This would be obtained from convergent beams which are focused almost to a spot on the photosensitive surface.
4) In the case of exactly parallel and normal incidence beams, the normalized power output decreases as the curvature of wavefronts increases. The required condition for negligible reduction in power output can be calculated by $|\varepsilon|L^2 \leqq 0.87\pi$.
5) The directional pattern is narrowed as the wavefronts become progressively more curved. In the case in which $\sqrt{2|\varepsilon|/\pi}\,L=0.2$, $\theta_1=0$, $\theta_2=\delta\theta$ and $L=3$ mm at 6000 Å, the directional pattern of converging wavefronts is about the same as that of diverging wavefronts. Their patterns are also much the same as that of plane waves.

Acknowledgment

The author is grateful to the staff of Electronic Engineering Department for their helpful suggestions. Also, the author wishes to thank Mr. K. Koyanagi for his valuable discussions. Finally there are all of those friends and colleagues whose discussions and criticisms have been invaluable, but whose contributions are so interwoven as to preclude individual acknowledgment.

References

1) Stroke, G. W.: Lectures on "Optics of coherent and non-coherent electromagnetic radiations" Department of Electrical Engineering, The University of Michigan, Ann Arbor, Michigan, 1964, and J. Opt. Soc. Am., vol. 47, pp. 1097–1103, 1957.
2) Siegman, A. E., Harris, S. E. and McMurtry, B. J.: "Optical heterodyning and optical demodulation at microwave frequencies", Proc. Symp. on Optical Maser, Polytechnic Institute of Brooklyn, New York, pp. 511–527, April 1963.
3) Sakuraba, I. and Chida, H.: "Directivity of linear-photoelectron-beam system in optical heterodyne detection processes", Bulletin of the Faculty of Engineering, Hokkaido University, Sapporo, Japan, No. 44, pp. 59–66, September 1967, (in Japanese).
4) DeLange, O. E.: "Optical heterodyne detection", IEEE Spectrum, vol. 5, pp. 77–85, October 1968.
5) Read, W. S. and Fried, D. L.: "Optical heterodyning with noncritical angular alignment", Proc. IEEE, vol. 51, p. 1787, December 1963.
6) Read, W. S. and Turner, R. G.: "Tracking heterodyne detection", Appl. Opt., vol. 4, p.

1570, December 1965.
7) Siegman, A. E.: "The antenna properties of optical heterodyne receiver", Proc. IEEE, vol. 54, pp. 1350–1356, October 1966.
8) Corcoran, V. J.: "Directional characteristics in optical heterodyne detection processes", J. Appl. Phys., vol. 36, pp. 1819–1825, June 1965.
9) Sakuraba, I. and Tsubo, T.: "Directional characteristics and forms of the detector surface in optical heterodyne detection processes", Memoirs of the Faculty of Engineering, Hokkaido University, Sapporo, Japan, No. 56, pp. 295–318, January 1969.
10) Hanbury Brown, R. and Twiss, R. Q.: "Correlation between photons in two coherent beams of light", Nature, vol. 177, pp. 27–29, January 7, 1956.
11) Corcoran, V. J. and Pao, Y. H.: "Detection of laser radiation", J. Opt. Soc. Am., vol. 52, pp. 1341–1350, December 1962.
12) Forrester, A. T.: "Photodetection and photomixing of laser output", Advances in Quantum Electronics, Columbia University Press, New York, pp. 233–238, 1961.
13) Born, M. and Wolf, E.: "Principles of optics", Third Edition, Pergamon Press, London, 1965.
14) Sakuraba, I.: "Analysis of O-type photoelectron beam demodulator for amplitud modulated light signals", Bulletin of the Faculty of Engineering, Hokkaido University, Sapporo, Japan, No. 32, pp. 175–186, October 1963, (in Japanese).
15) Sakuraba, I. and Rowe, J. E.: "Photodemodulation of coherent light signals in centrifugal electrostatic focusing systems", Technical Report No. 75, Electron Physics Laboratory, Department of Electrical Engineering, The University of Michigan, Ann Arbor, Michigan, September 1964.
16) Sakuraba, I. and Rowe, J. E.: "Partial conversion of current modulation in linear-photo-electron-beam systems", Trans. PTGED-IEEE, vol. ED-12, pp. 388–389, June 1965.
17) Rowe, J. E. and Sakuraba, I.: "Equivalent resistance of CEF photodemodulators", Proc. of the 6th International Conference on Microwave and Optical Generation and Amplification", Cambridge, England, pp. 458–463, September 1966.
18) Sakuraba, I.: "Directivity factors in optical heterodyne detection processes", Fall Meeting of IECE of Japan, Tokyo, September 1969.