Directional Characteristics of Gaussian Plane Waves in Optical Heterodyne Detection

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The fundamental properties of Gaussian plane wave in optical heterodyne detections, were presented. The derivation was based on Corcoran's analysis for a one-dimensional photocathode.

In the case in which the distribution length is comparable with the cathode length and two beams are exactly parallel and normal incident, the ratio of power output of Gaussian plane waves to that of uniform plane waves decreases as the distribution length decreases. In the case where the distribution length is small compared with the cathode, the directivity factor of a cathode of a width L is given by $\gamma^2 \exp(-\beta_c^2\gamma^2/2)$. The directivity of the normal incidence and non-parallel beams increases as the distribution length increases. The directional pattern of Gaussian plane waves is less sensitive to angles than to that of uniform plane waves.

1. Introduction

Angular selectivity properties in photomixing or heterodyning have been pointed out by Stroke[1], Siegman, Harris, McMurtry[2], Corcoran[3], Sakuraba[4] and DeLange[5]. The antenna properties of optical heterodyne detection have also been pointed out by Read, Fried, Turner[6),7)] and Siegman[8)]. The directional characteristics of 0-type microwave phototubes and two-dimensional photocathodes have recently been shown by Sakuraba, Chida[9)], Tsubo[10)] and Koyanagi[11)]. The wavefront curvature effects and quantum efficiency distribution effects on signal output power have more recently been given by Sakuraba[12)], Takajō[13)], Yoshida and Koyanagi[14)].

The directional properties arise because the photocathode is generally large compared to the optical wavelength. As a result, the phase difference between two light beams and thus the phase of the difference frequency signal, can vary widely over the photocathode. The maximum beat signal is obtained only when two light beams bear the same phase relationship over the complete coincidence. It is therefore implied that the optical phase must be uniform over the complete wavefront of each beam. DeLange[5)] has shown that these requirements are met only under the following conditions the two beams must have the same optical modes, the diameters of two beams must be coincident to provide maximum signal-to-noise ratio, the Poynting vectors of beams must be coincident, the beams must be identically polarized, and the wavefronts must have the same radius of curvature.

Boyd and Gordon[15)] have shown that the stable modes in Fabry-Perot resonators with spherical reflectors have a radial amplitude distribution which is the product of a Gaussian function and a Hermite polynomial. The lowest mode is pure Gaussian and the mode is observed in gas lasers. (see Fig. 1) Evtuhov and

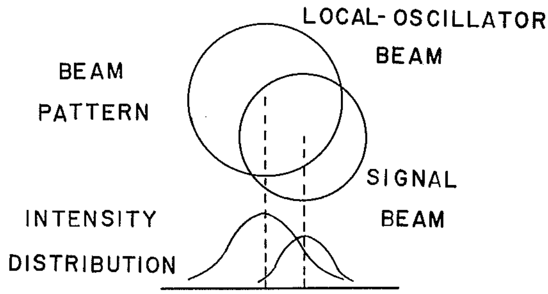

Fig. 1. Some simple heterodyne beams. Signal and local-oscillator beams are the TEM_{00} mode and the intensity distribution is Gaussian with a maximum intensity at the center of the beam. If two beams have the same polarization, photomixing takes place over the area of overlap.

Neeland[16)] have pointed out that the emisson from ruby lasers occurs in such modes, rather than those of a truly plane parallel resonator. A curve of effect of beam tilt in Gaussian plane waves on output current has been shown by DeLange[5)], but no analyses have been given in detail. These problems on the directional characteristics of illumination by plane waves of Gaussian amplitude distribution are discussed in the following paragraphs.

2. Output Current and Coupled Mode Description

Analyses of the photodetection mechanism for incoherent and coherent radiation have shown that the current output of the detection is proportional to the square of the total electric field at each point on the photocathode[17),18)]. For simplicity the proportionality is taken as unity. Then the current can be expressed as

$$i(P, t)=\frac{1}{T}\int_{t-T}^{t}\left[V^{(r)}(P, t')\right]^2 dt', \tag{1}$$

where $V^{(r)}(P, t')$ is associated with the real field of the light wave at P. The current is temporally averaged over a period which is long compared with that of any optical frequencies in the light wave and short compared with that of any microwave beats which may be produced. $V^{(r)}(P, t)$ is associated with an analytical signal $V(P, t)$[19)]. Hence the current becomes

$$i(P, t)=\frac{1}{4}<V^2(P, t)+V^{*2}(P,t)>+\frac{1}{2}<V(P, t)\,V^*(P, t)>, \tag{2}$$

where the brackets denote the time averaging over the period T. With these restrictions on T the first term on the right-hand side is zero. Even if they were not, we have no practical means of coupling an optical-frequency-modulated electron current. When the field at the photocathode is a superposition of monochromatic waves:

$$V(P, t)=\sum_n E_n(P)e^{-j\omega_n t}, \tag{3}$$

and the current is

$$i(P, t)=\frac{1}{2}\sum_n E_n(P)E_n^*(P)+\frac{1}{2}\sum_{\substack{nm\\ n\neq m}}<E_n(P)E_m^*(P)e^{j(\omega_m-\omega_n)t}>, \tag{4}$$

The time average is taken over a period $T<|\omega_m-\omega_n|^{-1}$, then the angular brackets can be neglected. Since ω_m and ω_n each occupy a range of values, the current becomes

$$i(P, t)=\frac{1}{2}\sum_n E_n(P)E_n^*(P)+Re\sum_{\substack{n,m\\ n\neq m}} E_n(P)E_m^*(P)e^{j(\omega_m-\omega_n)t}. \tag{5}$$

In particular when the field at P is the superposition of two waves,

$$i(P, t)=\frac{1}{2}\left[E_1(P)E_1^*(P)+E_2(P)E_2^*(P)\right]+Re\,E_1(P)E_2^*(P)\exp\left[j(\omega_2-\omega_1)t\right]. \tag{6}$$

It is only the ac term which is of interest and we can write for the complex current flowing from the photocathode:

$$J(z, t) = J(z)e^{j(\omega_2-\omega_1)t}, \tag{7}$$

where

$$J(z) = E_1(z)\cdot E_2^*(z), \tag{8}$$

and again, the unessential constant factor was ignored. This is excited by the light wave to produce the current at each point on the photocathode. Then the current of the detector can be expressed as

$$i(t) = e^{j(\omega_2-\omega_1)t}\int_A J(z)dA. \tag{9}$$

The simplest example we can consider is a photocathode which is a strip of width L in the z-direction and uniform in the x-direction. The current is therefore

$$i(t) = e^{j(\omega_2-\omega_1)t}\int_0^L J(z)dz. \tag{10}$$

A general photodetector consists of a photoelectric element, followed by an electron gun region and a microwave circuit to detect and mix the modulation placed on the electron beam or the carrier by the incident light-beam signal, as shown in Fig. 2. Many possible variations of this idea have been investigated

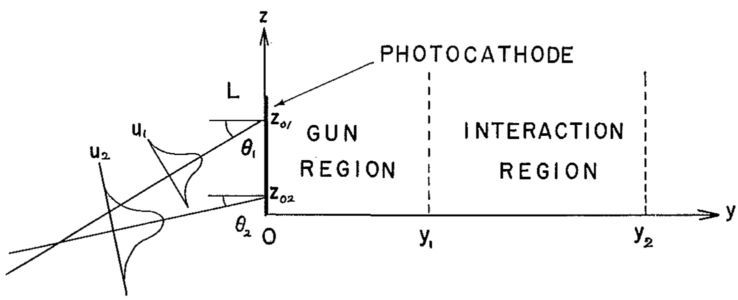

Fig. 2. Representation of detection process with Gaussian plane waves.

by various authors[20)~22)]. In the gun region a nonuniformity may be due to a potential variation and coupled-mode theory provides an elegant way of describing space-charge-wave propagation in the nonuniform region and it has been treated extensively in Haddad, Bevensee and Adair's literatures[23),24)]. Now consider the coupled-mode description of space-charge waves on a nonuniform electron beam in the gun region. Assuming a single velocity beam which is confined to flow in the y-direction by a homogeneous dc magnetic field and small-signal conditions where products of ac terms may be neglected, the following set of coupled-mode equations for space-charge-wave propagation on an electron beam in a nonuniform drift region may be derived[23]:

$$\left\{\frac{d}{dy}+j\left[\beta_e(y)\mp\beta_p(y)\right]\right\}a_{\mp}(y)=-\frac{1}{2}\left[\frac{d}{dy}\ln Z_b(y)\right]a_{\pm}(y), \tag{11}$$

where

$$a_{\mp}(y)=\frac{1}{4\sqrt{Z_b(y)}}\left[V_1(y)\pm Z_b(y)\left(-J_1(y)\right)\right], \tag{12}$$

$$Z_b(y)=\frac{2V_0(y)R(y)\,\omega_p(y)}{|J_0|\omega}, \tag{13}$$

$$\omega_p(y)=\left[\frac{|e||J_0|}{\varepsilon_0 m u_0(y)}\right]^{1/2}, \tag{14}$$

$$u_0(y)=\left[\frac{2e}{m}V_0(y)\right]^{1/2}, \tag{15}$$

$$V_1(y)=\frac{m}{|e|}u_0(y)u_1(y), \tag{16}$$

and $a_{\mp}(y)$ are the fast and slow space-charge modes, respectively, $Z_b(y)$ is beam impedance, $V_0(y)$ is the dc potential along the drift region, J_0 is the beam current density, ω is the radian frequency, $\omega_p(y)$ is the radian plasma frequency for an infinite beam, e and m are the electron's charge and mass, respectively, ε_0 is the free space permitivity, $u_0(y)$ is the dc beam velocity, $R(y)$ is the space-charge reduction factor, $V_1(y)$ is the beam kinetic potential, $u_1(y)$ is the ac beam velocity, $J_1(y)$ is the ac current density, $\beta_p(y)=\omega_p(y)/u_0(y)$ is the plasma propagation parameter and $\beta_e(y)=\omega/u_0(y)$ is the electronic propagation parameter. Next consider the case of pure current modulation at the input plane ($y=0$). This corresponds to the detection and photomixing of the modulation placed on the carrier by the incident light-beam signal. Recall that current and velocity modulation are related to the space-charge modes by[25]

$$J_1(y)=\frac{2}{\sqrt{Z_b(y)}}\left[a_-(y)-a_+(y)\right], \tag{17}$$

and

$$u_1(y)=\frac{|e|}{m}\frac{1}{u_0(y)}2\sqrt{Z_b(y)}\left[a_-(y)+a_+(y)\right]. \tag{18}$$

In the pure current modulation case it follows that

$$a_{\pm}(0) = \mp \frac{1}{4} \sqrt{Z_b(0)}\, J_1(0)\,. \tag{19}$$

and

$$J_1(0) = \int_0^L J(z) \cdot dz\,. \tag{20}$$

It is easy to calculate the velocity and current modulation in a nonuniform region. For instance, in the case where $\beta_p(0) = \beta_p(y)(1-\alpha y)$ and $|\alpha/2\beta_p(0)| < 1$, Haddad and Adair[24] have recently shown that

$$|V_1(y)|^2 = |J_1(0)|^2 Z_b^2(0) \left[\frac{1-\alpha y}{1-m^2} \sin^2\theta\right], \tag{21}$$

$$|J_1(y)|^2 = |J_1(0)|^2 \left\{\frac{1}{1-\alpha y}\left[\cos^2\theta + \frac{m^2}{1-m^2}\sin^2\theta + \frac{m}{\sqrt{1-m^2}}\sin 2\theta\right]\right\}. \tag{22}$$

If the slow-space charge wave can be interacted by a suitable mechanism, the ac power on the circuit at $y = y_2, P_{\text{out}}$, is given by

$$P_{\text{out}} = \frac{1}{2} |J_1(0)|^2 R_{\text{eq}}\,, \tag{23}$$

where R_{eq} is the equivalent resistance[20)~22)] that depends on the characteristics of nonuniform region and slow-wave circuits and its output connections and that can be calculated from Eqs. (19), (20), (21) and (22).

3. Signal Power Output by Plane Waves of Gaussian Amplitude Distribution

A schematic representation of the problem of a combination of Gaussian waves incident on a photocathode is shown in Fig. 2. The field at the photocathode is given by

$$E_1(z) = A_1 e^{-u_1^2/u_{01}^2} e^{jk_1 \sin\theta_1 z}\,, \tag{24}$$

$$E_2(z) = A_2 e^{-u_2^2/u_{02}^2} e^{jk_2 \sin\theta_2 z}\,, \tag{25}$$

where k_1 and k_2 are propagation constants and the signal beam is assumed to be directed towards a point z_{01} on the photocathode at the angle of incidence θ_1 and the local-oscillator beam is assumed to be directed towards a point z_{02} on the photocathode at the angle of incidence θ_2. It was assumed that $\theta_1 \neq \pi/2$ and $\theta_2 \neq \pi/2$. By substitution Eqs. (24) and (25) in Eqs. (8) and (10), it follows that

$$J_1(0) = A_1 A_2^* \int_0^L \exp\left[\frac{-(z-\alpha)^2}{\gamma^2} - \frac{\cos^2\theta_1 \cos^2\theta_2}{u_{02}^2 \cos^2\theta_1 + u_{01}^2 \cos^2\theta_2}(z_{01}-z_{02})^2 - j\beta_e z\right] dz\,, \tag{26}$$

where

$$\alpha=\frac{z_{01}u_{02}^2\cos^2\theta_1+z_{02}u_{01}^2\cos^2\theta_2}{u_{02}^2\cos^2\theta_1+u_{01}^2\cos^2\theta_2}\,, \tag{27}$$

$$\gamma^{-2}=\left[\frac{\cos^2\theta_1}{u_{01}^2}+\frac{\cos^2\theta_2}{u_{02}^2}\right], \tag{28}$$

and

$$\beta_c=k_2\sin\theta_2-k_1\sin\theta_1\,. \tag{29}$$

Again, the unessential constant factor was ignored. Now consider that two light beams are assumed to be directed towards a point $z_{01}=z_{02}=z_{0n}$ on the photocathode at the angle of incidence θ_n. Equations (26) and (27) become

$$J_1(0)=A_1A_2^*\int_0^L\exp\left[\frac{-(z-z_{0n})^2}{\gamma^2}-j\beta_c z\right]dz\,, \tag{30}$$

where

$$\alpha=z_{0n}\,. \tag{31}$$

Accordingly, the signal power output is obtained from Eqs. (23), (30) and (31). It is noted that the output current at $z=z$ is similarly Gaussian in distribution. In general this equation can only be integrated numerically.

4. Directional Characteristics

Consider the power output in the case where the distribution length is comparable with cathode length and $\theta_1=\theta_2=0$, namely $\beta_c=0$. From Eq. (30) the output current becomes

$$\begin{aligned}J_1(0)&=A_1A_2^*\int_0^L\exp\left[\frac{-(z-z_{0n})^2}{\gamma^2}\right]dz\,,\\&=\frac{1}{2}\sqrt{\pi}\,A_1A_2^*\gamma\left[Erf\left(\frac{L-z_{0n}}{\gamma}\right)+Erf\left(\frac{z_{0n}}{\gamma}\right)\right],\end{aligned} \tag{32}$$

where $Erf(x)$ is the error function whose mathematical definition is

$$Erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt\,. \tag{33}$$

Therefore the signal power output may be written

$$P_{\text{out}}=\frac{1}{8}\pi R_{\text{eq}}\gamma^2|A_1A_2^*|^2\left[Erf\left(\frac{L-z_{0n}}{\gamma}\right)+Erf\left(\frac{z_{0n}}{\gamma}\right)\right]^2. \tag{34}$$

Recall that the power output obtained for uniform plane wave illumination is[12]

$$P(0, L) = \frac{1}{2} R_{eq} L^2 |A_1 A_2^*|^2 \mathrm{sinc}^2(\beta_c L/2). \tag{35}$$

Thus, the expression for the ratio of the power output P_{out} to $P(0, L)$ takes the form

$$\frac{P_{out}}{P(0, L)} = \frac{\pi \gamma^2}{4L^2} \left[Erf\left(\frac{L - z_{0n}}{\gamma}\right) + Erf\left(\frac{z_{0n}}{\gamma}\right)\right]^2. \tag{36}$$

A plot of Eq. (36) as a function of u_{0n}/L is shown in Fig. 3. It was assumed that $\theta_1 = \theta_2 = 0$, $u_{01} = u_{02} = u_{0n}$ and $z_{0n} = L/2$. A study of Fig. 3 shows that the signal power output decreases when the distribution length decreases.

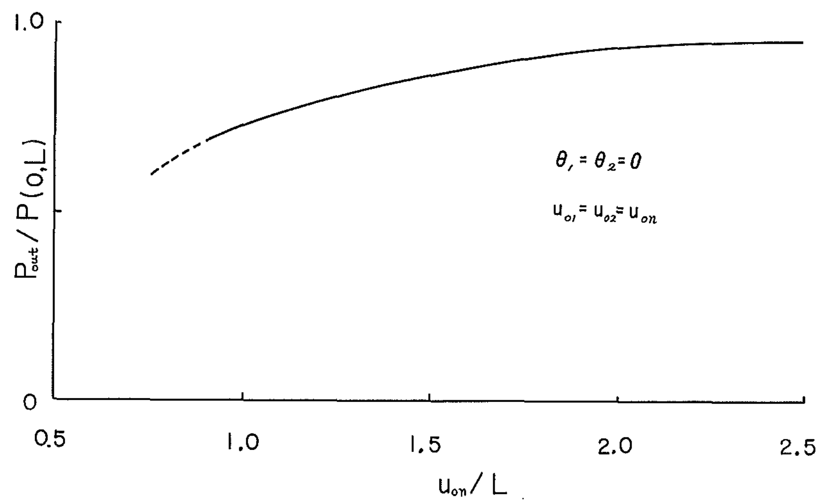

Fig. 3. $P_{out}/P(0, L)$ vs. u_{0n}/L in the case where $\theta_1 = \theta_2 = 0$ $u_{01} = u_{02} = u_{0n}$ and $z_{0n} = L/2$.

In the case of practical interest in which the length of distribution on the photocathode is small compared with the cathode length, this can be treated by putting the limits of integration as $\pm\infty$ with little error. Thus, the output current becomes

$$\begin{aligned} J_1(0) &= A_1 A_2^* \int_{-\infty}^{+\infty} \exp\left[\frac{-(z - z_{0n})^2}{\gamma^2} - j\beta_c z\right] dz \\ &= \sqrt{\pi}\, A_1 A_2^* e^{-j\beta_c z_{0n}} \gamma e^{-\beta_c^2 \gamma^2/4}, \end{aligned} \tag{37}$$

and the signal power output can be shown by

$$P_{out} = \frac{1}{2} \pi R_{eq} |A_1 A_2^*|^2 D_g^2, \tag{38}$$

where

$$D_g^2 = \gamma^2 e^{-\beta_G^2 \gamma^2/2} . \tag{39}$$

The quantity D_g specifies the directivity factor in heterodyning of plane waves of Gaussian amplitude distribution.

The special case of interest is that in which $\theta_1 = 0$ and $\theta_2 = \delta\theta$. This condition means normal incidence but non-parallel beams. In this case β_c reduces to

$$\beta_c = 2\pi\delta\theta / \lambda_2 .$$

When $u_{01} = u_{02} = u_{0n}$ and $L = 3$ mm at 6000 Å, plots of the normalized output as a function of $\delta\theta$ for various values of u_{0n} are presented in Fig. 4. This result

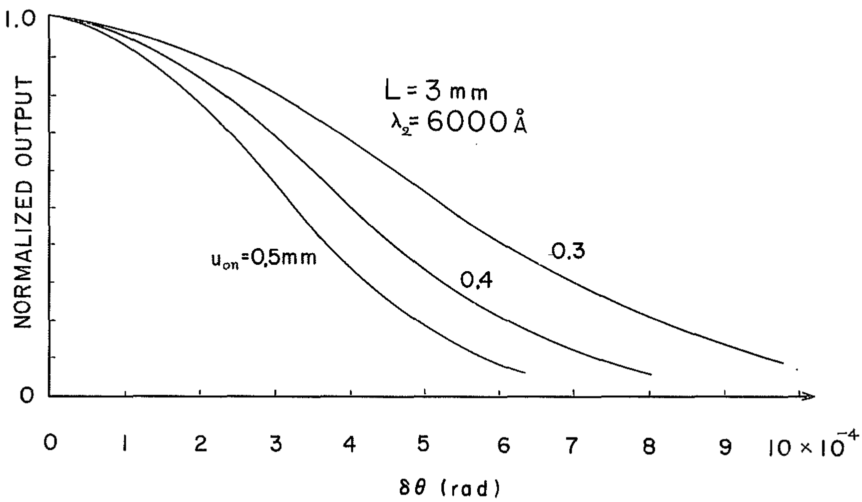

Fig. 4. The theoretical normalized ouptut vs. $\delta\theta$ in the case where $u_{01} = u_{02} = u_{0n}$ and $L = 3$ mm at 6000 Å.

shows that the directivity increases when the distribution length increases. It is apparent that the required condition for negligible reduction in power output can be calculated by

$$D_g^2 = \frac{1}{2} u_{0n}^2 e^{-(\pi\delta\theta u_{0n}/\lambda_2)^2} \geqq \frac{1}{2} . \tag{40}$$

For example, if two beams of distribution length u_{0n} transmit at 6000 Å and impinge on a photocathode of width L, the two beams must be parallel to within about $\delta\theta = 5.32 \times 10^{-5} L / u_{0n}$ radian. A plot of angular mismatch as a function of L / u_{0n} is given in Fig. 5. It should be noted that the required condition for uniform plane waves at 6000 Å is $\delta\theta \leqq 2.67 \times 10^{-7} / L$ radian. Thus the directional

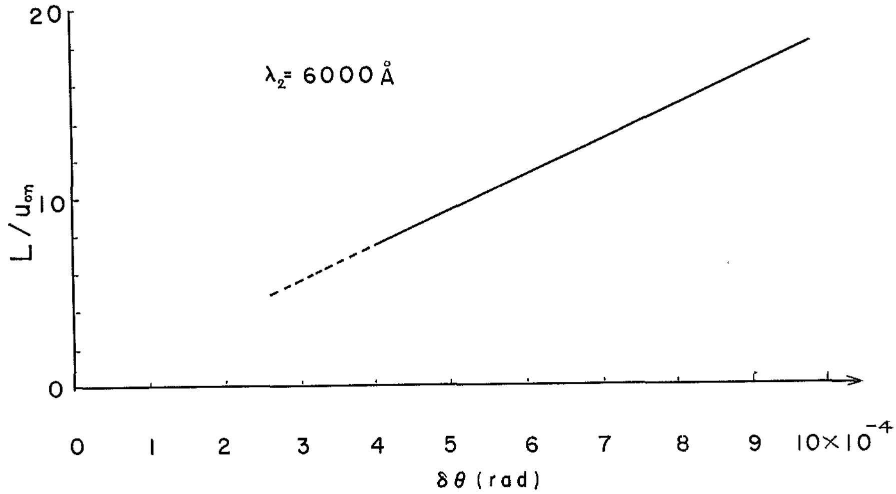

Fig. 5. The angular selectivity of $\delta\theta$ for $D_g=1/\sqrt{2}$ in the case in which $\theta_1=0$, $\theta_2=\delta\theta$, and $u_{01}=u_{02}=u_{0n}$ at 6000 Å.

pattern of Gaussian plane waves is less sensitive to angles than that of uniform plane waves[12]. Plots of normalized output with optical wavelengths as the parameter are shown in Fig. 6. It was assumed that $u_{0n}=0.3$ mm and $L=3$ mm. A study of Fig. 6 shows that the directional pattern is narrowed as the optical wavelengths decrease in the case of constant lengths of photocathode and distribution.

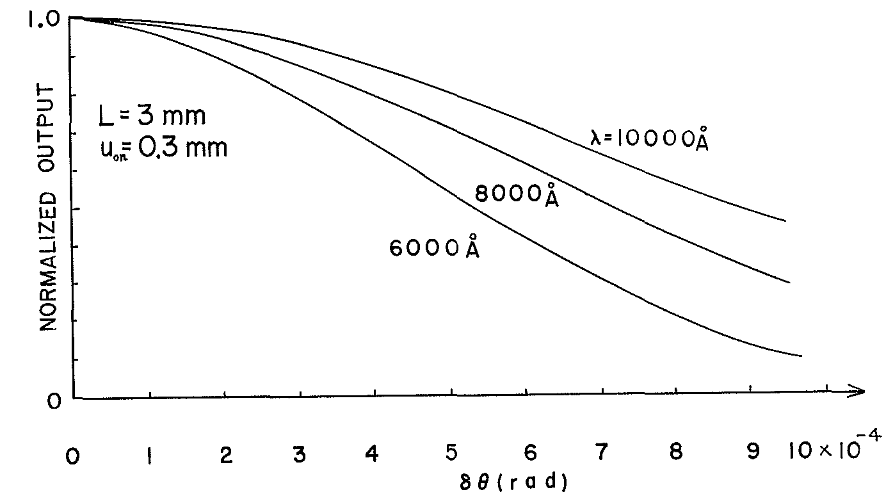

Fig. 6. The theoretical normalized output with optical wavelengths as the parameter. It was assumed that $u_{0n}=0.3$ mm, $L=3$ mm, $\theta_1=0$ and $\theta_2=\delta\theta$.

5. Conclusions

The fundamental properties of Gaussian plane waves in the optical heterodyne detection were calculated. The general characteristic could be summarized as follows;

1) In the case where the distribution length is comparable with the cathode length and the beams are exactly parallel and normal incident, the reduction factor of power output of Gaussian plane waves to that of uniform plane waves is given by $(\pi\gamma^2/L^2)\left[Erf\left(\frac{L-z_{0n}}{\gamma}\right)+Erf\left(\frac{z_{0n}}{\gamma}\right)\right]^2$. This ratio decreases when the distribution length decreases.

2) In the case in which the distribution length is small compared with the cathode length, the directivity factor of a photocathode of width L is given by $\gamma^2 \exp(-\beta_c^2\gamma^2/2)$. In the normal incidence and non-parallel beams, the directivity increases as the distribution length increases.

3) In the case of small distribution lengths, normal incidence and non-parallel beams, the directional pattern of Gaussian plane waves is less sensitive to angles than that of uniform plane waves.

4) The directional pattern is narrowed when the optical wavelength decreases in the case of constant ratio of the distribution length to the cathode width.

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