Stability of a Nuclear Reactor in a Three Dimensional State Space

Yuichi OGAWA*

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Abstract

This paper intends to clarify the exact figure and nature of the region in which a nuclear reactor may achieve a safe reactor state in a three dimensional state space. This region may be obtained as one of the four subregions which are produced by mutual crossing of a hyperbolic paraboloid and a plane characterized by parameters of the system. This region is expressed by an expanded form of that derived by Andreiev et al..

1. Introduction

Generally the power reactor possesses two kinds of internal feed back, one that is related mainly to the variation of the fuel temperature and the coolant density, and which responds to the variation of the reactor power relatively quickly; and the other which responds relatively slowly, which is chiefly governed by variation of the structure temperature.

To evaluate the stability of such a reactor, the criterions given by Popov or Lur'e have been established providing the conditions upon which the reactor is stable in a global space[1),2)]. But even if the criterions are applicable to a reactor system and the stability is obtained in a global space, it should be ascertained only in a specific limited region in which the state of the system is possible as a reactor state.

This paper intends to clarify the exact figure and nature of the possible region in two different state spaces, one of which is formed from three coordinates, reactor power n, its rate of change $\dot{n}$ and its acceleration $\ddot{n}$ (Space A), and the other is formed from reactor power n, delayed neutron precursor concentration c and reactor temperature T (Space B).

Andreiev et al. treated the problem of the stability and the possible region of a nonlinear reactor system on two dimensional plane (the coordinates of which are n and $\dot{n}$[3)]). They treated the problem under an assumption that the variation of the reactor power was carried out in quasi-statical manner and obtained the possible region which is bordered by two straight lines characterized by reactor

* Faculty of Engineering, Hokkaido University, Sapporo.

parameters. In this paper, the possible region is obtained without forcing any restriction against the variation of the reactor power. In the former space, the possible region is obtained as one of the four subregions which are produced by the mutual crossing of a hyperbolic paraboloid and a plane characterized by parameters of the system. In the latter space, the region does not depend on the value of c and is obtained as a region which is bordered by a plane which crosses the $n-T$ plane with right angle. Andreiev et al. pointed out that movement of a state between the zero power equilibrium point and the power equilibrium point could be sometimes prohibited by a band of impossible regions which lies between the two equilibrium points. But this phenomena arizes from the assumption of the quasi-statical variation of the reactor power, and does not arize in our treatment in which the assumption is not settled.

Numerial examples are shown on diagrams.

2. Dynamic Equations

Taking into account the two kinds of internal feedback as well as the effect of delayed neutrons, the reactor dynamic equations are

$$\frac{dn}{dt}=\frac{\rho-\beta}{l}n+\frac{1}{\tau_0}c$$

$$\frac{dc}{dt}=\frac{\beta}{l}n-\frac{1}{\tau_0}c$$

$$\frac{dT}{dt}=\frac{1}{K}n-\frac{1}{\tau}T$$

$$\rho=\rho_0+u+En+F'T \tag{1}$$

where n is the thermal output, c the delayed neutron precursors concentration, τ the thermal time constant, τ_0 the mean generating time of delayed neutrons, l the prompt neutron generating time, ρ_0 the cold reactivity, ρ the reactivity, β the fractional yield of the delayed neutrons, u the control reactivity, T the temperature of reactor, K the heat capacity, E the coefficient for relatively quick internal feedback, F' the coefficient for relatively slow internal feedback.

In Eq. (1), temperature T is measured as the deviation from the temperature of a reactor in a cold state. Hence the physically possible values of n, c and T must satisfy the following conditions

$$n\geq 0, \quad c\geq 0, \quad T\geq 0. \tag{2}$$

From Eq. (1), we obtain the following two equilibrium points

$$P_a: \quad n=0, \quad c=0, \quad T=0,$$

$$P_b: \quad n = n_p = -\rho_0/(E+\tau F), \quad c = c_p = \tau_0 \beta n_p / l$$
$$T = T_p = \tau n_p / K \tag{3}$$

where F is given by

$$F = F'/K. \tag{4}$$

To obtainthe positive value of n_p, we have two cases, one is the case in which ρ_0 is positive and $(E+\tau F)$ is negative, and in the other ρ_0 is negative and $(E+\tau F)$ is positive. But we could not accept the latter case from the standpoint of safety, because in this case the assembling of the smaller amount of fuel would invite the larger amount of n_p.

Introducing the new state variables

$$x = n, \quad y = \tau \dot{n}, \quad z = \tau \tau_0 \ddot{n}, \tag{5}$$

Eq. (1) is reduced into the nonlinear differential equations

$$\frac{dz}{dt} = \frac{Q(x, y, z)}{P(x, y)} \qquad \text{(a)}$$

$$\frac{dy}{dt} = \frac{1}{\tau_0} z \qquad \text{(b)}$$

$$\frac{dx}{dt} = \frac{1}{\tau} y \qquad \text{(c)} \tag{6}$$

where $P(x, y)$ and $Q(x, y, z)$ are the abbreviations for

$$P(x, y) = D_{12} y + D_{13} x \qquad \text{(a)}$$
$$Q(x, y, z) = D_1 z^2 + D_2 zy + D_3 zx + D_4 zx^2 + D_5 y^3 + D_6 y^2 x + D_7 y x^2 + D_8 x^3 + D_9 y^2 + D_{10} yx + D_{11} x^2. \qquad \text{(b)} \tag{7}$$

The coefficients Ds in Eq. (7) are given in Appendix A.

From Eq. (6) we obtain again two equilibrium points P'_a and P'_b which correspond to P_a and P_b respectively

$$P'_a: \quad x = 0, \quad y = 0, \quad z = 0,$$
$$P'_b: \quad x = n_p, \quad y = 0, \quad z = 0. \tag{8}$$

We proceed to examine the stability of the system in the vicinity of these equilibrium points and also seek for the optimum control which brings a phase point located near the power equilibrium point P'_b to the point most quickly.

2.1. Near point P'_b

In applying the Taylor's expansion in the vicinity of P'_b, Eq. (6) becomes

$$\frac{dz}{dt}=a(u)z+b(u)y+e(u)x+f(u),$$
$$\frac{dy}{dt}=\frac{1}{\tau_0}z$$
$$\frac{dx}{dt}=\frac{1}{\tau}y \tag{9}$$

where coefficients $a(u)$, $b(u)$, $e(u)$ and $f(u)$ are as given in Appendix B.

When the control u is absent, the above coefficients become

$$\begin{aligned} a(u)&=a=-(s\rho_0/\beta+1)/\Lambda-(1+d)/\tau_0 \\ b(u)&=b=-\left[(s+d)\rho_0/\beta+d\right]/\Lambda-d/\tau_0 \\ e(u)&=e=-\rho_0/\beta\Lambda \\ f(u)&=f=0, \end{aligned} \tag{10}$$

where s, d and Λ are defined by

$$s=E/(E+\tau F),\quad d=\tau_0/\tau,\quad \Lambda=l/\beta. \tag{11}$$

Then Eq. (9) becomes the linear equations

$$\frac{dz}{dt}=az+by+ex,\quad \frac{dy}{dt}=\frac{z}{\tau_0},\quad \frac{dx}{dt}=\frac{y}{\tau}, \tag{12}$$

from which the characteristic roots of the system λ_1, λ_2, λ_3 are obtained as the solution of the cubic equation

$$\lambda^3-a\lambda^2-(b/\tau_0)\lambda-e/\tau\tau_0=0. \tag{13}$$

From the well known relations which exist between the roots and coefficients

$$\begin{aligned} &\lambda_1+\lambda_2+\lambda_3=a \\ &\lambda_1(\lambda_2+\lambda_3)+\lambda_2\lambda_3=-b/\tau_0 \\ &\lambda_1\lambda_2\lambda_3=e/\tau\tau_0, \end{aligned} \tag{14}$$

and from the fact that the value e in Eq. (14) becomes always real and negative, it is found that there is at least one real and negative root[4]. If we let this root be λ_1, then the equilibrium point P_b' must be one of the following four types:

stable

$$\lambda_1<0,\quad \lambda_2<0,\quad \lambda_3<0 \qquad \text{(a)}$$
$$\lambda_1<0,\quad \lambda_2=v+jw,\quad \lambda_3=v-jw\ (v<0,\ w>0) \qquad \text{(b)}$$

unstable

$$\lambda_1<0\,,\quad \lambda_2=v+jw\,,\quad \lambda_3=v-jw\quad (v>0,\ w>0) \qquad \text{(c)}$$

$$\lambda_1<0\,,\quad \lambda_2>0\,,\quad \lambda_3>0\,. \qquad \text{(d)} \qquad (15)$$

The stable condition may be called respectively a stable node and stable node-focus. The unstable condition may be called respectively a focal saddle and nodal saddle.

It is clear that the transition from stability to instability must take place in the presence of the focal condition because the value $\lambda_1\lambda_2\lambda_3$ $(=e/\tau\tau_0)$ does not become zero. We can easily show that the transition condition $v=0$ can occur when

$$ab/\tau_0+e/\tau\tau_0=0\,. \qquad (16)$$

On the other hand, if we apply the Hurwitz condition to system (12), we obtain the following condition of stability

$$a<0\,,\quad b<0\,,\quad e/\tau\tau_0<0\,,\quad ab/\tau_0+e/\tau\tau_0>0\,. \qquad (17)$$

Condition (16) appears as an important part of Condition (17).

Substituting Eq. (11) into Eq. (17), we obtain the condition under which the system is stable near the equilibrium point P_b' as

$$(s\rho_0/\beta+1)+(1+d)\Lambda/\tau_0>0 \qquad \text{(a)}$$

$$\left[(s/d+1)\rho_0/\beta+1\right]+\Lambda/\tau_0>0 \qquad \text{(b)}$$

$$(s\rho_0/\beta+1)\left[(s/d+1)\rho_0/\beta+1\right]+\left[(2s+s/d+d)\rho_0/\beta+2+d\right]\Lambda/\tau_0$$
$$+(1+d)\Lambda^2/\tau_0^2>0\,. \qquad \text{(c)} \qquad (18)$$

The principal direction cosines of the characteristic root λ_1 are obtained as

$$\cos\alpha=\frac{1}{\sqrt{1+(\tau\lambda_1)^2+(\tau\tau\lambda_1^2)^2}}$$

$$\cos\beta=\frac{\tau\lambda_1}{\sqrt{1+(\tau\lambda_1)^2+(\tau\tau\lambda_1^2)^2}}$$

$$\cos\gamma=\frac{\tau\tau\lambda_1^2}{\sqrt{1+(\tau\lambda_1)^2+(\tau\tau\lambda_1^2)^2}}\,. \qquad (19)$$

These principal direction cosines are useful to examine the figure of the vector field in the vicinity of the equilibrium point.

Next, in order to estimate the optimum control which brings the state located near point P_b' to the point optimum time wise, Eq. (9) is considered again. In Eq. (9) the external control affects the force terms and also the coefficient sof the equation. If we only treat the case where the value of d is extremely smaller than unity, the parts of the coefficients which have to do with the external control are expressed approximately by

$$a_u = -d\left[u(t)+\tau\dot{u}(t)\right]/\beta\Lambda$$

$$b_u = 2d\left[u(t)+\tau\dot{u}(t)+(\tau\tau_0/2)\ddot{u}(t)\right]/\beta\Lambda$$

$$e_u = 2\ \left[u(t)+\tau\dot{u}(t)+)\tau\tau_0\ddot{u}(t)\right]/\beta\Lambda$$

$$f_u = n_p\ \left[u(t)+\tau\dot{u}(t)+\tau\tau_0\ddot{u}(t)\right]/\beta\Lambda \tag{20}$$

In Eq. (20), the control involved in the force term consists of a linear combination of control reactivity with its rate of change and its acceleration as $(u(t)+\tau\dot{u}(t)+\tau\tau_0\ddot{u}(t)=v)$. The same control is also involved in the coefficient $e(u)$. The control involved in $a(u)$ and $b(u)$ are different from v, but have a similar form. As the most effective control is that which is involved in the force term, we can assume that all controls consist of v in a sense of approximation. From the Pontryagin's optimal control theory, the optimum control to the linear system is estimated to be the so-called piecewise constant control with regard to the control parameter v.

2.2. Near point P'_a

For this case, assuming the existing of the relation $y=kx$ and accompanying relations $z=dk^2x$ and $dz/dt=dk^3x/\tau$ we obtain

$$K^3+a_0(u)K^2+b_0(u)K+e_0(u)=0\,. \tag{21}$$

from Eq. (6). Without control, the coefficients become

$$a_0 = \left[1+3\rho_0/\beta+(2-d)\Lambda/\tau_0\right]\tau/\Lambda$$

$$b_0 = \left[1-d-(2-d)\rho_0/\beta+(1-d)\Lambda/\tau_0\right]\tau/d\Lambda$$

$$c_0 = -(1-d)\tau\rho_0/\beta d^2\Lambda \tag{22}$$

It has been demonstrated that under any condition all coefficients cannot be positive simultaneously and the point becomes either the nodal saddle or the focal saddle. Although the above mentioned assumption does not hold generally, it is found from the numerical examination that for the variation of the reactor parameters the vector field indicates always the stable topological structure. Hence the system is always unstable at the zero power equilibrium point.

3. Possible reactor region

Except for specially designed reactors (for instance the pulse reactor), the usual power reactor should be operated at a state which is sufficiently below the prompt critical state. In this paper we do not include such a specially designed reactor and assume that the possible value of reactivity should be smaller than β.

3.1. Space A

Integrating Eq. (1) under the condition (2), we obtain the following form of the reactor dynamic equations

$$\frac{dn}{dt}=\frac{\rho-\beta}{\beta}n+\frac{1}{\tau_0}\int_{-\infty}^{t} n(t')\exp\left(-\frac{t-t'}{\tau_0}\right)dt' \tag{23}$$

$$\rho=\rho_0+u+En+F\int_{-\infty}^{t} n(t')\exp\left(-\frac{t-t'}{\tau}\right)dt'. \tag{24}$$

Next, substituting the relations

$$\int_{-\infty}^{t} n(t')\exp\left(-\frac{t-t'}{\tau}\right)dt'=U \tag{25}$$

$$\int_{-\infty}^{t} n(t')\exp\left(-\frac{t-t'}{\tau_0}\right)dt'=W. \tag{26}$$

into Eq. (23), we obtain the relations

$$W=\Lambda\tau_0 n(t)+\frac{\tau_0}{\beta}\left[\beta-\rho(t)\right]n(t) \tag{27}$$

$$\rho=\rho_0+u(t)+En(t)+FU. \tag{28}$$

Differentiating both sides of Eq. (27) with regard to t and substituting the following relations

$$\dot{W}=-\frac{1}{\tau_0}W+n(t)$$

$$\dot{U}=-\frac{1}{\tau}U+n(t), \tag{29}$$

we obtain the following expression

$$-\frac{1}{\tau_0}W+n=\tau_0\Lambda\ddot{n}+\frac{\tau_0}{\beta}(\beta-\rho)\dot{n}-\frac{\tau_0}{\beta}\dot{\rho}n. \tag{30}$$

We then substitute Eq. (27) into Eq. (30) and obtain

$$\tau_0 \Lambda \ddot{n} + \left(\tau_0 + \Lambda - \frac{\tau_0}{\beta}\rho\right)\dot{n} + \left(-\frac{\rho}{\beta} - \frac{\tau_0}{\beta}\dot{\rho}\right)n = 0\,. \tag{31}$$

Next, differentiating both sides of Eq. (28) we obtain

$$\begin{aligned}\dot{\rho} &= \frac{du}{dt} + E\frac{dn}{dt} + F\left(-\frac{1}{\tau}U + n\right)\\ &= E\dot{n} + (F + E/\tau)n + (\rho_0 - \rho)/\tau + u/\tau + \dot{u}\,.\end{aligned} \tag{32}$$

By the substitution of the above equation into Eq. (31), we finally obtain

$$\begin{aligned}&\tau_0\Lambda\ddot{n} + \left(1 + \frac{\Lambda}{\tau_0} - \frac{\rho}{\beta} - \frac{E}{\beta}n\right)\tau_0\dot{n}\\ &\quad - \left[\left(-1 + \frac{\tau}{\tau_0}\right)\frac{\rho}{\beta} + \frac{\rho_0}{\beta} + \frac{u}{\beta} + \frac{(E+\tau F)}{\beta}n + \frac{1}{\beta}\tau\dot{u}\right]\frac{\tau_0}{\tau}n = 0.\end{aligned} \tag{33}$$

Eq. (33) is a form of the dynamic equation of the reactor power which takes ρ as the parameter. By using relation (5), we can express the above equation by state variables x, y and z

$$\begin{aligned}&\frac{\Lambda}{\tau_0}z + \left(1 + \frac{\Lambda}{\tau_0} - \frac{\rho}{\beta} - \frac{E}{\beta}x\right)y\\ &\quad - \left[\left(-1 + \frac{1}{d}\right)\frac{\rho}{\beta} + \frac{\rho_0 + u}{\beta} + \frac{E+\tau F}{\beta}x + \frac{1}{\beta}\tau\dot{u}\right]x = 0.\end{aligned} \tag{34}$$

Eq. (34) represents a quadaratic surface in a three dimensional space which has x, y and z as the coordinates.

If we consider the case in which the external control u is absent, Eq. (34) becomes

$$\frac{\Lambda}{\tau_0}z + \left(1 + \frac{\Lambda}{\tau_0} - \frac{\rho}{\beta} - \frac{E}{\beta}x\right)y - \left[\left(-1 + \frac{1}{d}\right)\frac{\rho}{\beta} + \frac{\rho_0}{\beta} + \frac{E+\tau F}{\beta}x\right]x = 0. \tag{35}$$

From the above equation we can obtain two extreme surfaces, one is for $\rho = \beta$ and the other is for $\rho = -\infty$. They are for $\rho = \beta$,

$$\frac{\Lambda}{\tau_0}z + \left(\frac{\Lambda}{\tau_0} - \frac{E}{\beta}x\right)y - \left[\left(-1 + \frac{1}{d} + \frac{\rho_0}{\beta}\right) + \frac{E+\tau F}{\beta}x\right]x = 0\,, \tag{36}$$

and for $\rho = -\infty$,

$$y = (1 - 1/d)x\,. \tag{37}$$

Eq. (37) represents a plane which is in parallel with the z axis.

The obtained surfaces (36) and (37) are the boundary surfaces of the possible region of the reactor and have expanded forms of the following equations

$$-\frac{E}{\beta}y+\frac{E+\tau F}{\beta}x-\left(1-\frac{1}{d}-\frac{\rho_0}{\beta}\right)=0 \qquad \text{(a)}$$

$$y=(1-1/d)x\,, \qquad \text{(b)} \qquad (38)$$

which were derived by Andreiev et al. on the phase plane (x, y).

To find the tangible figure of surface (36), we transform the coordinates. By the relations

$$-(E+\tau F)/\beta=h_1,\qquad -E/\beta=h_2\,,$$
$$-(-1+1/d+\rho_0/\beta)=h_3\,,\qquad \Lambda/\tau_0=h_4 \qquad (39)$$

we can change Eq. (36) to a simpler form

$$h_1x^2+h_2xy+h_3x+h_4y+h_4z=0\,. \qquad (40)$$

Then we adopt the transformation of coordinates,

$$x=X\cos\theta-Y\sin\theta-h_4/h_2$$
$$y=X\sin\theta-Y\cos\theta+(2h_1h_4-h_2h_3)/h_2^2$$
$$z=Z+(h_2h_3-h_1h_4)/h_2^2\,. \qquad (41)$$

In this transformation, the origin of the new space (x_t, y_t, z_t) becomes

$$x_t=\beta\Lambda/E\tau_0$$
$$y_t=(1-1/d-\rho_0/\beta-2\Lambda/s\tau_0)\beta/E$$
$$z_t=(-1+1/d+\rho_0/\beta+\Lambda/s\tau_0)\beta/E\,. \qquad (42)$$

The rotational angle θ is selected between $-\pi/2$ and $\pi/2$, and is obtained by

$$\tan\theta=h_2/h_1=s \qquad (43)$$

or

$$\theta=(\tan^{-1}s)/2,\qquad (-\pi/2\leq\theta\leq\pi/2)\,. \qquad (44)$$

Applying the transformation, Eq. (40) is reduced to

$$Z=\frac{X^2}{A}+\frac{Y^2}{B}\,, \qquad (45)$$

where

$$A=-\left[\frac{1}{2h_1h_4}(h_1^2+h_2^2)\cos 2\theta+\frac{h_1}{2h_4}\right]^{-1}$$

$$B=-\left[\frac{-1}{2h_1h_4}(h_1^2+h_2^2)\cos 2\theta+\frac{h_1}{2h_4}\right]^{-1} \qquad (46)$$

To deal with the value of θ, we substitute Eq. (39) into the above equation and obtain

$$A=\frac{2\beta A}{(E+\tau F)[(-1)^{p}\sqrt{1+s^{2}}+1]\tau_{0}}$$

$$B=\frac{2\beta A}{(E+\tau F)[(-1)^{p+1}\sqrt{1+s^{2}}+1]\tau_{0}}$$

where

$$p=0 \quad (|\theta|<\pi/4),$$
$$p=1 \quad (\pi/4<|\theta|\leq\pi/2). \tag{47}$$

In the above equation, $|\theta|=\pi/4$ is omitted, because in this case the value $(E+\tau F)$ becomes zero and the infinite value of equilibrium power n_p is led forth. As mentioned before, the value $(E+\tau F)$ always has a negative value, and when $p=0$, A is negative and B is positive, and when $p=1$, A is positive and B is negative. Hence the surface (45) is a hyperbolic paraboloid.

It is easily shown that the entire space is separated into four subregions by mutual intersection of the hyperbolic paraboloid and the plane (37), because the plane is perpendicular to the $x-y$ plane. We can answer the question as to what part of the four subregions the possible region is situated, by the following procedure. Let $P_1(x, y, z_1)$ be a point which is located on the surface (35) and let $P_2(x, y, z_2)$ be on the surface (36). Then subtract z_2 from z_1. If this difference is positive, the possible region is situated over the boundary surface (36) (the direction of z positive), and if negative, it is under the surface. Executing the subtraction we obtain

$$z_1-z_2=\frac{\rho-\beta}{\beta}\cdot\frac{\tau_0}{A}\left[y+\left(-1+\frac{1}{d}\right)x\right]. \tag{48}$$

Eq. (48) indicates that, if it is the case where $y<(1-1/d)x$ the difference becomes positive, and vice versa. As the condition $y=(1-1/d)x$ coincides with Eq. (37), it is obvious that when we take the plane as the boundary, the situation for the existence of the possible region changes to the reverse. On one side of the plane the possible region is situated over the hyperbolic paraboloid, and on another side it is under the surface.

As a result, we get two possible subregions, one of which possesses the two equilibrium points P'_a and P'_b in it, and the other possesses only P'_a on the boundary. The two subregions contact each other on a specific line which is produced by the mutual intersection of the two extreme surfaces. In Fig. 1 these circumstances are shown.

Next we consider the possibility of moving of a state between the two subregions. If a state moves between the two subregions under the condition that the reactivity is $\rho(t)$, the movement must be carried out at a point which is situated on the line produced by the intersection of the quadaratic surface (35) and plane (37). We express the line on the $\xi-z$ plane (see Fig. 1) and substituting the relations

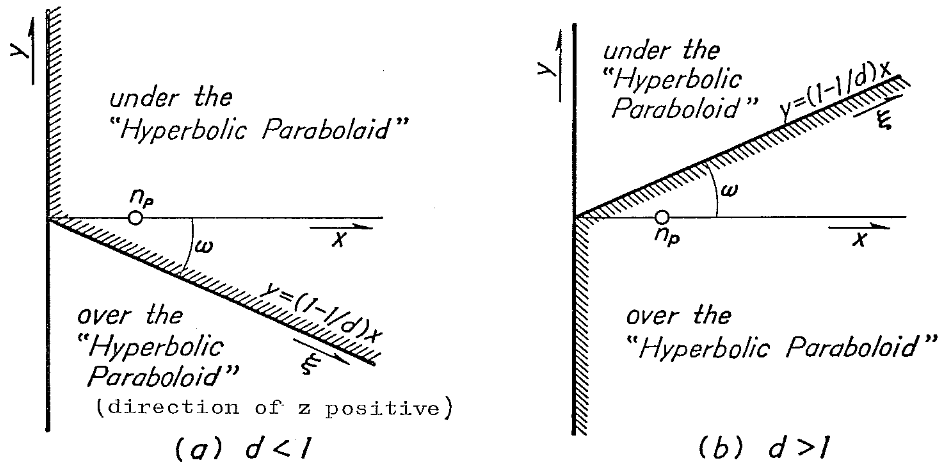


Fig. 1. Boundary Plane and Axis ξ

$$x=\xi\cos\omega\,,\qquad y=\xi\sin\omega\,, \tag{49}$$

and also substitutiry Eq. (35) into Eq. (33), we obtain the equation of the line

$$z=M\xi^2+N\xi \tag{50}$$

where

$$M=\frac{E\tau_0}{\beta\Lambda}\Big[(1-1\Lambda/d)+1/s\Big]\cos^2\omega$$

$$N=\frac{\tau_0}{\Lambda}\left[\frac{\rho_0}{\beta}-(1+\Lambda/\tau_0)(1-1/d)\right]\cos\omega\,. \tag{51}$$

It is remarkable that the above equation does not contain $\rho(t)$ in spite of dealing with the case in which the monement is conducted when the reactivity value is $\rho(t)$. This means that all quadaratic surfaces which correspond to innumerable values of the parameters ($\rho<\beta$), including the two extreme surfaces (36) and (37), are mutually crossed by the line (50). It is also noted that the quadaratic surfaces contain the zero power equilibrium point P_a' without exception.

If a state were possible to move between the two subregions, this movement must be carried out through a point located on the line (50), irregardless of value of $\rho(t)$ the system carries. It was found that at the moment of the movement, the values of the state variables render $P(x, y)$ in Eq. (6) zero, because the line (50) is situated on plane (6) the equation there of is expressed by $P(x, y)=0$. Thus in order that the movement is performed at a finite value of $\dot{z}$, it is at least necessary that at the moment of the movement, the values of the state variables also render $Q(x, y, z)$ zero. But after substitution of Eqs. (49) and (50) into the equation $Q(x, y, z)=0$, we obtain the value of ξ only as zero, which corresponds to the zero power equilibrium point P_a'. We have discussed in chapter 2 that the zero power equilibrium point is either the focal saddle or the nodal saddle and the movement of state between the two subregions could not be realized.

From the above discussion we conclude that the genuine possible region is limited to only one side of the subregions in which both the zero power and power equilibrium points are contained. As mentioned in the introduction, there is no evidence of an occurence of blocking of the movement of state by a band of the impossible region, which is contrary to Andreiev's conclusion. This contradiction arizes from the fact that Andreiev et al. treated the problem under the assumption that the variation of the reactor power was carried out in quasi-statical manner but in our treatment the assumption was not settled.

3.2. Space B

In this space, the possible region is obtained in a much simpler form applying the condition

$$\rho<\beta, \quad n\geq 0, \quad c\geq 0, \quad T\geq 0 \tag{52}$$

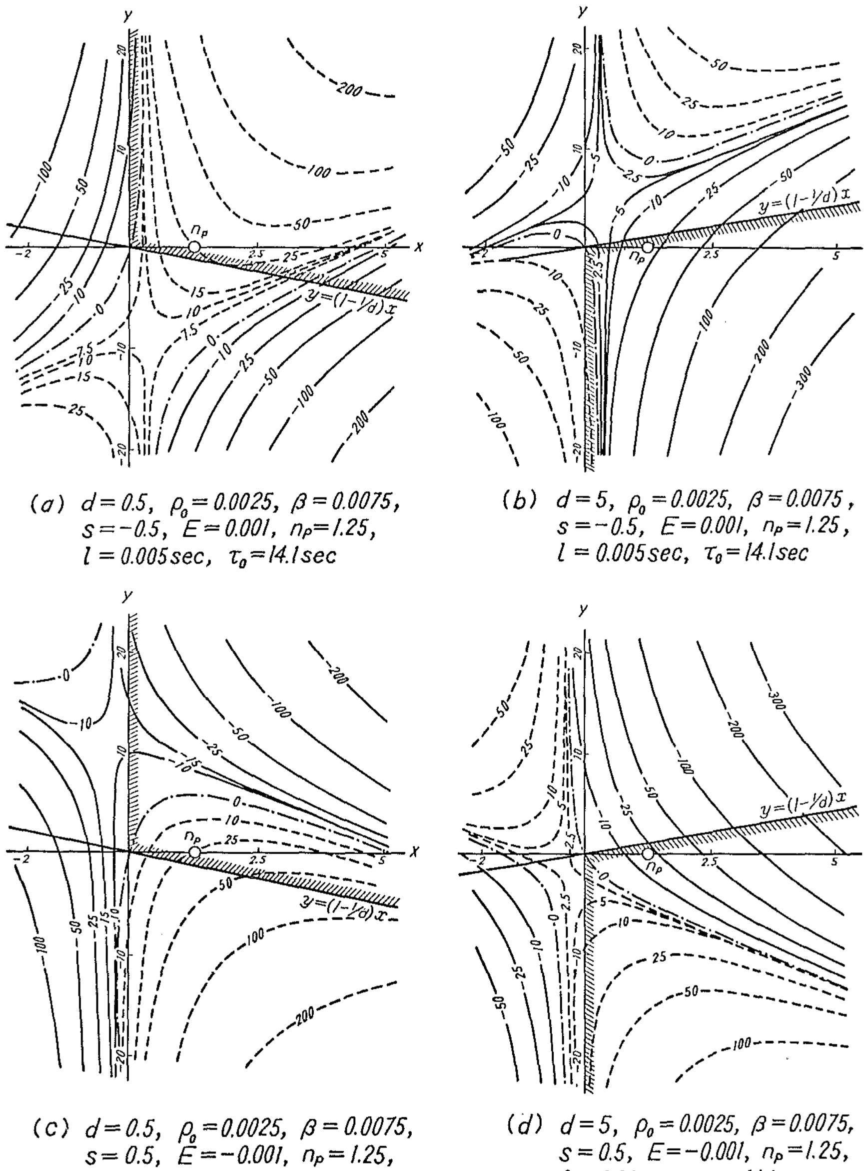
to Eq. (1). Considering again the case in which the control is absent, we obtain the boundary surface of the possible region which corresponds to $\rho=\beta$ as

$$En+F'T=\beta-\rho_0. \tag{53}$$

The possible region is situated on the possible side ($\rho<\beta$) of the boundary surface (53) satisfying the remaining conditions of Eq. (52). It is noted that the region is not restricted by the value of c and is given as a region which is bordered by a plane which crosses the $n-T$ plane at right angles.

4. Numerical Examples

In Fig. 2, the possible region as a reactor is shown for four typical examples in the three dimentional phase space (Space A) by means of a contour map. In these diagrams, the solid lines indicate that the height of contours are over the $x-y$ plane (direction of z positive) and the dotted lines indicate that they are



(a) $d=0.5$, $\rho_0=0.0025$, $\beta=0.0075$, $s=-0.5$, $E=0.001$, $n_P=1.25$, $l=0.005$ sec, $\tau_0=14.1$ sec

(b) $d=5$, $\rho_0=0.0025$, $\beta=0.0075$, $s=-0.5$, $E=0.001$, $n_P=1.25$, $l=0.005$ sec, $\tau_0=14.1$ sec

(c) $d=0.5$, $\rho_0=0.0025$, $\beta=0.0075$, $s=0.5$, $E=-0.001$, $n_P=1.25$, $l=0.005$ sec, $\tau_0=14.1$ sec

(d) $d=5$, $\rho_0=0.0025$, $\beta=0.0075$, $s=0.5$, $E=-0.001$, $n_P=1.25$, $l=0.005$ sec, $\tau_0=14.1$ sec

Fig. 2. Possible Region of a Reactor

below the plane. In all cases, taking the plane $y=(1-1/d)x$ as a boundary, on the upper side of the boundary (direction of y positive) the possible region is situated under the hyperbolic paraboloid, and on the lower side of the boundary, it is over the surface. In actuality, the region bordered by the oblique line is the genuine possible region in which the two equilibrium points are contained.

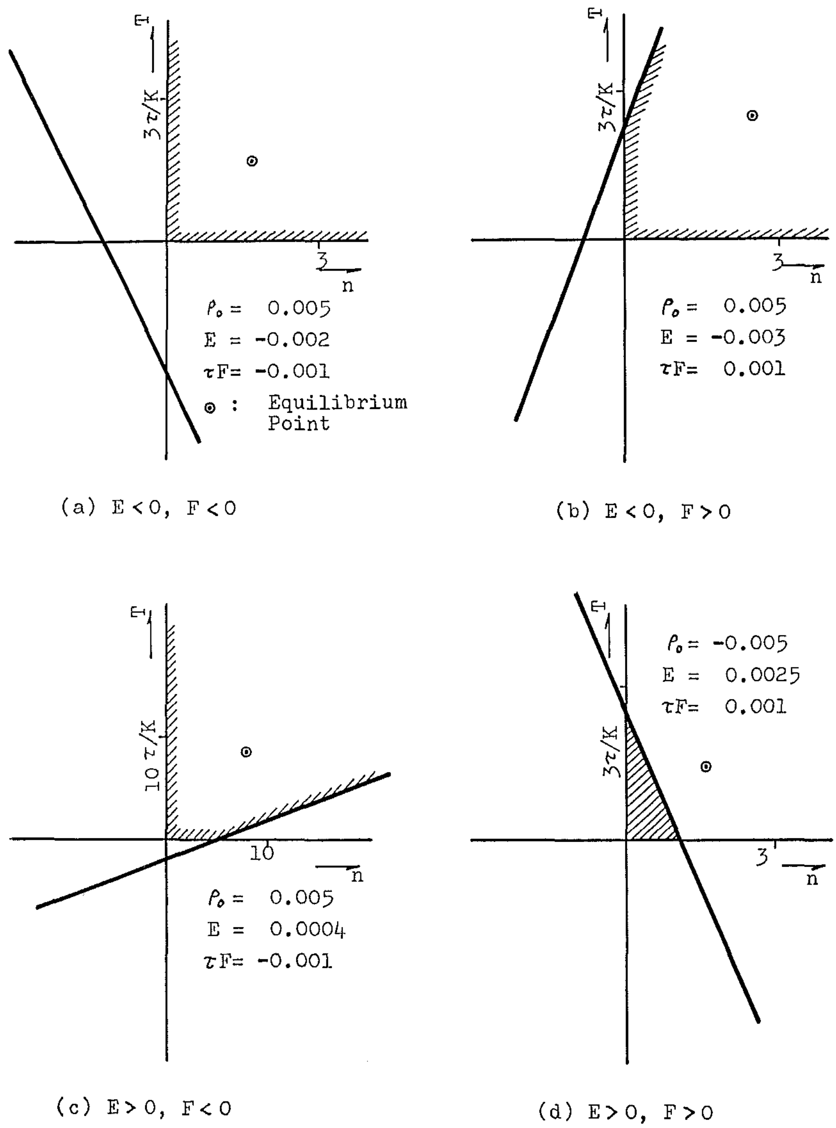


Fig. 3. Stable Region and Possible Region

In Fig. 3, the possible region as a reactor is shown for four typical examples in the three dimensional space (Space B). In each example the combination of the positiveness or negativeness of the internal feedback coefficients E and F is different from the others. The possible region is given as the region which is bordered by the oblique lines. In Fig. 3–(d), ρ_0 is negative and $(E+\tau F)$ is positive. This case should be excluded from the argument as mentioned before.

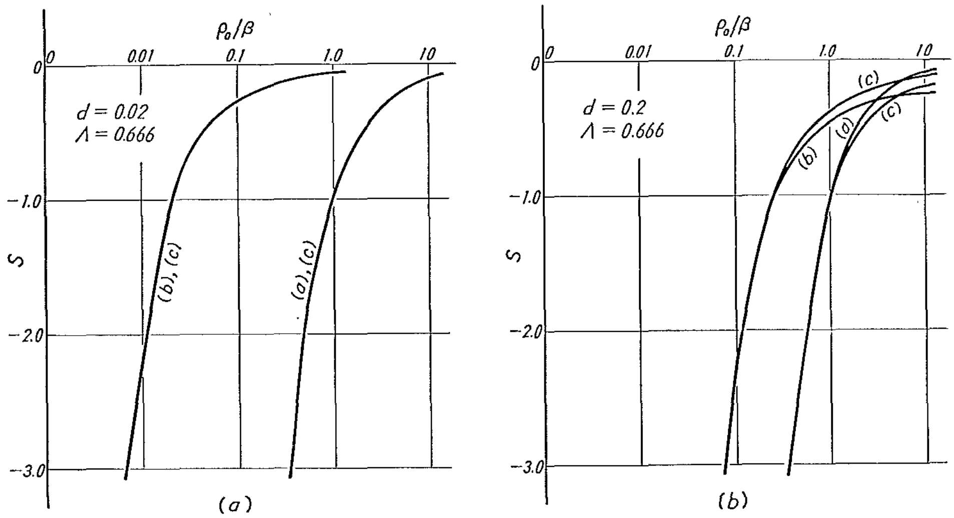


Fig. 4. Stable Region of a Reactor

Fig. 4 shows the results of caluculations of Eq. (18) for two examples. The condition of stability near point P_b' is shown by the relation of ρ_0/β versus s. The stability of the system is satisfied in the region which lies on the stable side (direction of s positive) of all the curves (a), (b) and (c) on the diagrams. In Fig. 6–(a), the curve (c) occasionally coincides almost with curves (a) and (b). This is caused by the fact that, in this example the value $e/\tau\tau_0$ in Eq. (17) is fairly small compared to that of ab/τ_0.

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Appendix A

$$D_1 = dA/\tau_0 \qquad \text{(app-1)}$$

$$D_2 = (1-2d)dA/\tau_0 \qquad \text{(app-2)}$$

$$D_3 = [-(1-d)-d\rho_0/\beta-(1-d^2)A/\tau_0]-(d/\beta)u(t)-(d/\beta)\tau\dot{u}(t) \qquad \text{(app-3)}$$

$$D_4 = (1-d-d/s)E/\beta \qquad \text{(app-4)}$$

$$D_5 = 2d^2E/\beta \qquad \text{(app-5)}$$

$$D_6 = [d(2-d)+3d^2/s]E/\beta \qquad \text{(app-6)}$$

$$D_7 = [1-d-d(2d-3)/s]E/\beta \qquad \text{(app-7)}$$

$$D_8 = (1-d)E/\beta s \qquad \text{(app-8)}$$

$$D_9 = [d(1-2d)-2d^2\rho_0/\beta+d(1-2d)A/\tau_0]+(2d^2/\beta)u(t)+(2d^2/\beta)\tau\dot{u}(t) \qquad \text{(app-9)}$$

$$\begin{aligned} D_{10} = {} & [d(1-2d)\rho_0/\beta]-d(1-d)-d(1-d)A/\tau_0 \\ & +[d(2-d)/\beta]u(t)+(2d/\beta)\tau\dot{u}(t)+(d/\beta)\tau\tau_0\ddot{u}(t) \end{aligned} \qquad \text{(app-10)}$$

$$D_{11} = (1-d)\rho_0/\beta+[(1-d)/\beta]u(t)+[(1-d^2)/\beta]\tau\dot{u}(t)+[(1-d)/\beta]\tau\tau_0\ddot{u}(t) \qquad \text{(app-11)}$$

$$D_{12} = dA \qquad \text{(app-12)}$$

$$D_{13} = (1-d)A \qquad \text{(app-13)}$$

Appendix B

$$a(u) = -[(s/\beta+1)/A+(1+d)/\tau_0]-[(1/\beta A)\cdot d/(1-d)](u(t)+\tau\dot{u}(t)) \qquad \text{(app-14)}$$

$$\begin{aligned} b(u) = {} & -[(s+d)\rho_0/\beta+d]/A+d/\tau_0 \\ & +(1/\beta A)[d/(1-d)][(2-d)u(t)+2\tau\dot{u}(t)+\tau\tau_0\ddot{u}(t)] \end{aligned} \qquad \text{(app-15)}$$

$$e(u) = -\rho_0/\beta+(2/\beta A)[u(t)+(1+d)\tau\dot{u}(t)+\tau\tau_0\ddot{u}(t)] \qquad \text{(app-16)}$$

$$f(u) = (n_p/\beta A)[u(t)+(1+d)\tau\dot{u}(t)+\tau\tau_0\ddot{u}(t)] \qquad \text{(app-17)}$$