Small-Signal Power Theorems and Radial Displacement Waves in CEF-Type Traveling-Wave Devices*

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Contents

Abstract

A small-signal power theorem of CEF-type traveling-wave devices was developed for systems in which a two-dimensional thin beam completes a single excursion through the interaction space of a coaxial-cylindrical structure wherein the inner conductor is replaced by a circuit that supports azimuthally slow waves. It is shown that the electromagnetic power delivered by the thin CEF-type electron beam is, within the assumption of small-signal theory, balanced by a decrease of the r–f azimuthal kinetic power, the r–f radial kinetic power and the r–f potential power and the radial-displacement waves are coupled out of the CEF-type traveling-wave device.

1. Introduction

CEF-type amplifiers and oscillators employing the electron beam being considered were studied extensively by Nunn and Rowe[1),2)], and the forward-wave growing- and beating-wave gain, as well as the backward-wave start-oscillation conditions were given. A small-signal efficiency estimate was also made, and it was found that CEF- and O-type devices are similar in this respect, i.e, there is generally little, if any, potential energy conversion. A small-signal analysis of the helitron oscillator was presented by Pantell[3)]. In contrast to the CEF-type device considered here, where the beam travel is limited to an angle of less than 360 spatial degrees, the ribbon beam drifts axially while it interacts with an azimuthal component of a transverse-electromagnetic wave in a helitron. That treatment was limited to considering the backward-wave starting conditions in the helitron where the electron follows a helical path parallel to the cylindrical axis. The existence of growing

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waves was also pointed out, even in the absence of a slow-wave circuit. Power carried by an E-type filamentary electron beam and dispersion curves were analyzed by Sakuraba[4),5)]. In that beam, the electron traces a helical path between coaxial cylinders and all electrons from a right circular hollow cylinder which is thin in the radial direction. A small-signal power theorem for a filament beam in arbitrary d–c electric and magnetic fields was presented by Haus and Bobroff[6)]. This power theorem is applicable to many beam devices, but the application in detail was limited to O- and M-type devices. Recently, Sakuraba and Rowe[7),8)] have shown that certain types of photodemodulators may make effective use of CEF-type electron beams. More recently, Sakuraba[9)] has proposed a microwave electron prism with an optical dispersing element. Microwave frequency modulation of an optical signal can be converted into spatial modulation by passing the light through an optical dispersing element. Analysis shows that the instantaneous center of gravity of the CEF-type electron beam from the photocathode oscillates transversely, exactly following the frequency modulation of the incident light. The transverse motion can be shown as the excitation of radial current waves on the CEF-type electron beam and these waves can then be amplified and coupled out of a microwave electron prism by interactions common in the CEF-type devices. One of these is shown in Fig. 1.

This paper is concerned with the development of a small-signal power theorem for CEF-type traveling-wave devices. The analysis employed here is patterned after

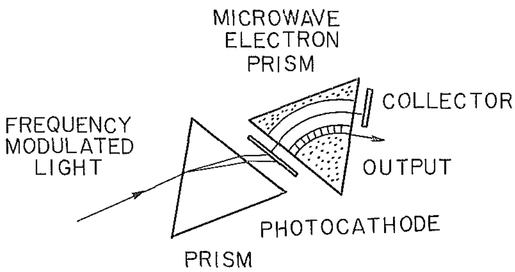

Fig. 1. Schematic of microwave electron prism.

the approach originally used by Haus[10)] for O- and M- type traveling-wave devices. The small-signal power theorem shows that the electromagnetic power delivered by the CEF-type beam is, under the small-signal theory, balanced by a decrease of the generalized r–f power in the beam.

2. Power Delivered by the Electron Beams

The basic model of the CEF-type traveling-wave device is considered as shown in Fig. 2. The interaction region consists of a coaxial-cylindrical structure with a slow-wave circuit situated along the inner concentric and a sole electrode along a tangent to the circle of motion in the cross-section plane of the system, and a static electric field is applied between the sole and circuit electrodes. The d–c potential of the circuit measured relative to the cathode electrode is chosen to be sufficiently larger than that of sole so that the centrifugal force of the electrons in the interaction region is balanced by an equal and opposite radial electric field force. It is assumed that the thin electron beam is infinite in extent in the z-direction. The d–c voltage between the electrode is $V_c - V_s$ and the distance $d = r_s - r_c$, where

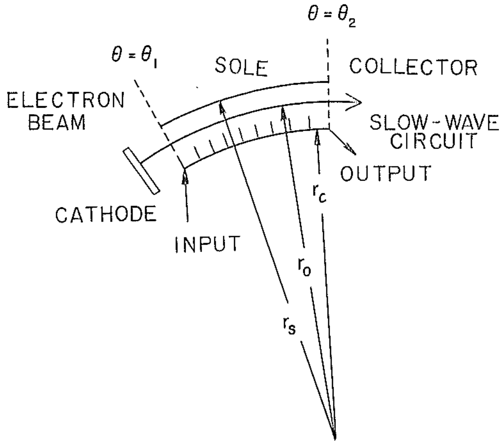

Fig. 2. The basic geometry of the CEF-type traveling-wave device.

V_c and V_s are the potential of the slow-wave circuit and the sole, measured relative to the cathode, respectively. In the d–c state, the electron beam has a radial thickness t and its average position is defined by r_0. For simplicity, it is assumed that $r_0 \gg d$ and $d \gg t$. Since a steady trajectory is possible with a constant angular velocity Ω_0, the d–c velocity must vary with r inside the electron beam. Then we assumed that the difference between fields is negligible, and that the field has everywhere the value of

$$E_0 = \left[\frac{1}{r_0} \frac{V_c - V_s}{\ln(r_s/r_c)}\right], \tag{1}$$

and all electrons have the same velocity

$$\eta\Omega_0 = \left[\frac{-\eta(V_c - V_s)}{\ln(r_s/r_c)}\right]^{1/2}, \tag{2}$$

where $\eta = -1.7589 \times 10^{11}$ coulomb/kg, the charge-to-mass ratio of the electron.

All r–f quantities possess the characteristic variation exp $j(\omega\tau - \beta\theta)$, where ω is the angular frequency of the r–f wave and β is the circular propagation constant. It follows that partial derivatives with respect to θ and time τ may be replaced with multiplication by $-j\beta$ and $j\omega$, respectively. Under the small-signal condition the electromagnetic power flow in the –r direction out of the region of the interaction as shown in Fig. 3, that enters the slow-wave circuit, is given within the second order of the small-signal amplitudes by

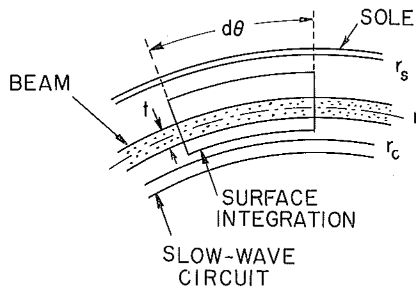

Fig. 3. Surface of integration.

$$-dP = \frac{1}{2} Re\left[E_{\theta+}H^*_{z+}(r_0 + t/2)d\theta - E_{\theta-}H^*_{z-}(r_0 - t/2)d\theta\right]$$

$$\approx \frac{1}{2} Re\left[\frac{1}{2}(E_{\theta+} - E_{\theta-})(H^*_{z+} + H^*_{z-}) + \frac{1}{2}(E_{\theta+} + E_{\theta-})(H^*_{z+} - H^*_{z-})\right] r_0 d\theta, \tag{3}$$

where $E_{\theta+}$ and $E_{\theta-}$ are azimuthal components of r–f electric field just above and just below the beam, respectively, and H_{z+} and H_{z-} are z-components of the r–f magnetic field just above and just below the beam, respectively, and it was assumed that $r_0 \gg t/2$. The region of interaction is bounded by $(r_0+t/2)d\theta$, $(r_0-t/2)d\theta$ and $t+\Delta$, $(t \gg \Delta > 0)$ and unit of length in the z-direction.

The discontinuity of magnetic field is given by

$$H_{z+} - H_{z-} = -K_\theta \tag{4}$$

where K_θ is the surface density of the r–f azimuthal current per unit length in the z-direction, which is given by

$$K_\theta = J_\theta t\,, \tag{5}$$

Under the small-signal theory, the volume density of r–f azimuthal current J_θ is

$$J_\theta \approx \Omega_0 r_0 \rho_1 + r_0 \Omega_0 \Omega_1 + \Omega_0 \rho_0 r_1\,, \tag{6}$$

where ρ_0 is the d–c component of volume charge density, ρ_1 is the r–f component of volume charge density, Ω_1 is the azimuthal velocity perturbation function and r_1 is the displacement of an electron of the excited beam from its unperturbed position, and it is assumed that r_1 is very small compared to the dimensions of the system, e. g., beam thickness t, center-of-the beam radius r_0 and wavelength.

Next consider the relation between H_{z+} and the z-component of r–f magnetic field just below the upper edge of the beam $H_{z\delta+}$. From Maxwell's equations it follows that

$$\frac{1}{r}\frac{\partial H_z}{\partial \theta} = j\omega E_r \varepsilon_0 + J_r\,, \tag{7}$$

where J_r is the volume density of r–f true current in the radial direction. This current is shown as

$$J_r = \rho_0 v_r\,, \tag{8}$$

where v_r is the r–f velocity in the radial direction, that is given by

$$v_r = \frac{dr_1}{d\tau} = \left(j\omega + \Omega_0 \frac{\partial}{\partial \theta}\right) r_1\,, \tag{9}$$

From the number of dipoles per unit volume, $\rho_0 r_1$, the boundary condition of the radial r–f electric field at the upper edge of the beam is

$$E_{r+} = E_{r\delta+} + \frac{\rho_0}{\varepsilon_0} r_1\,, \tag{10}$$

where E_{r+} is the radial r–f electric field just above the upper edge of the beam, $E_{r\delta+}$ is the radial r–f electic field just below the upper edge of the beam, and ε_0 is the permittivity of free space. Applying Eq. (7) to H_{z+} and $H_{z\delta+}$ and utilizing Eqs. (8), (9) and (10) the discontinuity of magnetic field at the upper edge of the beam is expressed by Eq. (11)

$$H_{z+} - H_{z\delta+} = -\rho_0 r_0 \Omega_0 \bar{r}_1\,, \tag{11}$$

where it was assumed that $r = r_0 + r_1 \approx r_0$ in the limit of the thin beam approximation r_1 was replaced by its mean value $\bar{r}_1$. In the same way, the magnetic field relation at the lower edge of the beam is developed as follows

$$H_{z-} - H_{z\delta-} = -\rho_0 r_0 \Omega_0 \bar{r}_1\,. \tag{12}$$

Using Eqs. (11) and (12) it follows that

$$\frac{1}{2}(H_{z+}+H_{z-})=\bar{H}_{zi}-\rho_0 r_0 \Omega_0 \bar{r}_1\,, \tag{13}$$

where $\bar{H}_{zi}$ is the average value of magnetic field inside the electron beam and is shown by Eq. (14)

$$\bar{H}_{zi}=\frac{1}{2}(H_{zi+}+H_{zi-})\,. \tag{14}$$

When the electron velocity is small compared with the velocity of light, the curl of the electric field is zero. Therefore the following equation is obtained for the electric field

$$E_{\theta+}-E_{\theta-}=\int_{r_0-t/2}^{r_0+t/2}\frac{\partial E_\theta}{\partial r}dr\approx\int_{r_0-t/2}^{r_0+t/2}\frac{1}{r_0}\frac{\partial E_r}{\partial \theta}ds\approx\frac{t}{r_0}\frac{\partial \bar{E}_{ri}}{\partial \theta}\,, \tag{15}$$

where it was assumed that the azimuthal r–f electric field is continuous at the edge of the beam and that $r_1 \ll r_0$. The quantity $\bar{E}_{ri}$ is the mean value of radial r–f electric fields inside the beam E_{ri}.

Using Eqs. (4), (7), (9), (13) and (15), Eq. (3) becomes

$$-dP=-\frac{1}{2}Re\Big[\bar{E}_\theta\cdot K_\theta^*+(E_{\theta+}-E_{\theta-})\rho_0 r_0\Omega_0\bar{r}_1^* + \bar{E}_{ri}\rho_0 t v_r^* - \frac{t}{r_0}\frac{\partial}{\partial\theta}(\bar{E}_{ri}\cdot\bar{H}_{zi}^*)\Big] r_0 d\theta\,, \tag{16a}$$

$$=-\frac{1}{2}Re\Big[\bar{E}_\theta\cdot K_\theta^*-j\omega r_1^*\sigma_0\bar{E}_{ri}+\sigma_0\Omega_0\frac{\partial}{\partial\theta}(\bar{r}_1^*\bar{E}_{ri}) - \frac{t}{r_0}\frac{\partial}{\partial\theta}(\bar{E}_{ri}\bar{H}_{zi}^*)\Big] r_0 d\theta\,, \tag{16b}$$

where σ_0 is the d–c surface charge density that is defined by $\rho_0 t$ and $\bar{E}_\theta=(E_{\theta+}+E_{\theta-})/2$. The first term on the right member of Eq. (16a) indicates the power expended by the azimuthal current against the impressed field $\bar{E}_\theta$. The second term indicates the power expended by the surface current $\pm\rho_0 r_0\Omega_0 r_1^*$ at the upper and lower edges of the beam against the electric fields $E_{\theta+}$ and $E_{\theta-}$. The third term gives the power expended by the radial current against the impressed field $\bar{E}_{ri}$. The last term gives the Poynting vector inside the electron beam.

3. Force and Continuity Equations

The d–c trajectory of an electron is given by

$$\theta=\theta_0=\Omega_0(\tau-\tau_0)\,, \qquad r=r_0\,,$$

where τ_0 is the injection time of an electron into the interaction space. If in addition an r–f electric field is present, the motion of an electron is perturbed and the following can be written as $\theta=\theta_0+\theta_1$, $r=r_0+r_1$ and $\Omega=\Omega_0+\Omega_1$. The equations of motion for an electron revolving about the center of the coaxial-cylindrical system are

$$2\dot{r}\dot{\theta}+r\ddot{\theta}=\eta\bar{E}_\theta\,, \tag{17}$$

and

$$\ddot{r}-r\dot{\theta}^2=\eta[\overline{E}_{ri}+E(r)]\,, \tag{18}$$

where $E(r)$ is the static radial electric field intensity which balances the centrifugal force of the electron at radius r. Therefore, for the d–c part of the force equation

$$\eta E_0+r_0\Omega_0^2=0$$

If the electron did not experience an r–f transverse displacement, the r–f electric field experienced by them would be simply $\overline{E}_\theta$ and $\overline{E}_{ri}$. However, if the d–c electric field is nonuniform as shown in Eq. (19)

$$E(r)=\frac{1}{r}\left[\frac{V_c-V_s}{\ln(r_s/r_c)}\right]. \tag{19}$$

An r–f displacement r_1 in a nonuniform d–c field simulates an r–f field of an amplitude which equals $\overline{r}_1$ times divergence of $E(r)$. Therefore, the small-signal r–f parts of force equations for an electron in terms of the polarization variables are

$$\overline{E}_\theta=\frac{1}{\eta}\left(j\omega v_\theta+\Omega_0\frac{\partial v_\theta}{\partial\theta}\right)+\frac{1}{\eta}2v_r\Omega_0\,, \tag{20}$$

$$\overline{E}_{ri}=\frac{1}{\eta}\left(j\omega v_r+\Omega_0\frac{\partial v_r}{\partial\theta}\right)-\frac{1}{\eta}2v_\theta\Omega_0-\overline{r}_1\frac{\rho_0}{\varepsilon_0}+E_0\frac{2\overline{r}_1}{r_0}\,, \tag{21}$$

where

$$v_\theta=r_0\Omega_1=r_0\dot{\theta}_1\,, \tag{22}$$

$$v_r=\left(j\omega+\Omega_0\frac{\partial}{\partial\theta}\right)\overline{r}_1\,, \tag{23}$$

and

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot E(r)\right]=\frac{\rho_0}{\varepsilon_0}. \tag{24}$$

Equation (24) can be obtained from Poisson's equation because the steady azimuthal electric field resulting from the beam charge is identically zero for ring charge[1].

For a cylindrical beam of infinite extent in the z-direction, the continuity equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}(rK_r)+\frac{1}{r}\frac{\partial K_\theta}{\partial r}=-j\omega\sigma_1\,, \tag{25}$$

where K_r is the surface density of true r–f radial current, that is given by $J_r t$, and σ_1 is the r–f component of the surface charge density. Utilizing the Eulerian fluid assumption $\partial r_1/\partial r=0$ and also $\partial i_r/\partial r=0$ and very small r–f displacement $r_1\ll r_0$, the continuity equation takes a simple form. Furthermore, in the case of a very large propagation constant of the beam ($\beta_e\gg 1$) and large center-of-the beam radius the azimuthal component of r–f convection current is more than ten times as large as the radial component. Therefore it follows that

$$\frac{\partial K_\theta}{\partial\theta}\approx -j\omega\sigma_1 r_0\,. \tag{26}$$

4. Power Conservation

Utilizing Eqs. (20), (21) and (26) the power flow of Eq. (16) becomes

$$-dP = -\frac{1}{2} Re \frac{1}{r_0} \frac{\partial}{\partial \theta} \left[\frac{1}{\eta} r_0 \Omega_0 (v_\theta K_\theta^*) + \frac{1}{\eta} \Omega_0 r_0 (-j\omega\sigma_0 \bar{r}_1^* v_r) + \frac{1}{\eta} 2\Omega_0^2 r_0 (\bar{r}_1 \cdot K_\theta^*) + \sigma_0 \Omega_0 r_0 \bar{E}_{r\delta} \bar{r}_1^* - t\bar{E}_{r\delta} \bar{H}_{z\delta}^* \right] r_0 d\theta . \quad (27)$$

Now, we define an r–f azimuthal kinetic voltage U_θ and an r–f radial kinetic voltage U_r as follows[6]:

$$U_\theta = \frac{1}{\eta} r_0 \Omega_0 v_\theta , \quad (28)$$

$$U_r = \frac{1}{\eta} r_0 \Omega_0 v_r , \quad (29)$$

where v_θ is the r–f tangential velocity of the wave at radius r_0 as shown in Eq. (22) and v_r is the r–f radial velocity as given by Eq. (23). The radial current K_R caused by the displacement of the electron beam from the its unperturbed position is given by

$$K_R = j\omega\sigma_0 \bar{r}_1 . \quad (30)$$

Since the displacement of the electron beam from the d–c position is $\bar{r}_1$, the polarization is given by $\rho_0 \bar{r}_0$. Then the displacement current of the electron beam of width t is given by Eq. (30). In the same way, the r–f potential is defined as[6]

$$\Phi = -\bar{r}_1 E_0 , \quad (31)$$

where E_0 is the d–c electric field as shown in Eq. (1). With these results the r–f power theorem becomes simply

$$-dP = -\frac{1}{2} Re \frac{1}{r_0} \frac{\partial}{\partial \theta} \Big[(U_\theta K_\theta^* + U_r K_R^* + 2\Phi K_\theta^*) + (r_0 \Omega_0 \sigma_0 \bar{r}_1^* \bar{E}_{r\delta}) - (\bar{E}_{r\delta} \cdot \bar{H}_{z\delta}^*) t \Big] r_0 d\theta . \quad (32)$$

As shown above, this $-dP$ means the power flow toward the slow-wave circuit that is delivered by the electron beam in the small length $r_0\ d\theta$.

The first term of the right side of Eq. (32) represents the azimuthal r–f kinetic power since it is the r–f kinetic power associated with the azimuthal small-signal motion of the beam. The second term is the radial r–f kinetic power since it is the r–f kinetic power carried by the radial displacement of the beam, It should be noticed that the power given by this expression is not a true radial power (strictly speaking, it is the energy transfer due to the radial fields) but the total power carried[11]. The third term is twice the r–f potential power. This potential power flow can be negative because the beam moved, on the average, into regions of lower potential (higher voltage, for a beam of negative charge). Now consider the fourth term in which $\sigma_0 \bar{r}_1^* \bar{E}_{r\delta}$ is the r–f static stored energy per unit area. The term $r_0 \Omega_0$ is the r–f linear tangential velocity at radius r_0. Therefore, $Re[r_0 \Omega_0 \sigma_0 \bar{r}_1^* \bar{E}_{r\delta}/2]$ represents the transport of the r–f static stored energy. The last term, $Re \cdot [-\bar{E}_{r\delta} \bar{H}_{z\delta}^* t/2]$, is the small-signal Poynting vector inside the electron beam in the $+\theta$ direction. However, in the usual operation of the CEF-type traveling-wave device with a thin electron beam, the last two terms are negligible.

The equation (32) can be written in the form

$$Re\left[-P+S_{\theta}(\theta)\Big|_{\theta_1}^{\theta_2}+S_b(\theta)\Big|_{\theta_1}^{\theta_2}\right]=0\ , \tag{33}$$

where

$$S_{\theta}(\theta)=-\frac{1}{2}(\bar{E}_{ri}\cdot\bar{H}_{zi}^{*})t\,, \tag{34}$$

and

$$S_b(\theta)=\frac{1}{2}\left[U_c K_{\theta}^{*}+U_r K_R^{*}+2\Phi K_{\theta}^{*}+(r_0\Omega_0\sigma_0\bar{r}_1^{*}\bar{E}_{ri})\right]. \tag{35}$$

The quantity $S_{\theta}(\theta)$ represents the azimuthal electromagnetic power in the cross sectional area of the electron beam within second-order terms of the small-signal amplitudes. The quantity $S_b(\theta)$ is the r–f power flow of the electron-beam. The quantity $-P$ is the electromagnetic power flowing out of the volume to the slow-wave circuit. Then the first and second terms of Eq. (33) represent the electromagnetic power flowing out of the volume bounded by planes θ_1 and θ_2. The third term is the power which is formed by the multiplication of small-signal excitation quantities in the beam. This power theorem establishes the fact that the electromagnetic power delivered by the CEF-type electron beam in the region $r_0(\theta_2-\theta_1)$ must be accompanied by a decrease of the four products on the right side of Eq. (35) with this length of the beam. There is a fair chance for detecting signals from radial-displacement-prebunching electron beams, because the r-f displacement $\bar{r}_1$ is included in the right side of Eq. (35).

5. Conclusions

A small-signal power theorem for two-dimensional CEF-type traveling-wave devices was developed and analyzed. The thin electron beam completes a single excursion through the interaction space of a coaxial- cylindrical structure. The results indicate that the electromagnetic power delivered by the CEF-type electron beam is balanced by a decrease of r–f azimuthal kinetic power, the r–f radial kinetic power and the r–f potential power. It is possible that the radial-displacement waves are coupled out of the CEF-type traveling-wave device.

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