Optimal Power Change of a Reactor with Constrained State Space

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Abstract

The time optimum control of a reactor system in which the internal feedback and the delayed neutrons are considered and the state space is constrained is obtained in two modes of the control. One is the control which transfers the system from the initial equilibrium state to the objecting equilibrium state in minimum time, and the other is the control which raises the reactor power to the objecting level as quickly as possible to follow the load change rapidly.

In the former mode of the control, the controllability of the nonlinear system is rigorously satisfied if the delayed neutrons are ignored, and even if the delayed neutrons are considered we can control the system optimally from a practical point of view.

In the latter mode of the control, we must apply a control after the objecting power level is reached in order to bring the system to the final equilibrium state keeping the power level constant.

It depends on the role of the reactor as to which mode of control should be employed.

1. Introduction

It is desirable that in nuclear reactors which are to be used for electric power generation or for propulsion purposes, the starting-up or the level change of power should be carried out as quick as possible. The time optimum control of the nuclear reactor has already been treated by some previous investigators[1),2),3)4),5),6),7),8)]. Moller, for instance, obtained the time optimum control of a nuclear reactor in which the rate of insertion or withdrawal of the externally applied reactivity was considered as a control parameter and the amount of the reactivity was constrained within some limits of value. In his study the effect of the delayed neutrons was neglected to facilitate easy analysis. Monta, employing the approximation of the prompt jump and the equivalent one group of delayed neutrons, obtained a time optimum control of a reactor in which the variational speed of the control rod, the amount of the reactivity and the overshoot of reactor power were constrained within some limits of values. In spite of the existence of these remarkable studies, it seems that there are hardly any reports concerning the optimum control of a reactor in which the feedback effect is considered and both the amount and rate of change of the external reactivity are constrained within some limits of values.

This paper intends to obtain the time optimum control of the above mentioned reactor in the following two control modes.

(i) The control which transfers the state from initial equilibrium point to objecting equilibrium point in a time optimum manner.

We designate this mode of control as "Control of mode A".

(ii) The control in which only the output of the reactor is required to be changed as quick as possible, although the delayed neutrons have not yet reached the final equilibrium state. In this mode of control it should be necessary to bring the system under control even after the objecting output is reached in order to transfer the system to the final equilibrium state. We designate this mode of control as "Control of mode B".

2. Dynamic Equations

Taking into account the effects of feedback and the delayed neutrons which are approximated by equivalent one group, the reactor dynamic equations are

$$\left.\begin{aligned} \frac{dn(t)}{dt} &= \frac{\rho(t)-\beta}{l}n(t)+\lambda c(t) && \text{(a)} \\ \frac{dc(t)}{dt} &= \frac{\beta}{l}n(t)-\lambda c(t) && \text{(b)} \\ \rho(t) &= \rho_c(t)-Fn(t)\,, && \text{(c)} \end{aligned}\right\} \quad (1)$$

where n is the thermal output, c the delayed neutron precursors concentration, ρ the reactivity, l the prompt neutron generating time, β the fractional yield of the delayed neutrons, λ the decay constant of the delayed neutron precursor, ρ_c the externally applied reactivity, F the coefficient for the internal feedback.

Defining the rate of insertion or withdrawal of the externally applied reactivity as a control parameter u, we have

$$\frac{d\rho_c(t)}{dt} = u(t)\,. \quad (2)$$

The problem is to obtain the optimum control u^0 that will transfer the system from some steady power level n_i to another power level n_f in a minimum time under the following constraints

$$\kappa \leq \rho_c \leq \gamma\,, \qquad |\dot{\rho}_c| \leq \eta\,. \quad (3)$$

We now transform the system (1), (2) by the following new variables which are obtained as the result of normalizing the state variables n, c, ρ_c by the respective values n_i, c_i, ρ_{ci} which correspond to the state variables of the initial equilibrium point P_i,

$$x_1(t) = \frac{n(t)}{n_i}\,, \quad x_2(t) = \frac{c(t)}{c_i}\,, \quad x_3(t) = \frac{\rho_c(t)}{\rho_{ci}}\,. \quad (4)$$

Then we obtain the following state equations

$$\left.\begin{aligned} \frac{dx_1}{dt} &= \frac{\rho_{ci}}{l}x_1x_3 - \frac{\rho_{ci}}{l}x_1^2 - \frac{\beta}{l}x_1 + \frac{\beta}{l}x_2 \\ \frac{dx_2}{dt} &= \lambda x_1 - \lambda x_2 \\ \frac{dx_3}{dt} &= \frac{1}{\rho_{ci}}u\,, \end{aligned}\right\} \quad (5)$$

where the relation $\rho_{ci}=Fn_i$, which is clearly obtained from Eq. (1)-(c), is used. In system (5), the constraint equation (2) becomes

$$\frac{\kappa}{\rho_{ci}} \leq x_3 \leq \frac{\gamma}{\rho_{ci}}, \qquad |\dot{x}_3| \leq \frac{\eta}{\rho_{ci}}. \tag{6}$$

3. Optimum Control

3.1. "Control of Mode A"

We obtain the control whitch transfers the system from initial equilibrium point $P_i(n_i,\ c_i,\ \rho_{ci})$ to objecting equilibrium point $P_f(n_f,\ c_f,\ \rho_{cf})$ in minimum time, under the fulfillment of the following inequality constraints

$$S(x) = x_3 - \frac{\gamma}{\rho_{ci}} \leq 0, \tag{7}$$

where we assume that n_f is larger than n_i and only the nonoscillatory system is dealt with.

As the state variable is constrained, the optimum control usually consisted from the following three segments (Fig. 1).

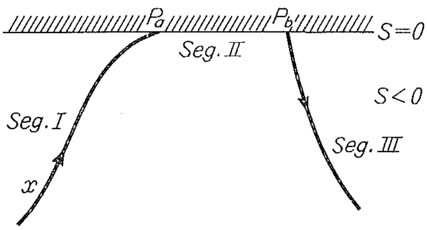

Fig. 1. Schematic representation of the state vector history.
Seg. I, III (Interior Segment) $S<0$
Seg. II (Boundary Segment) $S=0$

Segment I : From point P_i to point P_a where the state just arrives at the surface $S=0$.
Segment II : From point P_a to P_b where the state just leaves the surface $S=0$.
Segment III: From point P_b to Point P_f.

We assume that at time t_a the state just arrives at point P_a and at time t_b it leaves point P_b.

As we are dealing with the mimimum time problem, the cost function is expressed as

$$J = \int_0^t 1\, dt, \tag{8}$$

and we obtain the Hamiltonian of system (5)

$$\begin{aligned} H(x, p, u) &= 1 + p \cdot \dot{x} \\ &= 1 + p_1\left(\frac{\rho_{ci}}{l} x_1 x_3 - \frac{\rho_{ci}}{l} x_1^2 - \frac{\beta}{l} x_1 + \frac{\beta}{l} x_2\right) \\ &\quad + p_2(\lambda x_1 - \lambda x_2) + p_3 \frac{1}{\rho_{ci}} u. \end{aligned} \tag{9}$$

Because in order to have the system on the segment II, the values of S and all the derivatives of S must be zero, we apply the condition at the starting point of segment II.

Since the first one to contain u explicitly is the first derivative of S

$$S^{(1)} = \dot{x}_3 = \frac{u}{\rho_{ci}}, \tag{10}$$

the control which should be applied on the segment II is obtained from the condition $S^{(1)}=0$ as

$$u = 0. \tag{11}$$

Next, considering the constraint equation

$$S = x_3 - \frac{\gamma}{\rho_{ci}} = 0, \tag{12}$$

we obtain the following performance index

$$J' = \mu\left[x_3(t_a) - \frac{\gamma}{\rho_{ci}}\right] + \int_0^{t_f}\left[H - \sum_{i=1}^{3} p_i \dot{x}_i\right] dt, \tag{13}$$

where μ is a Lagrange multiplier.

Applying the variational technique to Eq. (13), it was found that at points P_a and P_b the following corner conditions must be satisfied.

at point P_a

$$\left.\begin{aligned} H(t_{a+}) &= H(t_{a-}) \\ p_1(t_{a+}) &= p_1(t_{a-}) \\ p_2(t_{a+}) &= p_2(t_{a-}) \\ p_3(t_{a+}) &= p_3(t_{a-}) - \mu, \end{aligned}\right\} \tag{14}$$

at point P_b

$$\left.\begin{aligned} H(t_{b+}) &= H(t_{b-}) \\ p_1(t_{b+}) &= p_1(t_{b-}) \\ p_2(t_{b+}) &= p_2(t_{b-}) \\ p_3(t_{b+}) &= p_3(t_{b-}). \end{aligned}\right\} \tag{15}$$

We also obtained the following costate equations of the system on the segment I and III (interior segment).

$$\left.\begin{aligned} \frac{dp_1}{dt} &= -\frac{\partial H}{\partial x_1} = -\frac{\rho_{ci}}{l} x_3 p_1 + \frac{2\rho_{ci}}{l} x_1 p_1 + \frac{\beta}{l} p_1 - \lambda p_2 \\ \frac{dp_2}{dt} &= -\frac{\partial H}{\partial x_2} = -\frac{\beta}{l} p_1 + \lambda p_2 \\ \frac{dp_3}{dt} &= -\frac{\partial H}{\partial x_3} = -\frac{\rho_{ci}}{l} x_1 p_1. \end{aligned}\right\} \tag{16}$$

On the segment II (boundary segment), we obtain the following costate equation in the vector notation

$$\dot{\boldsymbol{p}} = -\frac{\partial H}{\partial \boldsymbol{x}} + \frac{\partial H}{\partial u}\left(\frac{\partial S^{(1)}}{\partial u}\right)^{-1} \frac{\partial S^{(1)}}{\partial \boldsymbol{x}}, \tag{16$'$}$$

but the value $\partial S^{(1)}/\partial x$ is evidently zero from Eq. (10), the costate equation (16)′ becomes exactly the same as Eq. (16).

The optimum control which must be applied on the segment II is already obtained by means of the application of the Pontryagin's Maximum Principle to the Hamiltonian (9). We obtain the following optimum control,

$$\left.\begin{array}{l}\text{at segment I, III (interior segment)}\\ \quad u^0 = \eta\ \mathrm{sign}\,p_3\,,\\ \text{at segment II (boundary segment)}\\ \quad u^0 = 0\,.\end{array}\right\} \tag{17}$$

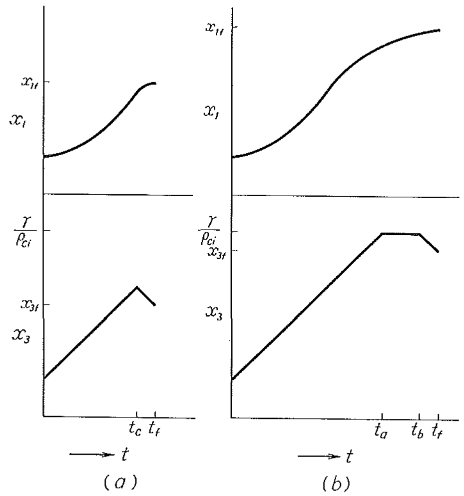

Fig. 2. Cotrol patten.

(a): the cace in which Seg. II is not existent in the optimum trajectory.

(b): the cace in which Seg. II is existent in the optimum trajectory.

As shown in Fig. 2, there are two patterns of control. One stands for the case in which the change of the sign of control is necessary before the state reaches the boundary surface $S=0$, and hence the segment II does not exist in the optimum trajectry, and the other stands for the case in which the boundary segment II exists. It depends on the amount of the change of the reactor power as to which of the control patterns should be applied.

In order to obtain the optimum control and corresponding trajectory, the unknown time $t_b(t_c)$ must be known, and which may be obtained by the following procedure. We first solve the equation (5) under the optimum control (17) for the time interval $0 \leq t \leq t_b (0 \leq t \leq t_c)$ assuming the positive value of p_3 and then solve the system for the time interval $t_b \leq t \leq t_f (t_c \leq t \leq t_f)$ with time reversed, assuming the negative value of p_3. Next from the condition that the state variables and their derivatives of the obtained two solutions must be respectively equal at the time $t_b(t_c)$, we can obtain the unknown time $t_b(t_c)$. The obtained time enables us to get the optimum trajectory.

From the application of the continuity of the state variables and the corner conditions (14), (15), we get

$$p_3(t_{a-})=0\,, \quad p_3(t_{b+})=p_3(t_{b-})=0\,. \tag{18}$$

Solving the costate system (16) under the boundary values (18), we get the costate value from which we can confirm the validity of the assumption given for the sign of p_3.

(a) System in which the delayed neutrons are ignored for this system, eliminating the terms which contain x_2 we obtain the following state equations

$$\left.\begin{aligned} \frac{dx_1}{dt} &= \frac{\rho_{ci}}{l}x_1x_3-\frac{\rho_{ci}}{l}x_1^2 && \text{(a)} \\ \frac{dx_3}{dt} &= \frac{1}{\rho_{ci}}u\,. && \text{(b)} \end{aligned}\right\} \tag{19}$$

(i) $1\leq t\leq t_a$ Assuming p_3 is positive and substituting Eq. (17)-(a) into Eq. (19)-(b), we integrate Eq. (19)-(b) and obtain

$$x_3=\frac{\eta}{\rho_{ci}}t+1\,. \tag{20}$$

Next, substituting Eq. (20) into Eq. (19)-(a), we obtain

$$\frac{dx_1}{dt}=\frac{\rho_{ci}}{l}\left(\frac{\eta}{\rho_{ci}}t+1\right)x_1-\frac{\rho_{ci}}{l}x_1^2\,. \tag{21}$$

Eq. (21) can be transformed to the following linear differential equation by the introduction of the new variables $v_1=1/x_1$

$$\frac{dv_1}{dt}+\frac{\rho_{ci}}{l}\left(\frac{\eta}{\rho_{ci}}t+1\right)v_1=\frac{\rho_{ci}}{l}\,. \tag{22}$$

The solution of Eq. (22) is easily obtained as

$$v_1=e^{-\int_0^t R(t')dt'}\left[\int_0^t Q(t')e^{\int_0^{t'}R(t'')dt''}dt'+K\right], \tag{23}$$

where

$$\left.\begin{aligned} R(t) &= \frac{\rho_{ci}}{l}\left(\frac{\eta}{\rho_{ci}}t+1\right) && \text{(a)} \\ Q(t) &= \frac{\rho_{ci}}{l}\,. && \text{(b)} \end{aligned}\right\} \tag{24}$$

In Eq. (23), we get $K=1$ because $v_1=1/x_{1s}=1$ at $t=0$, and also get the following relations

$$\left.\begin{aligned} &\int_0^t R(t')dt'=\int_0^t \frac{\rho_{ci}}{l}\left(\frac{\eta}{\rho_{ci}}t'+1\right)dt'=\frac{\eta}{2l}t^2+\frac{\rho_{ci}}{l}t\,, && \text{(a)} \\ &\int_0^t Q(t')e^{\int_0^{t'}R(t'')dt''}dt'=\frac{\rho_{ci}}{l}\cdot\exp\left(-\frac{\rho_{ci}^2}{2l\eta}\right)\int_0^t \exp\left[\frac{\eta}{2l}\left(t'+\frac{\rho_{ci}}{\eta}\right)^2\right]dt'. && \text{(b)} \end{aligned}\right\} \tag{25}$$

Introducing the new parameter

$$I_1(t)=\frac{\rho_{ci}}{l}\exp\left(-\frac{\rho_{ci}^2}{2l\eta}\right)\int_0^t\exp\left[\frac{\eta}{2l}\left(t'+\frac{\rho_{ci}}{\eta}\right)^2\right]dt', \tag{26}$$

Eq. (23) is reduced to

$$v_1=\exp\left[-\left(\frac{\eta}{2l}t^2+\frac{\rho_{ci}}{l}t\right)\right]\cdot\left[1+I_1(t)\right], \tag{27}$$

from which x_1 is obtained as

$$x_1(t)=\frac{\exp\left(\frac{\eta}{2l}t^2+\frac{\rho_{ci}}{l}t\right)}{1+I_1(t)}. \tag{28}$$

We can get the time at which the system transfers from the interior segment I to the boundary segment II, namely t_a, letting the value of x_3 be equal to γ/ρ_{ci} in Eq. (20)

$$t_a=\frac{\gamma-\rho_{ci}}{\eta}. \tag{29}$$

(ii) $\mathbf{t_a\leq t\leq t_b}$ In this interval the state must be on the boundary surface. Then we obtain

$$x_3=\frac{\gamma}{\rho_{ci}}. \tag{30}$$

Substituting Eq. (30) into Eq. (19)-(a), we integrate Eq. (19)-(a) by the same procedure employed in (i) and obtain

$$x_1(t)=\frac{\exp\left(\frac{\gamma}{l}t\right)}{\frac{\rho_{ci}}{\gamma}\cdot\exp\left(\frac{\gamma}{l}t'\right)\Big|_{t_a}^{t}+\frac{1}{x_{1a}}\cdot\exp\left(\frac{\gamma}{l}t_a\right)}, \tag{31}$$

where x_{1a} is the value of x_1 at time t_a and is obtained from Eq. (28).

(iii) $\boldsymbol{t_b\leq t\leq t_f}$ The expression of Eq. (19) with time reversed is

$$\left.\begin{aligned}\frac{d\bar{x}_1(\tau)}{d\tau}&=-\frac{\rho_{ci}}{l}\bar{x}_1(\tau)\bar{x}_3(\tau)+\frac{\rho_{ci}}{l}\bar{x}_1(\tau)^2 &&\text{(a)}\\ \frac{d\bar{x}_3(\tau)}{d\tau}&=-\frac{1}{\rho_{ci}}\bar{u}(\tau), &&\text{(b)}\end{aligned}\right\} \tag{32}$$

where τ is given by

$$\tau=t_f-t. \tag{33}$$

Assuming the negative value of p_3 for this interval and substituting Eq. (17)-(a) into Eq. (32), we integrate Eq. (32) and obtain

$$\bar{x}_3(\tau)=\frac{\eta}{\rho_{ci}}\tau+x_{3f} \tag{34}$$

$$\bar{x}_1(\tau)=\frac{x_{1f}\cdot\exp\left[-\left(\frac{\eta}{2l}\tau^2+\frac{\rho_{ci}}{l}x_{3f}\tau\right)\right]}{1-\bar{I}_2(\tau)}, \tag{35}$$

where

$$\bar{I}_2(\tau) = \frac{\rho_{ci}}{l} x_{1f} \cdot \exp\left(\frac{\rho_{ci}^2 x_{3f}^2}{2l\eta}\right)\int_0^{\tau} \exp\left[-\frac{\eta}{2l}\left(\tau' + \frac{\rho_{ci} x_{3f}}{\eta}\right)^2\right] d\tau' , \qquad (36)$$

and x_{1f}, x_{3f} are the objecting values.

Next, the unknown time t_b and t_f must be known. From Eq. (34) we obtain the time τ_b at which the system transfers from the interior segment III to the boundary segment II with time reversed, letting the value of $\bar{x}_3$ be equal to γ/ρ_{ci}

$$\tau_b = \frac{\gamma - \rho_{ci} x_{3f}}{\eta} . \qquad (37)$$

Substituting Eq. (37) into Eq. (35) we obtain

$$\bar{x}_1(\tau_b) = \frac{x_{1f} \cdot \exp\left[-\left(\frac{\eta}{2l}\tau_b^2 + \frac{\rho_{ci}}{l} \cdot x_{3f} \cdot \tau_b\right)\right]}{1 - \bar{I}_2(\tau_b)} . \qquad (38)$$

It is clear that at time t_b (hence τ_b) the value of $x_1(t_b)$ given in Eq. (31) must be equal to the value of $\bar{x}_1(\tau_b)$ given in Eq. (38). We can obtain the unknown time t_b from the above condition. We also obtain the unknown time t_f as

$$t_f = t_b + \tau_b . \qquad (39)$$

Hence we obtain the time advanced solution of the system in this interval as

$$\left.\begin{aligned} x_1(t) &= \frac{x_{1f} \cdot \exp\left[-\left\{\frac{\eta}{2l} \cdot (t_f - t)^2 + \frac{\rho_{ci}}{l} \cdot x_{3f}(t_f - t)\right\}\right]}{1 - I_2(t)} \\ x_3(t) &= \frac{\eta}{\rho_{ci}} \cdot (t_f - t) + x_{3f} , \end{aligned}\right\} \qquad (40)$$

Where

$$I_2(t) = \frac{\rho_{ci}}{l} x_{1f} \cdot \exp\left(\frac{\rho_{ci}^2 x_{3f}^2}{2l\eta}\right)\int_t^{t_f} \exp\left[-\frac{\eta}{2l}\left(t' + \frac{\rho_{ci} x_{3f}}{\eta}\right)^2\right] dt' . \qquad (41)$$

In Fig. 3 a numerical example is shown. In this example the externally applied reactivity is constrained to be less than ten times of the initial reactivity and the objecting power level is set at nine times the initial value.

(b) **System in which the delayed neutrons are considered** The optimum control of this system is obtained as a bang-bang control, but it is considered to be extremely difficult in this system to obtain the optimum time of the control change and also the optimum trajectory analytically as was readily done in the previous system.

It is considered that in the problem of the two point boundary value of the nonlinear system, the controllability of the system is not usually satisfied, and this fact is found to be true for our reactor system just considered from the observation of the results of the caluculations. An example of the caluculations is shown in Fig. 4. In this diagram, the deviations of the variables x_1 and x_2 from the objecting values are shown. If the time of the control change t_b is selected suitably, the three variables x_1, x_2, and x_3 can reach the respective values which are quite close to the objecting values at the time t_f. From the results of many calculations we can conclude that even if the controllability is not satisfied in a strict sense of the word for the system in which the delayed neutrons are considered, we can bring the system into a point located in a manifold, where all the points existing are very

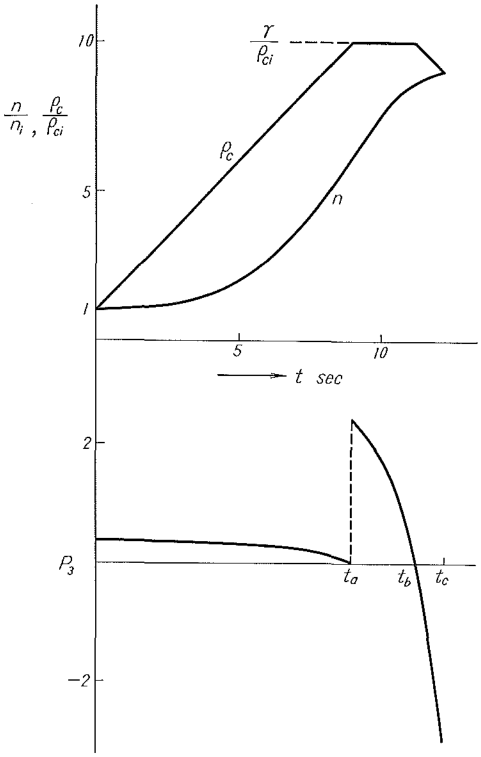

Fig. 3. Optimum control and corresponding neutron level, reactivity and adjoint variable.

$l=10^{-3}$ sec, $\beta=0.0065$, $\eta=1$, $\gamma=0.0065$ $\rho_{ci}=0.00065$

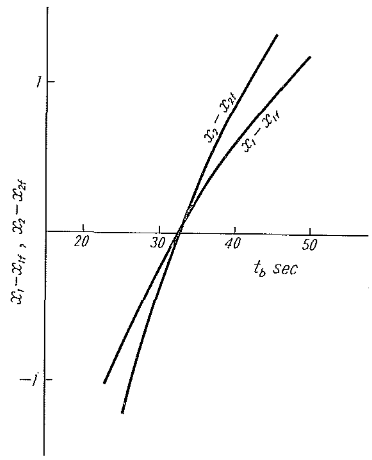

Fig. 4. Deviation of the state variables from the objective values vs. the time of switching the control t_b.

$l=10^{-3}$ sec, $\beta=0.0065$, $\lambda=0.08$ sec^{-1}, $\eta=0.04$

$\gamma=0.0052$, $\rho_{ci}=0.00052$, $\rho_c/\rho_{ci}=5$.

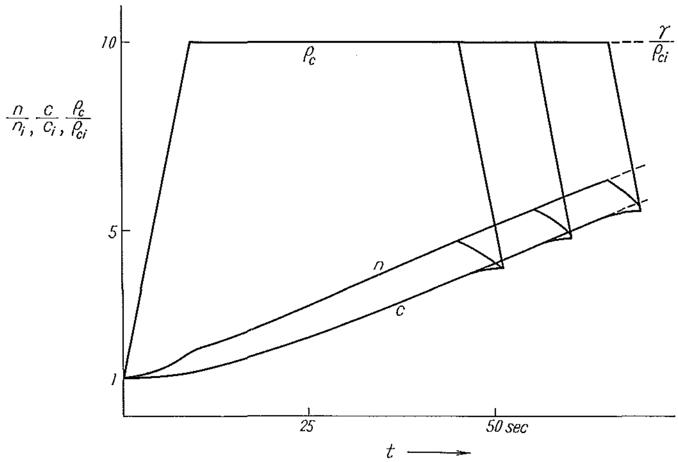

Fig. 5. Optimum Controls and corresponding trajectries of the several objective powers.
$l=10^{-3}$ sec, $\beta=0.0065$, $\lambda=0.08$ sec^{-1},
$\eta=0.04$, $\gamma=0.0026$, $\rho_{ci}=0.00026$.

close to the final equilibrium point and which can be regarded as the equilibrium point itself from a practical point of view. In Fig. 5 an numerical example is shown. In this example, the externally applied reactivity is constrained to be less than 40 cent (ten times larger than the initial reactivity) and the amount of the objecting power is taken to be free. It was found that in the course of the calculation with time reversed, quite large errors tended to appear, so the calculation was carried out by trial and error with time advanced.

In a case in which the deviations of the state values from the objecting ones

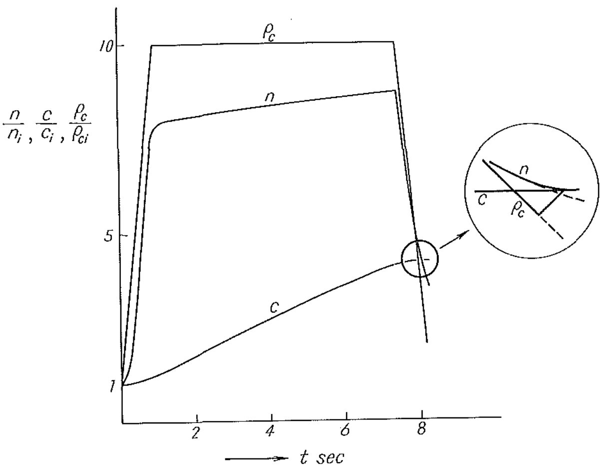

Fig. 6. Amandment of the trejectory near the final period of time by using a Bang-Bang control to bring the system in the final equilibrium state.
$l=10^{-3}$ sec, $\beta=0.0065$, $\lambda=0.08$ sec^{-1},
$\eta=4$, $\gamma=0.026$, $\rho_{ci}=0.0026$.

are large to some degree, we can bring the system to the equilibrium state relatively easily by applying a bang-bang control near the final period of time which is characterized by the short interval between the change of the control. This type of control is calculated and shown in Fig. 6.

As may be seen in Fig. 5, we raise the reactor power x_1 temporarily over the objecting value and then drop it to the objecting power level. This operation enables the density of the delayed neutron precursor to rise quickly and we can transfer the system to the final equilibrium state very rapidly.

If the coefficient of the internal feedback is large, the response of the system generally becomes slow, and then we must relieve the constraint applied to the external reactivity to some degree.

3.2 "Control of Mode B"

In the "Control of Mode A" no control is necessary after the system has reached the final objecting power. But if the reactor is required to follow the load change rapidly, we must be interested only in the behaviour of the reactor power x_1 for the present, and after the objecting reactor power is achieved we pay attention to the state of the density of the delayed neutron precursor x_2.

Under these circumstances, we raise the reactor power as quickly as possible until the objecting level is reached under the existence of the constraint condition, and after that achievement, we apply a control which bring the system to the final equilibrium state keeping the power level constant. The control is obtained analytically by the following procedure.

We assume that the objecting power level is reached at time t_d and at the time the values of the state variables x_1 and x_2 are x_{1d} and x_{2d} respectively. Because the rate of the change of power $\dot{x}_1$ must be zero at the time which is larger than t_d, Eq. (5) is reduced to

$$\left.\begin{aligned} \frac{dx_1}{dt} &= 0 = \frac{\rho_{ci}}{l}x_{1d}x_3 - \frac{\rho_{ci}}{l}x_{1d}^2 - \frac{\beta}{l}x_{1d} + \frac{\beta}{l}x_2 \qquad &\text{(a)} \\ \frac{dx_2}{dt} &= \lambda x_{1d} - \lambda x_2 \,. &\text{(b)} \end{aligned}\right\} \qquad (42)$$

Solving Eq. (42)-(b) under the condition that x_2 has the value of x_{2d} at time t_d, we obtain

$$x_2 = x_{1d} + (x_{2d} - x_{1d})e^{-\lambda(t-t_d)} \,. \qquad (43)$$

Substituting Eq. (43) into Eq. (42)-(a) we obtain the solution of Eq. (42)-(a) as

$$x_3 = x_{1d} + \frac{\beta}{\rho_{ci}}\left(1 - \frac{x_{2d}}{x_{1d}}\right)e^{-\lambda(t-t_d)} \,. \qquad (44)$$

Next differentiating Eq. (44) with time we obtain

$$\frac{dx_3}{dt} = \frac{u}{\rho_{ci}} = -\frac{\beta\lambda}{\rho_{ci}}\left(1 - \frac{x_{2d}}{x_{1d}}\right)e^{-\lambda(t-t_d)} \,, \qquad (45)$$

and the required control is obtained as

$$u = -\beta\lambda\left(1 - \frac{x_{2d}}{x_{1d}}\right)e^{-\lambda(t-t_d)} \,. \qquad (46)$$

As was be clearly noticed from Eq. (43) and (44), the system gradually reaches

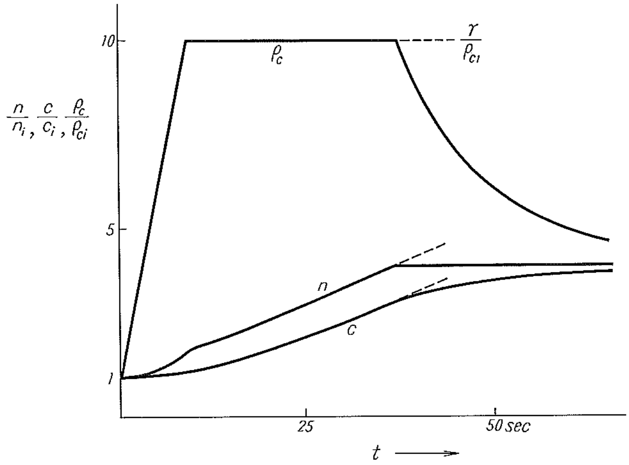

Fig. 7. Control applied to the system after the power level is time optimally brought to the objecting value.
$l=10^{-3}$ sec, $\beta=0.0065$, $\lambda=0.08\ \mathrm{sec}^{-1}$,
$\eta=0.04$, $\gamma=0.0026$, $\rho_{ci}=0.00026$.

the final equilibrium state exponentially with the time constant $1/\lambda$.

A numerical example is shown in Fig. 7.

4. Conclusions

In this paper the time optimum control of a reactor system in which the internal feedback and the delayed neutrons are considered and the state space is constrained was obtained in two modes of the control. One is the control which transfers the system from the initial equilibrium state to the objecting equilibrium state in minimum time and the other is the control which raises the reactor power to the objecting level as quickly as possible to follow the load change rapidly.

In the former mode of control, the controllability of the nonlinear system is rigorously satisfied if the delayed neutrons are ignored, and even if the delayed neutrons are considered, we can control the system from a practical point of view.

In the latter mode of the control, we must apply a control after the objecting power level is reached in order to bring the system to the final equilibrium state keeping the power level constant.

It would depend on the role of the reactor as to which mode of the control should be employed. If we are interested in the reactor operated in a electrical network, we employ either of the two modes of the control independently or use both of them jointly according to the part of the reactor in the electrical grid, namely the base load supplier or the load change follower.

As the internal feedback effect is considered in our discussion there would be no necessity of the application of the negative reactivity near the final period of time, which has been proposed by Moon in his reactor model[5].

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