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Author(s)	Sakuraba, Ichiro
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The Pulse Response of Growing Waves*

Ichiro SAKURABA**

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Contents

Abstract

The approach to obtain the general solution to the pulse response of an active forward-wave system was derived. The results are applicable to the calculation of pulse response in O–, M–, and CEF-type forward-wave amplifiers, bulk semiconductor amplifiers and the excitation of elastic waves arising from photoelastic coupling with high-intensity light.

1. Introduction

In this paper we consider the approach to obtain the general solution to the pulse response of a linear, uniform, lossless, active transmission system of infinite length in which two modes of propagation, with group velocities in the same direction, are coupled together. The type of coupling that occurs under these conditions is called "an active forward-wave system" or "co-flow skew-hermitian coupling"[1]. The behavior of this type of coupled-mode systems, for an input that varies sinusoidally with time, consists of growing waves. The power lost by the active mode is gained by the passive mode and the growth is exponential with distance, that is, such a system can lead to a convective instability[2),3] (see Fig. 1).

The impulse response of a passive forward-wave system of infinite length was given by Barnes[4]. Bobroff and Haus[5] showed the impulse response of active contraflow wave systems of infinite and finite lengths which describes the buildup of oscillations in backward-wave oscillators and the stimulated Brillouin scattering. The impulse response of a Kompfner null coupler was derived by Sakuraba[6], in which a forward wave and a positive-energy-carrying wave supported by an electron beam are passively coupled. A review of the literature reveals that the pulse response of growing waves has received relatively little analytical attention[7)~12].

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** Department of Electronic Engineering, Faculty of Engineering, Hokkaido University, Sapporo, 060, Japan.

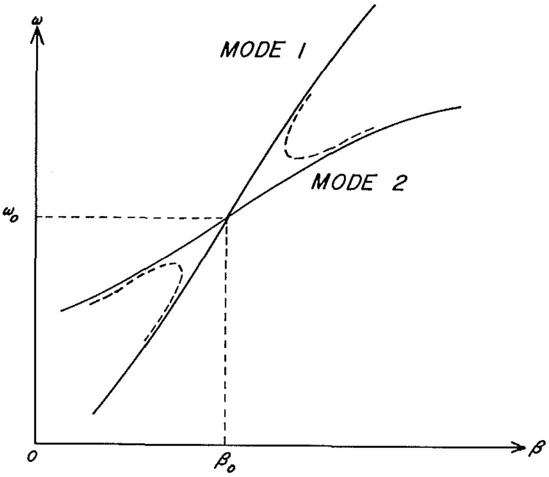

Fig. 1. Dispersion diagram for an active forward-wave system of modes 1 and 2. Solid lines repesent the dispersions of each mode before coupling and the dashed lines are the dispersion curves of the coupled system.

It is the purpose of this paper to point out the approach of the general solution to the pulse response that this forward-wave system exihibits when the input to the passive mode is an *r-f* signal of frequency ω_0 modulated by a rectangular pulse in time.

2. Analysis

When the passive mode is perfectly terminated in the forward-wave system, there is no possibility of oscillation in the forward-mode operation and the system will be stable. In this paper the active mode is considered to be in a steady-state condition at the time the *r-f* signal modulated by the rectangular pulse is applied at the input. Then the coupling can be considered to be a nontimevarying system. The input signal considered here is an *r-f* signal of frequency ω_0 modulated by a square wave extending from time zero to T. The time response of the growing wave is derived its frequency response by the Fourier transform relation

$$a(z, t) = \frac{1}{2\pi}\int_0^\infty \mathbf{a}(0, \omega)\, \mathbf{G}(0, z\,; \omega)\, e^{-j\omega t}\, d\omega + \text{c.c.}\,, \tag{1}$$

where $a(z, t)$ is the output mode amplitude in the time domain, $\mathbf{a}(0, \omega)$ is the input mode amplitude in the frequency domain, $\mathbf{G}(0, z\,; \omega)$ is the response function in the system and c.c. indicates the complex conjugate. The normalization is chosen so that the time average power per unit area carried by the wave is the product $a^*(z)\ a(z)$.

If the input time-domain amplitude is

$$a(0, t) = \frac{1}{2}\, a_0 (e^{j\omega_0 t} + e^{-j\omega_0 t}) \Big[U(t) - U(t-T) \Big], \tag{2}$$

where $U(t)$ is the unit-step function, we find that the frequency-domain amplitude $\mathbf{a}(0, \omega)$ can be written in the form

$$\mathbf{a}(0, \omega) = \frac{a_0}{2j}\left[\frac{1-e^{-jT(\omega-\omega_0)}}{\omega-\omega_0} + \frac{1-e^{-jT(\omega+\omega_0)}}{\omega+\omega_0}\right]. \tag{3}$$

When the response function of growing waves is given by

$$\mathbf{G}(0, z ; \omega) = \mathbf{A}(0, z ; \omega)\, e^{-j\theta(0,z;\omega)}, \tag{4}$$

the frequency-domain amplitude of the device at the output is

$$\begin{aligned} a(z, t) &= \frac{a_0}{4\pi j}\left[\int_0^\infty \frac{1-e^{-jT(\omega-\omega_0)}}{\omega-\omega_0}\,\mathbf{A}(0, z ; \omega)\, e^{-j\theta(0,z;\omega)+j\omega t} + \text{c.c.}\right] \\ &\approx \frac{a}{2\pi}\left[\int_0^\infty \frac{\mathbf{A}(0, z ; \omega)\sin[\omega t-\boldsymbol{\theta}(0, z ; \omega)]}{\omega-\omega_0}\, d\omega \right. \\ &\qquad \left. -\int_0^\infty \frac{\mathbf{A}(0, z ; \omega)\sin[\omega t-\boldsymbol{\theta}(0, z ; \omega)-(\omega-\omega_0)T]}{\omega-\omega_0}\, d\omega\right], \end{aligned} \tag{5}$$

where it was assumed that the carrier frequency is very high compared to the modulating components.

Let us expand $\boldsymbol{\theta}(0, z ; \omega)$ as a function of ω in Taylor series about the carrier frequency ω_0,

$$\boldsymbol{\theta}(0, z ; \omega) = \theta_0 + \left(\frac{d\boldsymbol{\theta}(0, z ; \omega)}{d\omega}\right)_0 (\omega-\omega_0) + \frac{1}{2}\left(\frac{d^2\boldsymbol{\theta}(0, z ; \omega)}{d\omega^2}\right)_0 (\omega-\omega_0)^2 + \cdots, \tag{6}$$

where $\boldsymbol{\theta}_0$ is the phase angle at the frequency ω_0 and the subscript zero means that the derivatives are evaluated at the frequency ω_0. The coefficient of the linear term is the transit time of the energy, that is, z/v_{g0}, where v_{g0} is the group velocity of the uncoupled passive mode at the frequency ω_0. Putting

$$\alpha \equiv \frac{1}{2}\left(\frac{d^2\boldsymbol{\theta}(0, z ; \omega)}{d\omega^2}\right)_0, \tag{7}$$

for the coefficient of the quadratic term and discarding high-order terms in the expansion, it is established that any well-behaved dispersion relation can be approximated by the general form

$$\boldsymbol{\theta}(0, z ; \omega) = \theta_0 + (z/v_{g0})(\omega-\omega_0) + \alpha(\omega-\omega_0)^2, \tag{8}$$

in the vicinity of a particular frequency and its related wave number. This expression is exact when $\boldsymbol{\theta}(0, z ; \omega)$ is a linear or quadratic function of ω. Then Eq. (5) can be written as

$$\begin{aligned} a(z, t) = a_0\Big[&\Big\{F_1(t-z/v_{g0}) - F_1(t-z/v_{g0}-T)\Big\}\cos(\omega_0 t-\theta_0) \\ &+ \Big\{F_2(t-z/v_{g0}) - F_2(t-z/v_{g0}-T)\Big\}\sin(\omega_0 t-\theta_0)\Big], \end{aligned} \tag{9}$$

where

$$F_1(t-z/v_{g0}) = \frac{1}{2\pi}\int_{-\omega_0}^{\infty} \frac{\mathbf{A}(0, z ; x+\omega_0)}{x}\sin\left[x(t-z/v_{g0})-\alpha x^2\right]dx, \tag{10}$$

$$F_2(t-z/v_{g0}) = \frac{1}{2\pi}\int_{-\omega_0}^{\infty} \frac{\mathbf{A}(0, z ; x+\omega_0)}{x}\cos\left[x(t-z/v_{g0})-\alpha x^2\right]dx, \tag{11}$$

and

$$x \equiv \omega - \omega_0 . \tag{12}$$

3. Discussions

The response function of the active forward-wave system can easily be obtained from the steady-state solutions of the coupled mode equations. For example, the frequency response of an 0-type forward-wave tube is

$$G(0, z ; \omega) = A_0 e^{j\phi} e^{x_1 C\omega z/v_p - j(1+Cy_1)\omega z/v_p}, \tag{13}$$

where A_0 and ϕ are the magnitude and phase of the growing wave induced on the slow-wave circuit by the input signal, x_1 and y_1 are the incremental propagation constants of the growing wave, C is the gain parameter, z is the length of the tube, and v_p is the phase velocity. Coefficients of A_0, ϕ, v_p, x_1 and y_1 are presented as functions of frequency, so that the approximations for response of the tube can easily be calculated from Eq. (13). In a dispersive mode the main variation in the response of growing waves is due to the change with frequency of the phase velocities of modes. This change reduces the degree of synchronization between two modes and hence reduces the gain. The buildup of $a(z, t)$ depends on how dispersive the mode is. The rise time varies directly as the difference between the transit times of the energy of the uncoupled passive and active modes.

4. Conclusions

A derivation of the pulse response of growing waves of an active forward-wave system was presented. It is hoped that this derivation will give new insights into the coupled mode theory.

It is necessary that further theoretical investigations be carried out to determine the pulse response of 0-, M-, and CEF-type forward-wave amplifiers, bulk semiconductor amplifiers and the excitation of elastic waves arising from photoelastic coupling with high-intensity light.

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