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Mode Selective Output-Coupling Apertures for Infrared Gaseous Laser

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Abstract

The confocal and Fabry-Perot resonators with an annular output-coupling aperture around the center of the reflector are considered for the purpose of mode selection by making use of diffraction-losses. For relatively large Fresnel numbers the percentage power losses arising from the annular output-coupling aperture are evaluated for the lower order modes, TEM_{00}, TEM_{10}, and TEM_{01}, on the basis of the field distribution functions for infinitely large mirrors. The annular aperture seems to be effective for selecting the lowest order mode TEM_{00} as the predominating mode, provided that it is made in somewhat more outer region than the location where the intensity of the next higher order circularly symmetric mode TEM_{10} becomes maximum. If a proper substratum for the mirror can not be prepared for a particular infrared or far infrared region, the mirror which is partially blocked out in the annular region is proposed and examined experimentally. A Fabry-Perot resonator with four output-coupling holes in the annular region is constructed for a CO_2 laser. The measured result of the degree of spatial coherence illustrates that the power output breaks into an approximately pure mode of oscillation in the present resonator.

1. Introduction

In the infrared or far infrared region it seems rather difficult to make partially transmitting dielectric mirrors for laser resonators, since there are few materials for reflection mirrors which are transparent to this wavelength region. Accodingly, the radiation energy has been effectively coupled out through an output-coupling hole$^{1)}$ in the center of the internal reflection mirror. Reflection mirrors made of metalic mesh$^{2)}$ can, however, be successfully available for the far infrared gaseous laser.

A rectangular confocal resonator with one reflector partially blocked out in rectangular shape has been firstly sketched analytically by Boyd and Kogelnik$^{3)}$. Theoretical calculations of the modes and diffraction losses for a symmetric cylindrical confocal resonator with a circular output-coupling hole in the center of each mirror have been made by McCumber$^{4)}$ and those with a circular ouput-coupling aperture in the center of only one end mirror have been made also by Moran$^{5)}$ for relatively small Fresnel numbers. The modes and eigenvalues of a symmetric cylindrical parallel plane resonator with a circular output-coupling hole in each mirror also have been computed by Li and Zucker$^{6)}$.

According to these theoretical calculations it is found for any type of the resonators that the transverse field distributions of the even symmetric modes,

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especially of the lowest order mode TEM_{00}, are considerably depressed around the center of the reflection mirror owing to the output-coupling loss out of the holes, while the field distributions of the odd symmetric modes, especially of the next higher order mode TEM_{10}, are less affected by the holes, since their field disiributions are zero at the center of the mirror. The power loss of the TEM_{00} mode prevails over that of the TEM_{10} mode in the certain region of coupling-hole size for a given Fresnel number and the losses of these modes tend to equalize, as the hole size increases. It follows that the predominating mode in any resonator changes with the coupling-hole size. According to Fox and Li[7] the mode that predominates in an active Fabry-Perot resonator in the absence of the output-coupling hole is the same lowest order TEM_{00} as in a passive resonator, while in an active confocal resonator the predominating mode changes with the Fresnel number.

Therefore, even if any type of resonators may be actually prepared, it will be almost impossible to select the lowest order mode TEM_{00} by means of the central coupling-hole except for relatively small Fresnel numbers. This is inconvenient for many applications which require a single mode operation. In order to avoid such inconveniences an annular output-coupling aperture has been proposed in an earlier paper[8] for the infrared or far infrared gaseous laser. The paper states that the predominating mode of a symmetric cylindrical resonator can be made to be the lowest order mode TEM_{00} up to the relatively large Fresnel number, provided that the annular aperture is made near the place where the intensity of the radial field distribution of the next higher order mode TEM_{10} is most intense on the reflection mirror. Because the TEM_{10} mode should be much affected due to the output-coupling loss as compared with the TEM_{00} mode by use of such an annular output-coupling aperture. In this method, however, substratum of the mirror with an annular aperture must be transparent to a given wavelength region of radiation for coupling power from the annular aperture. In view of practical situations it may be rather tedious to find out such substratum in the infrared or far infrared region.

In this paper multiple output-coupling holes which are arranged in the place corresponding to the annular output-coupling aperture are further proposed and demonstrated experimentally by means of a Fabry-Perot resonator with four output-coupling holes for a CO_2 laser.

2. Evaluation of Power Losses

The symmetric cylindrical confocal resonator and the cylindrical Fabry-Perot resonator are considered here as the typical resonators for the infrared or far infrared lasers. The resonators are characterized conventionally in terms of the Fresnel number

$$N = a^2/b\lambda, \qquad (1)$$

where a is the radius of the circular reflector, b the separation of two reflectors, and λ the wavelength of radiation. For the purpose of convenient analyses, let the Fresnel number be so large that the power loss may be coupled out mainly through the perfectly transmitting apertures for output coupling.

The field intensity is assumed to be circularly symmetric to the resonator axis[7]. The place where the intensity of the TEM_{10} mode is strongest on the mirror has been found to shift slightly towards the edge of the mirror[5],[6] when the aperture is centered at the resonator axis. However, that place also is assumed here to be invariable even though any coupling aperture may be arranged on the mirror. The present problems are thus discussed mainly on the basis of the percentage power loss for the TEM_{pq} mode. Here p and q denote angular and radial integral mode numbers, respectively, for a given longitudinal mode. If the field distribution functions for the TEM_{pq} mode extending to r_m are represented in terms of $f_{pq}(r)$, the percentage power loss can be given approximately by

$$L(pq)=\frac{\int_A r[f_{pq}(r)]^2\,dr}{\int_0^{r_m} r[f_{pq}(r)]^2\,dr}, \tag{2}$$

where r is the normalized radial coordinate and A denotes the total aperture area.

2.1 Confocal Resonator

The symmetrical field distribution functions for the TEM_{pq} mode can be expressed in terms of a function of the associated Laguerre polynominals $L_q^p(z)$[3],[4]. According to McCumber[4] the field distribution functions in the absence of any output-coupling aperture are expressed by

$$\begin{aligned}f_{pq}(r)=&\left[\frac{2q\,!}{(p+q)\,!}\right]^{1/2}(2\pi r^2)^{p/2}\exp(-\pi r^2)\,L_q^p(2\pi r^2)\\ &\times\left[1-\frac{q\,!}{(p+q)\,!}\int_{2\pi N}^{\infty}x^p\exp(-x)\,\{L_q^p(2\pi \mathrm{r}^2)\}^2\,dx\right]^{-1/2}\end{aligned} \tag{3}$$

with $r^2=\rho^2/b\lambda$, ρ being the radial coordinate.

The field distribution functions for the low order modes, TEM_{00}, TEM_{10}, and TEM_{01}, are especially taken into account here in view of practical interests. On the assumption mentioned above, the integral term in Eq. (3) can be regarded to be negligibly small as compared with unity for these order modes, if the Fresnel number N is considerably larger than unity. The field distribution functions for infinite mirrors will therefore be of use in the present considerations. The distribution functions for these order modes are thus derived from Eq. (3), dropping the integral term, as follows:

$$f_{00}(r)=\sqrt{2}\,\exp(-\pi \mathrm{r}^2), \tag{4}$$

$$f_{10}(r)=2\sqrt{\pi}\;r\exp(-\pi r^2), \tag{5}$$

and

$$f_{01}(r)=\sqrt{2}\,(1-2\pi r^2)\exp(-\pi r^2). \tag{6}$$

Now suppose the perfectly transmitting annular region symmetric to the center of the mirror which is surrounded by perfectly reflecting mirrors. Let $2\rho_0$ and W be the inner diameter and width of the annular apeture, and this output-coupling aperture can be characterized in terms of dimmensionless parameters that

$$\left.\begin{aligned}&r_0=\rho_0/\sqrt{b\lambda}\\ \text{and}\quad &d=W/\sqrt{b\lambda}\end{aligned}\right\}, \tag{7}$$

The integral upper limit r_m in the denominator of Eq. (2) should be replaced by infinity in this case. Calculations by substituting Eqs. (4), (5), and (6) into Eq. (2) give

$$L(00)=\exp(-2\pi r_0^2)-\exp\left\{-2\pi(r_0+d)^2\right\}, \tag{8}$$

$$L(10)=(1+2\pi r_0^2)\exp(-2\pi r_0^2)-\left[1+2(r_0+d)^2\exp\left\{-2\pi(r_0+d)^2\right\}\right], \tag{9}$$

and

$$L(01)=(1+4\pi^2 r_0^4)\exp(-2\pi r_0^2)-\left[1+4\pi^2(r_0+d)^4\exp\left\{-2\pi(r_0+d)^2\right]\right.. \tag{10}$$

It should be noted that r_0^2 corresponds to the Fresnel number of the central mirror surrounded by the annular aperture. The limiting case with respect to $r_0^2=0$ is to make a circular output-coupling hole at the resonator axis.

It is expected that the predominating mode changes with the value of r_0^2. The place where the annular aperture is mdae must be determined so that the TEM$_{00}$ mode may be always predominating. It seems reasonable to make the annular aperture with a required size near the location $r_0=R_0=1/\sqrt{2\pi}$, where the intensity of the next higher order mode TEM$_{10}$ becomes maximum in the absence of the aperture.

The percentage power losses for different values of r_0^2 are presented graphically in Figs. 1 to 4, which are based on Eqs. (8), (9), and (10). The power losses of the resonator with a central coupling hole, i.e., $r_0^2=0$, are shown in Fig. 4 for

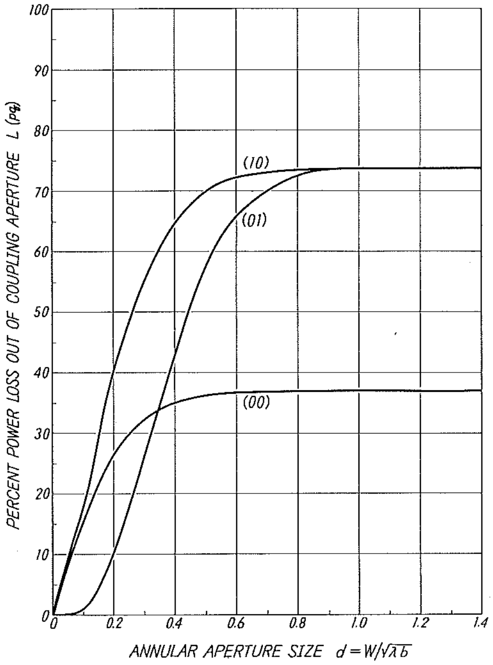


Fig. 1. Percent power loss versus annular aperture size for the TEM$_{00}$, TEM$_{10}$, and TEM$_{01}$ modes with $r_0^2=R_0^2=1/2\pi$.

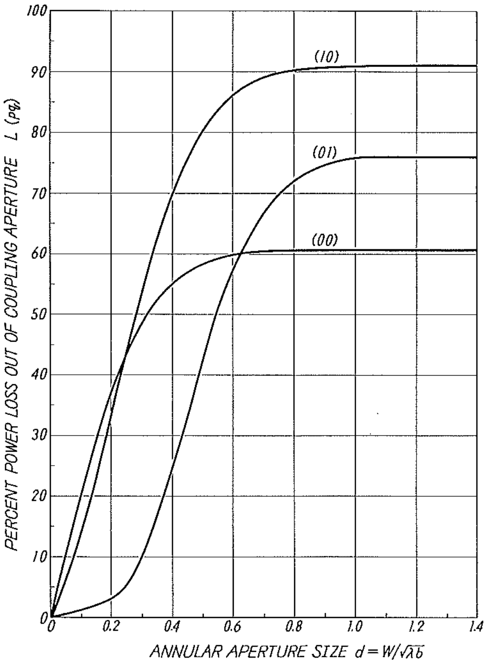


Fig. 2. Percent power loss versus annular aperture size for the TEM$_{00}$, TEM$_{10}$, and TEM$_{10}$ modes with $r_0^2=R_0^2/2=1/4\pi$.

the purpose of comparison with those having an annular coupling aperture. As may be seen from the figures, it is found that the TEM_{00} mode predominates, as expected, when both the Fresnel number r_0^2 of the central mirror and the width of the annular region d can be chosen just conveniently. In particular, the mode selection will be carried out fairly well, if r_0^2 is approximately 1.5 times as large as R_0^2 as shown in Fig. 3. Diffraction-coupled resonators[9] will be realized out of the periphery of the mirror, if d is taken to be relatively large as seen in Figs. 1 to 3.

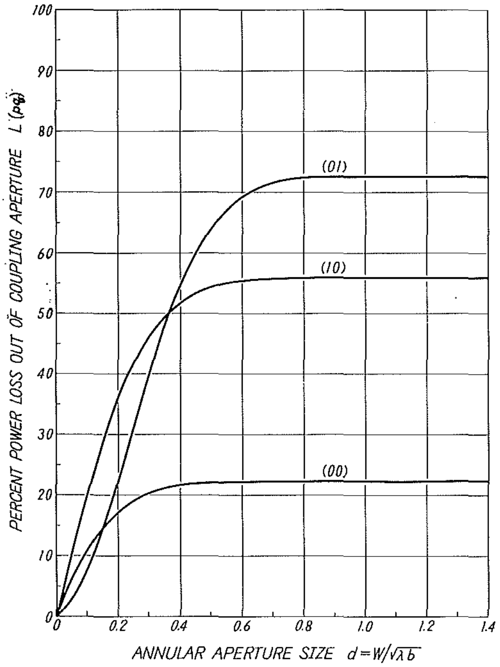


Fig. 3. Percent power loss versus annular aperture size for the TEM_{00}, TEM_{10}, and TEM_{01} modes with $r_0^2=3R_0^2/2=3/4\pi$

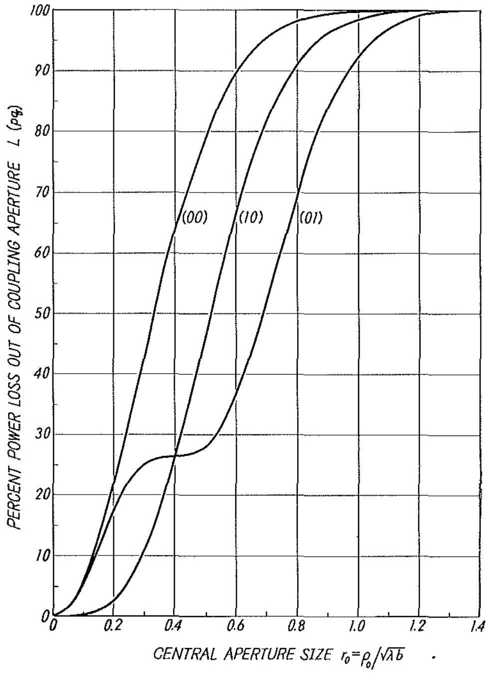
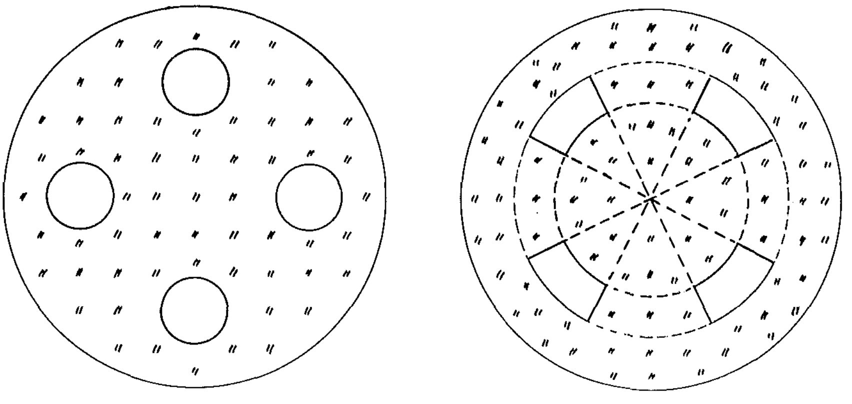


Fig. 4. Percent power loss versus central aperture size for the TEM_{00}, TEM_{10}, and TEM_{01} modes.



(a) (b)

Fig. 5. Coupling holes bored in the annular region.

Metalic films, for examle gold or aluminum, can be satisfactorily available except the annular region as highly reflective materials in the infrared or far infrared region. However, when there is not a proper substratum for the reflector in a particular wavelength region, the annular aperture region should be partially blocked out for output-coupling, for example, as shown in Fig. 5-a. In order to simply estimate the power losses for the given modes, it is convenient that the reflector is blocked out in the annular region as shown in Fig. 5-b. Let θ and n be an angle which an arc of the bored aperture makes to the center of the mirror and numbers of the blocked out apertures, repectively, and thus the percentage power losses can be estimated in terms of the quantity of $n\theta L(pq)/2\pi$.

2.2 Fabry-Perot Resonator

The theory of a Fabry-Perot resonator with a pair of plane parallel mirrors has been developed by several investigators[10)~13)]. According to Bergstein and Schachter[13)], the symmetric field distribution functions for relatively large Fresnel numbers can be written approxinately in terms of the Bessel function. The following considerations will be owed mainly to their interesting results. In their theory the Fresnel number H is twice as large as the conventional Fresnel number N and the order of radial mode number n in the designation of the set of modes begins from $n=1$. The Fresnel number H is replaced, for convenience, by the conventional Fresnel number, $N=H/2=a^2/b\lambda$. The order of radial mode number n also is replaced by $p+1$, so that p begins from zero as usual.

When the value of the Fresnel number N is above 2.5, the field distribution functions for the plane circular mirrors are, to the first order of approximation, given by

$$f_{pq}(r)=\left[\pi^{1/2}J'_p(\alpha_{pq})\right]^{-1}J_p(\alpha_{pq}r) \qquad (11)$$

$$\text{with} \quad r=\rho/a\,,$$

where $J_p(z)$ is the Bessel function of the pth order with respect to z. The roots of the Bessel function of the pth order are approximately given by

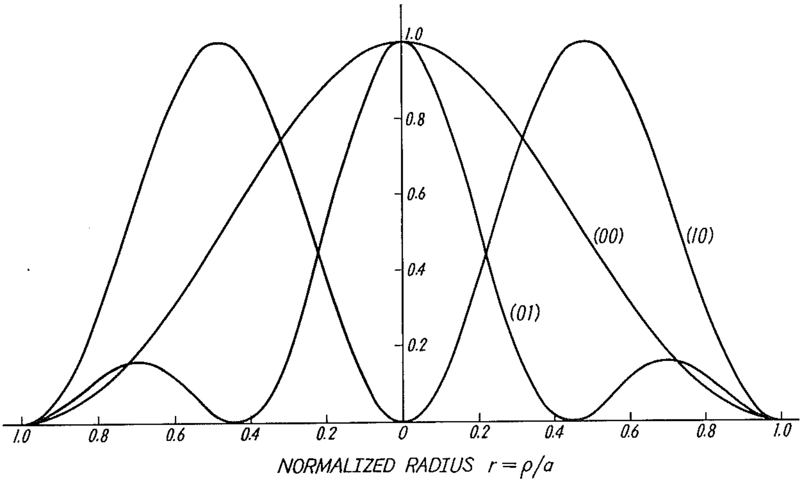


Fig. 6. Relative intensity distributions for the TEM$_{00}$, TEM$_{10}$, and TEM$_{01}$, modes.

$$\alpha_{pq}=(p/2+q+3/4)\pi,\quad q=0,1,2,\cdots. \tag{12}$$

The field distribution functions for the lower order modes, TEM$_{00}$, TEM$_{10}$, and TEM$_{01}$, are taken into account also in this case. The relative intensity distributions for these modes are sketched out in Fig. 6, in which the TEM$_{10}$ mode becomes maximum in its intensity at the place, $r=R_0=0.45$.

By making use of the Lommel integral formulae expressed by

$$\int_0^x\left[J_p(\alpha_{pq}r)\right]^2 rdr=\frac{x^2}{2}\left[\{J_p(\alpha_{pq}x)\}^2-J_{p-1}(\alpha_{pq}x)\,J_{p+1}(\alpha x_{pq})\right], \tag{13}$$

and if $p=0$,

$$\int_0^x\left[J_0(\alpha_{0q}r)\right]^2 rdr=\frac{x^2}{2}\left[\{J_0(\alpha_{0q}x)\}^2+\{J_1(\alpha_{0q}x)\}^2\right], \tag{14}$$

the percentage power losses $L(pq)$ cad be formulated as follows:

$$L(00)=\frac{(r_0+d)^2\,[(J_0\{3\pi(r_0+d)/4\})^2+(J_1\{3\pi(r_0+d)/4\})^2]}{\{J_1(3\pi/4)\}^2}$$

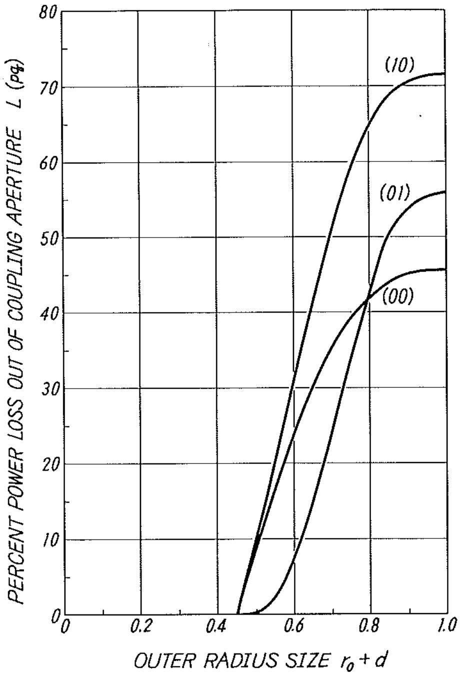


Fig. 7. Percent power loss as a function of outer radius of the annular aperture for the TEM$_{00}$, TEM$_{10}$, and TEM$_{01}$ modes with $r_0=0.45$.

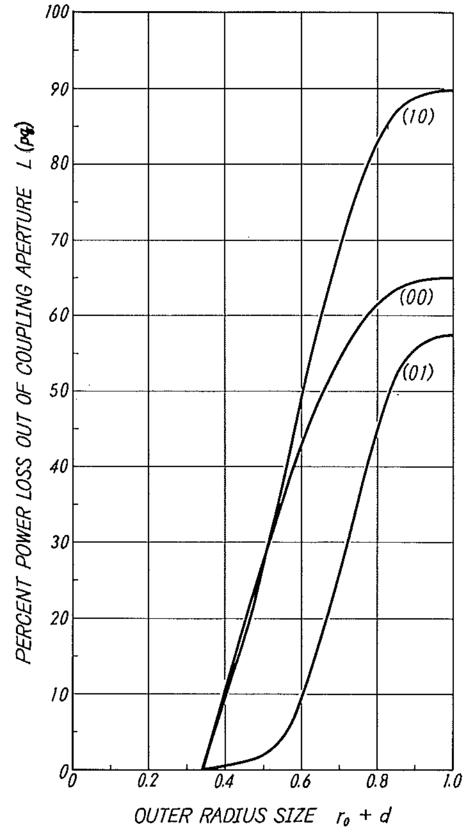


Fig. 8. Percent powes loss as a function of outer radius of the annular aperture for the TEM$_{00}$ and TEM$_{01}$ modes with $r_0=0.34$.

$$-\frac{r_0^2[\{J_0(3\pi r_0/4\}^2+\{J_1(3\pi r_0/4)\}^2]}{\{J_1(3\pi/4)\}^2}, \tag{15}$$

$$L(10)=\frac{(r_0+d)^2\,[[J_1\{5\pi(r_0+d)/4\}]^2-J_0\{5\pi(r_0+d)/4\}\,J_2\{5\pi(r_0+d)/4\}]}{\{J_1(5\pi/4)\}^2-J_0(5\pi/4)\,J_2(5\pi/4)}$$

$$-\frac{r_0^2[\{J_1(5\pi r_0/4)\}^2-J_0(5\pi r_0/4)\,J_2(5\pi r_0/4)]}{\{J_1(5\pi/4)\}^2-J_0(5\pi/4)\,J_2(5\pi/4)}, \tag{16}$$

and

$$L(01)=\frac{(r_0+d)^2[[J_0\{7\pi(r_0+d)/4\}]^2+[J_1\{7\pi(r_0+d)/4\}]^2]}{\{J_1(7\pi/4)\}^2}$$

$$-\frac{r_0^2[J_0(7\pi r_0/4)\}^2+\{J_1(7\pi r_0/4)\}^2]}{\{J_1(7\pi/4)\}^2}. \tag{17}$$

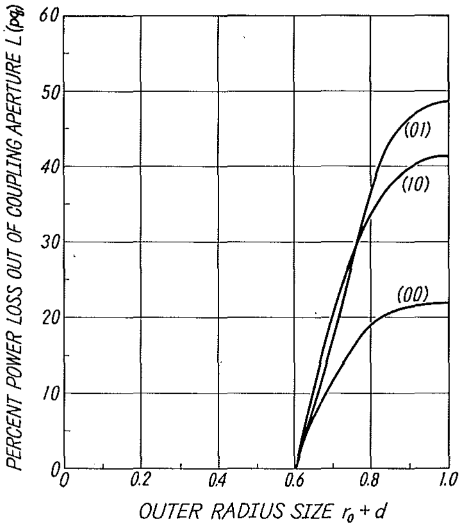


Fig. 9. Percent power loss as a function of outer radius of the annular aperture for the TEM_{00}, TEM_{10}, and TEM_{01} modes with $r_0=0.6$.

Numerical calculations for different values of r_0 denote the curves similar to those in the confocal resonator as shown in Figs. 7 to 10. In this case it seems somewhat difficult to separate the lower order modes by only the output-coupling loss in comparison with the case in the confocal resonator.

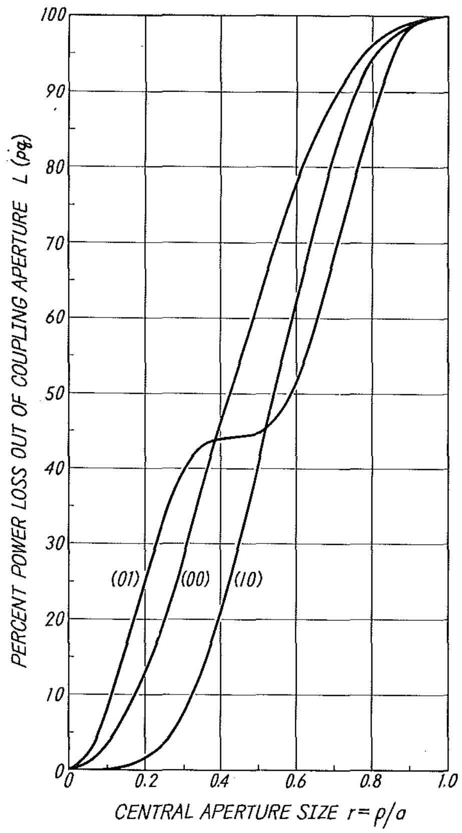


Fig. 10. Percent power loss as a function of radius of the central aperture for the TEM_{00}, TEM_{10}, and TEM_{01} modes.

2.3 Optimum Power Output-Coupling

The theory of homogeneous broadening line can be generally applicable to the infrared or far infrared molecular lasers. The maximum obtainable power output arising from the coupling aperture may be calculated according to this theory. Let us suppose that 1) the diffraction power losses split over the edge of the mirror is negligibly small as compared with the power losses arising from the output-coupling apertures, because an infinitely large mirror approximation holds for the present resonator, 2) the intensity of the particular mode can be determined only from the loss of the power coupling, and 3) the gain across the cross section of the laser tube is uniform. Accordingly, the power output for (pq) mode can be expressed in the form[14)]

$$P(pq)/P_0 = L(pq)\left\{\frac{2g_0 l}{L(pq)+2(1-R)} - 1\right\}, \tag{18}$$

where P_0 is a saturation parameter in power, l is the length of the resonator, g_0 is the unsaturated gain coefficicient, and R is the power reflection coefficient of the mirrors. Therefore, the maximum power output will be obtained from Eq. (18), when the percentage power loss is optimized to be

$$L_{0pt}(pq) = 2\left\{\sqrt{g_0 l(1-R)} - (1-R)\right\}. \tag{19}$$

3. Experimental

In a previous paper[15)] the annular output-coupling aperture made on a silicon reflection mirror has been reported to be useful for power coupling out of the He-Ne infrared laser from 3 to 25 μm region. In the present experiment four output-coupling holes bored in the annlar region on a circular plane glass-mirror have been used for power coupling out of a CO_2 infrared laser. It is further possible to enhance the mode volume[6)] by employing a pair of parallel plane mirrors.

A water-cooled resonator was 2,700 mm in length and 55 mm in diameter. Mixed gases, CO_2, N_2, and He, flowing through the resonator were excited by dc power supply. The photograph of the reflector with the four output-coupling holes is shown in Fig. 11. The aluminum-coated mirror is 53 mm in effective diameter and each coupling hole is 6 mm in diameter. The two holes lying on the line through the center of the mirror are 25 mm in separation. These coupling holes are sealed with NaCl window plates. This mirror gives the Fresnel number, $N=24.4$. According to Bergstein and Schachter[13)], when there are no output-coupling holes in the mirror, the power losses of all the lower order modes, TEM_{00}, TEM_{10}, and TEM_{01}, are less than 1 percent and especially that of the TEM_{00} mode is about 0.1 percent.

Now consider the power losses through the output-couping holes on the basis of the previous result. The location $R_0=0.45$, where the intesity of the TEM_{10} mode becomes maximum, gives the radial distance $\rho_0=12$ mm from the center of the

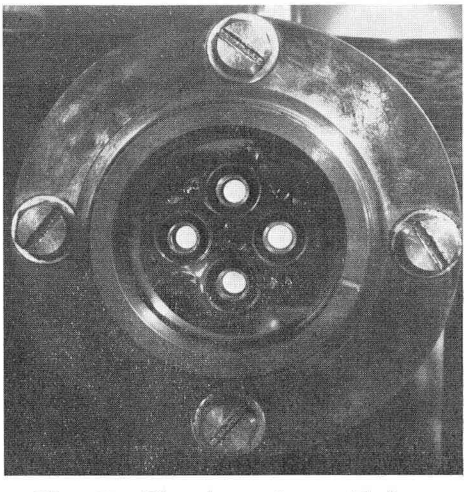


Fig. 11. The plane mirror with four coupling holes mounted on the laser tube.

mirror. Each of the four coupling holes shown in Fig. 11 is centered on the circle of 12.5 mm in radius. The percentage power loss for a given order mode will be roughly estimated here by the quntity $n\theta L(pq)/2\pi$ described in the previous section. The four output-coupling holes gives $n=4$.

On referring to Fig. 5-b, let us consider the aperture area bounded by two radii making an angle θ and two arcs of the concentric circles with 9 and 15 mm in radius, and then put this area to be equal to a circular aperture area located at the same place. As the mirror is 26.5 mm in radius, the value of r_0 and d are calculated to be 0.34 and 0.23, respectively. This geometrical situation yields $4\theta/2\pi=0.25$. As may be seen from Fig. 8, the coupling losses of the modes, TEM_{00}, TEM, and TEM_{01}, are thus found to be 9, 10.6, and 1.5%, respectvely. If the off-centered circular aperture were made in the somewhat more outerregion, the lowest order mode TEM_{00} might always predominate as seen from Fig. 9.

The output power was received by a thermocouple and recorded on a chart recorder. Two thermocouples were simultaneously used for detecting two beams of radiation emerging from two different output-conpling holes. They showed complex behaviors each other, after the dc power was supplied to the discharge tube. An example of the recorded results is shown in Fig. 12. Some difference about the signal to noise ratio in curves, a ond b, depends mainly on the properties of the thermocouples used. The correlative relationship between the two curves seems to be not obvious from the figure. The total output power emering from the four coupling holes was compared with that emerging from a central coupling hole of 6 mm in diameter. A typical result is shown in Fig. 13. It is found that the total output power from the four coupling holes is about 6 times as large as that from the central hole near the discharge current of 30 mA. It is not clear whether the central hole is optimum for output-coupling. However, the optimum size of the central hole has been found to be approximately 6 mm in diameter[16)]

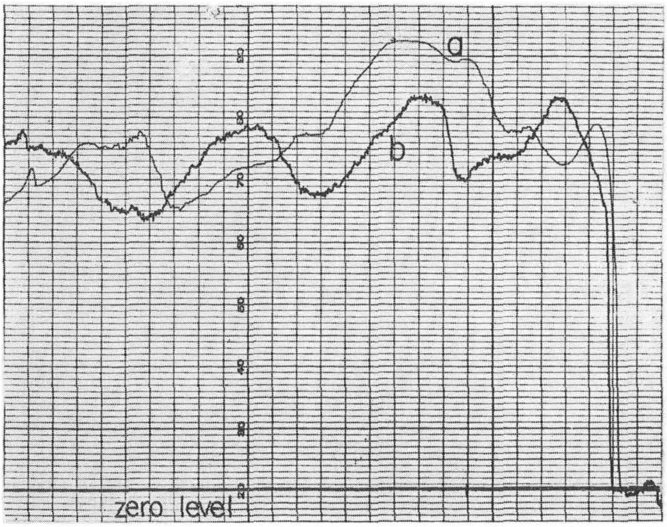


Fig. 12. Fluctuations of the output power emerging from different two coupling holes after begining of operation. The chart sheet proceeds from right to left with the speed of $2\,cm/min$.

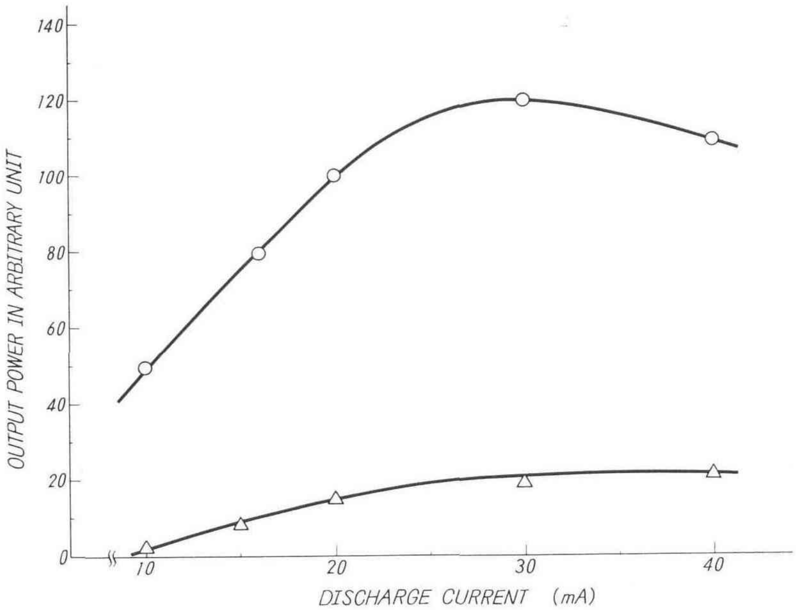


Fig. 13. The output power versus discharge current. The upper and lower curves denote the output power obtained from the four coupling holes and from only the one central coupling hole, respectively.

in the Q-switched CO_2 laser using a similar dimensional resonator.

According to Bertolotti, *et al.*[17] and Moreley, *et al.*[18] the degree of spatial coherence in a He-Ne visible gaseous laser has the value of unity when only a single transverse mode operation, while it has any value from zero to unity according to which two points are examined in the cross-section of the radiation beam when transverse multimode operation. These may be available for examination of the effects of mode selection by the off-centered coupling holes.

Conventional Young's interference experiments were made by double slits or pinholes. The intensity distribution across the interference fringes produced by the double pinholes, located in front of one of the coupling holes, gives rise to good visibility as shown in Fig.14. Here the degree of spatial coherence is difined as follows[19]:

$$|\mu_{12}| = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \times \frac{I_1 + I_2}{2\sqrt{I_1 I_2}}, \quad (20)$$

where $I_{\max}$ and $I_{\min}$ denote the maximum and minimum intensities of the interference

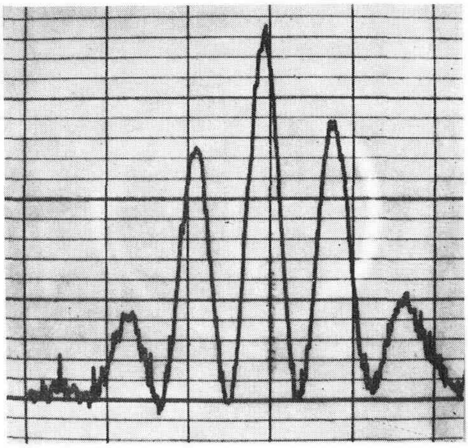


Fig. 14. The intensity distribution across the interference fringes. Two pinholes are spaced by 2.7 *mm* in front of one of the four coupling holes.

pattern, respectively, and I_1 and I_2 are the intensities of the radiation beams emrging from each slit or pinhole. It was found that $|\mu_{12}|$ was not decreased as the double slits were more separated each other in front of the coupling hole and moreover its value was held approximately to be unity over the whole area of the hole[20]. On the other hand, the degree of spatial coherence took the values from 0.5 to 0.8 in the case of only the central coupling hole under the same conditions as in the one of the four coupling holes. It may be concluded from these facts that the four off-centered output-coupling hole can produce nearly pure mode of laser oscillation.

4. Concluding Remarks

The resonator with a pair of internal reflectors, one of which has off-centered output-coupling holes or an annular aperture symmetric to the resonator axis, has been considered for the infrared or far infrared laser for the purpose of mode selection. For relatively large Fresnel numbers, the percentage power losses arising from an annular output-coupling aperture around the center of one of the mirrors are taken into consideration for the lower order modes, TEM_{00}, TEM_{10}, TEM_{01}, in the cylindrical circular confocal and Fabry-Perot resonators.

According to numerical calculations, the predominating mode changes with the size of the annular aperture. This aperture seems considerably desirable for selecting the lowest order mode TEM_{00} by differaction coupling as the predominating mode when it is arranged in the somewhat more outer region than the location where the intensity of the next higher order circularly symmetric mode TEM_{10} becomes maximum. If a proper substratum for the reflection mirror can not be prepared for a given infrared or far infrared region, the reflection mirror which is partially blocked out in the annular region around the center of the mirror is proposed and examined experimentally.

A Fabry-Perot resonator has been used for a CO_2 laser. One of the plain mirrors has four output-coupling holes bored circularly in the annular region. The total output power emerging from the four output-coupling holes is found to be approximately 6 times as large as the output power from the one coupling hole centered at the resonator axis. This central coupling hole has the same size as one of the four coupling holes. The degree of spatial coherence was measured in order to know how the off-centered output-coupling holes are efficient for selecting the operating mode. The results show that nearly perfectly coherent area extends over the area of one of the coupling holes. This fact illustrates that approximately pure mode of oscillation can be obtained from the Fabry-Perot resonator with the four off-centered output-coupling holes.

More precise computer calculations of the field distribution functions and diffiranction losses are under consideration on the basis of a usual integral equation for the resonators with the off-centered output-coupling apertures.

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