Beat Frequency Spectra of Light Diffracted by Successive Ultrasonic Waves

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Abstract

The use of double diffraction of coherent light formed by two successive ultrasonic light modulators (ULM) was suggested for optical heterodyning within the Raman-Nath regime. The light wave traversing a pair of ULM's has some discrete frequencies shifted by certain integral multiples of the first and second driving sound frequencies. From the double diffraction spectrum a pair of deflected light beams is chosen spatially and photomixed with one another at a detector. When a pair of standing or of standing and progressive sound waves is employed, the beating signal has various discrete carrier frequencies consisting of some combinations of the two sound frequencies. A beating signal required could be separated from the other by a suitably tuned electronic circuit. For a pair of progressive sound waves the beating signal has a difference or sum carrier frequency of two sound frequencies. The relative inclination angle which the two ULM's make with one another can be determined to a considerably less critical extent so that the beat output may attain its maximum.

1. Introduction

As is well known, an ultrasonic wave in a homogeneous and isotropic medium diffracts a light ray in the same manner as a moving phase grating. On account of this phenomenon[1] the light ray scattered by the sound wave splits into one or more directions in a diffraction spectrum and each deflected light beam undergoes some frequency shifts because of a Doppler like effect.

Optical heterodyne detection-techniques using a pair of coherent light beams such as lasers, one of which is frequency-shifted by the optical-acoustic interaction, have recently made it possible to produce a light beating signal whose carrier frequency is equal to the sound frequency[2-4]. The beating signal will include some significant informations in its amplitude, frequency, and/or phase. For example, some real-time signal processing can be carried out by virtue of the beating signal including modulation signals[5,6].

Since a certain beating signal required for experimental or physical situations can be picked up by means of an electronic circuit tuned to its beat frequency, it will be generally convenient to simultaneously generate beating signals. According to Raman and Nath[1] the deflected light ray of a given order has a number of discrete shifted frequencies or only one shifted frequency, according to whether the sound wave is standing or progressive. Accordingly, it will be possible to obtain light beating signals consisting of various kinds of beat frequencies if sound waves of more than one progressive wave are successively combined in an optical system. Based on the phase-lattice theory[1], this paper describes some features associated with the light beating signals arising from the double diffraction of coherent light by a set of successive ultrasonic waves: a pair of standing, a pair of progressive, and a pair of standing and progressive sound waves.

In addition spatial requirements for heterodyning, which are said to be very critical in a conventional method[7,8], will be discussed in the third section.

2. Analytical Description

For convenience we call an ultrasonic light modulator an ULM. An optical system with a pair of ULM's is shown schematically in Fig. 1. The first ULM is arranged at the front focal plane of the lens L_1 and inclined to the plane of the incident light wavefront at an angle α. The lenses L_1 and L_2 are separated by $2f$, f being a common focal length. The second ULM is placed at the back focal plane of the lens L_2 and is also tilted by an angle β. The image of the first ULM is, as is well known, formed so as to superimpose with the second ULM at a certain angle. Every deflected light ray emerging from the second

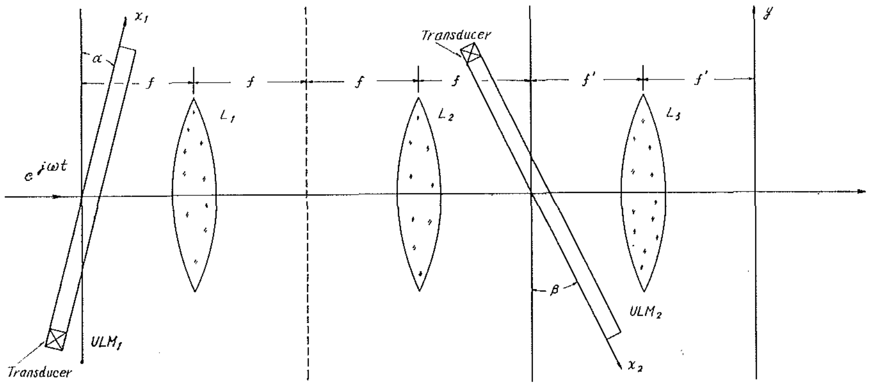

Fig. 1. Linear optical system with a pair of successive ultrasonic light modulators.

ULM is focused on the back focal plane of the lens L_3. The axes x_1, x_2, and y are taken along the first ULM, the second ULM, and the face of a square-law photodetector, respectively. The ζ-axis is taken along the direction of the light beam incident on the first ULM.

We shall first deal with the problems of the light diffraction by a pair of standing sound waves. Let F and K be the frequency and wave number of a plane sound wave, respectively. When there exists a simple standing sound wave in a transparent medium, the light beam will undergo a variation in the optical index of refraction as given by

$$n(x_i, t) = n_{0,i} - n_i\left\{\sin(2\pi F_i t - K_i x_i) + \sin(2\pi F_i t + K_i x_i)\right\},\quad i=1 \text{ or } 2 \qquad (1)$$

at a point inside the ULM, where the suffix $i=1$ or 2 distinguishes some physical quantities of the first ULM from those of the second, $n_{0,i}$ is the index of refraction without perturbation due to the sound wave, and n_i is the maximum variation of the index caused by the sound wave. The phase of the light wave is to be modulated sinusoidally in both space and time after its passage through the ULM. Let the light wave take the form $\exp(2\pi jft)$, f being the frequency of the monochromatic light wave, and be incident on the first ULM at an anlge α. This light wave will suffer successive dual Fourier transforms after it travels from the first ULM to the second.

Following the phase-lattice theory[1] and using the identity for the successive Fourier transforms

$$\mathscr{F}\left[\mathscr{F}\left\{G(x)\right\}\right]=G(-x), \tag{2}$$

$\mathscr{F}$ denoting a notation of the Fourier transform, the field of the light wave arriving at the entrance face of the second ULM can be described[9], ignoring constant phase terms, as

$$\begin{aligned} E_1 &= E_0 \exp j\left[2\pi ft+v_1\left\{\sin(2\pi F_1 t-K_1x_1)+\sin(2\pi F_1 t+K_1 x_1)\right\}+kx_1\sin\alpha\right] \\ &= E_0 \sum_r\sum_m J_r(v_1)J_{r+m}(v_1)\exp j\left[2\pi\left\{f+(2r+m)F_1\right\}t+kx_1\sin\phi_m\right] \end{aligned} \tag{3}$$

with $$v_1 = kn_1d_1\sec\alpha'\,\mathrm{sinc}\left\{(K_1d_1/2)\tan\alpha'\right\}, \tag{4}$$

where k is the wave number of light in vacuum, d_1 is the strip width of the sound column, α' is the refractive angle at the boundary, $J_s(v)$ is the Bessel function of the sth order, and the diffraction angle of the mth order satisfies the relation

$$\sin\phi_m = m(K_1/k)+\sin\alpha\,. \tag{5}$$

In the derivation of Eq. (3) the following mathematical identity is used:

$$\exp jB\sin W=\sum_s J_s(B)\exp jsW\,. \tag{6}$$

Note that the angles α and α' meet the Snell's law

$$\sin\alpha = n_{0,1}\sin\alpha'\,. \tag{7}$$

According to the geometrical situation in Fig. 1 the deflected light beam of the mth order emerging from the first ULM impinges upon the second ULM at an angle $(\phi_m+\beta-\alpha)$ after its passage through the two lenses L_1 and L_2. It follows that the phase term in Eq. (3) must be replaced, deleting constant phase factors, by $kx_2\sin(\phi_m+\beta-\alpha)$ when it reaches the second coordinate x_2. After traversing the second ULM, the wave field at the exit face of the second ULM may therefore be written as before in the form

$$\begin{aligned} E_2 &= E_0\sum_r\sum_m\sum_p\sum_q J_r(v_1)J_{r+m}(v_1)J_p(v_{2,m})J_{p+q}(v_{2,m}) \\ &\quad\times\exp j\left[2\pi\left\{f+(2r+m)F_1+(2p+q)F_2\right\}t+kx_2\sin\phi_{m,q}\right] \end{aligned} \tag{8}$$

with $$v_{2,m} = n_2kd_2\sec\beta'_m\,\mathrm{sinc}\left\{(k_2d_2/2)\tan\beta'_m\right\}, \tag{9}$$

where the angle $\phi_{m,q}$ satisfies the relation

$$\sin\phi_{m,q} = q(K_2/k)+\sin(\phi_m+\beta-\alpha)\,, \tag{10}$$

and the angle β'_m meets

$$\sin(\phi_m+\beta-\alpha)=n_{0,2}\sin\beta'_m\,. \tag{11}$$

It turns out that the spatial diffraction spectrum produced by a pair of sound waves is specified by the integral numbers m and q.

Let us now derive a formula for the output of a square-law photodetector located at the spatial frequency plane of the lens L_3. Consider a pair of typical and nearest spatial frequencies denoted by the diffraction angles $\phi_{m,q}$ and $\phi_{m',q'}$, and introduce a slit with the center lying at the mid point between these spatial frequencies. This slit should be suitably open so that only those spatial frequencies may be separated from the other and brought together onto the detector. Let a new coordinat be $z=y/\lambda f$, λ being the wavelength of light, and thus the following is derived:

$$k\sin\phi_{m,q}\simeq 2\pi y_{m,q}/\lambda f=2\pi z_{m,q}\,. \tag{12}$$

For brevity putting

$$A_{m,q}=\sum_r\sum_p J_r(v_1)J_{r+m}(v_1)J_p(v_{2,m})J_{p+q}(v_{2,m})$$
$$\times\exp 2\pi j\left\{f+(2r+m)F_1+(2p+q)F_2\right\}t\,, \tag{13}$$

Eq. (8) reduces to

$$E_2=\sum_m\sum_q A_{m,q}\exp(2\pi jx_2z_{m,q})\,. \tag{14}$$

If an aperture placed at the exit face of the second ULM restricts the light beam to its strip width of $2D$, the output of the detector will be proportional to

$$I(t)=\frac{1}{T}\int_{t-T}^{t}dt'\int_{-\infty}^{+\infty}\delta(z-z_c)\left|\int_{-\infty}^{+\infty}g(x_2)E_2(x_2,t')\exp(-2\pi jx_2z)dx_2\right|^2dz\,, \tag{15}$$

where $\quad z_c=(z_{m,q}+z_{m',q'})/2 \quad (16)$

and $\quad g(x_2)=\begin{cases}1 & \text{for } |x_2|\leqq D\\ 0 & \text{for } |x_2|>D\end{cases}. \quad (17)$

Care must be taken here to the fact that a time average must be taken over a period so that $T_1\ll T\ll T_2$, T_1 and T_2 being, respectively, reciprocal light and beat frequencies under consideration.

Applying a convolution theorem to the Fourier transform in Eq. (15), the straightforward calculation, substituting Eq. (14) into Eq. (15), leads finally to

$$I(m,q;m',q';t)=2D^2\sum_r\sum_p\sum_{r'}\sum_{p'}J_r(v_1)J_{r+m}(v_1)J_{r'}(v_1)J_{r'+m'}(v_1)J_p(v_{2,m})$$
$$\times J_{p+q}(v_{2,m})J_{p'}(v_{2,m'})J_{p'+q'}(v_{2,m'})\,\mathrm{sinc}^2\left\{\pi D(z_{m,q}-z_{m',q'})\right\}$$
$$\times\cos\left\{2\pi(2r-2r'+m-m')F_1+(2p-2p'+q-q')F_2\right\}t\,. \tag{18}$$

From this expression the $d.c.$ component is evidently given by

$$I_d=2D^2\sum_r\sum_p J_r^2(v_1)J_{r+m}^2(v_1)J_p^2(v_{2,m})J_{p+q}^2(v_{2,m}) \tag{19}$$

under the condition that

$$\left.\begin{array}{l} r=r', \quad m=m' \\ p=p', \quad q=q' \end{array}\right\}. \tag{20}$$

If two light beams specified by a pair of order numbers $m=0$, $q=1$ and $m'=1$, $q'=0$ are provided for optical heterodyning, the beat output will include such frequencies as $|F_1 \pm F_2|$, $|F_1 \pm 3F_2|$, $|3F_1 \pm F_2|$, $3|F_1 \pm F_2|$, $\cdots$and so on. Following experimental or physical requirements, any beating signal will be prepared by making use of an electronic tuned circuit.

Consider next the diffraction of light by a pair of progressive sound waves. For this case the index of refraction at a point inside the ULM can be expressed as

$$n(x_i, t) = n_{0,i} - n_i \sin(2\pi F_i t - K_i x_i) \qquad i=1 \text{ or } 2, \tag{21}$$

where the suffix i means the same as before. Note that K_2 is positive or negative, according to whether the sound wave in the second ULM propagates in the opposite direction or the identical direction from the one in the first ULM. As before, the wave field can be then described as

$$E_1 = E_0 \sum_m J_m(v_1) \exp j\left\{2\pi(f+mF_1)t + kx_2 \sin(\phi_m + \beta - \alpha)\right\} \tag{22}$$

at the entrance face of the second ULM, so that we obtain

$$E_2 = E_0 \sum_m \sum_q J_m(v_1) J_q(v_{2,m}) \exp j\left\{2\pi(f+mF_1+qF_2)t + kx_2 \sin \phi_{m,q}\right\} \tag{23}$$

at the exit face of the second ULM, where the diffraction angle $\phi_{m,q}$ still has the same formula as Eq. (10).

Setting

$$A_{m,q} = J_m(v_1) J_q(v_{2,m}) \exp 2\pi j(f+mF_1+qF_2)\, t\,, \tag{24}$$

we can get the same expression for the wave field as Eq. (14). For this case the intensity formula will be derived consequently from Eq. (15) as follows:

$$\begin{aligned} I(m, q; m', q'; t) = 2D^2 \operatorname{sinc}^2\left[\pi D(z_{m,q} - z_{m',q'})\right] J_m(v_1) J_{m'}(v_1) \\ \times J_q(v_{2,m}) J_{q'}(v_{2m'}) \cos 2\pi\left\{(m-m')F_1 + (q-q')F_2\right\} t\,. \end{aligned} \tag{25}$$

For the moment, the *d. c.* component will also be given by

$$I_d = 2D^2 J_m^2(v_1) J_q^2(v_{2,m}) \tag{26}$$

under the condition that

$$\left.\begin{array}{l} m-m'=0\,, \\ q-q'=0 \end{array}\right\}. \tag{27}$$

It should be noted that from the first order spectrum we can obtain a beating signal with $F_1 - F_2$ or $F_1 + F_2$, according as the second sound wave travels in the direction opposite or identical from the first sound wave.

Finally, consider the problems by a pair of standing and progressive sound waves. For this case we shall discuss simply the features by taking account of the previous two cases. Putting

$$A_{m,q}=\sum_p J_m(v_1)\,J_{p+q}(v_{2,m})\,J_p(v_{2,m})\exp 2\pi j\left\{f+mF_1+(2p+q)F_2\right\}t\,, \tag{28}$$

the intensity is expressed similarly by

$$\begin{aligned} I(m,q\,;m',q'\,;t)=2D^2\operatorname{sin c}^2\left[\pi D(z_{m,q}-z_{m',q'})\right]J_m(v_1)\,J_{m'}(v_1)\\ \times\sum_p\sum_{p'}J_{p+q}(v_{2,m})\,J_{p'+q'}(v_{2,m'})\,J_p(v_{2,m})\,J_{p'}(v_{2,m'})\\ \times\cos 2\pi\left\{(m-m')F_1+(2p-2p'+q-q')\right\}t\,. \end{aligned} \tag{29}$$

As a typical example, a set of $m=0$, $q=1$ and $m'=1$, $q'=0$ generates beat components consisting of $|F_1\pm F_2|$, $|3F_2\pm F_1|$, $|5F_2\pm F_1|$, $\cdots$ and so on. The *d. c.* component also is given by

$$I_d=2D^2J_m^2(v_1)\sum_p J_p^2(v_{2,m})\,J_{p+q}^2(v_{2,m})\,. \tag{30}$$

3. Discussion for Angular Alignment

Setting

$$G=\pi D(z_{m,q}-z_{m',q'})\,, \tag{31}$$

the beat output will be seen in any cases to be proportional to the product of $\sin c^2G$ by a set of Bessel functions of v_1 and v_2. The relative inclination angle $(\beta-\alpha)$ which the ULM's make with one another can be determined so that the factor $\sin c^2G$ may take the maximum value of unity. This occurs if the two wavefronts of the light wave under consideration are exactly superimposed on each other, that is, if the condition $z_{m,q}=z_{m',q'}$ holds.

Let us examine a permitted limit of the inclination angles α and β. For the angle α of the first ULM Eq. 5 gives rise to the condition

$$|mK_1/k+\sin\alpha|\leqq 1\,. \tag{32}$$

Combining Eq. (5) with Eq. (10) further reduces to

$$\left|qK_2/k+\sin\left[\sin^{-1}\{mK_1/k+\sin\alpha\}+\beta-\alpha\right]\right|\leqq 1\,. \tag{33}$$

From Eqs. (5), (10), (12), and (31) we have

$$G=(K_1D/2)\left\{(q-q')(K_2/K_1)+\left|(m-m')\operatorname{cosec}\delta\right|\cos(\beta-\alpha-\delta)\right\} \tag{34}$$

with $\delta=\tan^{-1}\left\{(\cos\phi_m-\cos\phi_{m'})(m-m')^{-1}(k/K_1)\right\}$, (35)

under the condition that

$$m-m'\neq 0\,,\quad q-q'\neq 0\,. \tag{36}$$

Since the spatially coincident condition with respect to a pair of deflected light waves under consideration takes $G=0$, the following relations are immediately derived from Eq. (34)

$$\left|\cos(\beta-\alpha-\delta)\right| = \left|(q-q')(m-m')^{-1}\cos\delta\right|(K_2/K_1) \leqq 1 \tag{37}$$

and if $\delta \ll 1$,

$$\left|\cos(\beta-\alpha)\right| = \left|(q-q')(m-m')^{-1}\right|(K_2/K_1) \leqq 1\,. \tag{38}$$

It is concluded that the two wavefronts of light can be accurately superimposed on one another at the face of the detector when the relation (37) or (38) holds under the conditions (32) and (33).

For the moment, care must be taken to the fact that the magnitude of the beat output could not be discussed only from the value of the factor $\mathrm{sinc}^2 G$, because Raman-Nath parameters v_1 and v_2 also depend on the inclination angles α and β. Let us consider the beat output associated with a pair of progressive sound waves and take a pair of deflected light waves specified by a set of $m=1$, $q=0$ and $m'=0$, $q'=\pm 1$, not only because of a special case but because of a practical interest. If the first ULM is fixed at some angle, the parameter v_1 can be regarded as a constant from Eqs. (4) and (7). In order to reduce the present discussion to the simplest one we shall take account of the situation where only the second ULM is inclined for the purpose of aligning the optical system. It follows that the magnitude of the beat output is proportional to

$$B = \mathrm{sinc}^2 G \left| J_0(v_{2,1}) J_1(v_{2,0}) \right|, \tag{39}$$

since $J_0(v_1)J_1(v_1)$ is to be regarded as a constant. Based on this expression, let us present an example in Fig. 2 which is calculated numerically from the following

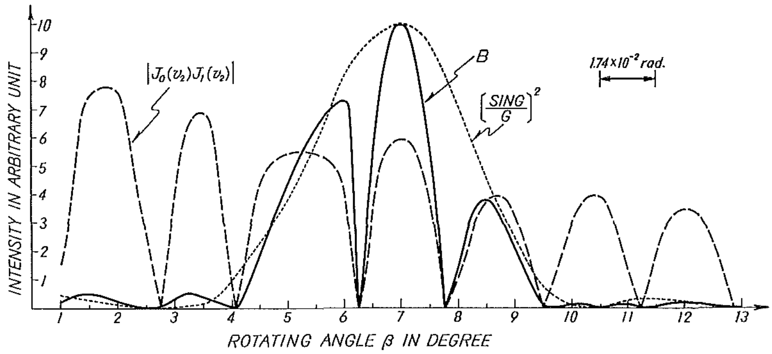

Fig. 2. The beat output B, its factors $\mathrm{sinc}^2 G$, and $|J_0(v_{2,1})J_1(v_{2,0})|$ as a function of the inclination angle β of the second ULM.

values: $F_1 = 10$ HMz, $K_2/K_1 = 0.97$, $\lambda = 6328$ Å, $D = 5$ mm, $d_2 = 5$ mm, $n_{0,2} = 1$, $n_2 = 10^{-4}$, and $\alpha = -7^0$. As may be seen from the figure, the beat output has to do so closely with both the two factors $\mathrm{sinc}^2 G$ and $|J_0(v_{2,1})J_1(v_{2,0})|$ and it is not symmetric on both sides of its maximum value. In general the maximum beat output does not coincide with the maximum of $\mathrm{sinc}^2 G$ in the case where two sound frequencies are appreciably different from each other, because $|J_0(v_{2,1})J_1(v_{2,0})|$ and $\mathrm{sinc}^2 G$ are at the same time a function of the inclination angle β.

4. Conclusions

The use of double diffraction of coherent light by a linear optical system, with a pair of ultrasonic light modulators (ULM) has been suggested for the surpose of generating light beating signals. For the moment, three combinations of the two ULM's i.e., standing–standing, progressive–progressive, and standing–progressive sound waves, are considered for the double diffraction. Since a certain beating signal for experimental or physical requirements is separately obtainable from the other signals by means of a suitably tuned electronic circuit, it will be convenient to generate at the same time some or more beating signals, with various kinds of discrete carrier frequencies. From this point of view two combinations of standing–standing and standing–progressive sound waves are especially taken into account. According to these systems a photodetector placed at a given spatial frequency plane will be able to simultaneously provide some beating signals, every carrier frequency of which consists of a sum or difference of the two sound frequencies multiplied by every diffraction order. A beating signal with only a difference or sum of those frequencies can be produced by means of a pair of progressive sound waves, according to whether the second sound wave travels in the opposite or identical direction from the first sound wave.

The relative inclination angle which the two ULM's make with one another can be determined within some permitted extent so that the beat output may take the maximum. A typical numerical example illustrates that angular alignment is considerably less critical than the usual method[10] employing such an optical system as Mach-Zehnder interferometer.

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