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Image Synthesis by Array Systems

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Abstract

A new technique for image formation by array systems is proposed, where the array elements act as active elements and synthesize images as their Fraunhofer radiation patterns. Various arrays such as the complex array, amplituded array, phased array and temporal array are discussed from the point of view of image synthesis. A numerical experiment is conducted by computer to simulate arrays of 64×64 elements and the Fraunhofer radiation patterns are calculated. Image synthesis in the Fresnel region is also discussed in connection with the Fresnel holograms.

1. Introduction

In the image formation by lens systems, a lens reproduces images to convert the wavefronts of the optical (or other kinds of) waves coming from the object. In this type of image formation, the lens acts as a passive element. On the other hand, recent investigations on computer synthesized holograms[1)], antenna arrays[2)], microwave[3)] and acoustical[4)] holographies suggest a method of image synthesis by array systems, where each array element acts as an active element. This image synthesis by active arrays, which may be called an active image formation, is a new technique in image technology.

It is not easy, in practice, to construct active array systems for the experiment of image synthesis. Fortunately, a recent advance in calculating the Fourier transform, the so called FFT (fast Fourier transform)[5)], allows us a numerical experiment of image synthesis by computer. In this paper basic discussions on the image synthesis by array systems are made and various methods to synthesize images are proposed. Following these methods numerical experiments are conducted to simulate arrays by computer. Computer simulation is the first step in realizing array systems and it gives us much information about image synthesis by this technique.

2. Some Aspects of Image Synthesis by Array Systems

The principle of image synthesis by array systems is that we synthesize images to describe the amplitude and phase of the waves radiated from, or transmitted through, the arrays. The waves coming from each array element must be correlated, that is, coherent waves are required in this type of image formation. Coherent waves such as microwaves, millimeter-waves, optical-waves, sound-waves and ultrasonic-waves can be used for this purpose. But these waves have their own properties; for example being visible or invisible, of long-wavelength or short-wavelength, easy or difficult in describing the amplitude and phase, etc. Therefore the array systems must be realized taking into account these properties. We consider various array systems relating them to the properties of waves.

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When both the amplitude and phase of the waves can be easily described, the arrays of type 1–1) can be realized, otherwise the arrays of type 1–2) or 1–3), where only the amplitude or phase of the waves is described, are available.

1–1) Complex arrays
1–2) Amplituded arrays
1–3) Phased arrays

The amplituded arrays, which describe only the magnitude of the amplitude, are named by analogy with the phased arrays which describe only the phase.

For waves of short-wavelength, such as optical or ultrasonic-waves, it is convenient to synthesize images in the Fraunhofer region (Fourier transform plane) 2–1). On the other hand, image synthesis in the Fresnel region 2–2) is suitable for waves of long-wavelength such as microwaves or sound-waves.

2–1) Image synthesis in the Fraunhofer region
2–2) Image synthesis in the Fresnel region

In microwaves or sound-waves, the antenna arrays or transducer arrays radiate waves and these arrays may be called radiation-type arrays 3–1). In optical or ultrasonic-waves it may be easier to construct transmission-type arrays 3–2) which control the amplitude and phase of the wave transmitted through the arrays.

3–1) Radiation-type arrays
3–2) Transmission-type arrays

If the array functions which describe the amplitude and phase conditions of each array element are time-independent, the arrays described by such array functions are called non-temporal arrays 4–1), whereas the arrays whose array functions are time-dependent are called temporal arrays 4–2).

4–1) Non-temporal arrays
4–2) Temporal arrays

In the following sections, we discuss image synthesis by the various array systems mentioned above, with reference to the results of numerical experiments by computer. In practice, we must consider the directivity and aperture size of each array element, coherency or spectrum band-width of waves, alignment of each array element, etc. But the present discussion is not concerned with the precise analysis or design of array systems, thus we adopt the following assumptions which allow for computer simulation of synthesizing images by array systems; 1) waves have one single spectrum, 2) the directivity of each array element is uniform, 3) each element is a point, 4) images are the Fraunhofer radiation patterns (Fourier transforms) of the arrays (except for the discussion on image synthesis in the Fresnel region), 5) mechanical defects, such as misalignment of array elements, difference of characteristics of each array element, etc., are neglected.

3. Image Synthesis by Complex Arrays

In Fourier optics, antenna theory and acoustical-wave diffraction theory, it is well known that the Fraunhofer diffraction or radiation patterns of apertures, antenna or transducer arrays can be described by the Fourier transforms of these source distributions. If the image is the Fraunhofer radiation pattern of an array, the image function $o(x, y)$ (in the present discussion we consider two-dimensional images) and the source distribution $A(\alpha, \beta)$ of the array are the Fourier transform pair,

$$o(x, y) = \mathscr{F}\left[A(\alpha, \beta)\right] \qquad (1)$$

$$A(\alpha, \beta) = \mathscr{F}^{-1}\left[o(x, y)\right] \qquad (2)$$

where $\mathscr{F}$ and $\mathscr{F}^{-1}$ represent the Fourier and inverse Fourier transform operators and x, y and α, β are the coordinates in the image and array planes respectively. As the function $A(\alpha, \beta)$ determines the amplitude and phase conditions of each array element, we call it an array function. Assumig that we realize an array by which we can describe both the amplitude and phase (that is the complex amplitude) of the waves radiated from the array. We would call such an array a complex array. Then we can synthesize images to excite the array according to the amplitude and phase distribution of the array function $A(\alpha, \beta)$ of Eq. (2).

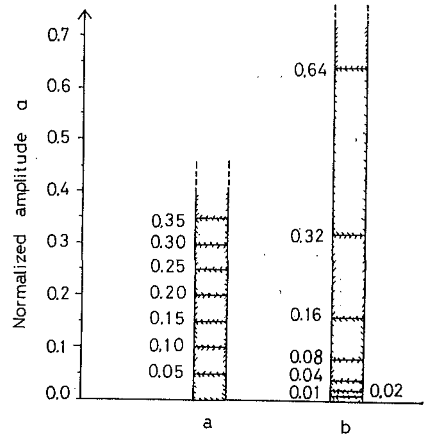


Fig. 1. Linear (a) and nonlinear (b) digitizations of amplitudes of array functions.

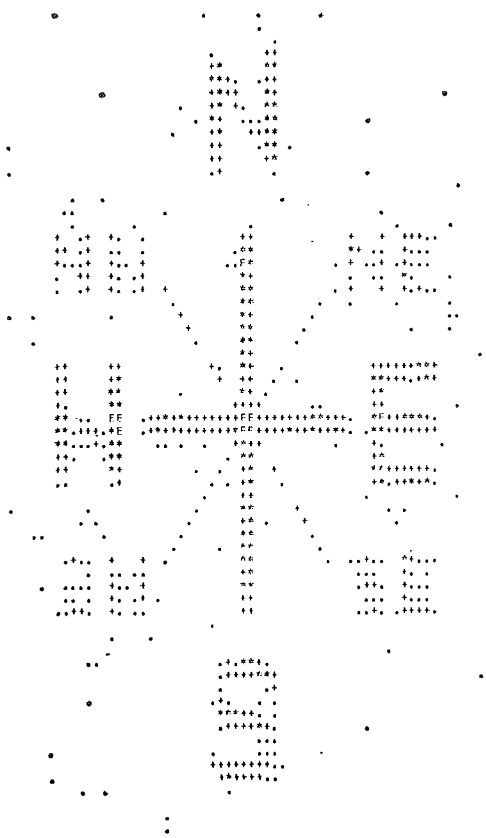


Fig. 2-a

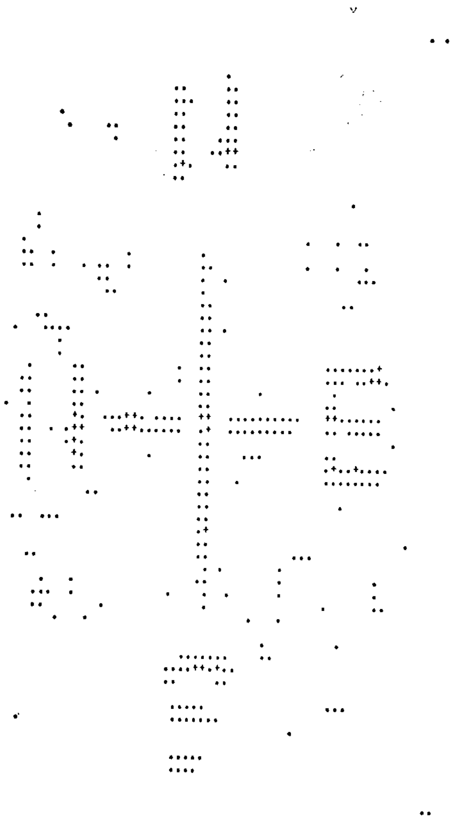


Fig. 2-b

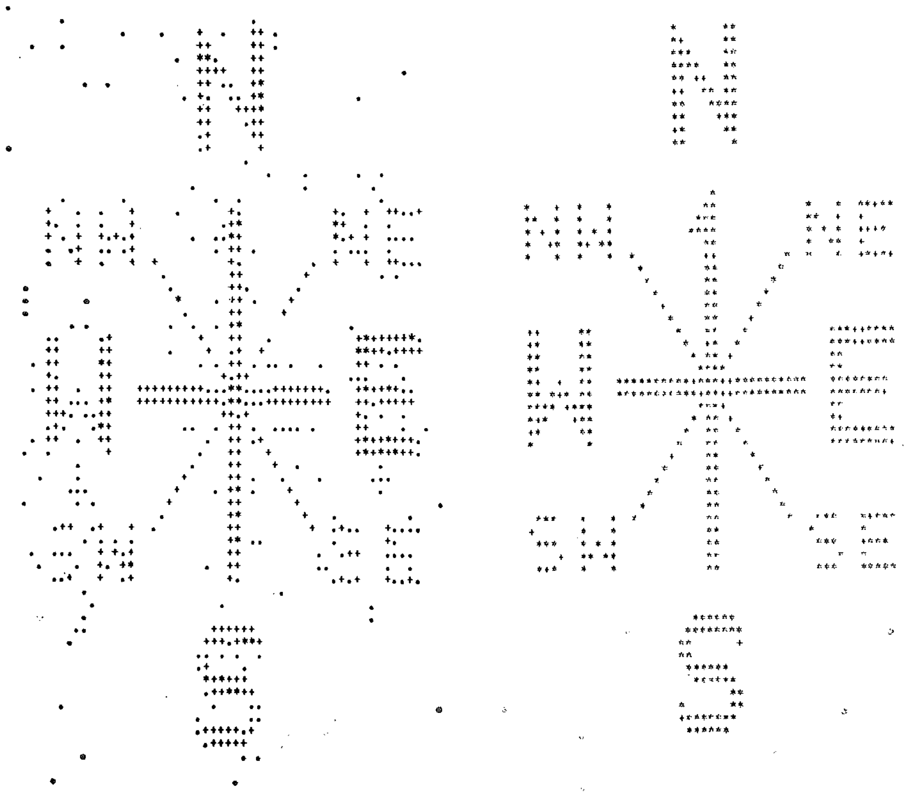


Fig. 2-c Fig. 2-d

Fig. 2. Numerically synthesized images, where a corresponds to the linear digitization of Fig. 2-a, and b, c and d correspond to the nonlinear digitization of Fig. 2-b. The digitized levels are as follows; a is eight (as shown in Fig. 2-a), b is two (0.0, 0.08) c is four (0.0, 0.04, 0.08, 0.16) and d is eight (as shown in Fig. 1-b). The intensity I (normalized) of the images is expressed by the letter $E\,(I\geqslant 0.8)$ and the symbols $*\,(0.8>I\geqslant 0.6)$, $+\,(0.6>I\geqslant 0.4)$, $\cdot\,(0.4>I\geqslant 0.2)$ and blank $(0.2>I)$.

It is difficult, in practice, to describe both the amplitude and phase precisely and it is not always necessary to do so. Therefore it is necessary and also possible to synthesize images with digitized amplitude and phase. In digitizing amplitude, there are two methods; the linear and nonlinear digitizations. In the linear digitization, we digitize the amplitude a (normalized) into linearly proportional levels as shown in Fig. 1-a. In the nonlinear digitization, the intervals between neighbouring levels are not linearly proportional. For example, we consider the nonlinear digitization of Fig. 1-b where each level is determined to double the level below.

A numerical experiment of image synthesis is conducted to simulate an array of 64×64 array elements and to calculate the Fraunhofer radiation patterns by computer. The process for the computer simulation of image synthesis is as follows; 1) Calculation of the Fourier transform of an image, 2) digitization of the amplitude of the obtained array function according to Fig. 1, 3) excitation of each array element according to the digitized array function, 4) calculation of the (inverse) Fourier transform of the excited array, 5) display of the synthesized images. The numerical synthesized images are obtained as shown in Fig. 2–a ~ d, where a corresponds to the linear digitization of Fig. 1–a and b, c and d correspond to the nonlinear digitization of Fig. 1–b respectively. Comparing the images of Fig. 2–a and d, we can see that the nonlinear digitization synthesizes images of better quality than that of the linear digitization. The images of Fig. 2–b, c and d are concerned with the roughness of digitizing levels respectively. From these images, we can see how the roughness of digitizing levels affects the synthesized images.

4. Image Synthesis by Digital Amplituded Arrays

An amplituded array can describe only the magnitude of the amplitude of waves radiated from or transmitted through the array. For example, we can imagine an array whose element, such as liquid crystals[6], can control the amplitude transparency for optical waves. In the image synthesis by such arrays, we must suppress or compensate for the phase information in describing the amplitude distribution in the arrays. In computer synthesized holograms, the phase information is translated into the position of each cell (each array element). It is difficult, in practice, to control the positions of array element except for computer synthesized holograms. Other methods, such as image synthesis in the Fresnel region and image synthesis by binary phased arrays, may be possible in synthesizing images by amplituded arrays. These methods will be discussed in the following sections.

Fig. 3. Images numerically synthesized from the real part of the complex array function. The images are displayed with the real part R (normalized) of the Fraunhofer radiation field and expressed by the symbols $*$ $(0.8>R\geqslant 0.6)$, $\cdot$ $(0.4>R\geqslant 0.2)$, blank $(0.2>R\geqslant -0.001)$ and $=$ $(-0.02>R)$.

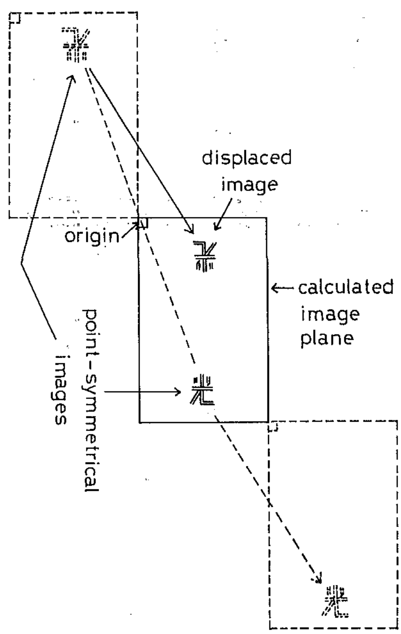


Fig. 4. Displacement of the numerically synthesized image due to use of the FFT algorithm.

Here a method to synthesize images using either the real or imaginary part of the complex array function is proposed. We divide the array function of Eq. (2) into two parts as follows,

$$A(\alpha, \beta) = \mathscr{R}_e A(\alpha, \beta) + i\left[\mathscr{I}_m A(\alpha, \beta)\right] \tag{3}$$

where $\mathscr{R}_e A(\alpha, \beta)$ and $\mathscr{I}_m A(\alpha, \beta)$ represent the real and imaginary parts. The Fourier transforms of the real and imaginary parts are described as follows[7]

$$2\,\mathscr{F}\left[\mathscr{R}_e A(\alpha, \beta)\right] = o(x, y) + o^*(-x, -y) \tag{4}$$

$$2\,\mathscr{F}\left[\mathscr{I}_m A(\alpha, \beta)\right] = o(x, y) - o^*(-x, -y) \tag{5}$$

where the asterisk * denotes the complex conjugate. From Eqs. (4) and (5), we can see that two images are synthesized from the real or imaginary part of $A(\alpha, \beta)$.

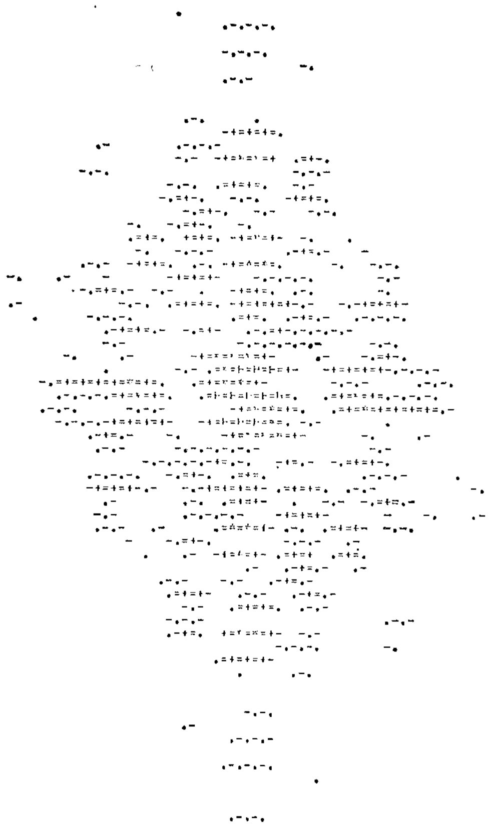


Fig. 5. Amplitude distribution of the array function. The amplitude a (normalized) is digitized into seven levels as expressed by the letter and symbols, $H\,(a \geqslant 1.2)$, $*\,(1.2 > a \geqslant 0.8)$, $+\,(0.8 > a \geqslant 0.6)$, $\cdot\,(0.6 > a \geqslant 0.5)$, blank $(0.5 > a \geqslant 0.3)$, $-\,(0.3 > a \geqslant 0.2)$ and $=\,(0.1 > a)$.

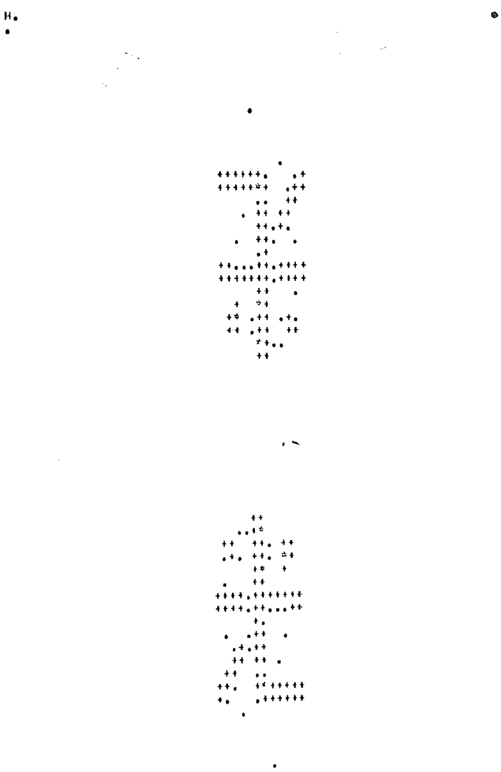


Fig. 6. Images numerically synthesized by the digital amplitude arrays of Fig. 5. The strong field distribution (displayed by the letter H) appears at the origin of the image plane (at the top left corner).

These images are point-symmetrical with respect to the origin of the image plane. For example, the computer simulated images from the imaginary part of $A(\alpha, \beta)$ are shown in Fig. 3, where the images are displayed with the real part of $[o(x, y) - o^*(-x, -y)]$ to point out the fact that the image $-o^*(-x, -y)$ has phase retardation (or advance) of $e^{-i\pi}$ with respect to the image $o(x, y)$. In Fig. 3 the image is a Japanese letter and the upper and lower images correspond to $-o^*(-x, -y)$ and $o(x, y)$ respectively. We should notice that the upper image is displaced, as shown in Fig. 4, because we adopt the FFT algorithm in performing the Fourier transform. If we can describe both positive and negative amplitudes by arrays, we can synthesize images from the real or imaginary part of the complex array function as shown in Fig. 3.

Now we consider the amplituded arrays by which we can describe only the positive amplitude. We add a bias term b to satisfy the following relations,

$$A_R(\alpha, \beta) = \mathscr{R}_e A(\alpha, \beta) + b \geqslant 0 \tag{6}$$

$$A_I(\alpha, \beta) = \mathscr{I}_m A(\alpha, \beta) + b \geqslant 0 \tag{7}$$

The new array functions $A_R(\alpha, \beta)$ and $A_I(\alpha, \beta)$ are real and positive. We can describe the amplituded array by these array functions. For example, the amplitude distribution $A_R(\alpha, \beta)$ is shown in Fig. 5, where the amplitude is digitized into seven levels. The computer simulated array is excited according to Fig. 5 and images are synthesized as shown in Fig. 6. In Fig. 6, we can see the same image as that of Fig. 3 and the strong field distribution at the origin of the image plane due to the bias term b of Eq. (6).

5. Image Synthesis by Digital Phased Arrays

We consider image synthesis by digital phased array, by which we can describe only the phase distribution on the array plane. Generally it is difficult to determine the array function without amplitude information in synthesizing images by such arrays. However, if the images are simple and are limited to a certain region within the image plane, it is possible to find such an array function. We assume an image made of finite and discrete points and express it as follows,

$$o(x, y) = \sum_n \delta(x - x_n, y - y_n) \tag{8}$$

where δ represents a delta function and (x_n, y_n) are the coordinates of each image point. The inverse Fourier transform of Eq. (8) is expressed as shown in Eq. (9),

$$A(\alpha, \beta) = \sum_n e^{i2\pi(x_n\alpha + y_n\beta)} \tag{9}$$

In Eq. (9) the exponential function has unit amplitude, so that the array function contains only phase information only when the image is a single point. When the image consists of many points, we must derive a method by which we can express the function $A(\alpha, \beta)$ as one exponential function. For this purpose, we sample the array elements and describe these elements with the phase of the exponential function of Eq. 9 for one image point. We continue this sampling and describing phase information for all image points. For example, Fig. 7 shows the procedure of sampling and describing phase information, where 16 image points are chosen and the array elements are numbered according to the number of the image points.

The numerical experiment is done following the method mentioned above. The phase is digitized by 20 steps from $-\pi$ to π. The computer simulated images are shown in Fig. 8, where the original image is a letter E. In Fig. 8, we can see many

Array

1	5	9	13	1	5	'	'	1	5
2	6	10	14	2	'	'	'	2	'
3	7	11	15	'	'	'	'	'	'
4	8	12	16	'	'	'	'	'	'
1	5	'	'	1	5	'	'	1	5
2	'	'	'	2	'	'	'	2	'
'	'	'	'	'	'	'	'	'	'
'	'	'	'	'	'	'	'	'	'
1	5	'	'	1	5	'	'	1	5
2	'	'	'	2	'	'	'	2	'

Fig. 7. Procedure for sampling and describing phase information in a phased array. The array elements of the same number are described by the phase information concerning the same image point.

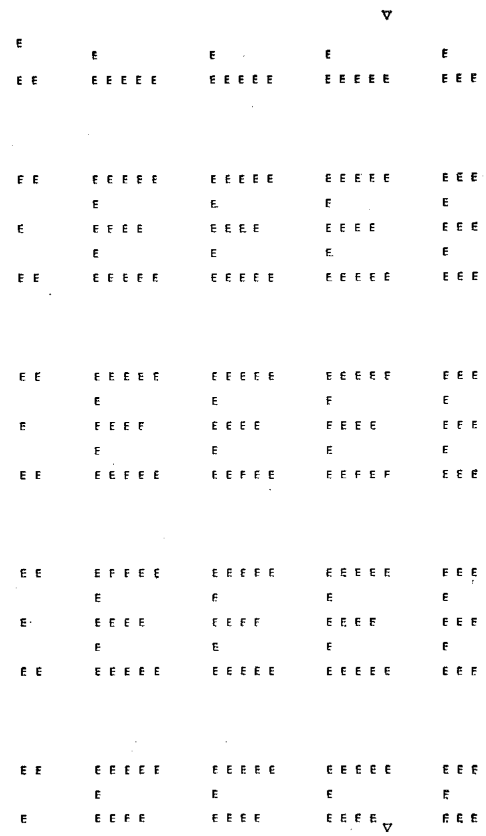


Fig. 8. Image numerically synthesized by the digital phased array, in which phase information is described according to the procedure as shown in Fig. 7.

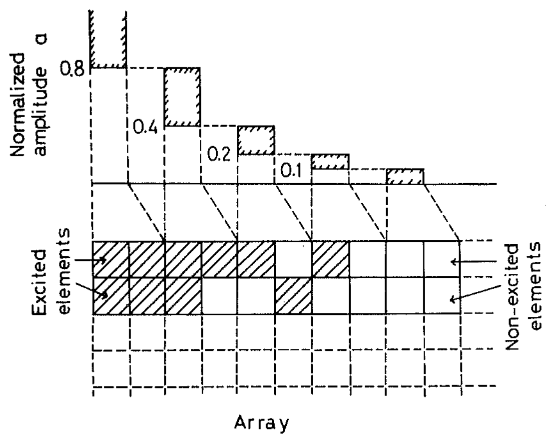


Fig. 9. Labelling of excited elements (as expressed by oblique lines) in a phased array.

higher-order images due to the samping of the array. These higher-order images have the same intensity because each array element is a point in the computer simulation. An actual array, whose elements have a finite aperture, synthesizes faint higher-order images. When the number of image points increases, we must sample the array more roughly, resulting in overlap of the synthesized zero and higher-order images. This means that the number of image points and the image size are limited in image synthesis by this method.

When we can control the on-off state, that is, excited or non-excited state, in addition to the phase state of the array elements, we can consider another method of image synthesis by digital phased arrays. We can describe the magnitude of the amplitude by the density of the excited array elements which radiate waves of unit intensity. For example, we can express five levels of digitized amplitude by the combination of four excited and non-excited array elements as shown in Fig. 9. The strongest level is expressed by four excited elements, the next level is expressed by three excited and one non-excited elements and so no. In this method it is necessary to sample the array function (or array elements) for the allocation of excited and non-excited elements and the sampling rate depends upon the number of levels of amplitude. For the numerical experiment, first we calculate the Fourier transform of an image and digitize the amplitude into five levels. Fig. 10 shows an example of the amplitude distribution of the Fourier transform of an image. Next we sample every other point (over an area) of Fig. 10 and label the excited elements of the array according to the level of the digitized amplitude as shown in Fig. 9. The phase is digized in steps of $\pi/4$ from $-\pi$ to π. Thus we obtain the final control map of the digital phased array as shown in Fig. 11, where the symbols ** show a non-excited element and the numerical values express the digitized phase of the excited elements. The computer synthesized image of the array of Fig. 11 is shown in Fig. 12. In Fig. 12 we can see an image of OPTICS and Japanese letters.

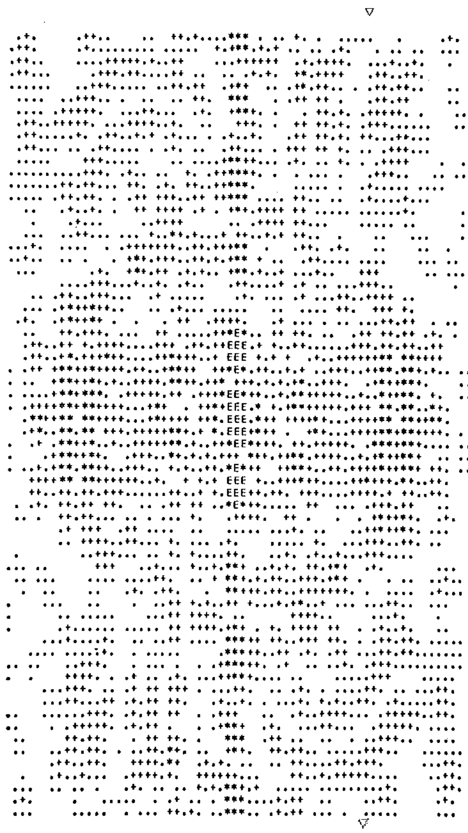


Fig. 10. Amplitude (absolute values) distribution of the Fourier transform of an image. The amplitude a (normalized) is digitized into five levels as expressed by the letter and symbols, $E(a \geqslant 0.8)$, $*(0.8 > a \geqslant 0.4)$, $+(0.4 > a \geqslant 0.2)$, $\cdot\ (0.2 > a \geqslant 0.1)$ and blank $(0.1 > a)$.

When the phase is digitized into two ranges, that is, 0 for the phase from $-\pi$ to 0 and π for the phase from 0 to π, the excited amplitude is 1 or -1 (we

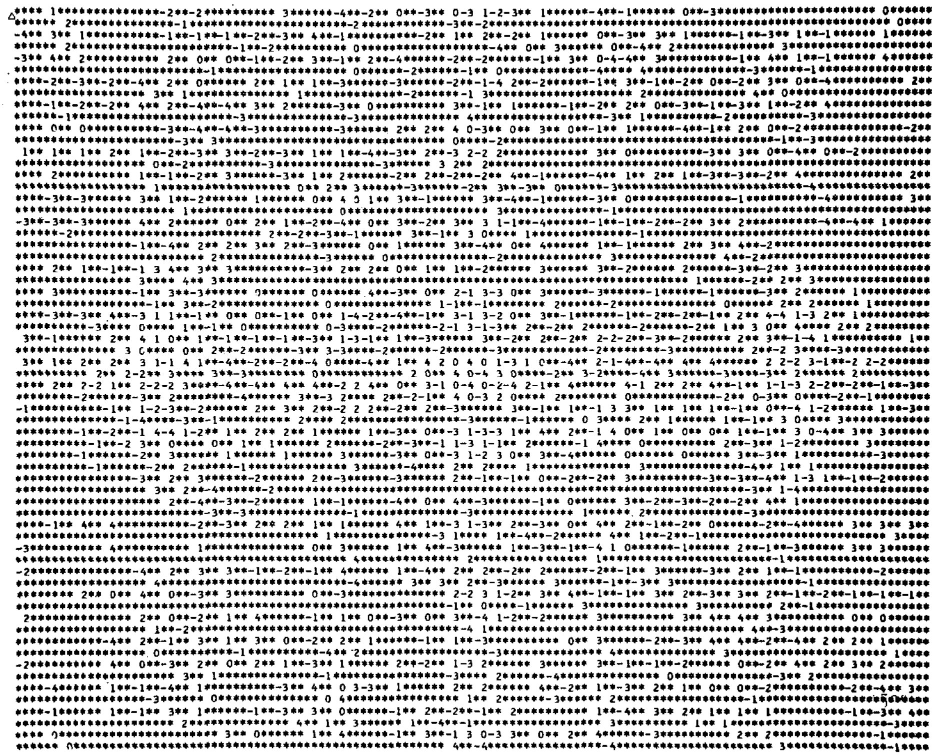


Fig. 11. Phase distribution of the digital phased array. The symbols ** express a non-excited array element. The numerical values n (positive and negative integers including 0 from 4 to -4) express the digitized phase $n\pi/4$ for the phase $n\pi/4+\pi/8>\theta>n\pi/4-\pi/8$.

express unit intensity by 1). In this condition also images are synthesized and one example is shown in Fig. 13. An extra image to the original one appears in Fig. 13, because of the ambiguity of the digitized phase. This image should be point-symmetrical with respect to the origin of the image plane, but it is displaced as explained in Fig. 4 due to use of the FFT algorithm. If we superpose the bias of unit intensity ($=1$) to the binary phased array, the array can be described by three digitized levels of amplitude, that is, 0, 1 and 2. This means that we can synthesize images by amplitude arrays following the above method. The computer simulation of the binary phased array with bias shows that images are synthesized with the strong field distribution located at the origin of the image plane.

6. Image Synthesis by Temporal Arrays

In the preceding sections we discussed arrays which are described by time independent array functions. If the receptors or display systems have an accumulative property for images which change rapidly with respect to time, we can synthesize images of better quality by describing the arrays by time-dependent array functions. We call such arrays temporal arrays for convenience of discussion. The sampling of the Fourier transform and the labelling of the excited array elements as shown

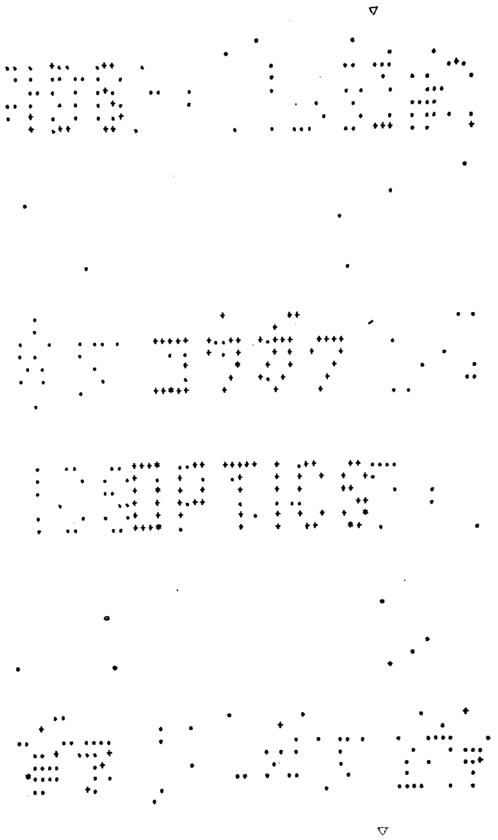


Fig. 12. Image numerically synthesized by the digital phased array of Fig. 11. The intensity I of the image is expressed by the symbols $*$ $(I \geqslant 0.6)$, $+$ $(0.6 > I \geqslant 0.4)$, $\cdot$ $(0.4 > I \geqslant 0.2)$ and blank $(0.2 > I)$.

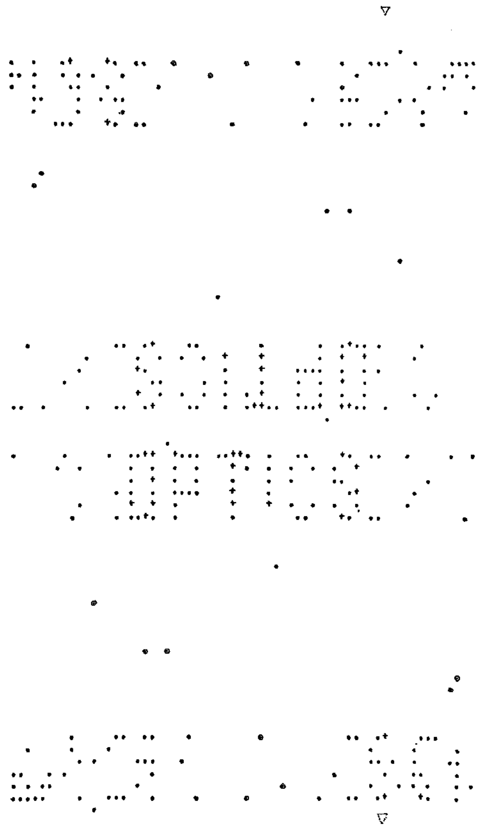


Fig. 13. Numerically synthesized image, where the digitized phase is binary. The image consists of the original image and an extra image of the letters OPTICS.

in Fig. 9 is not unique for the determination of the array function, that is, we can choose many ways for the sampling and labelling for one image. Let us assume that we determine N array functions by different sampling and labelling. Then we excite the array according to these array functions which are time dependent. The synthesized images are the time-sequential images. We can process these images statistically and detect an image from many indistinct images. We assume that the image receptor, for example the human eye, responds to the integrated field intensity with respect to time and the images are recognized as the time-average $\bar{o}(x, y)$ of the time-sequential images $o(x, y; t_n)$ as expressed in Eq. (10), for the convenience of discussion.

$$\bar{o}(x, y) = \frac{1}{N}\sum_{n=1}^{N} o(x, y; t_n) = \frac{1}{N}\sum_{n=1}^{N} \left[A(\alpha, \beta; t_n)\right] \tag{10}$$

In Eq. (10), the time-averaged image is expressed by the summation of images at discrete time intervals, divided by the number of images instead of the time period, for convenience of explanation of the numerical experiment. The assumption of a time-averaged image may not be correct for the actual image receptors. But we can consider image-display systems which display a time-averaged image of the time-sequential images.

For the numerical experiment, we choose six different combinations of sampling and labelling as shown in Fig. 14 and obtain six different array functions

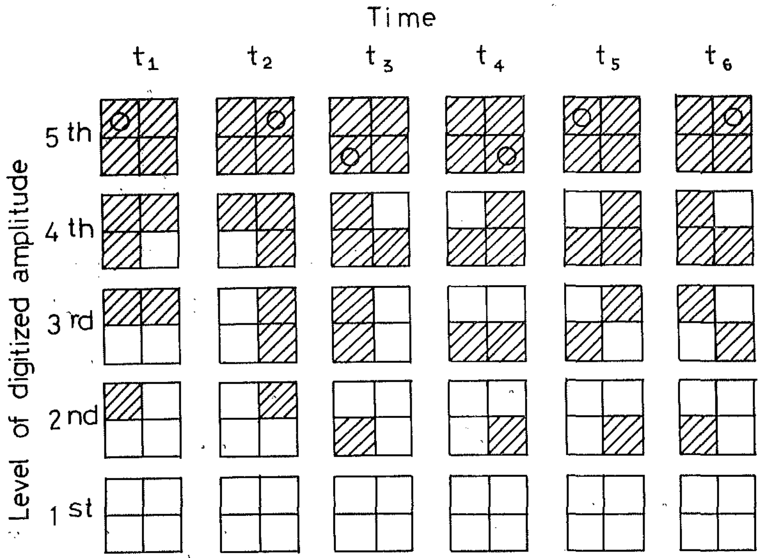
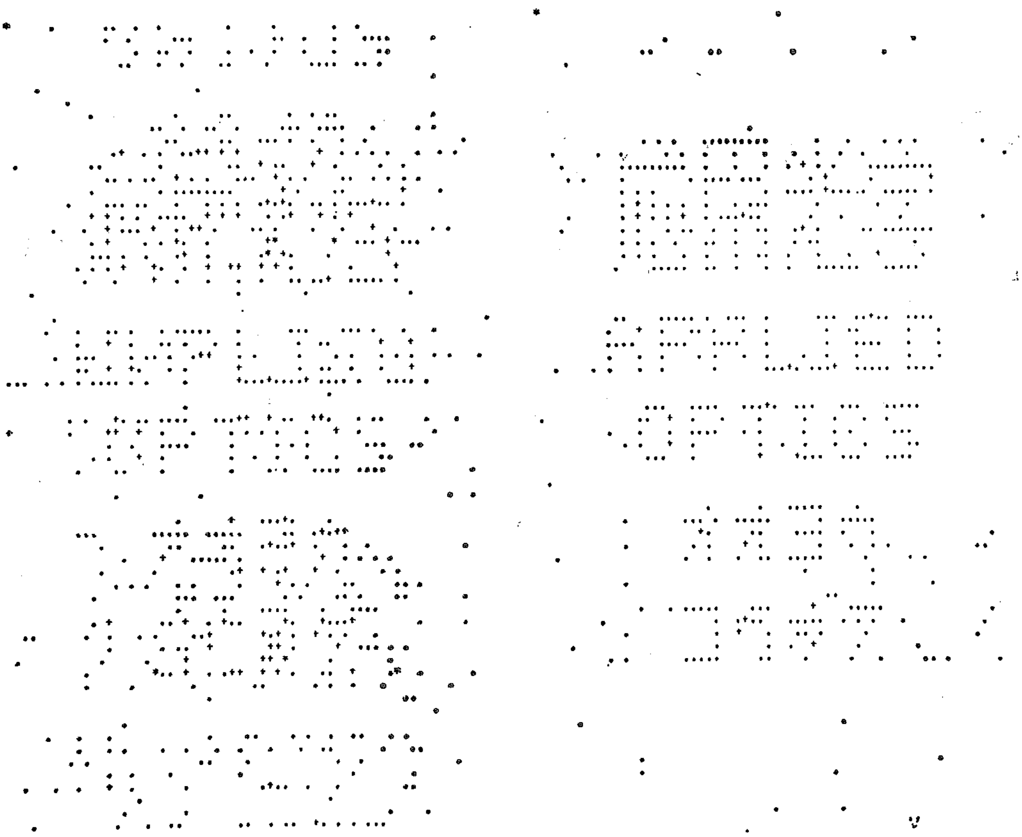


Fig. 14. Combination of sampling of the Fourier transform (as expressed by circles) and labelling of excited array elements (as expressed by oblique lines) in a temporal array, which is described by time-dependent array function.

for one image. We synthesize six images by different array functions and form one image as the intensity average of the six images. The image obtained is shown in Fig. 15–b. Fig. 15–a is the image synthesized according to one array function $A(\alpha, \beta; t_1)$. In Fig. 15–a, we can hardly recognize the image, whereas we can see the image of APPLIED OPTICS and Japanese letters in Fig. 15–b.

7. Image Synthesis in the Fresnel Region

The Fraunhofer region, where we can approximate the radiation (or diffraction) patterns of the array by its Fourier transform, is too far from the array to synthesize images in a laboratory system[8]. For waves of short-wavelength, such as optical or ultrasonic waves, we can see the Fraunhofer radiation pattern with the aid of optical or acoustical lenses. Lenses for waves of long-wavelength, such as microwaves or sound-waves, can be applied, but it is, in practice, difficult and inconvenient to use such lenses for the reproduction of Fourier transform images. Therefore it is practical to discuss image synthesis in the Fresnel region for waves of long-wavelength, except for antenna radiation pattern synthesis. In this section we discuss image synthesis in the Fresnel region in connection with Fresnel holograms.



(a) (b)

Fig. 15. Numerically synthesized images, where a is due to one array function and b is the intensity average of six images which are synthesized by the temporal array describing six different array functions obtained by the procedure of Fig. 14.

In Fresnel-transform holography, a hologram is constructed by recording the Fresnel diffraction pattern of the object. The Fresnel diffraction pattern $a(\alpha, \beta)$ of the object $o(x, y)$ is obtained by the Fresnel-Kirchhoff integral as follows,

$$a(\alpha, \beta) = f(\alpha, \beta) * o(\alpha, \beta) \tag{11}$$

where the asterisk $*$ denotes the convolution integral. The function $f(x, y)$ in Eq. 11 is a propagation function and it is expressed taking into account the paraxial-ray approximation as follows,

$$f(x, y) = e^{-i\gamma(x^2+y^2)} \tag{12}$$

where γ is a constant. Let us assume that the hologram is recorded by square-law detection. Then the hologram $h(\alpha, \beta)$ can be expressed as follows,

$$h(\alpha, \beta) = |b|^2 + |a(\alpha, \beta)|^2 + b^* a(\alpha, \beta) + b a^*(\alpha, \beta) \tag{13}$$

where b is the bias term. If we excite the array according to the intensity distribution of the hologram $h(\alpha, \beta)$ of Eq. (13), we can expect that a real image may be reconstructed at the Fresnel transform plane. We should notice that the real image reconstructed from the conjugate image term $ba^*(\alpha, \beta)$ can be displayed when the wavefront from the true image term $b^*a(\alpha, \beta)$, which forms a virtual image, is divergent at the conjugate image plane, resulting in no strong disturbance on the conjugate image. The above discussion shows that the hologram $h(\alpha, \beta)$ is an array function.

Now we calculate the array function according to the hologram of Eq. (13). First we calculate the inverse Fresnel transform $a(\alpha, \beta)$ of an image and derive the real part $\mathscr{R}_e a(\alpha, \beta)$ of it. Next we add a bias term b to the real part so as to satisfy the condition that the array function $a_R(\alpha, \beta)$ of Eq. (14) is positive.

$$a_R(\alpha, \beta) = b + \mathscr{R}_e a(\alpha, \beta) = b + \frac{1}{2}\left[a(\alpha, \beta) + a^*(\alpha, \beta)\right] \geqslant 0 \tag{14}$$

When we calculate the convolution of Eq. (11), we use the fact that the Fourier transform of a convolution is the product of the Fourier transforms of the two functions. The process of calculating the array function of Eq. (14) is shown in Fig. 16. The array function obtained is shown in Fig. 17, where $a_R(\alpha, \beta)$ is digitized into five levels. We excite the computer simulated array according to the intensity distribution of Fig. 17. The image to be synthesized in the Fresnel region is obtained by calculating the Fresnel transform following the process shown in Fig. 16. The image obtained is shown in Fig. 18. We can recognize that the same image as that of Fig. 3 is synthesized in Fig. 18 with the background field.

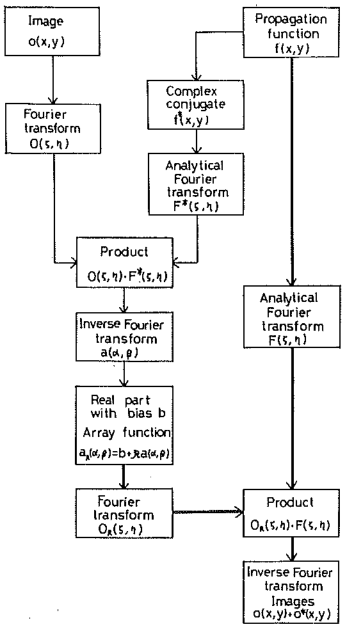


Fig. 16. Procedure for calculating the array function (as expressed by single flow lines) and calculating the Fresnel radiation pattern (image) of the array (as expressed by double flow lines).

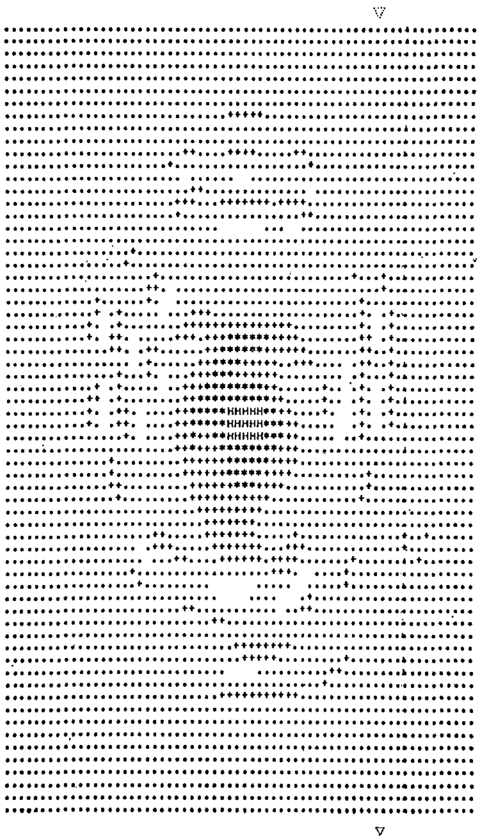


Fig. 17. Intensity distribution of the array function. The intensity a_R (normalized) is digitized into five levels as expressed by the letter and symbols, $H(a_R \geqslant 1.0)$, $*(1.0 > a_R \geqslant 0.75)$, $+(0.75 > a_R \geqslant 0.5)$, $\cdot\ (0.5 > a_R \geqslant 0.25)$, and blank $(0.25 > a_R)$.

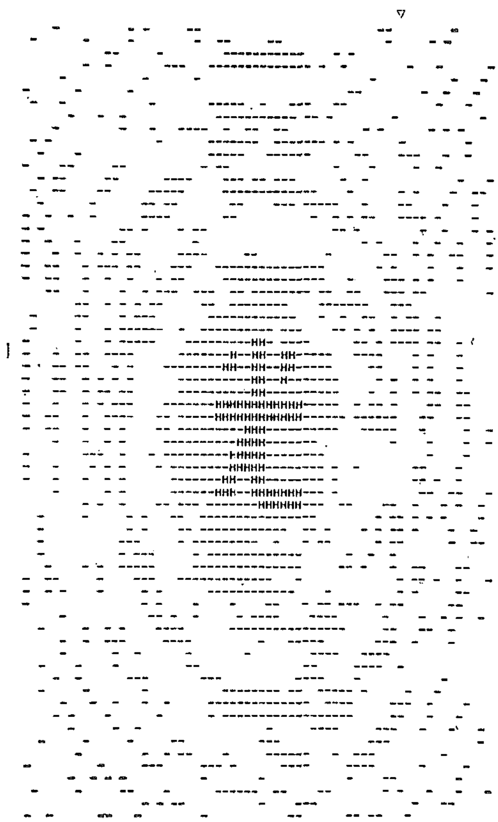


Fig. 18. Image numerically synthesized in the Fresnel transform plane by the array described by the array function of Fig. 17.

8. Conclusion

Image synthesis by array systems is discussed with the aid of computer simulation of arrays and the computer calculation of their radiation patterns. The present discussion should be considered as a conceptual one and a more precise and detailed discussion must be done for the design and construction of array systems for image synthesis. However it should be emphasized that computer simulation is a useful technique in studying image synthesis by array systems. Since image synthesis by this technique, as in radiation pattern synthesis by array antennas, is too complicated to be analyzed theoretically except for simple cases, it is imperative to adopt simulation techniques. Analogue simulation, such as optical simulation, is possible and convenient in certain cases, although it is difficult to simulate the complex array or phased array by optical procedures. If we can combine computer and optical simulations, we can expect more progress in the study of image synthesis by array systems.

The display of the simulated images is not good in the present numerical experiment, because a suitable output facility of the computer was not used for the display of calculated results. This should be improved by using a plotter or a CRT display.

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