A Practical Method for Calculating Room Temperature, Heating Load and Cooling Load of Multiroom

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It is possible to set up heat balance equations representing the dynamic thermal characteristics of a multiroom by using weighting functions, if the system is linear and invariable. With the advent of the digital computer, it is now possible to solve them directly, even if there are numerous rooms with different thermal behavior. However, as the number of rooms increases, the cost of calculation and the capacity of the computer required increase rapidly. Therefore, it may not be practical to solve the balance equations directly, when the number of rooms is large. This paper proposes a practical method for calculating room temperature, heating load and cooling load of a multiroom by means of a digital computer of relatively small capacity at reasonable cost. Rooms constituting a multiroom system may be divided into two groups; one has a stronger influence on the relation between the excitation and the system's response, the other has a lesser influence. Then, the response to the system consisting of only the former group may be good approximation to the response to the original undivided system. The accuracy is not known exactly at this stage, but it is shown that the effect of neglecting the latter group of rooms can be easily investigated on the principle of superposition or Thevenin's theorem. Also the improvement of accuracy can be made to any desired degree by a relaxation method on the same principle. Many examples for obtaining the room temperature of a multiroom by this method were studied and it has been found that, with an increasing number of rooms, the usefulness of the present method for analysis and synthesis increase rapidly.

Key Words: Multiroom digital computer, room temperature heat source, Thevenin's theorem.

1. Introduction

It is possible to set up heat balance equations representing the dynamic thermal behavior of a multiroom system by using the linear theory. Actually, it is difficult to obtain analytical solutions for room temperature because of the complexity. With the advent of the digital computer, it is now possible to obtain numerical solutions with any desired accuracy, even if there are numerous rooms with different thermal performances. It may not be worthy to get the solutions more accurate than required for practical problems. To avoid such worthless calculations, it is necessary to simplify the system's elements and the mathematical models. However, as the number of rooms increases, the cost of calculation and the capacity of the computer required increase rapidly. Therefore, it may not always be practical to solve the balance equations directly when the number of rooms with different thermal behavior is large. Our method is practical for engineering and application problems because calculations can be made by hand, by desk calculation or the digital computer of relatively small capacity. Also, it can be done at a reasonable cost, depending on the amount of calculations needed.

This method has been applied to obtain the room temperature of a multiroom system consisting of two rooms, five rooms in parallel, and fifteen rooms in series. To explain the present method clearly, the mathematical models are simplified by assuming a one dimentional heat flow, by uniform room air temperature, and by a combined heat transfer coefficient which would approximate the radiant heat interchange, betwaen the inside surfaces, and the convective heat interchange, between the inside surfaces and the room air. Attempt is made to calculate the indoor temperature variation caused by air conditioning, taking the initial temperature 0 ℃ and assuming the outdoor temperature always being 0 ℃. These calculations are carried out using an electronic digital computer with the capacity of 2-K words.

2. Practical Method

2.1 Connection and Division of Systems

There are two systems; one has some heat sources, and the other has none. Therefore, there can be temperature variation in the former. Now, we attempt to connect the latter to the former at any boundary position with a constant temperature of 0 ℃. Consequently, different responses will occur because of the effect of the connection. To investigate the effect, an artificial heat source is supplied to the combined position. The magnitude of this heat source is equivalent to the heat fluxes at the same position of the original system, which is caused by the original heat sources. The response of this artificial heat flow to the combined system represents the difference of the responses between the original system and the combined one, that is the effect of the connection. This will be described here in terms of the superposition principle as follows.

First, a couple of artificial heat sources, heating and cooling with the same thermal quantity, is supplied to the combined position of the total combined system, as shown in figure 1-A. Then, there is no change in the thermal performance, since their total heat fluxes supplied artificially become zero.

Second, a couple of heat sources is decomposed by the superposition principle. Two cases result; one is an artificial heat source with the original heat sources, the other is only an artificial heat source, as shown in figure 1-B,C. If the temperature of the divided position in the former case is kept at 0 ℃ at all times, this system should be equivalent to the original one shown in figure 1-D. The magnitude of the artificial heat source producing such a condition is equal to the heat fluxes at the marked position in the original system. Therefore, the effect of the connection is obtained by calculating the temperature distribution in the latter case shown in figure 1-D. When the artificial heat source is less than the original one, it can be very practically ignored. Thus, the response to the original system can be used as an approximation of the response of the combined one. This approximation can be directly applied to the system in which addition of rooms is made.

Furthermore, we can attempt to divide a system into two parts by the principle mentioned above. It is possible to simulate the divided system to the original one described above, so that the effect of the division can be obtained by the same method of the connection, as shown in figure 1.

2.2 Practical Method

Rooms constitution a multiroom are divided into two groups; one has a stronger influence on the response, the other has a lesser influence. Then, the response to the divided system of the former may be an approximation to the response to the total undivided system. The effect of neglecting the latter can be easily obtained by the method given in the foregoing paragraph. If the effect is negligible, the accuracy at this stage is sufficient for engineering purposes. In order to obtain an accurate solution, the improvement of accuracy must be made by calculating the response to the total system caused by the artificial heat source supplied for releasing another one at the divided position, as shown in figure 1-E. This system is also divided into two groups different from the previous ones. Then, the response to the divided system consisting only of the group having a stronger influence on the thermal behavior can be an approximation to the response to the total undivided system. The second approximation is obtained by adding the improved result to the first one. It is then decided whether or not to continue the improvement of accuracy. It is necessary to continue successively the process of division and connection, until the accuracy becomes good. Then, the final solution is obtained by adding the result of each process. If the improvement is continued to infinity, the complete solution is obtained.

The divided system should be as simple as possible. It is convenient to take a unit room, since there will be no trouble in how to divide a system, and since the thermal characteristics required for this method are only of a unit room, and not of a multiroom system.

Summarizing, calculation by this method may be performed in some very distinct stages as follows.

1. First stage

   a. The divided system is taken to be the heating or cooling room. The air temperature caused by the original heat source is calculated under the condition of the air temperature of 0 ℃ for all adjoining rooms.

   b. The magnitude of the artificial heat source for producing the temperature 0 ℃ must be calculated for every adjoining room. It is then decided whether or not to improve the accuracy.

2. Second stage

   a. The divided system is taken to be each adjoining room. The air temperature caused by the heat source mentioned in 1-b is calculated under the condition of the air temperature of 0 ℃ for all neighboring rooms.

   b. The magnitude of the artificial heat source for producing the temperature 0 ℃ must be calculated for every neighboring room. It is then decided whether or not to improve the accuracy.

3. Third stage

   a. The divided system is taken to be each neighboring room. The air temperature caused by the heat source mentioned in 2-b is calculated under the condition of the air temperature of 0 ℃ for all adjoining rooms.

   b. The magnitude of the artificial heat source for producing the temperature 0 ℃ must be calculated for every adjoining room. It is then decided whether or not to improve the accuracy.

4. Fourth stage

It is necessary to continue the procedure successively until the accuracy is sufficient for engineering purposes. Then, the final solution is obtained by adding up the result of each process.

The frequency of calculation required in this procedure depends upon the structure of a system and usually it may be a small number. The temperature caused by the artificial heat source may be negligible at early stages of the procedure, since the magnitude of the heat flow decreases radpidly with increase of the distance from the source.

2.3 Procedure of 5-Rooms in Parallel

In the case that the heat is supplied only to the center room as shown in figure 3, we will attempt to obtain the air temperature of 5 rooms in parallel by the method described above.

1. First stage

   a. Air temperature of heating room.

$$T_1(1) = G_1 \times Q \tag{1}$$

   b. Magnitude of the artificial heat source at every adjoining room.

$$Q_2(1) = F_{1\cdot 2} \times T_1(1) \tag{2}$$

$$Q_3(1) = F_{1\cdot3} \times T_1(1) \tag{3}$$

$$Q_4(1) = F_{1\cdot4} \times T_1(1) \tag{4}$$

$$Q_5(1) = F_{1\cdot5} \times T_1(1) \tag{5}$$

2. Second stage

a. Air temperature of every adjoining room

$$T_2(1) = G_2 \times Q_2(1) \tag{6}$$

$$T_3(1) = G_3 \times Q_3(1) \tag{7}$$

$$T_4(1) = G_4 \times Q_4(1) \tag{8}$$

$$T_5(1) = G_5 \times Q_5(1) \tag{9}$$

b. Magnitude of the artificial heat source at every neighboring room.

$$Q_1(2) = F_{2\cdot1} \times T_2(1) + F_{3\cdot1} \times T_3(1) + F_{4\cdot1} \times T_4(1) + F_{5\cdot1} \times T_5(1) \tag{10}$$

$$Q_2(2) = F_{3\cdot2} \times T_3(1) + F_{5\cdot2} \times T_5(1) \tag{11}$$

$$Q_3(2) = F_{2\cdot3} \times T_2(1) + F_{4\cdot3} \times T_4(1) \tag{12}$$

$$Q_4(2) = F_{3\cdot4} \times T_3(1) + F_{5\cdot4} \times T_5(1) \tag{13}$$

$$Q_5(2) = F_{2\cdot5} \times T_2(1) + F_{4\cdot5} \times T_4(1) \tag{14}$$

3. Third stage

a. Air temperature of every neighboring room.

$$T_1(2) = G_1 \times Q_1(2) \tag{15}$$

$$T_2(2) = G_2 \times Q_2(2) \tag{16}$$

$$T_3(2) = G_3 \times Q_3(2) \tag{17}$$

$$T_4(2) = G_4 \times Q_4(2) \tag{18}$$

$$T_5(2) = G_5 \times Q_5(2) \tag{19}$$

b. Magnitude of the artificial heat source at every adjoining room.

Final results.

$$T_1 = T_1(1) + T_1(2) + T_1(3) + \text{-----} \quad (20)$$

$$T_2 = T_2(1) + T_2(2) + T_2(3) + \text{-----} \quad (21)$$

$$T_3 = T_3(1) + T_3(2) + T_3(3) + \text{-----} \quad (22)$$

$$T_4 = T_4(1) + T_4(2) + T_4(3) + \text{-----} \quad (23)$$

$$T_5 = T_5(1) + T_5(2) + T_5(3) + \text{-----} \quad (24)$$

Where;

- Ti(J) : air temperature of No.i room at J stage of calculation.
- Qi(J) : magnitude of the artificial heat source of No.i room at J stage of calculation.
- Gi : transfer function between the magnitude of No.i room heat source and No.i room air temperature.
- Fik : transfer function between No.i room air temperature and the heat fluxes from No.i to No.k room.
- i,k : subscript denoting room number. No.1 is the center room, No.2, 3,4,5 are the others.
- J : subscript denoting the frequency of calculation in the procedure given in the foregoing paragraph.
- Q : magnitude of the original heat source at No.1 room.
- T,Q,G,F : these functions are represented by the Laplace transformation.

3. Calculation and Solution of Examples

3.1 Description of Examples

This method is applied to some multirooms; two rooms, five rooms in parallel, and fifteen rooms in series. The module of each room is an office, 10×10×4 meters, without any windows and any furniture. It is constructed of concrete with a thickness of 0.15 meters. The thermal condition and structure of every story are similar, with a ceiling and floor slub which seems to be insulated. The heat source with a unit step function is supplied to the corner of two rooms, fifteen rooms, and the center of five rooms. The inside combined heat transfer coefficient for these calculations is taken to be 8 Kcal in $hr^{-1}m^{-2}$deg C and the outside one to be 20 Kcal in $hr^{-1}m^{-2}$deg C.

3.2 Results

The results of these calculations are given in figure 2,3 and 4, where the room temperature is plotted versus time at every stage of this method. In figure 2 and 3 the curves are represented for the grade of improvement for every room, and in figure 4 it is represented for the frequency of calculation by this method described in foregoing paragraph.

The results for two rooms shown in figure 2 indicate that as the frequency of successive improvement increases, the temperature degree to be improved becomes smaller remarkably. For instance, the second improvement ratio between the true temperature of each room and the improved supplementary one, X3/X and Y3/Y in figure 2, is less than one per cent. In the first stage of improvement, good accuracy is obtained. It is thus possible to finish the calculation when it reaches the fourth stage described in foregoing paragraph. The approximation is given by summing up the first and second temperatures, X1 and X2 or Y1 and Y2 in figure 2.

The computer capacity required for approximation by this method is about one-half as small as the one required for the complete solution by working out the balance equations directly. In the case of a multiroom with just a few rooms, the time necessary for calculation does not differ greatly from that of the complete solution.

The results for five rooms in parallel shown in figure 3 indicate that the approximation with good accuracy is obtained when the frequency of calculation reaches the fourth stage, so that it is given by summing up the first and second temperatures, X1 and X2 or Y1 and Y2 in figure 3.

The computer capacity required for approximation by this method is about two-fifths as small as the one required for the complete solution by working out the balance equations directly. The time necessary for calculation of this method is about one-fiftieth as short as that of the balance equations. It is possible to calculate them by hand or by a desk computer.

The results for fifteen rooms in series shown in figure 4 indicate that as the frequency of successible calculation increases, the temperature degree to be improved decreases remarkably. For instance, the temperature at the fourth stage, curve 4 in figure 4, is about one-hundredth as small as that at the first stage, curve 1 in figure 4. Therefore, the temperature seems to be zero for rooms with a room number larger than 5, and the approximation of the rooms numbering from 1 to 4 is obtained by summing up two curves representative of each room.

The computer capacity required is about one-fifteenth as small as that for solving the balance equations, and the time necessary for calculation may be about one-two hundredths as short as the latter. It is also possible to calculate them by hand or by a common desk computer.

4. Summary

The practical method for obtaining room temperature of a multiroom has been proposed. Many examples; two rooms, five rooms in parallel, and fifteen rooms in series, were studied, and it has been shown that the present method is excellent for engineering and application purposes because these calculations can be made by hand, by a desk computer, or by a digital computer of relatively small capacity. This can be down at a reasonable cost.

It has also been demonstrated that with an increasing number of rooms, the usefulness of the present method increases remarkably. For instance, in the case of fifteen rooms in series, the time necessary for calculation of this method is about one-two hundredths as short as that of the balance equations method. The computer capacity for the former is about one-fifteenth as small as the latter.

It has also been found that this method is useful to obtain the effect produced by an addition of rooms.

It is obvious that this method is also useful for obtaining the heating load and cooling load.

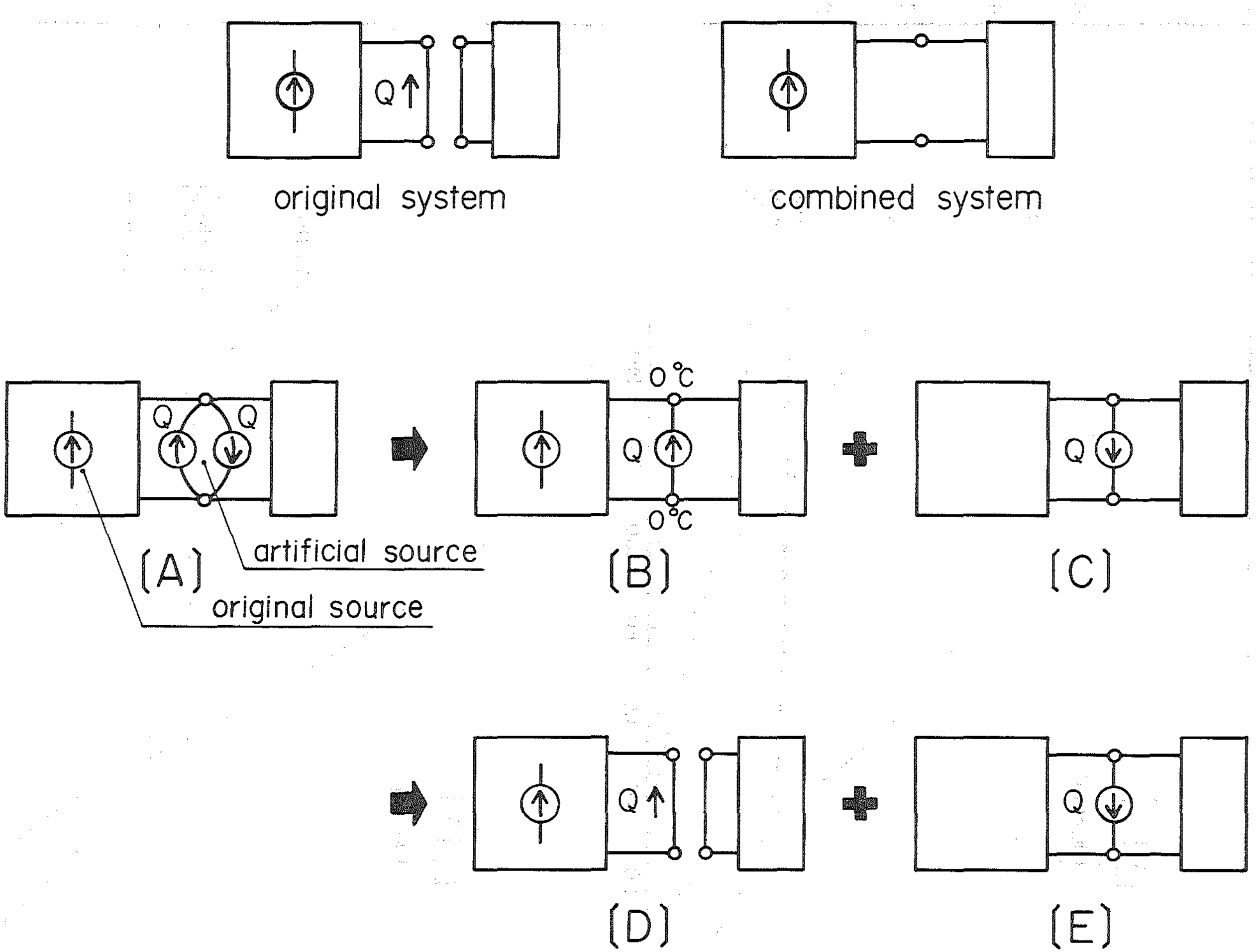

Figure 1 Schematic interpretation of connection and division of systems.

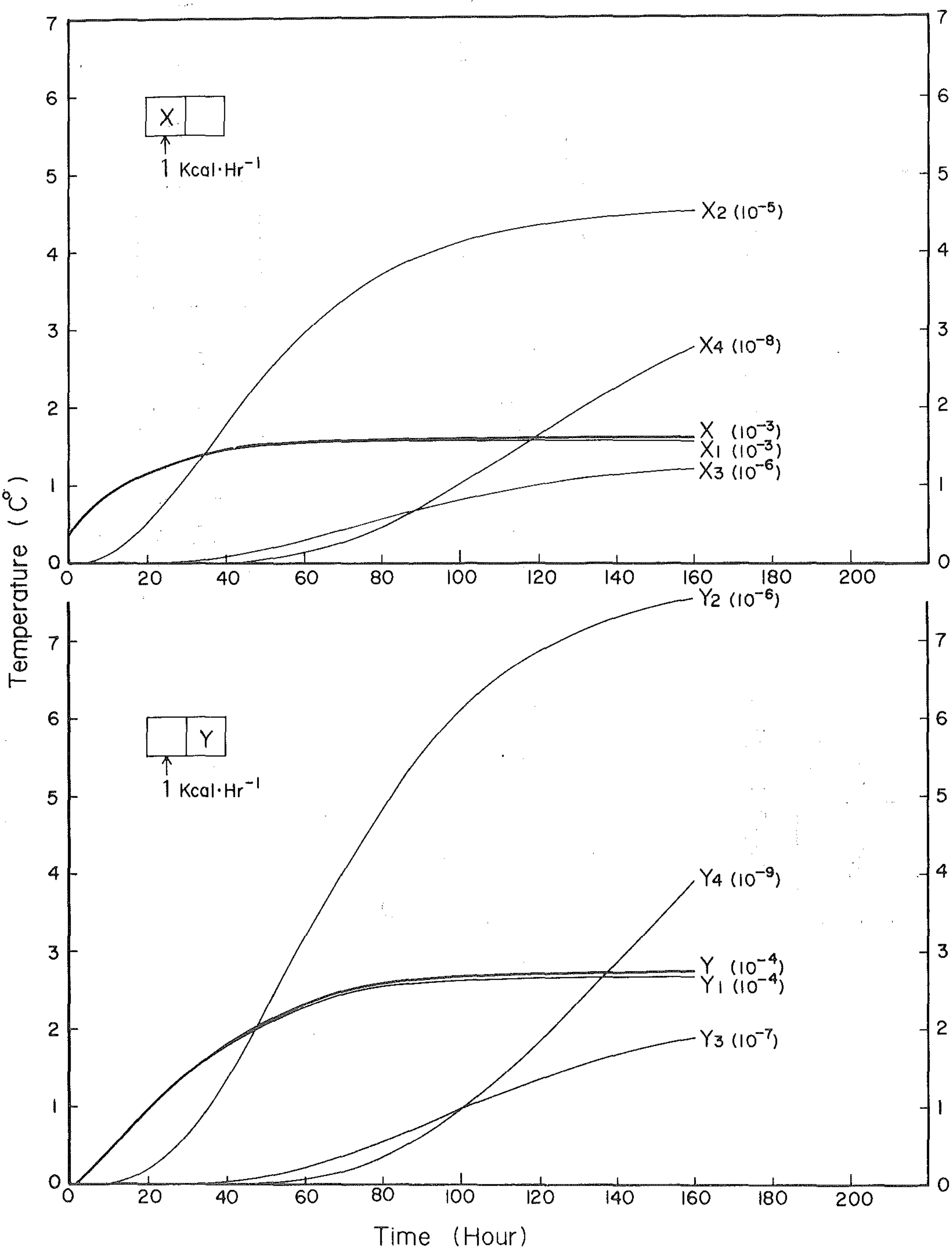

Figure 2 Room air temperature of 2-rooms resulted from heating in the form of a unit step function at X-room.

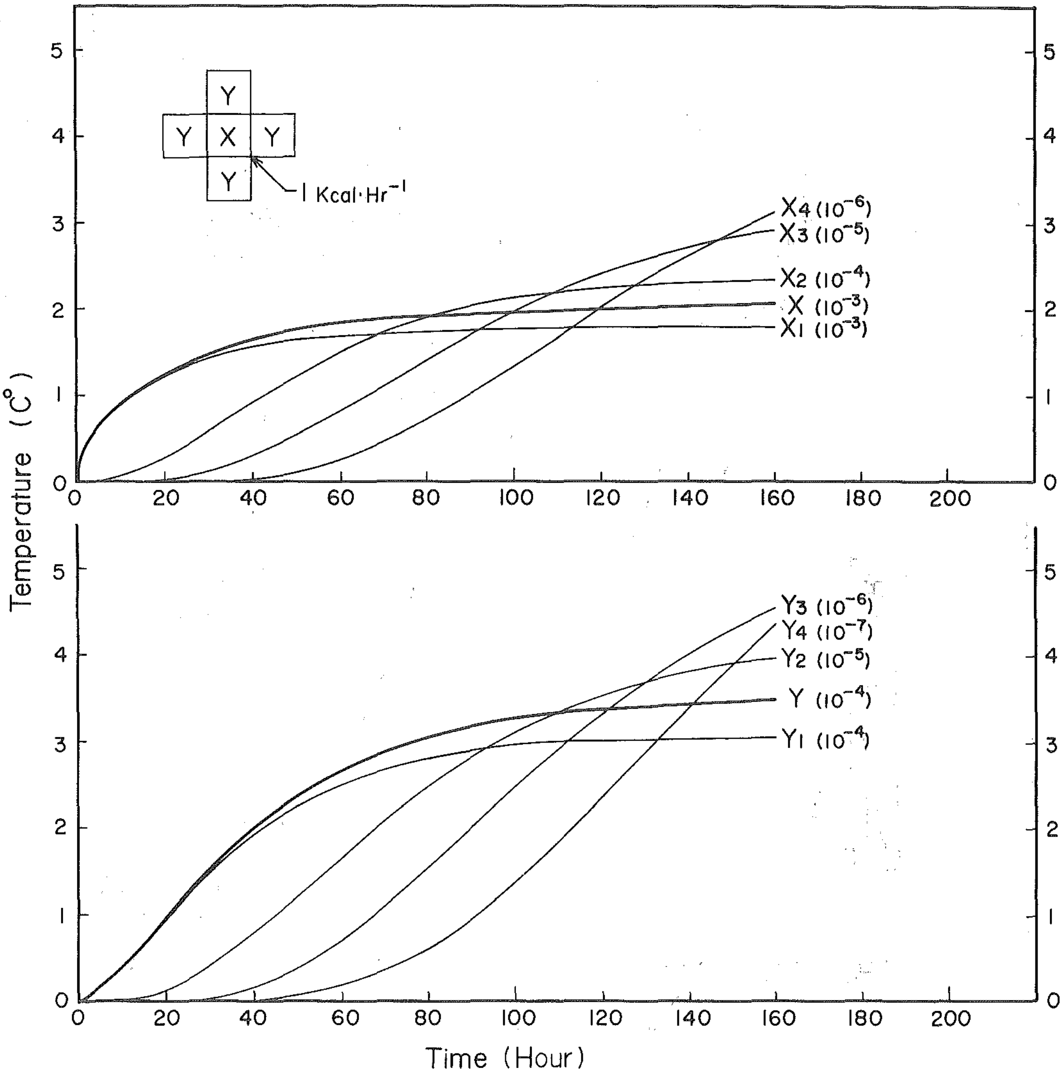

Figure 3 Room air temperature of 5-rooms in parallel resulted from heating in the form of a unit step function at X-room.

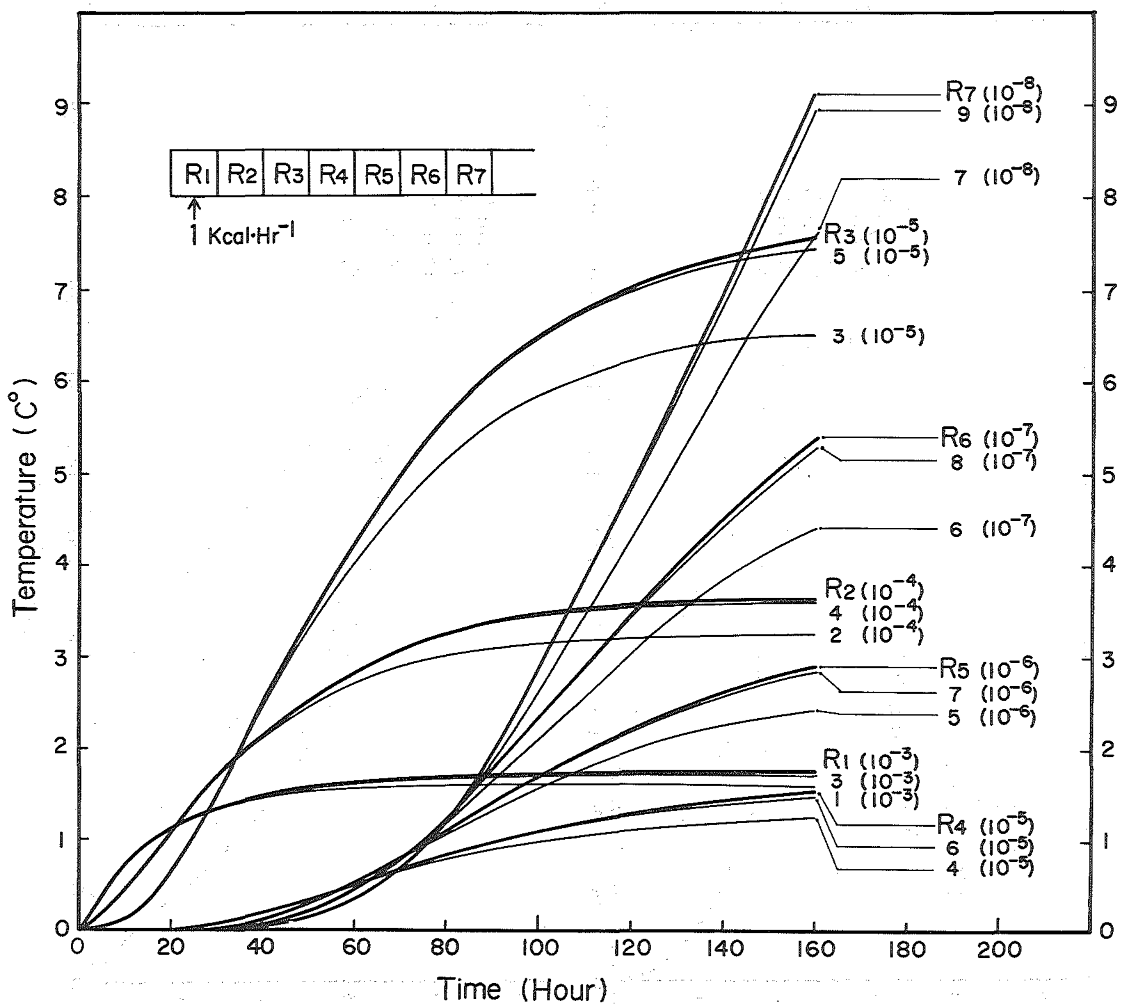

Figure 4 Room air temperature of 15-rooms resulted from heating in the form of a unit step function at R1-room.